ESTIMATING THE CHANGE IN ASYMPTOTIC DIRECTION DUE TO SECULAR CHANGES
IN THE GEOMAGNETIC FIELD

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ABSTRACT
The concept of geomagnetic optics, as described by the asymptotic
directions of approach, is extremely useful in the analysis of
cosmic radiation data. However, when changes in cutoff occur
as a result of evolution in the geomagnetic field, there are
corresponding changes in the asymptotic cones of acceptance. We
introduce here a method of estimating the change in the asymptotic
direction of approach for vertically incident cosmic ray particles
from a reference set of directions at a specific epoch by consider-
ing the change in the geomagnetic cutoff.

1. Introduction. Cosmic ray particles must travel along specific allowed
trajectories through the geomagnetic field to reach a location on or near
the earth. In order to relate cosmic ray intensity variations observed
at different cosmic ray stations to the cosmic ray flux in space the
concept of asymptotic directions of approach was developed (see McCracken
et al., 1968, for a review). By application of the asymptotic directions
of approach the user need n⌄t be concerned about the specific details of
the allowed cosmic ray trajectories and can relate any specific cosmic
ray particle with a unique direction in space. For a cosmic ray particle
with rigidity R, arriving at a specific location (characterized by the
geographic latitude Λ and the geographic longitude Φ) from a direction of
incidence (described by the zenith angle θ and the azimuthal angle ϕ) the
asymptotic direction of approach is given by the unit vector $\underline{A}(R,\Lambda,\Phi,\theta,\phi)$
pointing in the reverse direction to the particle's velocity vector prior
to the particle's entry into the geomagnetic field. For the purposes of
this paper and for a specific location the vector $\underline{A}$ is specified for ver-
tical incidence in terms of the geocentric coordinate system as asymptotic
latitude, $\lambda(R) = \lambda(R,R_1(\Lambda,\Phi), \theta = 0°)$ and asymptotic longitude, $\psi(R) =$
$\psi(R,R_1(\Lambda,\Phi), \theta = 0°)$ where R_1 is the rigidity corresponding to the first
discontinuity in asymptotic longitude as defined below.

The allowed rigidity spectrum of cosmic ray particles arriving from
a specific direction at any location in the geomagnetic field contains
distinct fiducial marks: R_1, the rigidity associated with the first dis-
continuity in asymptotic longitude occurring as the trajectory calcula-
tions are progressing down through the rigidity spectrum, and R_U, the
rigidity at and above which the trajectory calculations yield allowed

orbits. The rigidity value R_1 is always greater than or equal to R_U if both are determined by employing the same uniform discrete (usually 0.01 GV) rigidity intervals in the trajectory calculations. R_1 is, in general, a value extremely close to the main cone cutoff rigidity as defined by Lemaitre and Vallarta (1936). A change in the geomagnetic field has an almost equivalent effect on both the rigidity corresponding to the first discontinuity and the vertical upper cutoff, and results in a similar effect on the vertical effective cutoff rigidity (Fluckiger et al., 1983a, 1983b).

Fluckiger et al., (1983b) have shown that geomagnetic disturbances reduce the cutoff rigidity in a predictable manner dependent on the strength and longitudinal structure of the magnetic perturbation and the longitudinal difference between the magnetic perturbation and the observing location. Furthermore, the change in asymptotic longitude (down to the first discontinuity) also behaves in a similarly predictable manner. Therefore the asymptotic directions of approach during perturbed geomagnetic conditions can be deduced with considerable accuracy from the asymptotic directions computed using the quiescent geomagnetic field if the associated change in cutoff rigidity is known. In this paper we extend these concepts to include the time evolution of the geomagnetic field on the asymptotic direction of approach for cosmic ray particles arriving at a particular location.

2. Method. We will define the terms $\delta\lambda^*(R)$ and $\delta\psi^*(R)$ as $\delta\lambda^*(R) = \lambda'(R) - \lambda(R - \delta R_1)$, and $\delta\psi^*(R) = \psi'(R) - \psi(R - \delta R_1)$, where $\delta R_1 = R_1' - R_1$, and the primed values indicate the evolved geomagnetic field and the unprimed values indicate the reference geomagnetic field. When these values are plotted as a function of rigidity, it has been found that there are practically no changes for $\delta\lambda^*$ down to the rigidity value of R_1'. Therefore, we may set $\delta\lambda^* = 0°$ (Fluckiger et al., 1983b). For $\delta\psi^*$ only small residual changes on the order of several degrees are found down to rigidities approaching the value of R_1. For any particular location and for rigidities up to several GV above the main cutoff the following expressions can be used to describe the correlation between the asymptotic directions in an evolved geomagnetic field and the asymptotic directions in a reference geomagnetic field:

$$\lambda'(R) \approx \lambda(R - \delta R_1), \text{ and } \psi'(R) \approx \psi(R - \delta R_1) + C \cdot \delta R_1,$$

where C is a measure of the residual change $\delta\psi^*$. This procedure is valid only for rigidities larger than R_1 or R_1', respectively.

At rigidities below R_1 no similar relation has been found, although coherent clusters of trajectories may be distorted uniformly by magnetic changes. It has been shown that the main features of allowed and forbidden regions in the penumbra are conserved to a certain extent in a perturbed geomagnetic field (Fluckiger et al., 1979, 1982). However, the asymptotic longitudes of the allowed trajectories of the fine detailed structure in the cosmic ray penumbra continue to be quasi-random.

3. Application. We have applied this procedure by comparing the asymptotic directions calculated for the International Geomagnetic Reference Field Epoch 1965.0 with those calculated for epoch 1980.0 for cosmic ray stations and world grid locations. To illustrate this application, we

the evolved values. We would expect a close comparison between the asymptotic directions above the first discontinuity (R_1) such that $\psi(R_1^*) + \Delta R \approx \psi'(R_1^{*'}) + \Delta R$ where ΔR represents an arbitrary rigidity value above R_1^* and $R_1^{*'}$. Here ψ denotes the asymptotic longitude in the reference field and ψ' the asymptotic longitude in the evolved field. R^* and $R^{*'}$ are approximations to the rigidity value of the first discontinuity obtained by examining the gradient in the change of the asymptotic direction with rigidity as the first discontinuity is approached from rigidity values above the main cutoff. The values selected to approximate the first discontinuity in asymptotic direction, R_1^* and $R_1^{*'}$ were the rigidity values where the gradient in asymptotic direction was greater than $1000°$ per GV and increased by more than 1.5 times in the next 0.01 GV increment. Examination of these results and comparison with other calculations have shown that this approximation is close to and slightly greater than the rigidity of the first discontinuity calculated using very small rigidity intervals.

For the examples given in the following tables, the increment of rigidity added to the approximation of the first discontinuity value was the change in rigidity of the first discontinuity between the reference field and the evolved field. This value was used because it was sure to be in the set of continuous asymptotic directions above the main cutoff in both data sets. In Table 1 we illustrate the results for cosmic ray stations at locations where the geomagnetic cutoff is decreasing with time. In Table 2 we show results for cosmic ray stations at locations where the cosmic ray cutoff is increasing with time. An inspection of the asymptotic longitudes given in the second and third columns from the right in these tables indicates that the asymptotic longitudes for the specified rigidity values are quite similar.

4. Conclusions. We have illustrated that the asymptotic directions for an evolved geomagnetic field for rigidity values above the R_1 value (the first discontinuity in asymptotic direction progressing down through the rigidity scale) can be obtained from a "known" reference set of asymptotic directions if the change in cutoff is known.

References.
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TABLE 1.
Changes of Asymptotic Longitude for Cosmic Ray Stations Where the Cutoff is Decreasing.

ψ for 1980 at $R_1^{*'} + |\Delta R| \approx \psi$ for 1965 at $R_1^* + |\Delta R|$, where $\Delta R = (R_1^{*'}$ for 1980$) - (R_1^*$ for 1965$)$

STATION NAME	GEOGRAPHIC LAT	LONG	EPOCH 1965 R_U	R_1^*	ψ	EPOCH 1980 R_U'	$R_1^{*'}$	ψ'	ΔR_1^*	1965 RIG	ψ	1980 ψ'	RIG
Ahmedabad	23.01	72.61	15.90	15.92	46	15.77	15.79	51	-0.13	16.05	329	327	15.92
Alma Ata	43.20	76.94	6.92	6.93	36	6.87	6.90	30	-0.03	6.96	358	355	6.93
Brisbane	-27.50	153.01	7.39	7.42	121	7.22	7.26	116	-0.16	7.58	31	30	7.42
Buenos Aires	-34.58	301.50	10.59	10.61	266	10.12	10.15	246	-0.46	11.07	130	124	10.61
Chacaltaya	-16.31	291.85	12.85	12.87	257	12.53	12.54	263	-0.33	13.20	144	144	12.87
Climax	39.37	253.82	3.14	3.24	146	3.12	3.22	153	-0.02	3.26	124	128	3.24
Gulmarg	34.07	74.42	12.33	12.35	44	12.24	12.26	45	-0.09	12.44	336	337	12.35
Hermanus	-34.42	19.22	5.02	5.06	307	4.83	4.86	311	-0.20	5.26	215	214	5.06
Hobart	-42.90	147.33	2.10	2.12	44	2.06	2.08	43	-0.04	2.16	13	11	2.12
Huancayo	-12.05	284.67	13.24	13.25	266	12.91	12.93	253	-0.32	13.57	142	140	13.25
Mexico City	19.33	260.82	9.57	10.24	257	9.27	9.94	250	-0.30	10.54	121	123	10.24
Mildura	-34.23	142.22	4.56	4.59	97	4.43	4.46	87	-0.13	4.72	11	11	4.59
Mt. Wellington	-42.92	147.24	2.03	2.11	49	1.99	2.07	49	-0.04	2.15	14	12	2.11
Palestine	31.75	264.35	4.74	4.90	185	4.69	4.86	194	-0.04	4.94	145	147	4.90
Potchefstroom	-26.70	27.10	7.68	7.72	346	7.49	7.53	337	-0.19	7.91	247	245	7.72
Sydney	-33.60	151.10	5.16	5.19	86	5.06	5.09	84	-0.10	5.29	21	21	5.19
Tbilisi	41.72	44.80	6.96	7.00	357	6.95	6.97	14	-0.03	7.03	322	321	7.00

TABLE 2.
Changes of Asymptotic Longitude for Cosmic Ray Stations Where the Cutoff is Increasing.

ψ for 1980 at $R_1^{*'} + |\Delta R| \approx \psi$ for 1965 at $R_1^* + |\Delta R|$, where $\Delta R = (R_1^{*'}$ for 1980$) - (R_1^*$ for 1965$)$

STATION NAME	GEOGRAPHIC LAT	LONG	EPOCH 1965 R_U	R_1^*	ψ	EPOCH 1980 R_U'	$R_1^{*'}$	ψ'	ΔR_1^*	1965 RIG	ψ	1980 ψ'	RIG
Athens	37.97	23.72	8.98	8.99	355	9.06	9.08	340	0.09	9.08	280	275	9.17
Bologna	44.50	11.33	5.41	5.44	297	5.52	5.55	295	0.11	5.55	231	231	5.66
Budapest	47.50	18.90	4.74	4.77	309	4.82	4.83	309	0.06	4.83	254	253	4.89
Calgary	51.08	245.91	1.16	1.22	123	1.17	1.24	125	0.02	1.24	95	94	1.26
Deep River	46.10	282.50	1.13	1.19	170	1.25	1.32	150	0.13	1.32	80	85	1.45
Dourbes	50.10	4.60	3.42	3.44	298	3.57	3.60	294	0.16	3.60	208	215	3.76
Durham	43.10	289.16	1.67	1.69	197	1.84	1.86	179	0.17	1.86	93	103	2.03
Fukushima	37.75	140.48	11.36	11.38	104	11.45	11.47	100	0.09	11.47	40	38	11.56
Irkutsk	52.47	104.03	3.92	3.95	34	3.95	3.98	42	0.03	3.98	359	4	4.01
Jungfraujoch	46.55	7.98	4.81	4.82	300	4.91	4.94	289	0.12	4.94	222	220	5.06
Kerguelen Is.	-49.35	70.22	1.24	1.31	299	1.15	1.31	315	0.00	1.31	299	315	1.31
Kiel	54.33	10.13	2.39	2.50	298	2.59	2.61	307	0.11	2.61	224	226	2.72
Kiev	50.72	30.30	3.74	3.77	335	3.79	3.80	332	0.03	3.80	290	289	3.83
Leeds	53.82	358.45	2.26	2.35	265	2.41	2.48	267	0.13	2.48	191	204	2.61
Lomnicky Stit	49.20	20.22	4.21	4.24	314	4.28	4.31	329	0.07	4.31	256	257	4.38
Magadan	60.11	151.01	2.22	2.33	52	2.34	2.36	45	0.03	2.36	9	6	2.39
Morioka	39.70	141.13	10.47	10.51	122	10.61	10.63	109	0.12	10.63	35	32	10.75
Moscow	55.47	37.32	2.60	2.61	320	2.50	2.62	313	0.01	2.62	302	297	2.63
Mt. Norikura	36.12	137.56	12.02	12.04	09	12.09	12.11	107	0.07	12.11	45	46	12.18
Mt. Washington	44.30	288.70	1.41	1.50	165	1.55	1.66	214	0.16	1.66	89	96	1.82
Mussala	42.18	25.58	6.50	6.51	346	6.56	6.57	343	0.06	6.57	281	280	6.63
Pic du Midi	42.93	0.25	5.58	5.61	294	5.80	5.81	306	0.20	5.81	204	209	6.01
Predigtstuhl	47.70	12.88	4.59	4.60	309	4.68	4.70	299	0.10	4.70	235	232	4.80
Rome	41.90	12.52	6.35	6.37	318	6.50	6.54	324	0.17	6.54	235	236	6.71
Tokyo-Itabashi	35.75	139.72	12.12	12.13	115	12.19	12.21	99	0.08	12.21	45	41	12.29
Yakutsk	62.02	129.72	1.74	1.85	22	1.78	1.87	32	0.02	1.87	352	356	1.89
Zugspitze	47.42	10.98	4.62	4.64	314	4.72	.75	298	0.11	4.75	230	227	4.86