HADRONIC COMPONENTS OF EAS BY RIGOROUS SADDLE POINT METHOD IN THE ENERGY RANGE BETWEEN 10^5 AND 10^8 GEV.

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Introduction : The study of hadronic components in the high energy range between 10^5 and 10^8 Gev exhibits by far the strongest mass sensitivity[1] since the primary energy spectrum as discussed by Linsley[2] and measured by many air shower experimental groups[3] indicates a change of slope from -1.7 to 2.0 in this energy range. This change of slope may be due to several reasons such as due to a genuine spectral feature of astrophysical origin, a confinement effect of galactic component or a rather rapid change of mass which problem has been attempted here to study in details.

Several attempts have already been done as discussed by Grider[3]. Here, we shall analyse the hadronic components of EAS at this energy range within the scheme of a scale breaking model of multiparticle production[4]. We have used a modified rigorous saddle point method[5] to calculate the inverse Mellin transform of diffusion equations. The hadron spectra has been calculated for primary energy range from 10^5 to 10^8 Gev at sea-level as well as at mountain height at 720 gm/m^2.

Method : The integro-differential cascade equation for nucleon N and pions N_π are respectively given by

$$\frac{\partial N}{\partial y}(E, y) = -\frac{N(E, y)}{\lambda_{in}} + \int_E^{E_0} \frac{N(E', y)}{\lambda_{in}} W_{NN}(E', E)\, dE' \qquad (1)$$

$$\frac{\partial N_\pi}{\partial y}(E, y) = -\left(\frac{1}{\lambda_{in}} + \frac{B}{Ey}\right) N_\pi(E, y) + \int_E^{E_0} \frac{N(E', y)}{\lambda_{in}} W_{N\pi}(E, E')\, dE'$$

$$+ \int_E^{E_0} \frac{N_\pi(E', y)}{\lambda_{in}} W_{\pi\pi}(E', E)\, dE' \qquad (2)$$

For simplification, we have considered (i) the nucleon interaction length $\lambda_N \approx$ pion interaction length λ_π at high energy[6]. (ii) Limiting fragmentation is assumed to hold for F_{NN}. The unknown function $N_\pi(E, y)$ can be expressed in the form

$$N_\pi = n_\pi(E, y)\, e^{-y/\lambda_\pi} = n_\pi(E, y)\, e^{-y'} \qquad (3)$$

Let $\sigma_{in} = \sigma_{in}\ (1 + a \ln E/100)$ (4)

i.e. $\frac{1}{\lambda_\pi} = (1 + a \ln E/100)$ where $a \simeq .037$.

Initially, this rigorous saddle point method was proposed for scaling model(5) where

$$n_\pi + N_n = \frac{C_0}{E}\sqrt{\frac{\pi}{y'}}\ e^{y'\ (g(s) - 1) - \bar{s}_0 \ln w} \quad (5)$$

So is given by $\left.\frac{dg}{ds}\right|_{s = s_o} = \frac{\ln w}{y'}$ and $\bar{s}_o$ by $= s_o - ay' + 1$

But according to our model the inclusive cross-section and the average multiplicity are given by

$$\frac{1}{\sigma_{in-el}} E \left.\frac{d\sigma}{d^3p}\right|_{pp \to \pi \bar{x}} \simeq \frac{9}{\langle n \rangle \pi^-} \cdot \frac{25}{4} \cdot \frac{1}{4\pi} \frac{f_\rho\ \pi w}{16\pi^2} \exp\left[- \frac{26.88}{\langle n \rangle_{\pi^-}} \cdot \frac{p_T^2}{(1-x)}\right]$$

$$\exp\left[- 2.38\ \langle n \rangle_{\pi^-}\ x \right] \frac{mb}{Gev^2} \quad (6)$$

where $\langle n \rangle_{\pi^+}\ _{pp} = \langle n \rangle_{\pi^-}\ _{pp} = \langle n \rangle_{\pi^0} = 0.35\ S^{\frac{1}{3}}$ (Sin Gev^2)

Thus $F_{N\pi} = 1.06\ e^{-2.38 \langle n \rangle_\pi \cdot x}$

$F_{\pi\pi} = 1.062\ e^{-2.38 \times 5/9 \langle n \rangle_\pi \cdot x}$

Now at high energy we can write $F_{N\pi} = 1.06\ e^{-2.38 . 2^{\frac{1}{3}} x^{\frac{1}{3}} E_0^{\frac{1}{3}}}$ (7)

where $E/E' \sim E/E_0$.

Here x be the Feynman scaling variable as used in the previous paper on saddle point method(5). So we can use the previous method to our case also. Now after some calculations it can be shown that s_o admits of an approximate representation in terms of gamma function i.e.

$$\left.\frac{dg}{ds}\right|_{s = s_o} = \frac{\ln w}{y'}$$

Here the following approximation has been taken for gamma function

$$(s) = 1 + a_1(s - 1) + a_2\ (s - 1)^2 + a_3\ (s - 1)^3 + a_4\ (s - 1)^4 + a_5(s - 1)^5$$

where $a_1 = 0.5748$, $a_2 = 0.95123$, $a_3 = - 0.699$, $a_4 = 0.4245$
$a_5 = - 0.10106$.

Results : In Fig.1, the several data are shown together with the hadron spectrum obtainted by us from our model and also with that calculated by Grieder. These data cover a large size range centred around Ne $= 10^5$ and this corresponds to primary energy 10^6 Gev. So different types of detecting system with large size window, explain this large dispersion of data.

But even with this dispersion we can emphasize from the comparison of these date with our prediction for iron and proton initiated showers of same total energy i.e. 10^6 Gev, that this energy range of primary spectrum can not be dominated by iron nuclei.

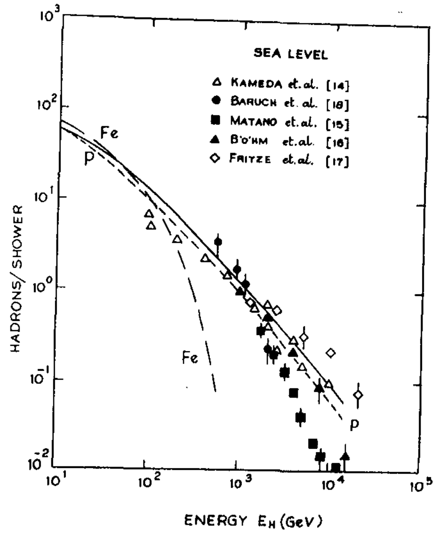

FIG.1

Hadron energy spectra at sea level in proton initiated showers of 10^6 Gev primary energy obtained from our present work (solid line). Dashed and dot lines are from Grieder [1].

In Fig.2, we have compared the data obtained by Dubovy et al(7) at 720 gm/m^2 which shows a nice agreement with our predicted hadron spectra with proton as primary particles whereas the spectra expected by taking heavy particle (Fe = 56) as primary is showing a sharp cut off supporting our conclusion.

Now it is evident from the above results that in the critical energy region under consideration i.e. between 10^5 and 10^8 Gev, in particular around 10^6 Gev, is not dominated by iron primaries.

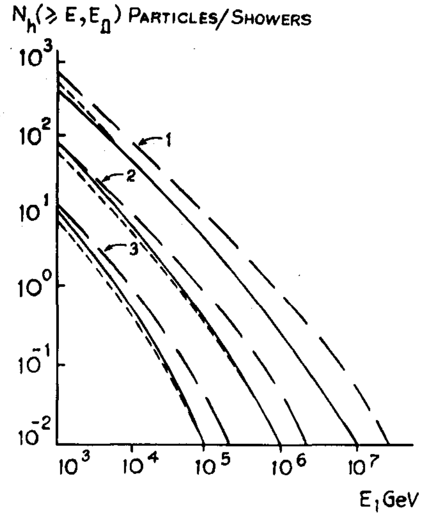

FIG. 2

Integrated energy spectra of hadrons in EAS as calculated for y = 720 gm/cm^2 and primary energy E_0 (Gev) 1-10^8, 2-10^7, 3-10^6 and compared with those from Boyadzhyn et. al. [22] for $a_n = a_\pi = 0$ (dashed curves), $a_n = a_\pi = 0.03$ (dot curves) and prediction from present model (solid curves).

References :

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