# EXPECTED RATES WITH MINI-ARRAYS FOR AIR SHOWERS

Wayne E. Hazen
Randall Laboratory of Physics
University of Michigan
Ann Arbor, MI 48109, USA

1. Introduction. As a guide in the design of mini-arrays used to exploit the Linsley effect in the study of air showers, it is useful to calculate the expected rates. The results can aid in the choice of detectors and their placement or in predicting the utility of existing detector systems. Furthermore, the potential of the method can be appraised for the study of large showers. Specifically, we treat the case of a mini-array of dimensions small enough compared to the distance of axes of showers of interest so that it can be considered a point detector.

The input information is taken from the many previous studies of air showers by other groups. The calculations will give: (a) the expected integral rate, $F(\sigma,\rho)$, for disk thickness, $\sigma$, or rise time, $t_{1/2}$, with local particle density, $\rho$, as a parameter; (b) the effective detection area $A(N)$ with $\sigma(\min)$ and $\rho(\min)$ as parameters; (c) the expected rate of collection of data $F_L(N)$ versus shower size, N. The latter is flatter than the number spectrum because the detection area increases with N.

2. Method. The required input relations are:
(a) the shower disk thickness, $\sigma \simeq Br^{\beta}$ (1), which is a passable form for the large values of core distance, r, of interest here;
(b) the particle density distribution, $\rho \simeq CNr^{-n}$ (2), where N is the total number of particles and, again, the simple form is adequate for our purposes;
(c) the number spectrum, $F(N) = DN^{-\gamma}$, or $f(N) = -\gamma DN^{-\gamma-1}$ (3), where a constant value of $\gamma$ is an approximation adequate to our purposes.

(A) The frequency of Linsley events is obtained from

$$F(\sigma,\rho) = \int_{N(\min)}^{\infty} A(N)f(N)dN \tag{4}$$

where

$$A(N) = \pi\left(r^2_{(\max)} - r^2_{(\min)}\right) \tag{5}$$

and $r(\min) = (\sigma/B)^{1/\beta}$ from (1) (6) and $r(\max) = (CN/\rho)^{1/n}$ from (2) (7) and, finally, $N(\min) = (1/C)B^{-n/\beta}\,\sigma^{n/\beta}\,\rho$ (8) which comes from $A(N(\min)) = 0$ (see (5) above).

The result is:
$F(\sigma,\rho) = \pi C^{\gamma}B^{(n\gamma-2)/\beta}\,D(1-\gamma/(\gamma-2/n))\,\rho^{-\gamma}\,\sigma^{(2-n\gamma)/\beta}$ (9), where the constants are defined in (1), (2), and (3) above.

(B) The effective detecting area, $A(N)$, obtained from (5), (6), and (7), is $A(N) = \pi C^{2/n}\rho^{-2/n}N^{2/n} - \pi B^{-2/\beta}\,\sigma^{2/\beta}$ (10).

(C) The expected rate of collection of data versus shower size is

$$F_L(N) = \int_N^{\infty} A(N)f(N)dN \tag{11}$$

which gives: $F_L(N) = \pi C^{2/n} D(\gamma/(\gamma-2/n))\rho^{-2/n} N^{-\gamma+2/n} - \pi B^{-2/\beta} D\sigma^{2/\beta} N^{-\gamma}$ (12).

The Haverah Park Group reduce their data to energy of the incident primary and, therefore, give an energy spectrum, F(E), rather than a shower size spectrum, F(N). The rate calculations above are directly applicable with N replaced by E, since the approximations are about equally valid.

3. Numerical Results. In order to get a tentative idea of actual rates, values for the constants were obtained from a brief survey of the literature.

For (1), we have Linsley's fit to his Volcano Ranch data[1], which has been corroborated by others as far as average values are concerned. Thus $\sigma = 2.6(1 + r/30)^{1.5}$ (nsec) gives $\sigma \simeq 1.58\times10^{-2}\, r^{1.5}$ (nsec) that is, $B = 1.58\times10^{-2}$ and $\beta = 1.5$.

For (2), the Akeno Highlands data with scintillators reported at Bangalore[2] seem appropriate since they are for large showers and extend out to r > 1000m. The constants in (2), for large r, are C = 853 and n = 3.8 for $\rho$ to be in particles/$m^2$.

For (3), the reviews by Hillas[3] give summaries of determinations of the frequency shower-size spectrum. The constants obtained for (3) for large showers are D = 318 and $\gamma = 1.7$.

The above are plotted in Figs. 1 and 2. Some rates for accidentals (see below) are also shown.

In the case of the deep water Cerenkov detectors of Haverah Park, $t_{1/2}$ replaces $\sigma$ in (1). The constants in (1), determined from the data in the World Catalogue[4], are B = 0.14 and $\beta$=1. The constants obtained[5] for (2) are $C = 7.8\times10^{-9}$ and n = 3.5. As stated earlier, it is convenient to do (3) in terms of E (of the primary) in place of shower size, N. The constants become[5] $D = 3.5\times10^{27}$ and $\gamma = 2.17$, when E is in eV. The results are also plotted in Figs. 1 and 2.

4. Accidental Rate. A basic limitation to the sensitivity of the method is due to accidental time clustering of pulses that are unrelated. The accidental rate, R, of occurrence of m pulses within a time window $\tau$ when the rate is f can be written $R = f^m \tau^{m-1}$ .

The particle rate J×S (where J is the omni-directional flux and S the total detector area) can be a good approximation to f with the usual sort of detectors used in shower studies. There is generally a shoulder in the free running pulse height distribution and consequently a minimum pulse height, $P_v$, can be chosen such that the particle detection efficiency is high and signal/noise is also high (Fig. 3). The pulse rate is then approximately equal to the particle rate. Since $\sigma$ is the FWHM value, the effective window width, $\tau$, is, say, ~1.5$\sigma$.

We then have $R = (JS)^m (1.5\sigma)^{m-1}$ (13). In Fig. 1, we show examples for m=4 and two areas, S=2 or $4m^2$, that we are using in exploratory runs. The particle densities are then $\rho$=1 or $2m^{-2}$, and the singles rates $f \simeq 300$ or 600 $s^{-1}$. (The accidental rate for m=3 is dominated by $\mu\to e$ plus a single!)

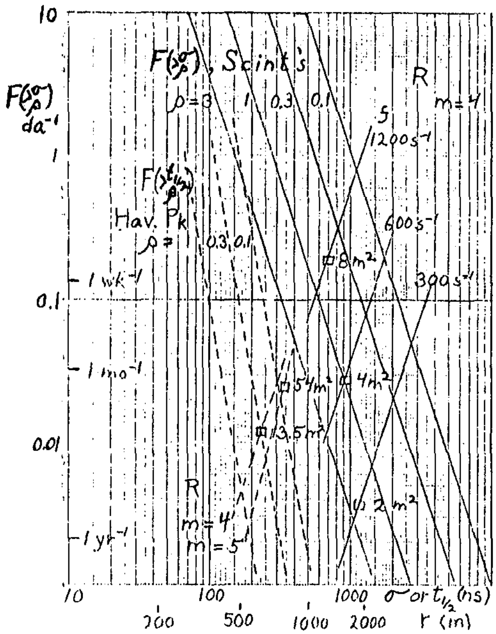


Fig. 1

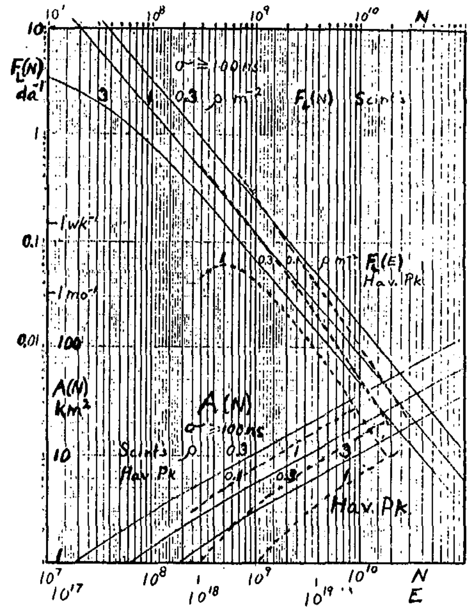


Fig. 2

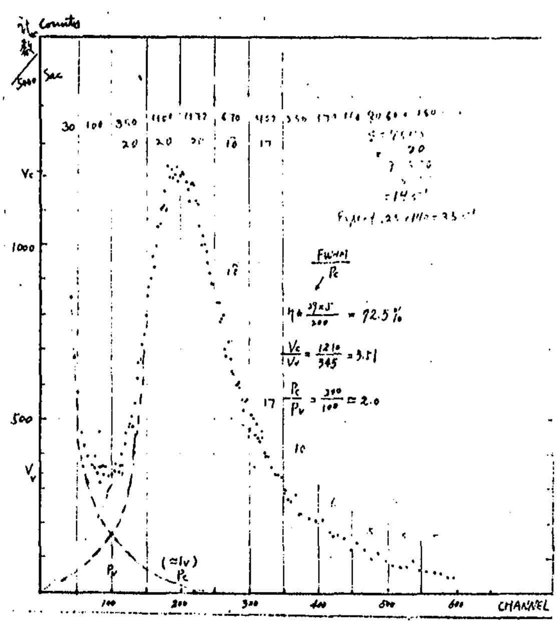


Fig. 3

5. Conclusions. The above predictions show that it is necessary to go down to low particle densities, ρ, in order to get a good rate for large showers. But, to limit the uncertainties, the size of the particle sample, ≃m, cannot be too small. Thus, the total detector area, S (=m/ρ) must be as big as feasible. However, as the area is increased, the "singles" rate increases with consequent increase in accidental rate.

The relationships among the above are implicit in Fig. 1. It may be more graphic to use total detector area, S, as abscissa with m and σ as parameters. This is done in Fig. 4, with shower rate F(S), solid lines, and accidental rate R(S), dashed lines, as ordinates. Numerical values of the parameters, m and σ, are written near the lines.

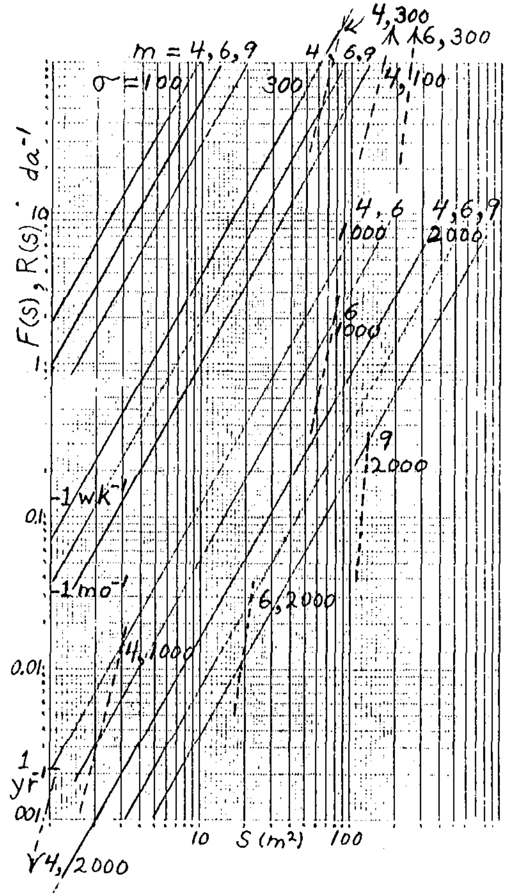


Fig. 4

It is seen that there is a good prospect for a useful "window" for exploratory observations with running times of a few weeks even with an area of a few square meters.

From Fig. 1, we see that scintillators give data collection rates larger than deep Cerenkov water tanks by about a factor three out to the distances of a few kilometers where shower structure has been measured. In view of the relative economy of water detectors, it will be desirable to develop medium depth tanks with added wave-length shifter, aiming for low-energy electron and photon sensitivity comparable to their muon sensitivity.

## References

1. Linsley, J. (1983) Research Report UNML 6/83.
2. Hara, T. et al. (1983) ICCR 11, 276.
3. Hillas, A.M. (1975) Phys. Reports 20c, 79.
4. Haverah Park (1980) Catalogue of Cosmic Rays, IPCR Itabashi, Tokyo.
5. Edge, D.M. et al. (1973) J. Phys. A 6, 1612.