

VARIABILITY OF RAINFALL OVER SMALL AREAS

N86-14098

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ABSTRACT

A preliminary investigation was made to determine estimates of the number of raingauges needed in order to measure the variability of rainfall in time and space over small areas (approximately 40 mi²). The literature on rainfall variability was examined and the types of empirical relationships used to relate rainfall variations to meteorological and catchment-area characteristics were considered. Relations between the coefficient of variation and areal-mean rainfall and area have been used by several investigators. These parameters seemed reasonable ones to use in any future study of rainfall variations. From a knowledge of an appropriate coefficient of variation (determined by the above-mentioned relations) the number of rain gauges needed for the precise determination of areal-mean rainfall may be calculated by statistical estimation theory. The number of gauges needed to measure the coefficient of variation over a 40 mi² area, with varying degrees of error, was found to range from 264 (10% error, mean precipitation = 0.1 in) to about 2 (100% error, mean precipitation = 0.1 in).

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INTRODUCTION

This paper discusses some preliminary considerations necessary for planning a more-extensive study on the determination of the spatial and temporal variability of rainfall over small areas. These variations may be important in affecting the variability of soil moisture that, in turn, may affect crop yield.

The forecasting of crop yield by the remote sensing of the status of crops can provide routine assessments over larger geographical areas than measurements made at a few sample plots on the ground. Relations are sought between the amount and frequency of energy reflected, scattered, and emitted (the signature) back to the sensor from the crop at a particular stage of crop growth. The factors that can cause variations in signatures are numerous and the reduction of these factors to measureable proportions is an essential aim of remote-sensing research. Some of these factors are planting date, row direction, row width, leaf area index, spectral properties of individual leaves, the geometry of the canopy, soil background, crop stage, and crop condition (Pitts et al., 1983).

Weather-sensitive factors may produce significant correlations between yield and particular meteorological elements but these factors may not act in a direct cause-effect relation (Munn, 1970). Insect infestations and crop diseases are usually related to weather conditions but their impact may not be immediate. Many investigators feel that soil moisture is the best environmental predictor of crop yield (Munn, ibid.). However, once adequate rainfall has occurred, variation in rainfall amount seems to have a small effect on crop yield. Pitts et al. (1983) compared rainfall amounts with the prediction of the date when spring wheat would be ripe. These dates were obtained from a spring-wheat phenology model developed by Doraiswamy and Thompson (1982). Across small areal segments (5X6 nautical miles) the influence of rainfall on the phenology of spring wheat did not vary appreciably when rainfall amounts accumulated for periods of a day or longer were used. Variations, of the elapsed days to the ripe stage were only 1 to 2 days out of approximately 225 days necessary to attain the ripe stage. However, some slight variability, found within a few segments, did appear to be related to variations in rainfall. Changnon and Neill (1968) compared the association of corn yields from 60 farms with 10 weather and 4 agricultural variables. Nine years of data were used and the authors found that, among the meteorological variables, monthly rainfall amounts did not seem to be too

important. For a growing season extending from May until August, the best rainfall-related correlation coefficient was between August rainfall and yield. The value of this coefficient was 0.29. Rainfall amounts during July, June, and the pre-season months correlated more poorly.

The increasing use of sensors that can resolve spectral signatures of ever smaller areas suggests the desirability of understanding variations of rainfall within areas smaller than 40 square miles. A preliminary understanding of these variations is the object of this study.

Previous studies of rainfall variability were examined and the empirical relationships, from these studies, were chosen that showed the association among rainfall variability and mean precipitation, size of sampling area, season, type of precipitation, and synoptic weather type. When these relationships were combined with statistical estimation theory, a simple expression was obtained that indicates the number of rain gauges needed in order to measure rainfall variability, within certain error bounds, at a predetermined level of confidence.

THEORY

In order to find approximate empirical relationships between rainfall variability and meteorological quantities, the extensive studies performed by the Illinois State Water Survey (Huff and Neill, 1957, Huff and Shipp, 1968, and Huff, 1970) were chosen as being representative ones of rainfall during the growing season over relatively flat terrain. Catchment areas ranged from 10 to 550 square miles and they have been studied by this organization since 1955. Gauge densities ranged from 0.12 to 0.50 gauges per square mile. Uniform spacing between gauges was used. Such spacing is preferable when rain is produced by convective motions (Rodda, 1972). Convective rainfall is the type most likely to occur during growing seasons.

The coefficient of variation (V) is a useful statistic for showing both variation about the mean and variability relative to the mean.

$$V = \frac{s}{P}$$

where s is the sample standard deviation and P is the sample mean rainfall amount.

Huff and Neill (1957) have shown the following relations involving V:

$$V \propto 1/P,$$

$$V \propto A,$$

where A is area,

$$V \propto \text{rainfall gradient},$$

and

$$V \propto 1/D,$$

where D is the duration of the rain event.

Rainfall gradient and duration are strongly proportional to mean rainfall. Thus, to a first approximation, the natural variation of rainfall, expressed by V, is

$$\log V = \log k + a \log P + b \log A, \quad (1)$$

where k, a, and b are constants obtained by regression analysis. Logarithms of P and A were used since they range over several orders of magnitude. Typical values, for shower-type precipitation, are $k = 9$, $a = -0.44$, and $b = 0.15$ when V is in per cent, P is in inches, and A is in square miles. The correlation coefficient of Eq. (1) is about 0.7.

To relate the coefficient of variation, obtained by Eq. (1), with the minimum number of gauges needed for adequate measurements, the following expression for confidence intervals, for V, is used, viz.,

$$V \pm Z_{\alpha/2} \frac{V}{\sqrt{2N}} \sqrt{1 + 2V^2}, \quad (2)$$

where Z is the standardized value of V and N is the number of observations (rain gauges) (Spiegel, 1961). This expression gives the interval for $(1 - \alpha)100\%$ confidence.

Solution of Eq. (2) for N gives

$$N = \frac{1}{2} \left[\frac{Z_{\alpha/2}}{d} \right]^2 \sqrt{1 + 2V^2}, \quad (3)$$

where d is the percentage uncertainty in the measurement of V that is acceptable. To be 95% confident, $1 - \alpha = 0.95$, and $Z_{0.025} = 1.96$.

RESULTS

An illustration of the results anticipated by the method explained above is shown in Table 1. An area of 40 square miles was chosen to correspond to the sizes of areas recently observed by the Landsat Multi-Spectral Scanner (Pitts et al., 1983).

Table 1. Required number of gauges (N) and gauge density (G) to measure the variation of rainfall over an area of 40 square miles with a confidence of 95%.

P	V	N	G	
inches	-	-	$\frac{\text{Miles}^2}{\text{Gauge}}$	
0.1	0.43	264	0.15	d = 0.1V
0.5	0.21	209	0.19	
1.0	0.15	202	0.20	
2.0	0.11	198	0.20	
0.1	0.43	66	0.61	d = 0.2V
0.5	0.21	53	0.75	
1.0	0.15	51	0.78	
2.0	0.11	49	0.82	
0.1	0.43	11	3.6	d = 0.5V
0.5	0.21	9	4.4	
1.0	0.15	8	5.0	
2.0	0.11	8	5.0	
0.1	0.43	3	13.3	d = 1.0V
0.5	0.21	2	20.0	
1.0	0.15	2	20.0	
2.0	0.11	2	20.0	

CONCLUSIONS

The relationship between V and P and A, from the Illinois State Water Survey, is consistent with results of other studies of the characteristics of shower-type rainfall. The effect of mean rainfall is greater than the effect of area in the empirical determination of V. However, area of rainfall is not independent of rainfall mean. Eq. (1) with a correlation coefficient of 0.7 and a coefficient of determination of 0.49 means that only 49% of the variation is explained by a linear relation of logarithms. It remains to be seen whether or not non-linear relations and/or the incorporation of other variables will produce higher correlations.

Huff(1970) indicates that Eq. (1) may have an uncertainty of 2 orders of magnitude. This range of uncertainty is not unusual for many geophysical measurements but it may be large enough to affect the estimate of the minimum number of gauges determined by Eq. (1).

The regression coefficients for Eq. (1) vary, to some extent, with locality. This variation, in the coefficients, suggests a survey of rainfall characteristics in a variety of crop-growing regions during the growing season.

Continuation of this research will consist of an investigation of rainfall data collected during concomitant surveys of crops by remote sensing from Landsat. Sites from several climatic zones were surveyed with each site having dimensions of 5 x 6 n mi. Terrain is relatively smooth and the number of rain gauges varies from 1 to 23 per site. The rainfall data can be stratified in several ways and regression relationships other than linear will be investigated. Since the rainfall-producing processes are complex, an initial investigation, as suggested here, is needed to detect some of the predominant relations between these processes and the resulting rainfall characteristics.

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