## THE LONGITUDINAL DEVELOPMENT OF MUONS IN COSMIC RAY AIR SHOWERS AT ENERGIES $10^{15}-10^{17}$ eV

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## ABSTRACT

The relationship between longitudinal development of muons and conventional equi-intensity cuts is carefully investigated. The development of muons in EAS has been calculated using simulation with a scaling violation model at the highest energies and mixed primary composition. Profiles of equi-intensity cuts expected at observation altitudes of 550, 690 and 930 gcm<sup>-2</sup> can fit the observed data very well.

1. Introduction In recent years estimates of the longitudinal development of muons in EAS have been presented by several groups, based on equi-intensity cuts of muon size spectra following the method introduced 20 years ago by the BASJE group [1]. As is well known the relationship between such equi-intensity cuts and the profile of longitudinal development of the muons is not as straightforward as in the case of the electromagnetic component, and is particular to the altitude of observation. This arises because the muon longitudinal development itself depends on the zenith angle and the nature of the primary particle, moreover observations are made in general above a muon threshold energy which is zenith angle dependent. Some reported calculations when compared with measurements appear to show significant discrepancies [2],[3]. We report here calculations of equi-intensity curves, based on simulated showers, for different fractions of Fe nuclei in an otherwise pure proton beam using a model for hadronic interactions consistent with the degree of scaling violation discussed in [4].

2. Model Used in the Simulations A model for p-p collisions was adopted which incorporates scaling violation for interactions of the leading particle at  $E > 2 \ 10^{13} \text{ eV}$  to the extent proposed in [4], with radial scaling assumed for all other interactions. Above  $10^{11} \text{ eV}$  the cross-section for hadron air-nucleus interactions was assumed to rise as  $\sigma^{\text{inel.}} = \sigma_{\varrho} (1+a\ln^2 E)$ , with values of the interaction lengths below this energy given by  $\lambda_{p-air} = 90 \text{ gcm}^{-2}$  and  $\lambda_{\pi-air} = \lambda_{k-air} = 120 \text{ gcm}^{-2}$ . For heavy nuclei incident we assumed  $\sigma^{\text{inel.}} = \pi r_{\varrho} (A^{1/3} + A_{air}^{-1/3} - \delta)^2$ . The smaller elasticity [5] and larger multiplicity [6] reported for p-air nucleus collision was allowed for by taking  $\eta_{p-air} = 0.31$  and  $\langle m_{p-air} \rangle = 1.58 \langle m_{pp} \rangle$ . Although these additional particles arising from a huclear target are in the central region and contribute little to the shower size they make a noticable contribution to the muon component at low energies. A  $\Gamma$  distribution was taken for the transverse momentum, whose mean,  $\langle p_t \rangle$ , was assumed to vary with the density of particles in rapidity space, consistent with recent accelerator data [7].

Three dimensional Monte Carlo simulations were made for proton and Fe nucleus primaries at different energies, at the vertical and at zenith angles of  $30^{\circ}$ ,  $40^{\circ}$ ,  $50^{\circ}$  and  $60^{\circ}$ . In fig.1 we show the average



Expected muon longitudinal development at fixed primary energies initiated by protons (solid curves) and iron dashed curves). The dot-dashed curve is for radial scaling model.

vertical longitudinal development of muons,  $N_{\mu}(E_0, \theta=0, x, \varepsilon > 0.6 \text{GeV})$  at fixed energies, the case for protons where radial scaling is assumed at all energies is also shown for comparison;  $\varepsilon$  here is the muon threshold energy. These curves are generally similar to the calculations reported in [3].

3. Calculation of Equi-intensity Cuts To replicate the method used to derive equi-intensity cuts in the experiments would require the derivation of muon size spectra at different depths and zenith angles arising from particles selected from a primary spectrum of given slope and composition. This would involve an inordinate amount of computation, instead an approximate method is used. Because the muon decay probability depends on zenith angle, vertical longitudinal profiles cannot be used for constructing equi-intensity curves for comparison with experiment, unlike the case for the electron component. Longitudinal developments at different zenith angles at each depth must be used, in addition, as already mentioned, the relevant muon threshold energy must, in general, be taken as  $\epsilon_0 Sec\theta$  where  $\epsilon_0$  is the threshold for vertical muons. At a vertical depth  $x_0$ , when there is only one primary species present the experimental equi-intensity cuts correspond to  $N_{L}(E_0, \theta, x = x_0 Sec\theta, \varepsilon > \varepsilon_0 Sec\theta)$  — assuming a unique relationship between

the energy of the primary  $E_0$  and muon size at  $\theta$ ,  $J(N_{\mu},x) = J(E_0(N_{\mu},x))$  (fluctuations may be allowed for by considering instead root mean square sizes [8]). If we consider a primary beam containing two species, protons and Fe nuclei say, we have

$$J(N_{\mu}, x) = J_{p}(E_{o}^{(p)}(N_{\mu}, x)) + J_{Fe}(E_{o}^{(Fe)}(N_{\mu}, x)).$$

If the primary spectrum  $J(E_0) = AE_0^{-\gamma}$  is assumed to contain protons, with a constant fraction k, and Fe nuclei only an equi-intensity curve is defined by

 $AE_{o}^{(p)-\gamma}[k+(1-k)(E_{o}^{(p)}/E_{o}^{(Fe)})^{\gamma}] = J_{\mu}$ , a constant.....(1)

Since the range of primary energies of one species contributing to any one equi-intensity curve will not be very great we approximate  $N_{\mu}^{(p)}(E_{o},x) = E_{o}^{\omega}g_{p}(x), N_{\mu}^{(Fe)}(E_{o},x) = E_{o}^{\omega}g_{Fe}(x)$ , where  $\omega$  may depend on depth or zenith angle. Defining  $\Lambda(x) = (g_{p}(x)/g_{Fe}(x))^{1/\omega}$  and using (1) we may write  $N_{\mu}(x,J_{\mu}) = E_{o}^{\omega}g_{p}(x)$ , where  $E_{o}^{\dagger} = E_{o}^{(p)}(N_{\mu},x)[k+(1-k)\Lambda^{-\gamma}(x)]^{1/\gamma}$  $E_{o}^{(p)}(N_{\mu},x) = (A/J_{\mu})^{1/\gamma}$ . Thus when k is not 0 or 1 equi-intensity curves' should be constructed from the simulated longitudinal developments, not at a fixed  $E_{o}$  but at values of  $E_{o}^{\prime}$  which depend on the depth.

4. Results Using the profiles simulated at different zenith angles values of the function  $\Lambda(x)$  could be obtained. Taking A = 2.5 x 10<sup>6</sup>,



 $\gamma = 2$  [8], expected equi-intensity curves for  $J_{\mu} = 2.5 \ 10^{-8} \ m^{-2} \ s^{-1} \ sr^{-1} \ at$ Chacaltaya, Tien Shan and Akeno have been calculated and are shown in fig.2, where a significant difference from the profiles at constant energy can be observed. Curves for 75%p, 25%Fe, normalised in each case to give a best fit to the data, are compared to experimental data from these experiments in fig.3. The relevance of these curves to the primary composition is noted in another paper (OG 5.2-12) at this conference.

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Fig.3 Normalised muon development curves (75%p, 25%Fe) compared with observations.