

OPTIMAL DESIGN APPLICATION ON THE ADVANCED  
AEROELASTIC ROTOR BLADE

Fu-Shang Wei  
Senior Aeromechanics Engineer  
and  
Robert Jones  
Assistant Director of Aeromechanics  
Kaman Aerospace Corporation  
Bloomfield, Connecticut 06002-0002

Abstract

The vibration and performance optimization procedure using regression analysis has been successfully applied to an advanced aeroelastic blade design study. The major advantage of this regression technique is that multiple optimizations can be performed to evaluate the effects of various objective functions and constraint functions. In this application, the data bases obtained from the rotorcraft flight simulation program C81 and Myklestad mode shape program are analytically determined as a function of each design variable. Those predicted results from regression equations, such as performance, vibration, and modal parameters, when compared with C81 and Myklestad outputs, correlate exceptionally well. The regression equations also predicted the minimum of 4/rev total vertical hub shear based on the coefficients of each equation. This approach has been verified for various blade radial ballast weight locations and blade planforms. This method can also be utilized to ascertain the effect of a particular cost function which is composed of several objective functions with different weighting factors for various mission requirements without any additional effort. Utilization of this technique can significantly reduce the engineering efforts and computer time to optimally design a high performance and low vibration blade.

Introduction

It is highly desirable for most helicopter engineers to design a vehicle having high performance and low vibrations (References 1 - 13). With a best dynamic blade as an input to the airloads program, the blade having minimum vibration and maximum performance under certain constraints could be determined by using an existing optimization code; or vice versa, from an optimized airloads distribution to find a desired blade planform. Blade

dynamic and aerodynamic effects are coupled within the design range of interest. Separation of these effects during the design procedure may not be possible to obtain the best result that one expects. Therefore, the approach which can be utilized to optimize dynamic and aerodynamic effects is strongly recommended.

Vibration and performance data generated from C81 and the coefficients of modal participation factor (CMPF) of hub shear and hub moment generated from Myklestad can be analytically expressed as a function of each design variable using regression analysis (References 14 - 20). Regression equations not only directly provide the sensitivity of each blade design variable, but also combine both dynamic and aerodynamic effects within the overall design procedure. Furthermore, regression technique need not be performed in a continuous run; it may be carried out individually or in groups, as convenient. This technique can also treat numerous design variables, objective functions, constraint functions, and various combinations of several objective functions in a convenient manner. After the data base is obtained from the technique program, the optimization criteria can be varied, based on various mission requirements. Therefore, a significant savings on computer time and engineering efforts have been achieved.

The optimization procedure of the regression analysis was first used at Kaman in its analytical studies of the Controllable Twist Rotor in developing secondary control requirements to minimize vibration, with constraints on horse-power, angle of attack, and blade bending moments. This control optimization was done for both steady and one-per-rev controls, as well as for higher harmonic controls (Reference 20). Blade controls on the full scale Multicyclic Controllable Twist Rotor with higher harmonics were optimized experimentally by using wind tunnel results for the data base (Reference 17).

The optimization procedure was also used to investigate the effects of several blade design parameters as independent

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variables in a study of an advanced flight research rotor (Reference 18). All previous results are obtained either from Kaman's program, 6F, or from the wind tunnel test, on the hinged blade. The input mode shapes for the 6F are uncoupled modes with pitch horn control and servo flap control degrees of freedom.

The main rotor in this study is a hingeless, 4-bladed General Purpose Research Rotor (GPRR) (References 18, 19) which weighs 287.5 lbs per blade and has 27 ft radius, 25.5 in. thrust weighted chord, 256 rpm angular speed, and 723.8-ft/sec tip speed. Bingham RC airfoil tables are used to determine blade aerodynamic coefficients. The fuselage has 18,400 lbs total gross weight and 23 square ft flat plate drag area. C81 was modified to incorporate variable sweep stations along the blade radial direction.

Thirty-six C81 quasi-static (QS) trim cases as a function of blade built-in twist, sweep angle, percent tip taper, and taper ratio have been generated to find the regression equations for performance analysis at five different airspeeds from hover to 160 knots. The predictions of the horsepower from regression equations, which are not included in those 36 QS trim cases, compared with C81 are within 1.5% of the total range of interest.

The regression equations of the modal parameters also have been generated using 84 Myklestad cases by adding blade ballast running weights along the blade radial stations. The predicted results from regression equations, compared with the Myklestad, are in excellent agreement up to the first six modes.

Thirty-five C81 QS Trim, followed by Time-Varying Trim (TVT), cases as a function of blade built-in twist, percent tip taper, and taper ratio are used for vibration analysis. The multiple correlation factors for horsepower, 4/rev vertical hub shear, oscillatory beamwise and chordwise bending moments, and torsional moments are correlated at least 95.4%. The excellent predictions from regression equations for the vibration data are also presented.

With the exceptionally well-fitted regression equations from C81 and Myklestad, the blade can be dynamically controlled by controlling each individual CMPF, or its product with MPF, to achieve the design goal under certain constraints.

#### Performance Analysis

In order to determine blade characteristics for the performance analysis, blade physical parameters from Reference

18 are used for the baseline blade. Blade torsional control spring is pre-determined as an input to Myklestad coupled mode shapes program such that the blade clamped torsional frequency is 25.6 Hz. Only the first out-of-plane mode shape is used as an input in C81 for the performance study. Bingham RC 8%, 10%, and 12% airfoil tables are used to look up blade local aerodynamic lift, drag, and pitching moment coefficients. Blade built-in twist, sweep angle, percent tip taper, and taper ratio are treated as independent variables input to C81 to vary blade airloads distribution. QS trim in C81 uses the first flapping mode; therefore, blade sweep angle gives no dynamic coupling effects and only has aerodynamic effects on Mach No. reduction; and aerodynamic effects on pitching moment variation due to aerodynamic center shift. The blade sweep station starts at that point at which the Mach No. is the same as the Mach No. on the blade tip. There are four independent variables in the analysis, and the range of interest of these variables is listed in Table 1.

Table 1. Independent variables for performance analysis.

Independent Variables	Levels
Built-in Twist*	-8°, -12°, -14°, -16°
Sweep Angle**	20°, 0°, -20°, -30°
Percent Tip Taper	15%, 25%, 50%
Taper Ratio	1.1:1, 2:1, 3:1
*Built-in Twist: + Nose Up	
**Sweep Angle: + Forward Sweep	

The quadratic regression equation of the independent variables is written as follows:

$$Y = A_0 + \sum_{i=1}^N A_i \delta_i + \sum_{i=1}^N A_{ii} \delta_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^N A_{ij} \delta_i \delta_j$$

Where Y is the dependent variable;  $\delta$  is the independent; and A is the coefficient of regression equation.

There are 144 different combinations for these variables. Only 36 combinations are randomly selected as inputs for C81 QS trim at each flight speed. The regression equations having linear and quadratic terms which are generated from these data are shown in Table 2 for 5 different speeds. These equations give multiple correlation coefficients of 97.5% or better at each different speed from hover to 160 knots. With the existing data

Table 2. Regression equations for performance analysis at 5 different flight speeds.

Coefficient	Variable	Hover	40 Knots	80 Knots	120 Knots	160 Knots
A <sub>0</sub>		1975.08	1324.90	1038.82	1316.16	2054.82
A <sub>1</sub>	δ <sub>1</sub>	---	0.85 (9)	---	-0.45 (7)	-1.31 (7)
A <sub>2</sub>	δ <sub>2</sub>	12.13 (5)*	17.96 (4)	18.89 (7)	32.23 (3)	44.88 (3)
A <sub>3</sub>	δ <sub>3</sub>	-77.59 (7)	-47.36 (8)	-27.60 (8)	-28.33 (10)	-27.88 (4)
A <sub>4</sub>	δ <sub>4</sub>	---	---	-11.31 (3)	20.65 (5)	---
A <sub>11</sub>	δ <sub>1</sub> *δ <sub>1</sub>	-0.067 (2)	-0.04 (2)	-0.04 (1)	-0.05 (1)	-0.08 (1)
A <sub>22</sub>	δ <sub>2</sub> *δ <sub>2</sub>	---	0.52 (6)	0.74 (6)	1.46 (2)	2.078 (2)
A <sub>33</sub>	δ <sub>3</sub> *δ <sub>3</sub>	19.43 (6)	10.91 (7)	5.45 (9)	2.84 (11)	---
A <sub>44</sub>	δ <sub>4</sub> *δ <sub>4</sub>	---	130.55 (11)	63.80 (10)	106.75 (12)	140.25 (9)
A <sub>12</sub>	δ <sub>1</sub> *δ <sub>2</sub>	---	0.04 (10)	-0.01 (4)	-0.05 (6)	-0.10 (6)
A <sub>13</sub>	δ <sub>1</sub> *δ <sub>3</sub>	---	-0.11 (12)	---	-0.08 (13)	---
A <sub>14</sub>	δ <sub>1</sub> *δ <sub>4</sub>	---	---	---	0.43 (14)	0.61 (10)
A <sub>23</sub>	δ <sub>2</sub> *δ <sub>3</sub>	1.26 (1)	0.10 (1)	-0.29 (5)	-1.39 (9)	-1.84 (5)
A <sub>24</sub>	δ <sub>2</sub> *δ <sub>4</sub>	-7.0 (3)	0.75 (5)	---	6.68 (8)	9.12 (8)
A <sub>34</sub>	δ <sub>3</sub> *δ <sub>4</sub>	-106.88 (4)	-77.88 (3)	-38.14 (2)	-22.17 (4)	---
M.C.C.**		0.987	0.991	0.982	0.975	0.978
S.E.E.***		11.7	5.8	4.7	7.2	9.1
δ <sub>1</sub> Sweep      δ <sub>2</sub> Built-in Twist      δ <sub>3</sub> Taper Ratio      δ <sub>4</sub> % Tip Taper * Sensitivity      ** Multiple Correlation Coefficient      *** Standard Error of the Estimate						

Table 3. Regression equation for performance analysis with airspeed as an independent variable.

Coefficient	Variable	Horsepower	Coefficient	Variable	Horsepower
A <sub>0</sub>		2063.68			
A <sub>1</sub>	δ <sub>1</sub>	---	A <sub>12</sub>	δ <sub>1</sub> *δ <sub>2</sub>	---
A <sub>2</sub>	δ <sub>2</sub>	36.55 (8)*	A <sub>13</sub>	δ <sub>1</sub> *δ <sub>3</sub>	---
A <sub>3</sub>	δ <sub>3</sub>	-21.58 (4)	A <sub>14</sub>	δ <sub>1</sub> *δ <sub>4</sub>	---
A <sub>4</sub>	δ <sub>4</sub>	---	A <sub>15</sub>	δ <sub>1</sub> *δ <sub>5</sub>	---
A <sub>5</sub>	δ <sub>5</sub>	-22.76 (2)	A <sub>23</sub>	δ <sub>2</sub> *δ <sub>3</sub>	---
A <sub>11</sub>	δ <sub>1</sub> *δ <sub>1</sub>	-0.06 (3)	A <sub>24</sub>	δ <sub>2</sub> *δ <sub>4</sub>	2.01 (6)
A <sub>22</sub>	δ <sub>2</sub> *δ <sub>2</sub>	1.06 (11)	A <sub>25</sub>	δ <sub>2</sub> *δ <sub>5</sub>	-0.12 (7)
A <sub>33</sub>	δ <sub>3</sub> *δ <sub>3</sub>	---	A <sub>34</sub>	δ <sub>3</sub> *δ <sub>4</sub>	-57.54 (10)
A <sub>44</sub>	δ <sub>4</sub> *δ <sub>4</sub>	---	A <sub>25</sub>	δ <sub>3</sub> *δ <sub>5</sub>	0.23 (5)
A <sub>55</sub>	δ <sub>5</sub> *δ <sub>5</sub>	0.13 (1)	A <sub>45</sub>	δ <sub>4</sub> *δ <sub>5</sub>	1.03 (9)
Multiple Correlation Coefficient: 0.999 Standard Error of the Estimate: 18.4					
δ <sub>1</sub> Sweep      δ <sub>3</sub> Taper Ratio      δ <sub>5</sub> Airspeed δ <sub>2</sub> Built-in Twist      δ <sub>4</sub> % Tip Taper      * Sensitivity					

base, the regression equation for performance as a function of airspeed has also been analyzed. The multiple correlation coefficient from the equation with airspeed as an independent variable is correlated at 99.9%, shown in Table 3.

The sensitivity results from regression equations show that each design variable has a clear performance trend at each airspeed and for the airspeed sweep. The independent variables in these regression equations have not been normalized. Therefore, the physical parameters are treated as the input to these regression equations. From Table 2, blade sweep angle squared, built-in twist squared, and built-in twist are the three most important terms at 160 knots from the performance regression equation sensitivity result. Also, the product of taper ratio and built-in twist, sweep angle squared, the product of built-in twist and percent tip taper, and the product of percent tip taper and taper ratio are the four most important terms in hover. Blade sweep angle squared, the product of taper ratio and percent tip taper, percent tip taper and the product of sweep angle and built-in twist are the four most important terms in the regression equation at 80 knots. From Table 3, the regression equation shows that airspeed squared, airspeed, blade sweep angle squared, and taper ratio are the four most important terms in the whole airspeed sweep region. Also from Table 2, the regression equation shows that the constant term has the minimum value at 80 knots. All the design variables have either an increased or a decreased contribution to the constant term at each flight speed, depending on the combination of each individual design variable.

In order to gain a better understanding of the effects each independent variable contribution to performance, the plots of horsepower vs each independent variable at different speeds (Fig. 1 to 4) are described as follows:

1. For a blade having  $-10^\circ$  built-in twist and 25% tip taper, results show that a 3:1 taper ratio blade saves 20 HP over a 1.1:1 taper ratio blade at 160 knots; saves 25 HP at 80 knots; and saves 80 HP in hover.

2. Results also show that a  $30^\circ$  aft sweep blade saves 75 HP and 35 HP at 160 knots, 60 HP and 35 HP in hover, and 35 HP and 25 HP at 80 knots over a non-swept blade and a  $20^\circ$  forward sweep blade, respectively.

3. The 3:1 taper ratio blade has better performance than the 2:1 and 1.1:1

taper ratio blades at all flight speeds of interest, except at 160 knots.

4. For the blade having  $-16^\circ$  built-in twist and 50% tip taper, the 3:1 taper ratio blade uses slightly more HP than a 1.1:1 taper ratio blade at 160 knots, and saves 150 HP in hover and 40 HP at 80 knots.

5. For a high negative built-in twist blade,  $-16^\circ$ , the best performance is at hover, with very little effect on performance at 160 knots. The best performance at 160 knots is with the blade which has approximately  $-10^\circ$  built-in twist.

The prediction of the horsepower from regression equations compared with C81 trim results is exceptionally good. The difference between the two results is within 1.5% of the total range of interest. The comparison is shown on Tables 4 and 5.

The regression equations for horsepower at 160 knots, 80 knots, and hover are used for the performance optimization study. Power limits from C81 QS trim are treated as constraints at 160 knots and 80 knots. Those constraints for maximum power available are assumed to be 1740 HP at 160 knots and 840 HP at 80 knots. The minimum horsepower from 36 QS trim cases used as the starting point for optimization is the blade having a plan. rm  $30^\circ$  aft sweep,  $-16^\circ$  built-in twist, 3:1 taper ratio, and 50% tip taper. The optimization code KAOPT (Reference 21) is used for performance optimization. There are two minimum points detected using the KAOPT volume search technique. The first point is the blade having  $30^\circ$  aft sweep,  $-15.8^\circ$  built-in twist, 50% tip taper, and 3:1 taper ratio. The second point is  $20^\circ$  forward sweep,  $-10.4^\circ$  built-in twist, 44% tip taper, and 3:1 taper ratio. The performance results are 1740 HP, 822 HP, 1500 HP for point 1; and 1740 HP, 841 HP, and 1616 HP for point 2 at 160 knots, 80 knots, and hover, respectively (also shown in Table 4). The contour plots of power at hover, with and without constraints, are shown in Fig. 5. For 1740 HP available constraint applied to 1 g thrust, 160 knots and 1.5 g thrust, 120 knots, level flight conditions, the minimum power at hover within constraints is 1516 HP, and the blade has  $30^\circ$  aft sweep  $-14.54^\circ$  built-in twist, 3:1 taper ratio, and 50% tip taper.

#### Modal Analysis

The elastic rotor uses seven independent modes representation in the C81 airloads analysis. The time history of rotor

Table 4. Performance predictions for regression equation vs C81 at three different speeds.

$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	V = 160 KNOTS		V = 80 KNOTS		V = 0 KNOTS	
SWEEP	BUILT IN TWIST	TAPER RATIO	% TIP TAPER	REGRESSION (HP)	C81 (HP)	REGRESSION (HP)	C81 (HP)	REGRESSION (HP)	C81 (HP)
- 20°	- 13°	2.5:1	30%	1753.65	1776.25	849.69	839.81	1624.21	1618.44
- 30°	- 9°	2.5:1	30%	1712.66	1715.82	838.76	839.81	1643.57	1629.31
- 11°	- 9.4°	3:1	46%	1766.20	1764.91	851.47	847.61	1624.22	1627.86
- 30°	- 11.8°	3:1	46%	1698.0	1709.82	816.0	814.16	1559.0	1540.03
- 30°	- 13°	2.5:1	30%	1711.29	1714.56	830.20	829.73	1590.81	1581.85
- 20°	- 9°	2.5:1	30%	1750.94	1752.02	857.88	861.98	1676.96	1678.20
20°	- 13°	2.5:1	30%	1761.53	1768.76	854.52	852.32	1624.21	1618.01
20°	- 9°	2.5:1	30%	1742.53	1748.47	861.22	862.08	1676.96	1677.67
- 30°	- 15.8°	3:1	50%	1740.0	1743.76	822.0	820.66	1500.0	1508.61
- 30°	- 13°	1.5:1	30%	1715.24	1715.92	843.64	839.34	1639.15	1616.54
- 30°	- 9°	1.5:1	30%	1723.97	1724.64	853.37	848.83	1686.86	1662.44
- 20°	- 13°	1.5:1	30%	1757.60	1776.04	863.13	865.69	1672.55	1662.93
- 20°	- 9°	1.5:1	30%	1762.26	1760.44	872.50	875.68	1720.26	1700.64
20°	- 10.4°	3:1	44%	1740.0	1749.87	841.0	839.35	1616.0	1604.04
20°	- 9°	1.5:1	30%	1753.85	1755.93	875.84	875.93	1720.26	1719.92
20°	- 13°	1.5:1	30%	1765.49	1767.46	867.96	865.14	1672.55	1662.34

Table 5. Performance predictions for regression equation vs C81 with airspeed as an independent variable.

$\delta_1$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_5$	HORSEPOWER	
SWEEP	BUILT IN TWIST	TAPER RATIO	% TIP TAPER	AIRSPEED (KNOTS)	REGRESSION (HP)	C81 (HP)
- 30°	- 9.0°	1.5:1	30%	160.0	1716.38	1724.64
- 20°	- 13.0°	2.5:1	30%	160.0	1765.17	1776.25
- 16°	- 13.6°	3:1	44%	147.0	1507.30	1518.51
12°	- 14.4°	3:1	46%	80.0	845.81	850.02
- 11°	- 9.4°	3:1	46%	114.0	1034.92	1023.69
20°	- 10.4°	3:1	44%	0.0	1624.73	1604.04
- 20°	- 9.0°	1.5:1	30%	0.0	1733.29	1720.64
- 30°	- 15.8°	3:1	50%	160.0	1761.64	1743.76
12°	- 14.4°	3:1	46%	57.0	887.20	880.20
- 20°	- 13.0°	1.5:1	30%	0.0	1677.99	1662.93
- 16°	- 13.6°	3:1	44%	111.0	1001.60	1013.50
20°	- 9.0°	2.5:1	30%	80.0	875.04	862.08
- 16°	- 13.6°	3:1	44%	63.0	859.24	853.93
20°	- 13.0°	1.5:1	30%	160.0	1766.93	1767.46
12°	- 14.4°	3:1	46%	137.0	1344.81	1357.36
- 30°	- 11.8°	3:1	46%	160.0	1719.79	1709.82
- 11°	- 9.4°	3:1	46%	97.0	909.68	900.60

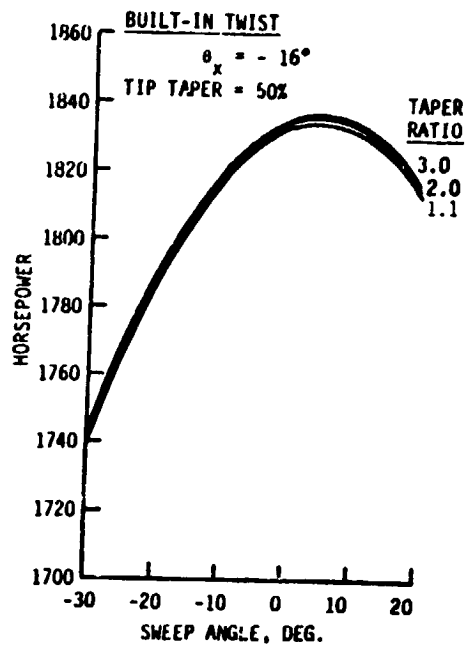
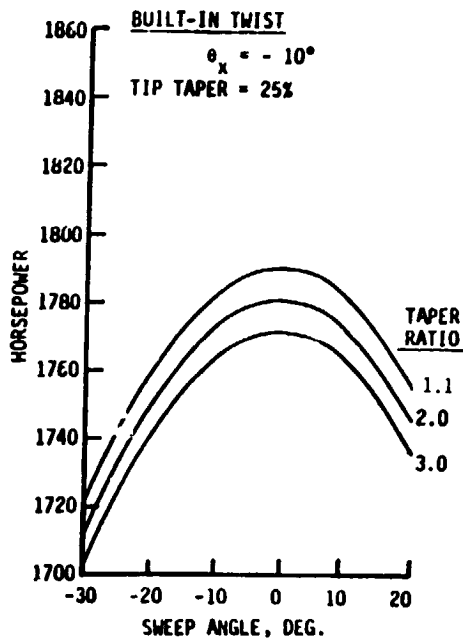


Fig. 1. Regression equation - performance vs sweep angle at 160 knots.

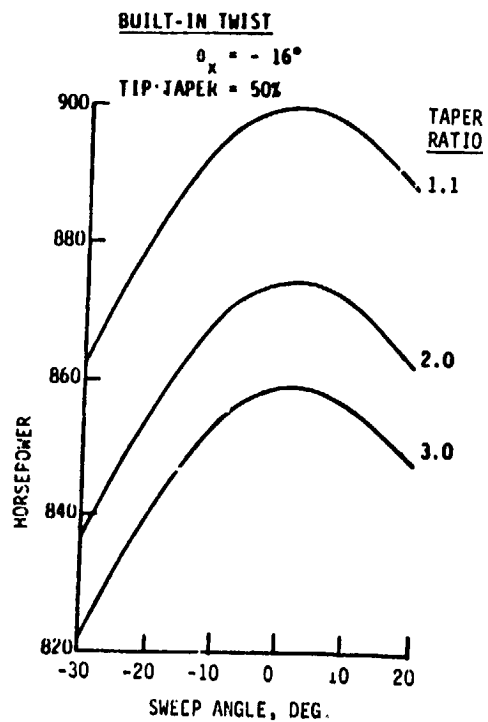
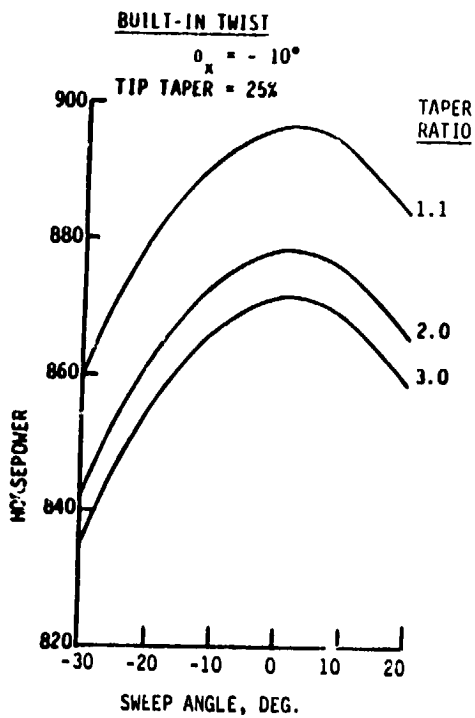


Fig. 2. Regression equation - performance vs sweep angle at 80 knots.

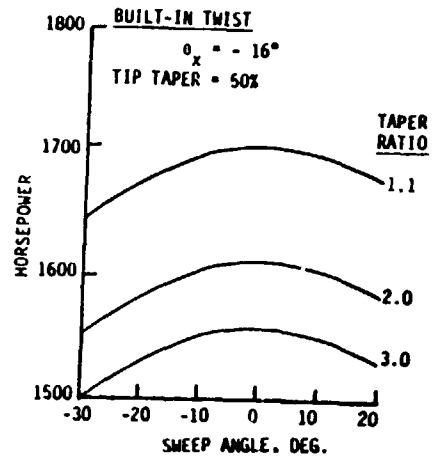
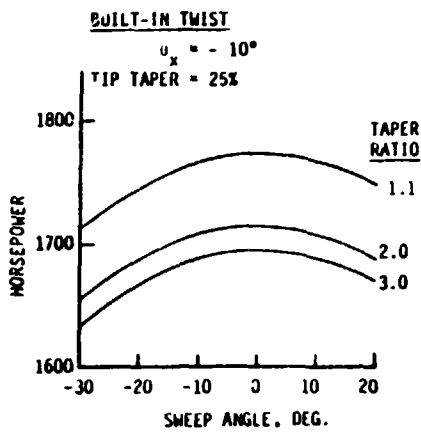


Fig. 3. Regression equation - performance vs sweep angle at hover.

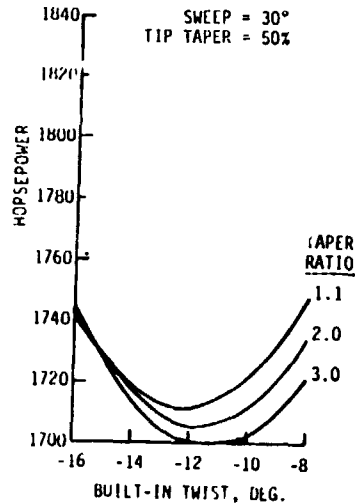
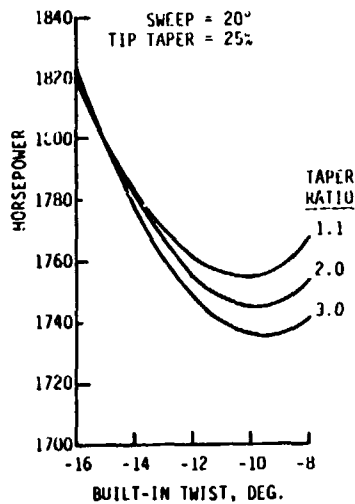


Fig. 4. Regression equation - performance vs built-in twist at 160 knots.

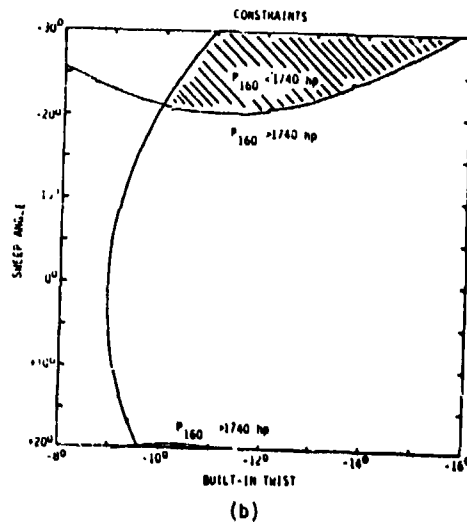
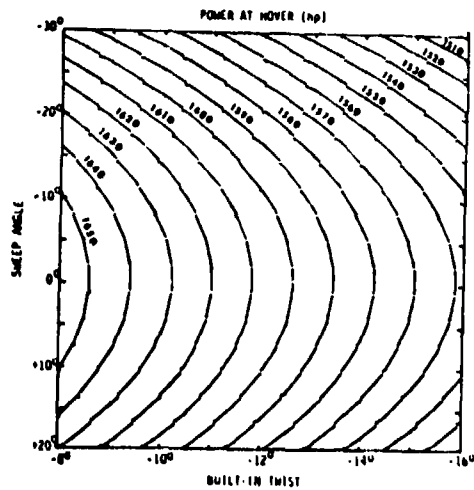


Fig. 5. Contours of power at hover; (a) without constraints (b) with constraints.

hub shear and hub moment at any given station can be computed from the modal participation factors (MPF) for the last rotor revolution in C81. Multiply the MPF for each given mode by the hub shear or the hub moment coefficient of that mode at any station and sum over all modes to get the value at that time point. Those coefficients of MPF can be obtained from the Myklestad coupled mode shape program. Regression analysis can be used to tune the coefficient of the modal participation factor (CMPF) or its product with MPF for aeroelastic blade design technique.

The baseline blade is divided into nineteen 13-inch-long equal segments, with each segment treated as an independent variable in the regression modal analysis. The regression equations of the first three out-of-plane (OP) frequencies, second and third OP deflections, static moment, flapping inertia, Lock number, and the CMPF of hub shears and moments of the first seven independent modes have been generated by adding blade ballast running weights of 1, 2, or 3 lb/in., with a total constant ballast weight of 39 lbs on each baseline blade.

There are 6,859 possible combinations for putting ballast weight in a blade with 19 independent variables and 3 levels for each. 84 cases are randomly selected to provide enough data for linear regression analysis. The linear regression equation with 19 independent variables is written as follows:

$$Y = A_0 + \sum_{i=1}^{19} A_i \delta_i$$

The out-of-plane components of the CMPF of hub shear and moment have been curve fitted up to 7 independent modes based on a 1-inch tip deflection, or 10° tip torsion. Since CMPF of hub shear of the first in-plane mode is either 0 or 1, from Myklestad, no regression analysis is needed for that mode. At least 250 more cases are required if quadratic regression equations are considered in the modal analysis.

The regression equations for the modal analysis are shown in Table 6. The multiple correlation coefficients (MCC) from the regression equations are extremely well-fitted and correlated from 94.5% to 99.9% for Myklestad modal data. For the first three OP frequencies, MCC correlates those frequencies from Myklestad output at least 98.7%. For static moment, flapping inertia, and Lock no., the MCC correlates no less than 99.7%. The mode shape deflections of second

and third modes correlate better than 98.5%. For CMPF of hub shear and moment of the first four OP modes, the MCC correlates at least 96.1%, and correlates torsion mode at least 95.1%.

The regression equation sensitivity results are also concluded as follows:

1. Blade outboard stations 16, 17, 18, and 19 are very sensitive to the first three OP frequencies. The intercepts of these OP frequencies are 1.0896 P, 2.5074 P, and 4.5889 P, respectively.

2. Adding ballast weight in these four stations (16, 17, 18, 19) will decrease the first OP frequency and increase the second and third OP frequencies. However, adding weight at the first blade station will increase the first three OP frequencies; and adding weight at station 8 will decrease first and second OP frequencies and increase the third OP frequency.

3. The values of static moment and flapping inertia are increased by adding ballast weight in blade outboard stations 18 and 19. However, by adding weight at inboard blade stations 1, 2, and 3, these values are decreased. Reverse trend is obtained for Lock number by adding the same ballast weight at the same stations.

4. For the second and third OP mode shape deflections, putting ballast weight at stations 18 and 19 will make minimum deflections of these modes more negative and maximum deflection of the third OP mode more positive. However, adding ballast weight at stations 11, 12, 13, and 14 gives the reverse trend of the second and third OP modes minimum deflections and the same trend of the third OP mode maximum deflection.

5. The CMPF of hub shear and moment of the first OP mode are decreased by adding blade ballast weight. Adding ballast weight at stations 17, 18, and 19 gives the second and fourth OP mode CMPF of hub shear and moment more negative and the 1st torsion mode less negative. Also, adding ballast weight at stations 18 and 19 increases the CMPF of hub shear and moment of the third OP mode.

The predicted results from the regression equations, compared with the Myklestad, are extremely well as shown on Table 7. The first three out-of-plane frequencies, static moment, flapping inertia, and Lock no. are within 1%. The predicted second and third OP deflections are within 2.5%. The predicted coefficients of hub shear and moment for the first 6 independent modes are in excellent





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agreement, except for the fourth OP hub shear and moment. Higher order terms in the regression equation are required in order to have better prediction for the modal parameters higher than the seventh mode. However, the seventh mode, or higher than seventh modes, normally gives very little effect on blade performance and vibration analysis; therefore, the linear regression analysis is an appropriate approach for future blade design study.

Vibration Analysis

Thirty-five C81 QS Trim followed by Time-Varying Trim (TVT) cases, each having two ballasting configurations, as a function of blade built-in twist, percent tip taper, and taper ratio are used for vibration analysis at 160 knots.

During the TVT, only flapping angles of the time-variant rotor are allowed to vary; the fuselage and control positions are held fixed at the values determined by the QS trim. The hub shear, hub moment, horsepower, and modal participation factor are obtained after the rotor reaches steady state within 8 rotor revolutions. Linear and quadratic terms are adapted to determine regression equations for horsepower, 4/rev vertical hub shear, blade

root oscillatory beamwise and chordwise bending moments, and torsional moments.

The coefficients of the regression equations and the multiple correlation coefficients are shown on Table 8. The multiple correlation coefficients for horsepower, 4/rev vertical hub shear, blade root oscillatory beamwise and chordwise bending moments, and torsional moments are correlated at least 95.4%.

The predictions between regression equations and C81 TVT results are shown in Table 9. The prediction of performance, bending moment and 4/rev vertical hub shear are correlated very well with C81 TVT and the regression equation results.

The best performance blade obtained from the regression equation prediction is a 3:1 taper ratio, -10° built-in twist, and 50% tip taper blade. The 1.1:1 taper ratio blade has lower 4/rev total vertical hub shear than those blades which have 2:1 or 3:1 taper ratio at 160 knots, from the regression analysis.

Also, three different planforms combined with various ballasting configurations along the blade span have been investigated. There are twelve different ballast weight locations chosen from the

Table 7. Regression equation prediction vs Myklestad for modal results.

	BASELINE + .0 lb/in. @ Sta 103, 142 + 1.4 lb/in. @ Sta 285		BASELINE + .5 lb/in. @ Sta 116, 220, 246 + 1.5 lb/in. @ Sta 272		BASELINE + .7 lb/in. @ Sta 142, 220, 246 + 0.9 lb/in. @ Sta 285		BASELINE + .8 lb/in. @ Sta 103, 129 + 1.4 lb/in. @ Sta 272		BASELINE + .5 lb/in. @ Sta 116, 207, 233 + 1.5 lb/in. @ Sta 259		BASELINE + .4 lb/in. @ Sta 103, 142, 220 + 1.8 lb/in. @ Sta 233	
	MYKLESTAD	REGRESSION EQUATION	MYKLESTAD	REGRESSION EQUATION	MYKLESTAD	REGRESSION EQUATION	MYKLESTAD	REGRESSION EQUATION	MYKLESTAD	REGRESSION EQUATION	MYKLESTAD	REGRESSION EQUATION
1st OP Freq. <sup>1</sup>	1.0791	1.080	1.0767	1.077	1.0776	1.078	1.0805	1.081	1.0784	1.079	1.0811	1.082
2nd OP Freq.	2.8544	2.831	3.0018	2.989	2.893	2.910	2.8593	2.857	2.9202	2.915	2.7849	2.777
3rd OP Freq.	5.2696	5.273	5.1519	5.175	5.1992	5.255	5.0185	5.046	5.0794	5.064	4.8965	5.008
1st OP H. S. <sup>2</sup>	214	214.109	224	224.183	223	222.565	217	211.705	221	221.385	216	216.085
1st IP H. S.	0	---	0	---	0	---	0	---	0	---	1	---
2nd OP H. S.	-605	-588.461	-477	-479.024	-458	-461.022	-559	-532.534	-411	-411.061	-358	-344.066
1st Tor H. S.	-1774	-1786.013	-1810	-1810.984	-1807	-1813.136	-1785	-1796.245	-1829	-1829.988	-1855	-1851.268
3rd OP H. S.	839	828.265	902	898.196	964	1041.823	705	690.476	950	1008.5	1105	1181.704
2nd I <sup>1</sup> S.	559	560.664	590	597.694	543	547.634	578	576.658	556	559.369	509	502.102
4th OP . S.	-1971	-2184.809	-2212	-2361.017	-2520	-2811.589	-2018	-2033.212	-2465	-2782.17	-2944	-2798.805
1st OP H. M. <sup>3</sup>	6860	6860.998	7041	7040.226	7014	7021.024	6818	6804.477	6995	6995.117	6899	6897.413
1st IP H. M.	-367	-366.102	-364	-363.947	-364	-363.62	-367	-366.414	-364	-364.064	-364	-364.514
2nd OP H. M.	-17883	-17241.222	-13810	-13806.906	-13387	-13364.227	-16596	-15679.67	-17020	-12043.57	-10617	-10296.459
1st Tor H. M.	-18666	-18900.591	-19441	-19454.057	-19422	-19492.453	-19038	-19335.908	-20058	-19981.741	-20959	-20746.979
3rd OP H. M.	23873	23578.438	25843	25576.833	27544	29398.054	20726	20441.421	27744	29242.885	33142	34627.764
2nd IP H. M.	13819	13840.247	14336	14320.965	13296	13349.15	14315	14304.599	13543	13580.914	12437	12173.862
4th OP H. M.	-45679	-50321.515	-50900	-54496.604	-57117	-63894.336	-47303	-47719.696	-57038	-63516.643	-66730	-63250.904
2nd OP Min Def <sup>4</sup>	-0.6313	-0.615	-0.5055	-0.512	-0.5111	-0.507	-0.5749	-0.562	-0.4525	-0.451	-0.412	-0.407
3rd OP Max Def	0.4574	0.467	0.5454	0.539	0.5844	0.617	0.4198	0.421	0.6028	0.625	0.7329	0.754
3rd OP Min Def	-0.5597	-0.555	-0.4469	-0.441	-0.5207	-0.519	-0.4929	-0.497	-0.461	-0.475	-0.5428	-0.546
S <sub>b</sub> (slug-ft)	93.383	93.058	96.882	97.214	96.313	96.108	92.42	92.728	95.788	95.742	93.733	93.601
I <sub>b</sub> (slug-ft <sup>2</sup> )	1667.83	1670.272	1758.884	1759.854	1735.098	1736.989	1632.549	1633.57	1714.527	1713.713	1642.375	1642.611
Y	9.226	9.41	8.749	8.663	8.869	8.925	9.426	9.448	8.975	9.019	9.369	9.5

(1) per rev (2) lb (3) in-lb (4) in.

Table 8. Regression equation for C81 vibration results.

Coefficient	Variable	Horsepower	Chordwise Bending Moment (in-lb)	Beamwise Bending Moment (in-lb)	Torsional Moment (in-lb)	4/REV. Hub Shear (lb)
$A_0$		2709.16	100148	-3109	3866	-1543
$A_1$	$\delta_1$	795.00 (8)*	- - -	236752 (6)	24108 (5)	16042 (4)
$A_2$	$\delta_2$	113.56 (9)	16775 (7)	99279 (5)	6232 (1)	4121 (2)
$A_3$	$\delta_3$	121.88 (6)	-1157 (4)	-9539 (7)	- - -	- - -
$A_{11}$	$\delta_1 * \delta_1$	-1499.25 (2)	40948 (2)	-514400 (3)	-45008 (4)	-22194 (1)
$A_{22}$	$\delta_2 * \delta_2$	-65.48 (5)	-4845 (6)	-5238 (8)	-626 (6)	-805 (3)
$A_{33}$	$\delta_3 * \delta_3$	4.35 (1)	-144 (3)	187 (9)	- - -	-11 (6)
$A_{12}$	$\delta_1 * \delta_2$	-409.90 (3)	-46025 (1)	75817 (1)	2911 (3)	-3457 (5)
$A_{13}$	$\delta_1 * \delta_3$	-58.48 (7)	-2241 (8)	-6583 (7)	-307 (7)	-161 (8)
$A_{23}$	$\delta_2 * \delta_3$	-21.79 (4)	-612 (5)	6121 (4)	171 (2)	-79 (7)
Multiple Correlation Coefficient		0.965	0.966	0.972	0.959	0.954
Standard Error of Estimate		45.64	2673	8362	615	334
$\delta_1$ % Tip Taper $\delta_2$ Taper Ratio $\delta_3$ Built-in Twist      * Sensitivity						

Table 9. Regression equation prediction vs C81 TVT results.

	$\delta_1 = 50\%$ $\delta_2 = 3:1$ $\delta_3 = -10^\circ$		$\delta_1 = 15\%$ $\delta_2 = 2.5:1$ $\delta_3 = -14^\circ$		$\delta_1 = 50\%$ $\delta_2 = 2.5:1$ $\delta_3 = -14^\circ$		$\delta_1 = 25\%$ $\delta_2 = 2.5:1$ $\delta_3 = -12^\circ$		$\delta_1 = 25\%$ $\delta_2 = 1.5:1$ $\delta_3 = -10^\circ$	
	C81	REGRESSION EQUATION	C81	REGRESSION EQUATION	C81	REGRESSION EQUATION	C81	REGRESSION EQUATION	C81	REGRESSION EQUATION
HORSEPOWER	2094	2030.2	2526.8	2546.4	2400.3	2411.4	2445.7	2425	2342	2372.3
4/REV <sup>1</sup> VERTICAL HUB SHEAR	2959.7	2971.8	4890.2	5349.3	3299.4	3678.8	5547.4	5507.5	4693.9	4673.3
OSCILLATORY <sup>2</sup> BEAMWISE BENDING MOMENT	261804	286507	200099.7	206572	270584	271004	259239	236343	198050	200266
OSCILLATORY <sup>2</sup> CHORDWISE BENDING MOMENT	72751.5	74817	107091.9	109607	89002	89630	101425	103857	111012	111684
OSCILLATORY <sup>2</sup> TORSIONAL MOMENT	17054.8	18489	13463.5	13874	15413	16124	17791	16349	13621	14308
(1) lb (2) in.-lb										

no ballast blade by putting the ballasting at maximum and minimum deflections and nodal points of the OP modes.

The regression equations obtained from the combination of CMPF from Myklestad and MPF from C81 provides the sensitivity of each design variable, and also predicts two local minimum points of 4/rev total vertical hub shears from the coefficients of each equation, shown in Fig. 6 and 7. From these figures, the inboard minimum 4/rev vertical hub shear ballasting location is between station 129 and 155, and the outboard minimum hub shear ballasting location is between station 246 and 272.

Because the GPRR blade has a large third OP modal component contribution to the 4/rev total vertical hub shear from the modal analysis, the inboard ballasting location does not have strong coupling between modal forces and mode shapes. Therefore, the results show that the best vibration and performance blades for each of the three inboard ballasting configurations have converged to the same blade planforms, respectively, for each ballasting location. For the inboard converged point, the untapered blade predicts a higher power requirement and less vibration, compared with tapered blades. However, this trend is reversed for the 50% tip taper, 3:1 taper ratio, and -10° built-in twist blade.

For the outboard minimum, the data shows that there is a strong modal force and mode shape coupling which significantly reduce the third OP modal components. For the outboard minimum point, the best performance blade has a similar vibration level compared with the untapered blade, but the performance is 15% better than the untapered blade. Further study is required to investigate other possible local minimum vibration locations.

With the exceptionally well-fitted regression equations from C81 and Myklestad, the blade can be dynamically controlled by controlling each individual CMPF, or its product with MPF, to achieve the design goal under certain constraints. The best performance blade, obtained from the best ballasting configuration in this study, has at least 2.5 times the reduction of vibration level compared with the original ballasting configuration with various planform and the power requirement is at least 15% better than the untapered blade.

#### Conclusions

From the performance, modal, and vibration analysis of the advanced

aeroelastic blade design study, the following conclusions can be obtained from the results:

1. With the exceptionally well-fitted regression equations from C81 and Myklestad, regression technique can be used for vibration analysis, modal analysis, and performance analysis for designing future advanced aeroelastic rotor blades.

2. Multiple optimizations can be performed to evaluate the effects of various objective functions and constraint functions, or to evaluate the combinations of several objective functions with different weighting factors for various mission requirements.

3. Regression technique can directly determine the sensitivity of each blade design variable and analyze the dynamic and aerodynamic effects during the entire design process.

4. The predicted results from regression equations for performance analysis, modal analysis, and vibration analysis are exceptionally good when compared with C81 and Myklestad outputs.

5. For the GPRR blade, the combination of CMPF from Myklestad and MPF from C81 predicts the same converging points for different blade planforms and different ballast weight configurations along the blade.

6. The best performance blade obtained from the best ballasting configuration has at least 2.5 times the reduction of vibration level when compared with original configurations and the power requirement is at least 15% better than the untapered blade.

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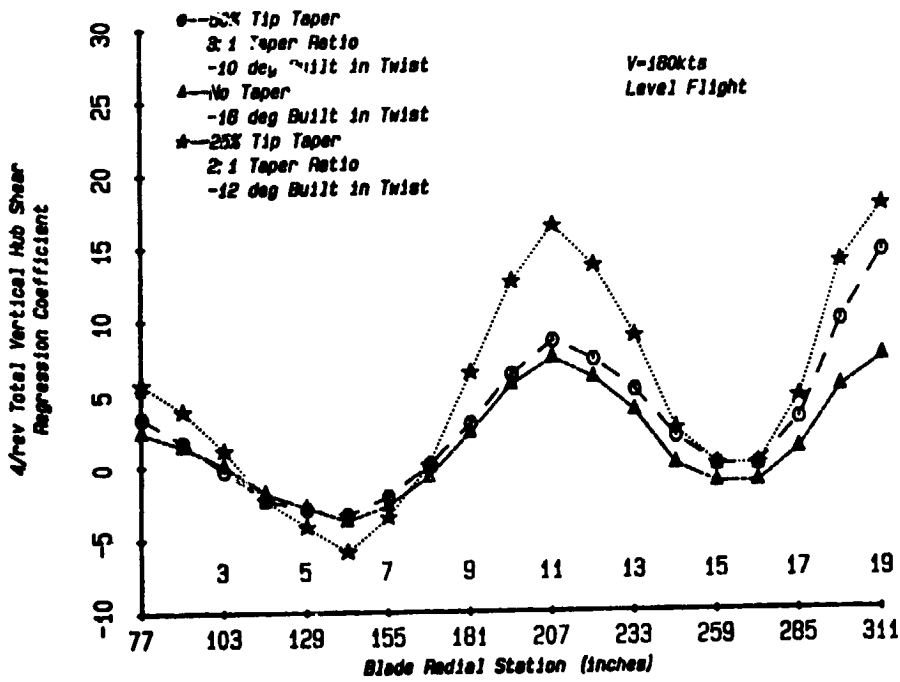


Fig. 6. 4/rev total vertical hub shear regression coefficients for ballast weight at 168.

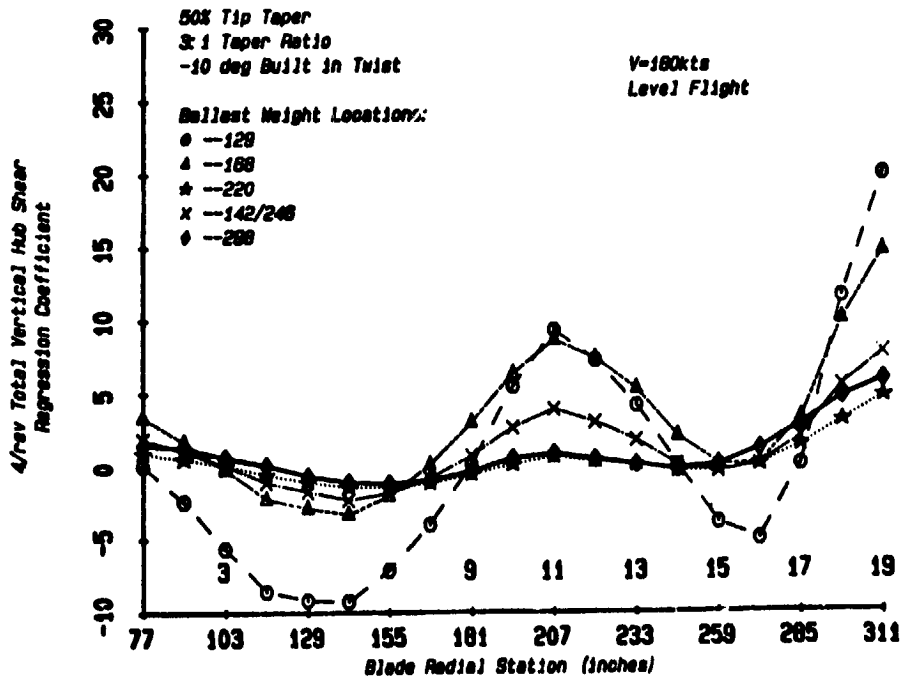


Fig. 7. Radial distribution of 4/rev total vertical hub shear regression coefficients.

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DISCUSSION  
Paper No. 10

OPTIMAL DESIGN APPLICATION ON THE ADVANCED AEROELASTIC ROTOR BLADE  
Fu-Shang Wei  
and  
Robert Jones

Dave Peters, Washington University: Do you have some feel for a comparison like this: How many times would I have to run, say, C81 or Myklestad to get the regression analysis as opposed to how many times I would have to run it if I just hooked it up to an optimization program and just reran it every time? Do you understand the question?

Wei: I'll tell you. It depends upon how many design variables you are using. Right now we are using four independent variables. Normally we are using the quadratic regression analysis and here we have 36 cases. I personally believe that if we have less design variables and directly hook onto the analyzer combined with the optimizer, we are going to save time. If you have a tremendous [number of] design variables the regression analysis could be beneficial. I think the tradeoff here in independent variables is around seven; [this] would be a nice number.

Bob Blackwell, Sikorsky Aircraft: I might ask if you could comment on whether the blade model and the inflow model and so forth that are used for your study are really sufficient for prediction of vibratory shears and prediction of blade response. Is it your [opinion] that a model as simple as this and able to be run for 36 times is sufficient or does the model have to get so detailed it just becomes cumbersome even with that?

Wei: I personally feel that the present model still has to be improved so that we can use it for future design. Right now we only deal with four different independent variables and more independent variables are required in the future if we are going to do more in a real study. However, one thing that I can mention is that the people at Kaman [are] using the optimization technique to design for the SH-2 and they are using it now. How good are the results going to be? I don't have any answer at this moment. But we are going to see.

Bob Taylor, Boeing Vertol: Just a quick question. Do you have any plans to do any testing to back up your theory?

Wei: That's what I am saying. We are going to do the SH-2 composite rotor to hook on the SH-2 helicopter.

Taylor: That's how you are going to prove your theory? Build a full-scale blade?

Wei: No, I can't give you an answer for that.

Bob Goodman, Sikorsky Aircraft: It seems that the only way that you can really check this kind of thing is to run a variety of cases--isn't that true? I mean, really you need a baseline.

Wei: We need a data base to generate equations. I think, Bob, you can give more details.

Bob Jones, Kaman: The regression equations are never going to be any better than the data base. If you have no faith in C81 then this is lousy. If you have no faith in something else then it is lousy. What you are doing is fitting statistical [variables to the] data base. If you have a good fit then it's a good equation, but it's no better than your data base, however. And you can do this with testing. I can get a data base with testing, fit a curve, [and do some interpolating]. This [fit] is really what it's based on. So there is no proof of theory if you want to look at it from that standpoint. We are working on methods where we have our regression equations based upon analysis and change them as we get testing results.

Jing Yen, Bell Helicopter: John, I am just curious to ask you what kind of inflow model you used here.

Wei: We just used the simple one that you see in the C81.

Yen: You did not use the Dr. Gene Sadler's free wake [analysis]?

Wei: No.