THE LONGITUDINAL DEVELOPMENT OF MUONS IN COSMIC RAY AIR SHOWERS AT ENERGIES $10^{15}-10^{17} \mathrm{eV}$

T. Cheung and P.K. MacKeown Department of Physics University of Hong Kong HONG KONG


#### Abstract

The relationship between longitudinal development of muons and conventional equi-intensity cuts is carefully investigated. The development of muons in EAS has been calculated using simulation with a scaling violation model at the highest energies and mixed primary composition. Profiles of equi-intensity cuts expected at observation altitudes of 550,690 and $930 \mathrm{gcm}^{-2}$ can fit the observed data very well.


1. Introduction In recent years estimates of the longitudinal development of muons in EAS have been presented by several groups, based on equi-intensity cuts of muon size spectra following the method introduced 20 years ago by the BASJE group [1]. As is well known the relationship between such equi-intensity cuts and the profile of longitudinal development of the muons is not as straightforward as in the case of the electromagnetic component, and is particular to the altitude of observation. This arises because the muon longitudinal development itself depends on the zenith angle and the nature of the primary particle, moreover observations are made in general above a muon threshold energy which is zenith angle dependent. Some reported calculations when compared with measurements appear to show significant discrepancies [2],[3]. We report here calculations of equi-intensity curves, based on simulated showers, for different fractions of Fe nuclei in an otherwise pure proton beam using a model for hadronic interactions consistent with the degree of scaling violation discussed in [4].
2. Model Used in the Simulations A model for p-p collisions was adopted which incorporates scaling violation for interactions of the leading particle at $\mathrm{E}>210^{13} \mathrm{eV}$ to the extent proposed in [4], with radial scaling assumed for all other interactions. Above $10^{11} \mathrm{eV}$ the cross-section for hadron air-nucleus interactions was assumed to rise as $\sigma^{\text {inel. }}=\sigma_{0}\left(1+\operatorname{aln}^{2} E\right)$, with values of the interaction lengths below this energy given by $\lambda_{\text {p-air }}=90: \mathrm{gcm}^{-2}$ and $\lambda_{\pi-\operatorname{air}_{2}}=\lambda_{\mathrm{k}_{-}-\operatorname{air}_{1}}=120 \mathrm{gcm}^{-2}$. For
 smaller elasticity [5] and larger multiplicity [6] reported for p-air nucleus collision was allowed for by taking $\eta_{p-a i r}=0.31$ and $<m_{p-a i r}>=1.58<m_{p p}>$. Although these additional particles arising from a huclear target are in the central region and contribute little to the shower size they make a noticable contribution to the muon component at low energies. A $\Gamma$ distribution was taken for the transverse momentum, whose mean, $\left\langle p_{t}\right\rangle$, was assumed to vary with the density of particles in rapidity space, consistent with recent accelerator data [7].

Three dimensional Monte Carlo simulations were made for proton and Fe nucleus primaries at different energies, at the vertical and at zenith angles of $30^{\circ}, 40^{\circ}, 50^{\circ}$ and $60^{\circ}$. In fig. 1 we show the average


Fig. 1 Expected muon longitudinal. development at fixed primary energies initiated by protons (solid curves) and iron dashed curves). The dot-dashed curve is for radial scaling model.
vertical longitudinal development of muons, $N_{\mu}\left(\mathrm{E}_{\mathrm{o}}, \theta=0, \mathrm{x}, \varepsilon>0.6 \mathrm{GeV}\right)$ at fixed energies, the case for protons where radial scaling is assumed at all energies is also shown for comparison; $\varepsilon$ here is the muon threshold energy. These curves are generally similar to the calculations reported in [3].
3. Calculation of Equi-intensity Cuts To replicate the method used to derive equi-intensity cuts in the experiments would require the derivation of muon size spectra at different depths and zenith angles arising from particles selected from a primary spectrum of given slope and composition. This would involve an inordinate amount of computation, instead an approximate method is used. Because the muon decay probability depends on zenith angle, vertical longitudinal profiles cannot be used for constructing equi-intensity curves for comparison with experiment, unlike the case for the electron component. Longitudinal developments at different zenith angles at each depth must be used, in addition, as already mentioned, the relevant muon threshold energy must, in general, be taken as $\varepsilon_{0} \operatorname{Sec} \theta$ where $\varepsilon_{0}$ is the threshold for vertical muons. At a vertical depth $\mathrm{x}_{\mathrm{O}}$, when there is only one primary species present the experimental equi-intensity cuts correspond to $N_{\mu}\left(E_{0}, \theta, x=x_{0} \operatorname{Sec} \theta, \varepsilon>\varepsilon_{0} \operatorname{Sec} \theta\right)$ - assuming a unique relationship between
the energy of the primary $E_{o}$ and muon size at $\theta, J\left(N_{\mu}, x\right)=J\left(E_{o}\left(N_{\mu}, x\right)\right)$ (fluctuations may be allowed for by considering instead root mean square sizes [8]). If we consider a primary beam containing two species, protons and Fe nuclei say, we have

$$
J\left(N_{\mu}, x\right)=J_{p}\left(E_{o}^{(p)}\left(N_{\mu}^{\xi}, x\right)\right)+J_{F e}\left(E_{o}^{(F e)}\left(N_{\mu}, x\right)\right)
$$

If the primary spectrum $J\left(E_{O}\right)=A E_{o}^{-\gamma}$ is assumed to contain protons, with a constant fraction $k$, and Fe nuclei only an equi-intensity curve is defined by

$$
\begin{equation*}
\mathrm{AE}_{\mathrm{o}}^{(\mathrm{p})-\gamma}\left[\mathrm{k}+(1-\mathrm{k})\left(\mathrm{E}_{\mathrm{o}}^{(\mathrm{p})} / \mathrm{E}_{\mathrm{o}}^{(\mathrm{Fe})}\right)^{\gamma}\right]=J_{\mu}, \text { a constant. } \tag{1}
\end{equation*}
$$

Since the range of primary energies of one species contributing to any one equi-intensity curvewill not be very great we approximate $N_{\mu}^{(p)}\left(E_{0}, x\right)=E_{o}^{\omega} g_{p}(x), N_{\mu}^{(F e)}\left(E_{O}, x\right)=E_{O}^{\omega} g_{F e}(x)$, where $\omega$ may depend on depth or zenith angle. Defining $\Lambda(x)=\left(g_{p}(x) / g_{F e}(x)\right)^{1 / \omega}$ and using (1) we may write $N_{\mu}\left(x, J_{\mu}\right)=E_{o}^{\omega} g_{p}(x)$, where $E_{o}^{\prime}=E_{0}(p)\left(N_{\mu}, x\right)\left[k+(1-k) \Lambda^{-\gamma}(x)\right]^{1 / \gamma}$, $\mathrm{E}_{\mathrm{O}}^{(\mathrm{p})}\left(\mathrm{N}_{\mu}, \mathrm{x}\right)=\left(\mathrm{A} / \mathrm{J}_{\mu}\right)$. Thus when k is not 0 or 1 equi-intensity curves' should be constructed from the simulated longitudinal developments, not at a fixed $E_{O}$ but at values of $E_{o}^{\prime}$ which depend on the depth.
$\frac{\text { 4. Results }}{\text { values of the }}$ function the profiles simulated at different zenith angles values of the function $\Lambda(x)$ could be obtained. Taking $A=2.5 \times 10^{6}$,


Fig. 2 Expected muon longitudinal development for an integral intensity $\mathrm{J}=2.5 \times 10^{-8} \mathrm{~m}^{-2} \mathrm{~s}^{-1} \mathrm{sr}^{-1}$
at Chacaltaya, Tien Shan and Akeno. The dot and dot-dashed curves correspond. to $50 \%$ and $25 \%$ of iron in primary cosmic ray.
$\gamma=2$ [8], expected equi-intensity curves for $J_{\mu}=2.510^{-8} \mathrm{~m}^{-2} \mathrm{~s}^{-1} \mathrm{sr}^{-1}$ at Chacaltaya, Tien Shan and Akeno have been calculated and are shown in fig. 2, where a significant difference from the profiles at constant energy can be observed. Curves for $75 \%$ p, $25 \% \mathrm{Fe}$, normalised in each case to give a best fit to the data, are compared to experimental data from these experiments in fig. 3 . The relevance of these curves to the primary composition is noted in another paper ( $O G$ 5.2-12) at this conference.

Acknowledgments We are grateful to Director of Computer Services, H.K. University Centre of Computer Studies and Applications for the provision of computing facilities.

## References

[1] Bradt H., Clark, G., La Pointe, M., Domingo, V., Escobar, I., Kamata, K., Murakami, K., Suga, K., and Koyoda, Y., (1965), Proc. 9th Int. Conf. Cosmic Rays, London, 2, 715-717.
[2] Aguirre, C. et al., (1979), J. Phys. G: Nucl. Phys. 5, 151-157.
[3] Honda, M. et al., (1983), 18th Int. Conf. Cosmic Rays, Bangalore, Conference Papers, 11, 350-353.
[4] Wdowczyk, J. and Wolfendale, A.W., (1984), J. Phys. G: Nucl. Phys. 10,


Fig. 3 Normalised muon development curves ( $75 \%$ p, $25 \% \mathrm{Fe}$ ) compared with observations. 257-272.
[5] Jones, L.W., (1983), 18th Int. Conf. Cosmic Rays, Bangalore, Conference Papers, 5, 17-20.
[6] Tasaka, S. et al., (1982), Phys. Rev. D25, 1765-1785.
[7] Breakstone, A. et al., (1983), Phys. Lett. 132B, 463-466.
[8] Gaisser, T.K. and Hillas, A.M., (1977) 15th Int. Conf. Cosmic Rays Plovdiv, Conference Papers, 8, 353-357.

