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NONSTATIONARY MODULATION OF GALACTIC COSMIC RAYS IN A NONLINEAR MODEL

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ABSTRACT

An automodel equation for the solar wind velocity is obtained in the self-consistent model. The solution of the convection-diffusion equation is obtained for the density of galactic cosmic rays at a definite dependence of the diffusion coefficient and solar wind velocity on the rigidity and distance.

In studying the propagation of galactic cosmic rays (GCR) in the interplanetary space, properties of the solar wind are considered, independent of the CR intensity. Though it should be noted that the influence of GCR on the supersonic solar wind is comparable with that of other factors (galactic magnetic field etc.).

Then, while investigating the GCR modulations by the solar wind, one should take into account the fact that with the variation of the GCR intensity in time and space, the influence of GCR on the solar wind varies, which in its turn results in the variation of the solar wind velocity. That is, the self-consistent problem should be solved.

In the spherically symmetrical case nonstationary equations of hydrodynamics for the solar wind with account of the GCR influence have the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\gamma}\left(\frac{\partial c^2}{\partial r} + c^2\frac{\partial}{\partial r}\ln\rho\right) + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0$$

$$\frac{\partial}{\partial t}(\ln\rho) + u\frac{\partial}{\partial r}(\ln\rho) + \frac{\partial u}{\partial r} + \frac{2u}{r} = 0 \qquad (1)$$

$$\frac{\partial c^2}{\partial t} + u\frac{\partial c^2}{\partial r} + (\gamma-1)c^2\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0$$

where u, ρ are the velocity and the density of the solar wind, c^2 is the sound velocity, γ is the polytropic index, r is the heliocentric distance, and P is the GCR pressure equal to $P = \frac{1}{3}\int_0^\infty N(r,R,t)\,p v\,dR$ (here N is the concentration of particles with the rigidity R ; p, v are the momentum and the velocity of GCR particles, respectively).

The variation of the spectrum N is determined by the character of GCR modulation in the interplanetary space, i.e. it is defined in the end by the velocity of solar wind and diffusion coefficient via nonstationary equation of diffusion with account of convection and GCR energy variation /1,2/

$$\frac{\partial N}{\partial t} - \vec{\nabla}(\varkappa\nabla N) + (\vec{u}\,\vec{\nabla})N - (\vec{\nabla}\,\vec{u})\frac{R}{3}\frac{\partial N}{\partial R} \qquad (2)$$

Let us introduce as in /3/ new dimensionless dependent variables $\varkappa$ and y by means of relations $u = \varkappa\frac{r}{t}$, $c^2 = y\frac{r^2}{t^2}$ and dimensionless independent variables $\ln t$ and $\ln r$, then the system (1) will have the form

$$\dot{x} - x + x^2 + x' + \frac{1}{\gamma}\left[y' + 2y + y(\ln\rho)'\right] + yp' = 0$$

$$(\ln\rho) + x(\ln\rho)' + x' + 3x = 0 \qquad (3)$$

$$\dot{y} + xy' + 2(x-1)y + (\gamma-1)y(x' + 3x) = 0$$

where, in calculating the last term in the first equation, it is approximately assumed $(\gamma - 1) \approx 1$.

Let us assume that X and y are the function of one independent variable $Z = \tau t^{-a_1}$, where a - const. Let us require that τ and t do not enter explicitly in (2), then the density should have the form $\rho = t^{a_2}\xi(Z)$. Since $\partial/\partial(\ln t) = -a_1 d/d(\ln Z)$; $\partial/\partial \ln\tau = d/d\ln Z$ the system (3) will take the form

$$x'(X-a_1)+x^2-x+\frac{1}{\gamma}\left(2y+y'+y(\ln\xi)'\right)+yP'=0$$

$$(\ln\xi)'(X-a_1)+a_2+X'+3x=0 \qquad (4)$$

$$y/y'(x-a_1)+(\gamma-1)x'+[2(\gamma-1)+(\gamma+1)]X-2=0$$

where the dash means $\partial/\partial\ln Z$.

Eliminating $d(\ln\xi)$ from the first two equations of the system, one may rewrite (4) as

$$\frac{d\ln Z}{dx}=\frac{(a_1-x)\frac{d\ln y}{dx}-(\gamma-1)}{(3\gamma-1)x-2}=$$

$$=\frac{(a_1-x)^2-y}{y\left[\frac{2(a_1-1)+a_2}{\gamma}+3x+F(a_1-x)\right]-x(1-x)(a_1-x)} \qquad (5)$$

where it is assumed $F=dP/d\ln Z$

As it is seen, the problem of integration of the initial set of equations in partial derivatives of the first order is reduced to the integration of a first order ordinary differential equation of the form

$$\frac{dy}{dx}=y\,\frac{F_1(x,y)}{F_2(x,y)}$$

and to two quadratures.

To define N one should solve eq.(2) which in the spherically symmetric case in dimensionless variables has the form

$$\frac{\partial N}{\partial \tau} - bx\frac{\partial^2 N}{\partial \rho^2} + \left(au - \frac{2\varkappa b}{\rho} - b\frac{\partial \varkappa}{\partial \rho}\right)\frac{\partial N}{\partial \rho} - \frac{2}{3}\,\frac{au}{\rho}\,\frac{dN}{\partial \rho} - \frac{1}{3}a\xi\frac{\partial N}{\partial \xi} = 0 \quad (6)$$

here $\rho = r/r_o$, $\xi = R/R_o$, $\tau = t/T$, $a = T/r_o$, $b = T/r_o^2$
where r_o is the dimension of the modulating volume, R_o is the minimum rigidity of particles, T is the time which the solar wind spends to reach the boundary of the modulating volume.

In order to obtain the analytical solution of eq.(6), one should choose the dependence of the diffusion coefficient and the solar wind velocity in the form

$$\varkappa = \varkappa_o\,\rho^{2(1-\alpha)}\,\xi^{2P} \;; \qquad u = u_o\,\rho^{-2-\frac{3\alpha}{\beta}}$$

where α and β are the arbitrary constants.

Let us search for a solution with boundary conditions

$$N(\rho,\xi,\tau)/_{\rho=0} < \infty \;; \qquad N(\rho,\xi,\tau)/_{\rho=1} = K\,\xi^{-\gamma}$$

Passing to the variable $\mathcal{Z} = \rho^{\alpha}\xi^{-\beta}$ and using the separation of the variables $N(\mathcal{Z},\tau) = \varphi(\tau)\psi(\mathcal{Z})$ we have from (6)

$$\frac{\partial \varphi}{\partial \tau} = -K^2\varphi\,, \qquad \varphi = c\,e^{-K^2\tau}$$

$$\mathcal{Z}\frac{\partial^2\psi}{\partial \mathcal{Z}^2} + \left(\frac{3}{\alpha} - 1\right)\frac{\partial \psi}{\partial \mathcal{Z}} + \frac{K^2}{\alpha^2 b\,\varkappa_o}\psi = 0 \quad (7)$$

where K^2 is the separation constant.

The solution of eq.(7) is

$$\psi = c_1\mathcal{Z}^{\nu}J_\nu\left(\sqrt{1/\alpha^2 b\varkappa_o}\;K\mathcal{Z}\right) + c_2\mathcal{Z}^{\nu}Y_\nu\left(\sqrt{1/\alpha^2 bx}\;K\mathcal{Z}\right) \quad (8)$$

where J_ν and Y_ν are the Bessel functions of type 1 and 2, and $\nu = 1 - \frac{3}{2\alpha}$. After satisfying the boundary conditions we have

$$N = \frac{2}{[Y_{n+1}(\mu_n)]^2}\sum_{l=1}^{\infty}\mathcal{Z}^{-n}e^{\alpha\sqrt{b\varkappa_o}\,(1-\tau)}J(\mu_e\mathcal{Z})\left(\int_0^1 \mathcal{Z}_n(\mu_e\eta)\,\eta^{\frac{\gamma}{\beta}+n+1}d\eta\right)$$

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