# TOPOLOGICAL ASPECTS OF AGE PARAMETER

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## 1. Introduction

The well known NKG function is a very useful tool to describe the lateral extension of the electromagnetic component in EAS ; however, in spite of non negligible qualities (simplicity, normalization by beta-function), it doesn't correspond exactly to the natural shape of the lateral electron distribution. Several bias may occur in size estimation if NKG is used without correction, for instance, contradiction between lateral and longitudinal development the lateral parameter $s_\perp$ being quite lower than the longitudinal one (1)(2). We emphasize here how the longitudinal age parameter $s_t$ can be correlated with the information obtained from the lateral electron densities according to the conditions of use of the NKG function.

## 2. Local age parameter phenomenology

The theoretical age parameter $s_t$ illustrates the declining stage of a shower and is determined as the saddle point in the inverse Mellin transformation. We have postulated that the NKG function, derived from diffusion equations fails to describe EAS data, mainly because a uniqueness of parameter s was assumed, and we have admitted local agreement with NKG function in small bands of distance by introducing the local age parameter

$$s(r) = \frac{1}{2x+1}\left((x+1)\frac{\partial \mathrm{Ln}\, f}{\partial \mathrm{Ln}\, x} + (6.5\, x + 2)\right) \qquad (f \equiv \mathrm{NKG})$$

$(x = r/r_o)$, $r_o$ Molière radius. From two neighbouring points $x_i$, $x_j$, the lateral age parameter $s_{ij}$ in $[r_i, r_j]$ is given by

$$s_{ij} = \mathrm{Ln}\,(F_{ij} X_{ij}^2 Y_{ij}^{4.5}) / \mathrm{Ln}\,(X_{ij} Y_{ij})$$

where $F_{ij} = f(r_i)/f(r_j)$ , $X_{ij} = r_i/r_j$ , $Y_{ij} = (x_i+1)/(x_j+1)$.

If $r_i \to r_j$ , $s_{ij} \to s(r)$ when $r = (r_i+r_j)/2$.

Different behaviour of lateral structure function were proposed from different analytical treatment of diffusion equations (3) or Monte-Carlo simulation of e.m. cascades (4). A convenient formula was advanced consisting to use NKG formula with a Molière radius reduced by a factor $m = 0.78-0.21\, s_t$ (for individual e.m. cascade). We observed for single e.m. cascade that s(r) had, versus (3) or (4) a behaviour similar to fig. 1 ; incorporating those results in EAS-3D simulation, we ascertained that this typical behaviour of s(r) survived in EAS lateral distribution for all sizes and levels.

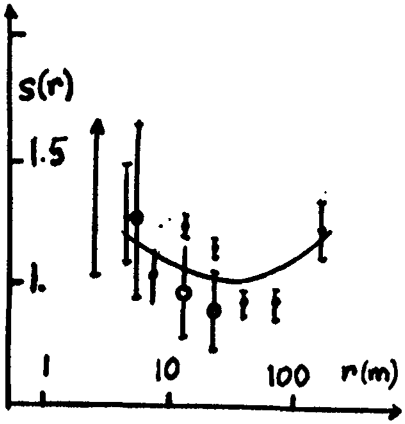


Fig. 1

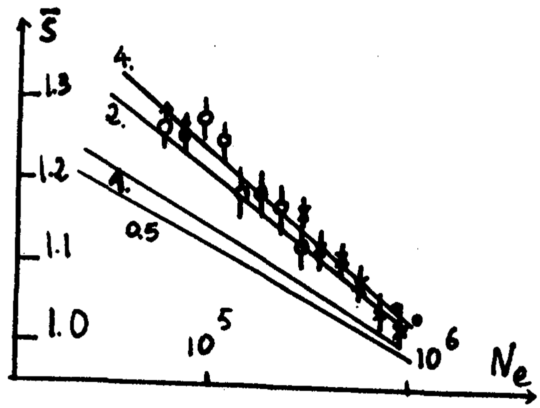


Fig. 2

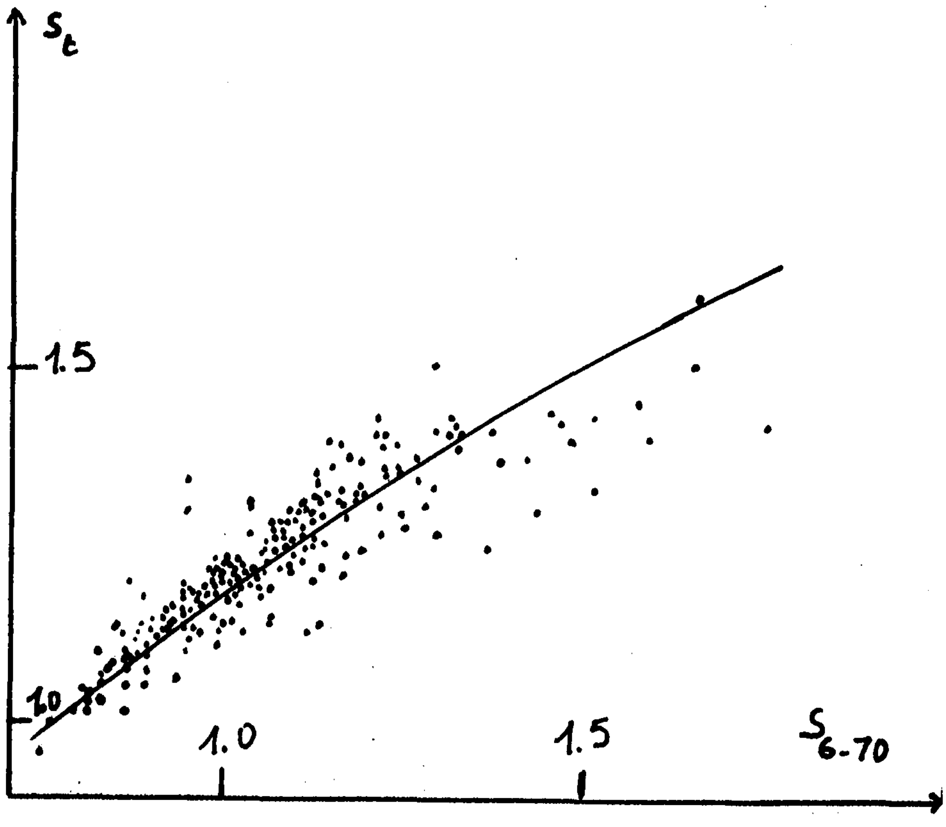


Fig. 3

Recently we developed a very detailed Monte-Carlo 3D-simulation, including all possible causes of deviation (for instance, the double body decay of each $\pi^0$, is completely described to produce energy and director cosines of each outgoing $\gamma$, $< p_t >$, is correlated with central rapidity density...) ; the model used for nuclear interaction is the multicluster phenomenological one described in HE 4.1-9,10. A surprising agreement is obtained with Akeno data (5) in favour of the parametrization (fig. 1)

$$s(r) = \alpha\xi^2 + \beta\xi + \gamma \quad , \text{ where } \xi = \text{Ln}\,(r/r_o)$$

For a uniform density of detectors, the answer of an experimental array will be averaged on all the shower disk seen by the array. The radius R of this shower disk depends on the detector area (or density threshold) and it comes :

$$\bar{s} = \frac{1}{R}\int_0^R s(r)\,dr = \frac{1}{X}\,(\alpha F_1(X) + \beta G_1(X)) + \gamma \quad \text{with } X = R/r_o$$

where $F_1(X) = X\text{Ln}^2 X - 2X(\text{Ln}\,x - 1)$ , $G_1(X) = X\text{Ln}\,X - X$. According to the shape of fig. 1, any attempt to estimate $\bar{s}$ in [0 - 50 m] and [50 - 150 m] will give the same value of the integral (6).

## 3. Comparison with experiment

The variation of $\bar{s}$ versus size has been calculated for MPM (nucleon primaries, $\theta = 22°5$) at Akeno level (fig. 2). Four values of R have been taken corresponding to 4 $e^-/m^2$, 2, 1, 0.5 $e^-/m^2$. The first value corresponding to 0.25 $m^2$ detector is in very good agreement with the experimental data. At Tian-Shan level, we have plotted the correlation between $s_{6-70}$ and $s_t$, the first parameter being estimated by NKG function from numerical values simulated at 6 and 70 m (fig. 3) for different sizes between $10^4$ - $10^6$ particles. A general correlation appears : $s_t = 1.434\, s_{6-70}^{1/2} - 0.243$.

The correspondance giving $s_t$ at Akeno and Moscow altitudes has been also obtained as $s_t = 1.246\, \bar{s}^{1/2} + 0.044$ and $s_t = 1.157\bar{s}^{1/2} + 0.183$. We note that with the present assumption included for e.m. cascade, the correlation is not independent on level (fig. 4).

## 4. Discussion

The behaviour versus r of s(r) is not smeared out by the hadronic cascade and survives in EAS (fig. 1). The agreement obtained previously with high multiplicity model is now obtained with the multicluster phenomenological model

(describing $\bar{p}$-p data) up to $5.10^6$ GeV (limit of our Monte-Carlo simulation) as well at Akeno for $\bar{s}$ and s(r) as in Tian-Shan for $s_{6-70}$. The age parameter data doesn't suggest between $2.10^4$ - $5.10^6$ GeV any increase of primary mass and supports better a nucleon dominance in primary cosmic rays.

## References

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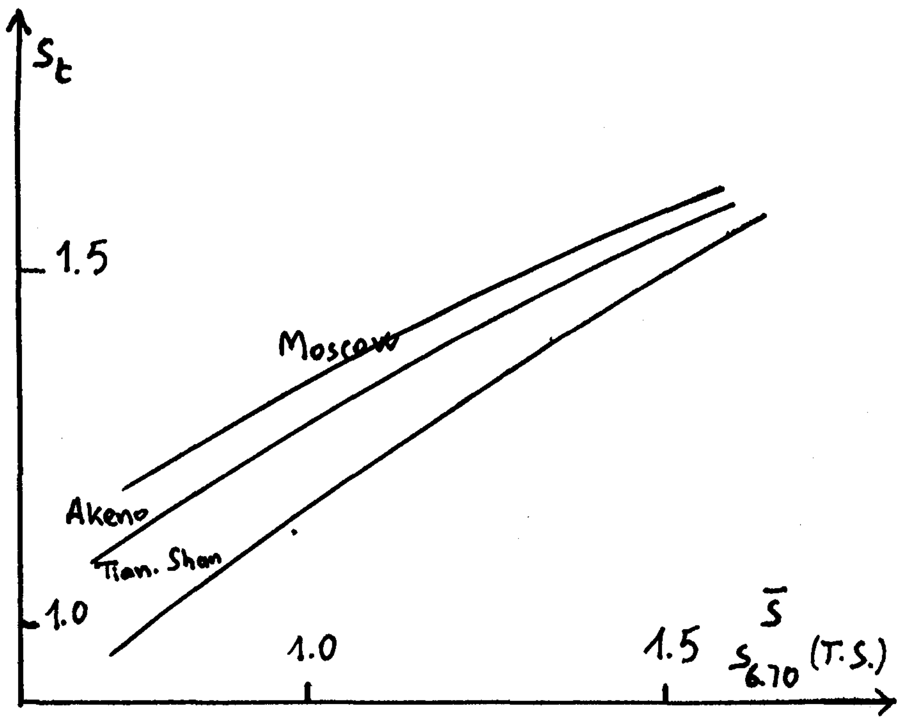


Fig. 4