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7.1D CONSTRUCTION OF COMPLEMENTARY CODE SEQUENCE SETS

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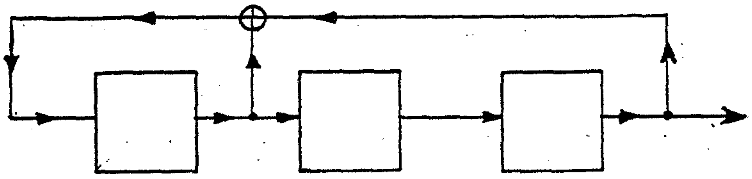
A set of code sequences is said to be a complementary code sequence set if the sum of the aperiodic autocorrelation functions for the sequences is zero everywhere except at the origin. This note discusses a simple construction for sets of complementary code sequences. Suppose we have a linear feedback shift register whose feedback polynomial is irreducible (PETERSON and WELDON, 1972). For example, according to the tables in (PETERSON and WELDON, 1972, Appendix C), the polynomial $x^3 + x^2 + 1$ is irreducible and corresponds to the linear feedback shift register shown below (SARWATE and PURSLEY, 1980).



A feedback shift register with n stages produces 2^n different sequences corresponding to the 2^n initial loadings. The length (or period) of the sequences is N where $N = 2^n-1$ if the feedback polynomial is primitive, and N is a proper divisor of 2^n-1 if the polynomial is nonprimitive (PETERSON and WELDON, 1972). For example, if we choose a primitive polynomial of degree 6, we can get 64 sequences of length 63, while if we choose a nonprimitive polynomial of degree 6, we can get 64 sequences of length 21 or 9. These 2^n sequences form a complementary code sequence set. For example, the 8 sequences of length 7 generated by the shift register shown above form a complementary code sequence set. The sequences are as follows:

0000000
0011101
0111010
1110100
1101001
1010011
0100111
1001110

It will be noted that the all-zeroes sequence is always one of the sequences obtained thus. Since this may not be convenient for some applications, we consider the following modification. Choose an arbitrary binary sequence of length N and add it to all the sequences obtained from the shift register. Here, addition means bit-by-bit EXCLUSIVE OR addition of sequences. The resulting set of sequences is still a complementary code sequence set. For example, if we choose the sequence 0011010 and add it to the sequences above, we obtain the set of sequences shown:

0011010
0000111
0100000
1101110
1110011
1001001
0111101
1010100

The aperiodic autocorrelation function for a binary sequence is computed by first converting the sequence from the alphabet (0,1) to the alphabet (+1,-1) and then using the formula

$$C_x(\ell) = \sum_{i=0}^{N-\ell-1} x_i x_{i+\ell}, \quad 0 \leq \ell \leq N-1 .$$

The results of such computations for the 8 sequences are as shown below. It is clear that these sequences do indeed form a complementary code sequence set.

ℓ	0	1	2	3	4	5	6
	7	-2	-1	0	-1	0	1
	7	4	1	-2	-3	-2	-1
	7	2	3	2	1	0	1
	7	0	-1	0	3	0	-1
	7	2	-3	-2	1	2	1
	7	-2	-3	4	-1	-2	1
	7	0	1	0	-1	2	-1
	7	-4	3	-2	1	0	-1

In general, we can construct 2^{N-n} different complementary code sequence sets from a given shift register. Some of these may be more suitable for applications than others. The construction given in this note can be generalized to produce polyphase sequences also. Details are given in (SARWATE, 1983).

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