MODELING CURRENTS AT SATELLITE ALTITUDES

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In a conducting medium, the magnetic field cannot be written $\vec{B}=-\vec{\nabla}\psi$ because $\mu_o\vec{J}=\vec{\nabla}\times\vec{B}\neq\vec{0}$. However, $\vec{B}$ is still solenoidal, and any solenoidal field can be written $\vec{B}=\vec{\nabla}\times\vec{\Lambda}p+\vec{\Lambda}q$ where $\vec{\Lambda}=\vec{r}\times\vec{\nabla}$. Let $S(r)$ be the spherical surface of radius r centered on the origin, and let $\langle f\rangle_r$ be the average of the function f on $S(r)$. For each r the poloidal and toroidal fields, $\vec{\nabla}\times\vec{\Lambda}p$ and $\vec{\Lambda}q$, are uniquely determined on $S(r)$ by $\vec{B}$ on $S(r)$, and p and q are determined up to arbitrary additive constants. These can always be chosen uniquely by demanding $\langle p\rangle_r=\langle q\rangle_r=0$. A toroidal field is a solenoidal field without radial component. In the solid spherical shell between $S(a)$ and $S(b)$, $\vec{J}=\vec{0}$ if and only if $q=0$ and $\nabla^2p=0$. Thus a vacuum field is poloidal, and its poloidal scalar is harmonic. In a source-free shell, the poloidal scalar p represents $\vec{B}$ as economically as does the magnetic potential ψ; and p has the advantage that it continues to be physically meaningful where $\vec{J}\neq\vec{0}$.

Being solenoidal, the current density $\vec{J}$ can be analyzed into poloidal and toroidal parts, and in fact $\mu_o\vec{J}=\vec{\nabla}\times\vec{B}=\vec{\nabla}\times\vec{\Lambda}q-\vec{\Lambda}(\nabla^2p)$. Thus the toroidal currents are the source of the poloidal magnetic field and the poloidal currents are the source of the toroidal magnetic field. For each r, the radial component of $\vec{J}$ on $S(r)$ determines the toroidal part of $\vec{B}$ on $S(r)$. If $\vec{B}$ is known on $S(r)$ when J_r is determined there, as are the poloidal magnetic fields produced by the toroidal currents inside $S(r)$ and by those outside $S(r)$. The sources of the poloidal magnetic field on $S(r)$ are inside and outside $S(r)$, while the sources of the toroidal magnetic field on $S(r)$ are on $S(r)$ itself. If $\vec{B}$ is known on $S(a)$ and $S(b)$ then the radial averages in $a<r<b$ of the toroidal current and the tangential component of the poloidal current can be determined.

Analysis of p and q into surface spherical harmonics can replace the conventional Gaussian analysis of the vacuum field. However, the radial dependence of the spherical harmonic coefficients for p and q is arbitrary in current-carrying regions unless some further physical hypothesis is introduced. At MAGSAT altitudes, a reasonable hypothesis is $\vec{J}\times\vec{B}=\vec{0}$ (field-aligned currents, or a force-free plasma). This hypothesis greatly reduces the space of field-models to be considered, and at MAGSAT altitudes it can be implemented by linear iteration with a vacuum field as the first step. One must recognize, however, that even with $\vec{J}\times\vec{B}=\vec{0}$ the magnetic effects of $\vec{J}$ are non-local. Polar currents produce equatorial magnetic fields.