N86-29395

ANALYSIS OF SILICON STRESS/STRAIN RELATIONSHIPS

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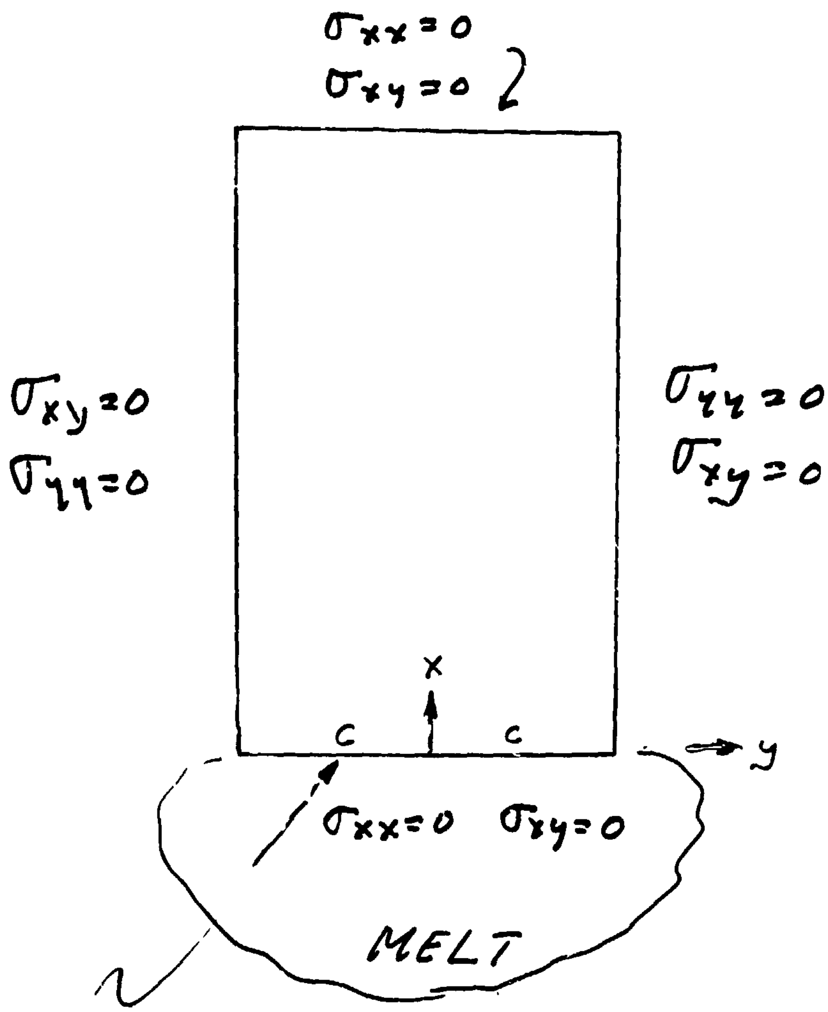
$$\dot{\varepsilon}_{ij} = \frac{(1+\nu)}{E}\dot{\sigma}_{ij} - \frac{\nu\dot{\sigma}_{KK}\delta_{ij}}{E} + \dot{\varepsilon}_{ij}^{PL} \tag{1}$$

where the inelastic strain rate tensor $\dot{\varepsilon}_{ij}^{PL}$ is the plastic strain rate and is

$$\dot{\varepsilon}_{ij}^{PL} = f\,S_{ij} \tag{2}$$

where S_{ij} are the deviatoric stress components and where

$$f = \frac{bB}{\tau_o^m} N_m(\sqrt{J_2} - D\sqrt{N_m})\,\frac{e^{-Q/kT}}{\sqrt{J_2}} \tag{3}$$



Note
$\sigma_{yy} \neq 0$

PRE BUCKLING - IN PLANE STRESSE
ASSUMES $t > t_{cr}$
(NO BUCKLING)

$$\nabla^2(\sigma_{xx}+\sigma_{yy}) = -\alpha E\nabla^2 T + \frac{1}{v}\int_0^x \left(\frac{\partial^2\dot{\varepsilon}_{xx}^{PL}}{\partial y^2} + \frac{\partial^2\dot{\varepsilon}_{yy}^{PL}}{\partial x^2} - 2\,\frac{\partial^2\dot{\varepsilon}_{xy}^{PL}}{\partial x\partial y}\right) E\,du \qquad (8)$$

$$\sigma_{xx} = \frac{2}{f}\left[\dot{\varepsilon}_{xx}^{oc} + \frac{1}{2}\,\dot{\varepsilon}_{yy}^{oc} - z\,\frac{\partial^2\dot{w}^c}{\partial x^2} - \frac{z}{2}\,\frac{\partial^2\dot{w}^c}{dy^2}\right]$$

where w^c is the creep (viscoplastic) portion of the transverse displacement. The moment intensity is related to the stress by the basic definition.

$$M_{xx} = -\int_{-h/2}^{h/2} \sigma_{xx}\, z\, dz$$

The "elastic moment" M_{xx} is then

$$M_{xx} = \frac{Eh^3}{12(1-\nu^2)}\left[\frac{\partial^2 w^e}{\partial x^2} + \nu\,\frac{\partial^2 w^e}{\partial y^2}\right] \qquad (5)$$

while the corresponding "inelastic" moment component is

$$M_{xx} = \frac{1}{f}\left[\frac{\partial^2\dot{w}^c}{\partial x^2} + \frac{1}{2}\,\frac{\partial^2\dot{w}^c}{\partial y^2}\right]\frac{h^3}{12} \qquad (6)$$

Since the moments are the same, the displacements are clearly related by

$$\frac{\partial^2\dot{w}^c}{\partial x^2} = \frac{fE}{(1-\nu^2)}\,\frac{\partial^2 w^e}{\partial x^2} \qquad (7)$$

$$D\,\nabla^4\dot{w}^e = N_{xx}^o\,\frac{\partial^2\dot{w}^e}{\partial x^2} + 2N_{xy}^o\,\frac{\partial^2\dot{w}^e}{\partial x\partial y} + N_{yy}^o\,\frac{\partial^2\dot{w}^e}{\partial y^2}$$

$$+\,\frac{f\,E\,h^3}{12(1-\nu^2)}\left[N_{xx}^o\,\frac{\partial^2 w^e}{\partial x^2} + 2N_{xy}^o\,\frac{\partial^2 w^e}{\partial x\partial y} + N_{yy}^o\,\frac{\partial^2 w^e}{\partial y^2}\right] \qquad (9)$$

$$w^e(x,y,t) = g(t)W(x,y) \tag{10}$$

and obtain

$$\dot{g} - \lambda^2 g = 0 \tag{11}$$

for the time part and

$$D \nabla^4 W = N^o_{\alpha\beta} \left(1 + \frac{f\,F}{12(1-\nu^2)\lambda^2}\right) \frac{\partial^2 W}{\partial x_\alpha \partial x_\beta} \tag{12}$$

Hence one can see that the inelastic material behavior results in buckling very much like the elastic case but with the pseudo in plane forces given by

$$N^o_{\alpha\beta} \left(1 + \frac{f\,E}{12(1+\nu^2)} \frac{1}{\lambda^2}\right) \tag{13}$$

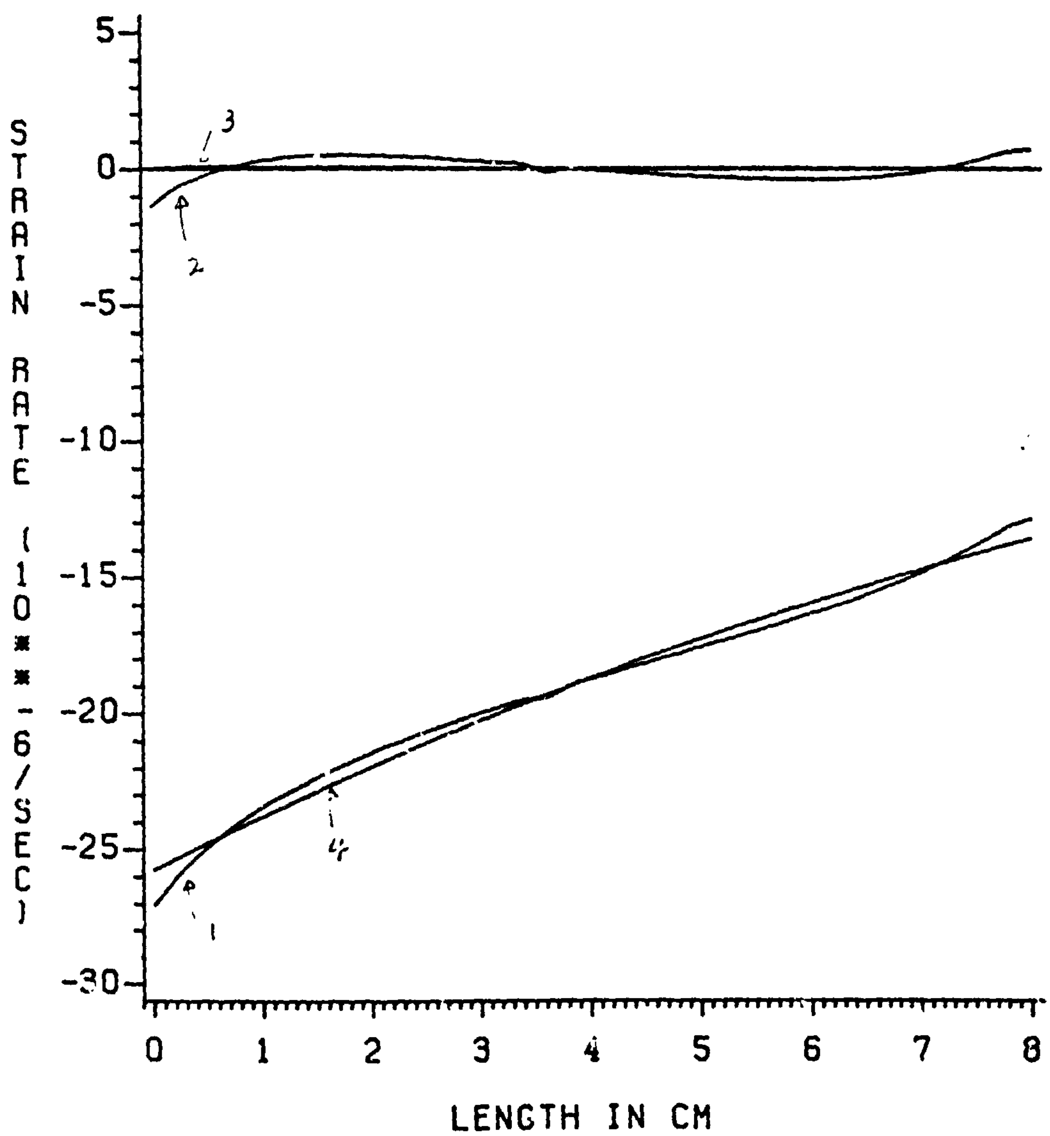
The separation parameter λ^2 in Eq (13) reflects how "fast" the lateral deflections grow from some initial value. Clearly the presence of $f(x,y)$ in the numerator of Eq (13) makes simple interpretation impossible for λ^2 except as given in Eq (11).

To obtain values for λ^2, we use a Galerkin method on Eq (12) and find that

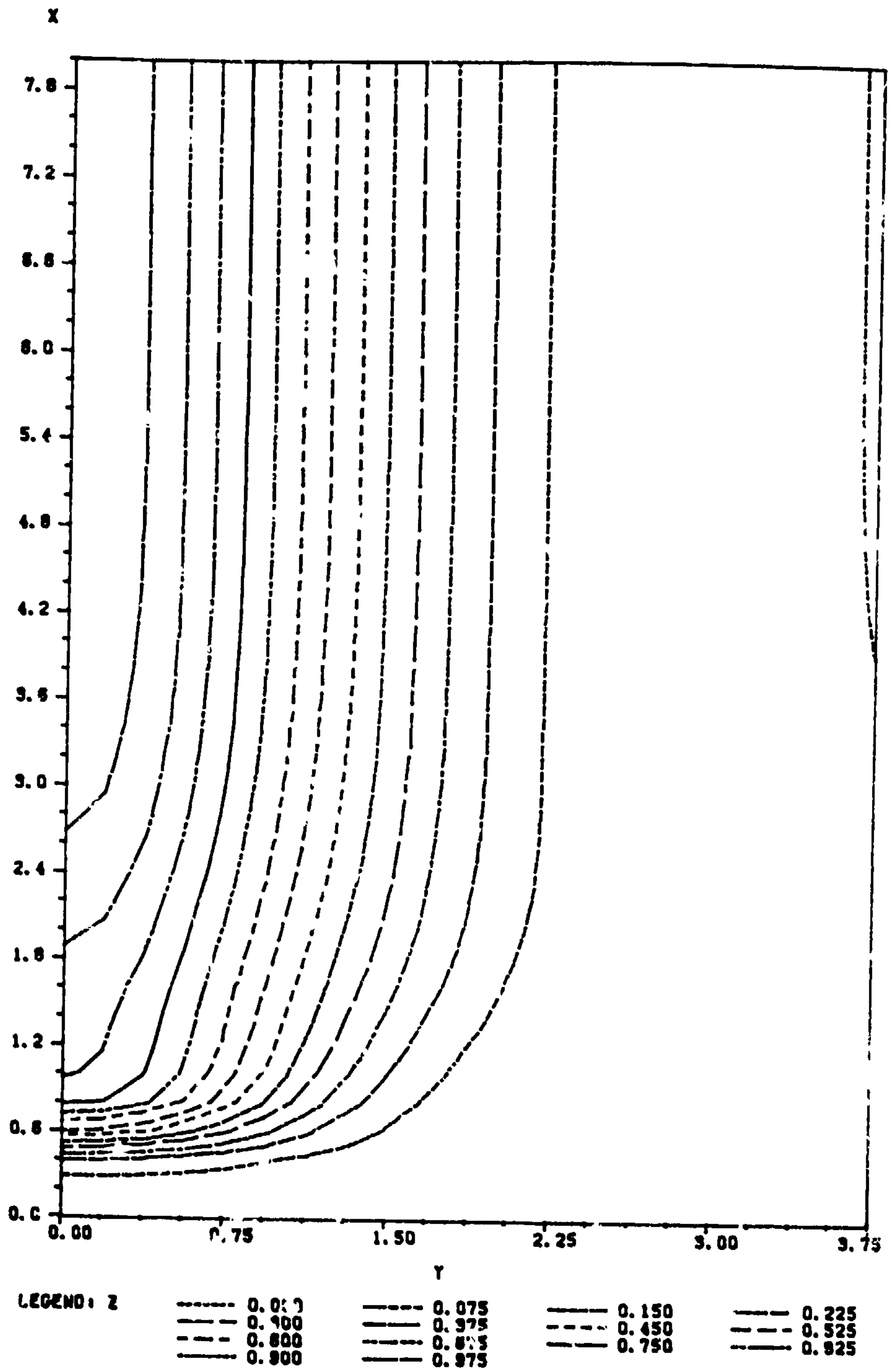
$$\lambda^2 = \frac{2\,h_{cr}^3 \iint f E^2\, \nabla^4 W W \, da}{3(h^3 - h_{cr}^3) \iint E\, \nabla^4 W W \, da} \tag{14}$$

Normal Strain Rate XX Along Y · 0 (Centerline)
for T = 1440*Exp (-0.08X) Width = 6.0 CM

LINE 1 IS TOTAL, LINE 2 IS ELASTIC, LINE 3 IS PLASTIC AND LINE 4 IS THERMAL STRAIN RATE



The Dislocation Density Contour Plot
for T = 1440·Exp (−0.08X)
Unit of X and Y = CM, Unit of Z = 10^3 Per CM^2



6cm x 6cm
T = Tw

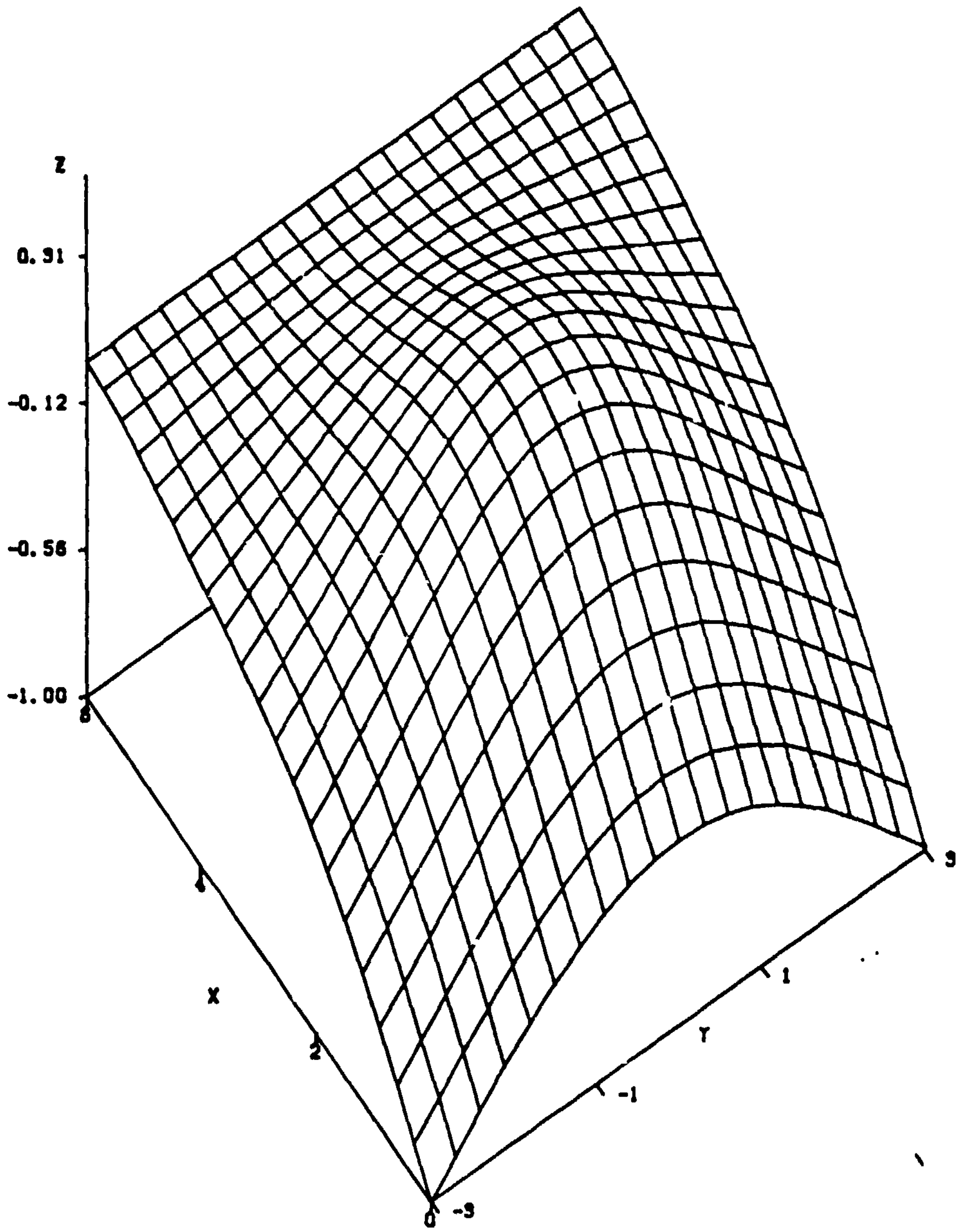
The results are

N_{i}/cm^2	N_f max $/cm^2$	σ_{yy}max MPa	σ_{xx}max MPa	$t_2$cr mm	$t_2$cr m
3	2497	-22.64	15.23	.1936(c)	.1375(t)
.15	1092	-22.65	15.99	.1965(c)	.1388(t)
.01	266	-23.69	17.1	.1982(c)	.1394(t)
	D→10D		Larger backstress		
.3	1811	-22.64	15.41	.1948(c)	.1381(t)

Table I

C - 7

The 20th Elastic Buckling Mode
for Westinghouse Temperature Profile
Critical Thickness = 0, 19817 MM
Unit of X and Y = CM



Mode	hcr (mm)	λ^2 (sec^{-1})	
1	.1936	+.06668	critical case
2	.1375	+.01124	
3	.1132	+.00742	
4	.1031	+.00452	
5	.0893	+.00365	
6	.08503	+.00153	
7	.07565	+.00698	
8	.06755	+.00117	
9	.06644	+.006555	
10	.04983	-.0000416	
11	.04086	+.0000832	

$T = T_W$

$No = .3/cm^2$

6cm x 6cm

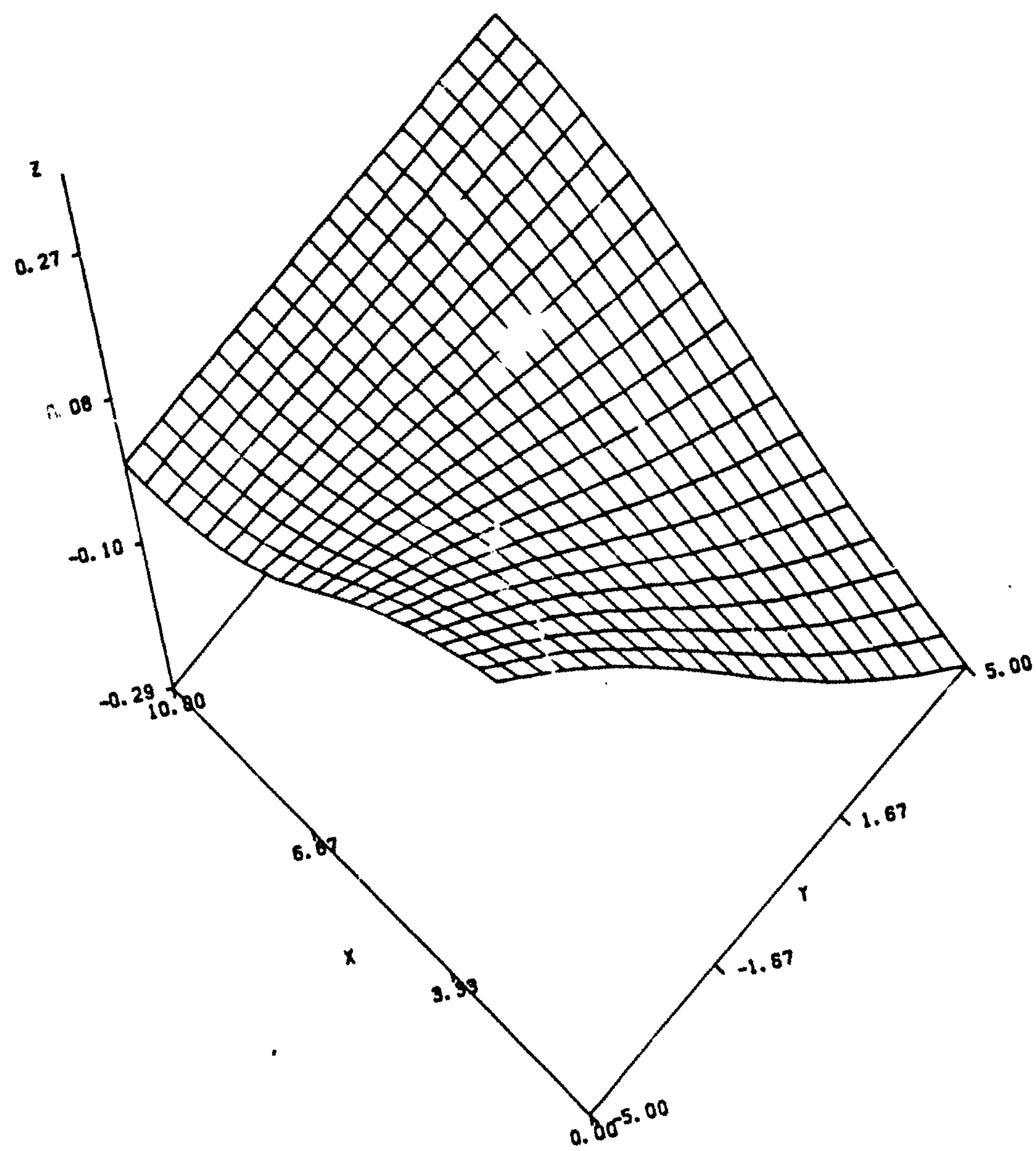
Mode	hcr (mm)	λ^2 (sec^{-1})	
1	.1965	+.02937	critical case
2	.1388	+.004919	
3	.1099	+.003094	
4	.1074	+.0018414	
5	.09146	+.0016733	
6	.08562	+.000734	
7	.076924	+.001504	
8	.069803	+.000597	
9	.06800	+.000367	
10	.062011	+.006800	

$T = T_w$

$N_o = .15/cm^2$

6cm x 6cm

The First Positive Buckling Mode
for Westinghouse Temperature Profile
Critical Thickness = 0.76154 MM
Unit of X and Y = CM



ADVANCED SILICON SHEET

Mode	hcr (mm)	$\lambda^2 sec^{-1}$	
1	.4204	-1.706×10^{-3}	
2	.3872	-7.164×10^{-3}	
3	.3346	-5.623×10^{-4}	
4	.3080	$+2.8909 \times 10^{-3}$	
5	.2789	$+4.9641 \times 10^{-3}$	critical case
6	.2663	-7.8054×10^{-4}	
7	.2337	$+2.3418 \times 10^{-4}$	
8	.2281	-1.1527×10^{-3}	
9	.2044	-6.0790×10^{-4}	
10	.1974	$+1.2900 \times 10^{-4}$	
11	.1801	-2.0870×10^{-4}	
12	.1724	$+1.0930 \times 10^{-4}$	

T = Modified EFG

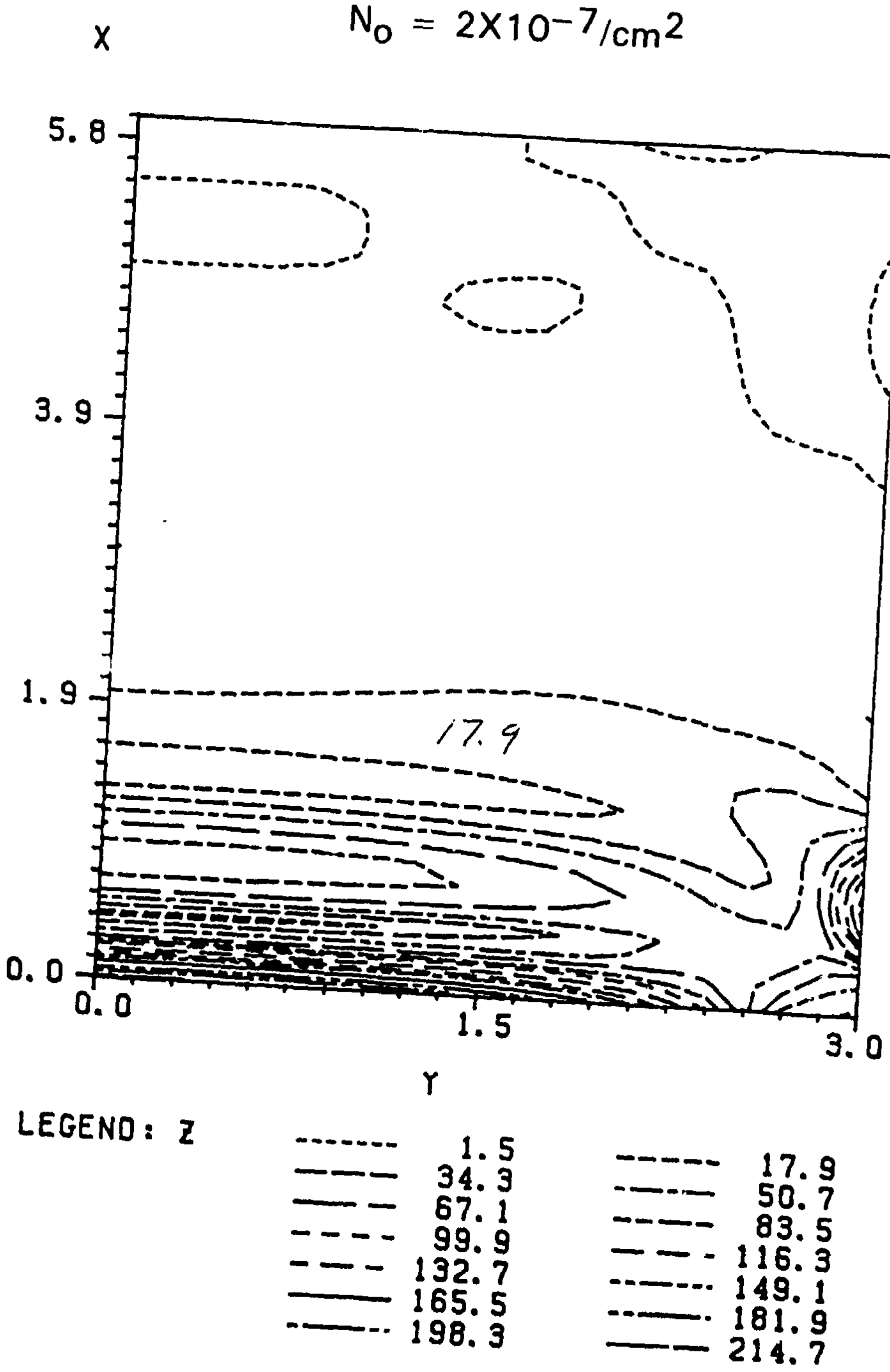
No = $2 \times 10^{-7}/cm^2$

6cm x 6 cm

Run V-0545
8 June

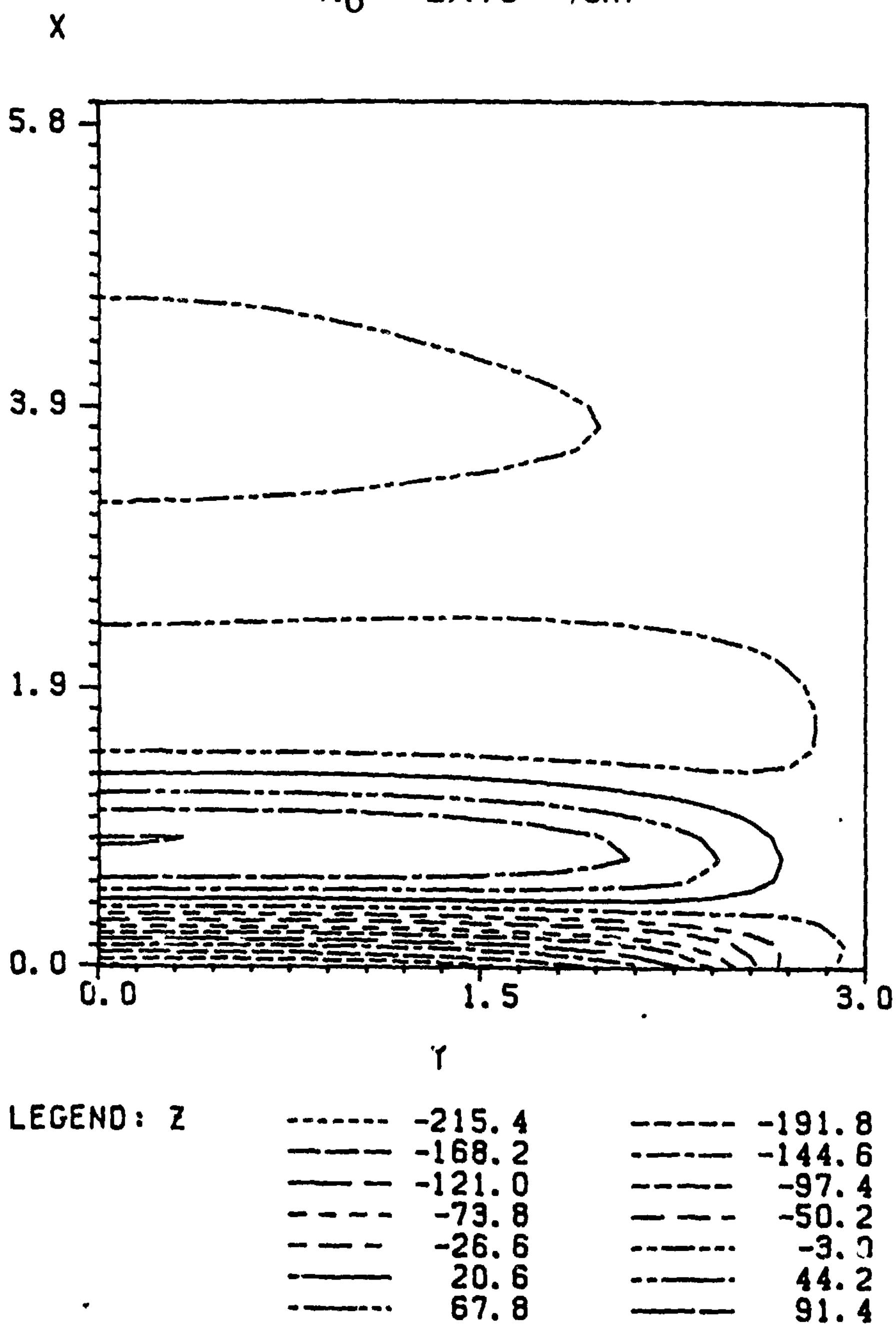
The Effectiveness Stress Contour Plot
for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

$N_o = 2X10^{-7}/cm^2$



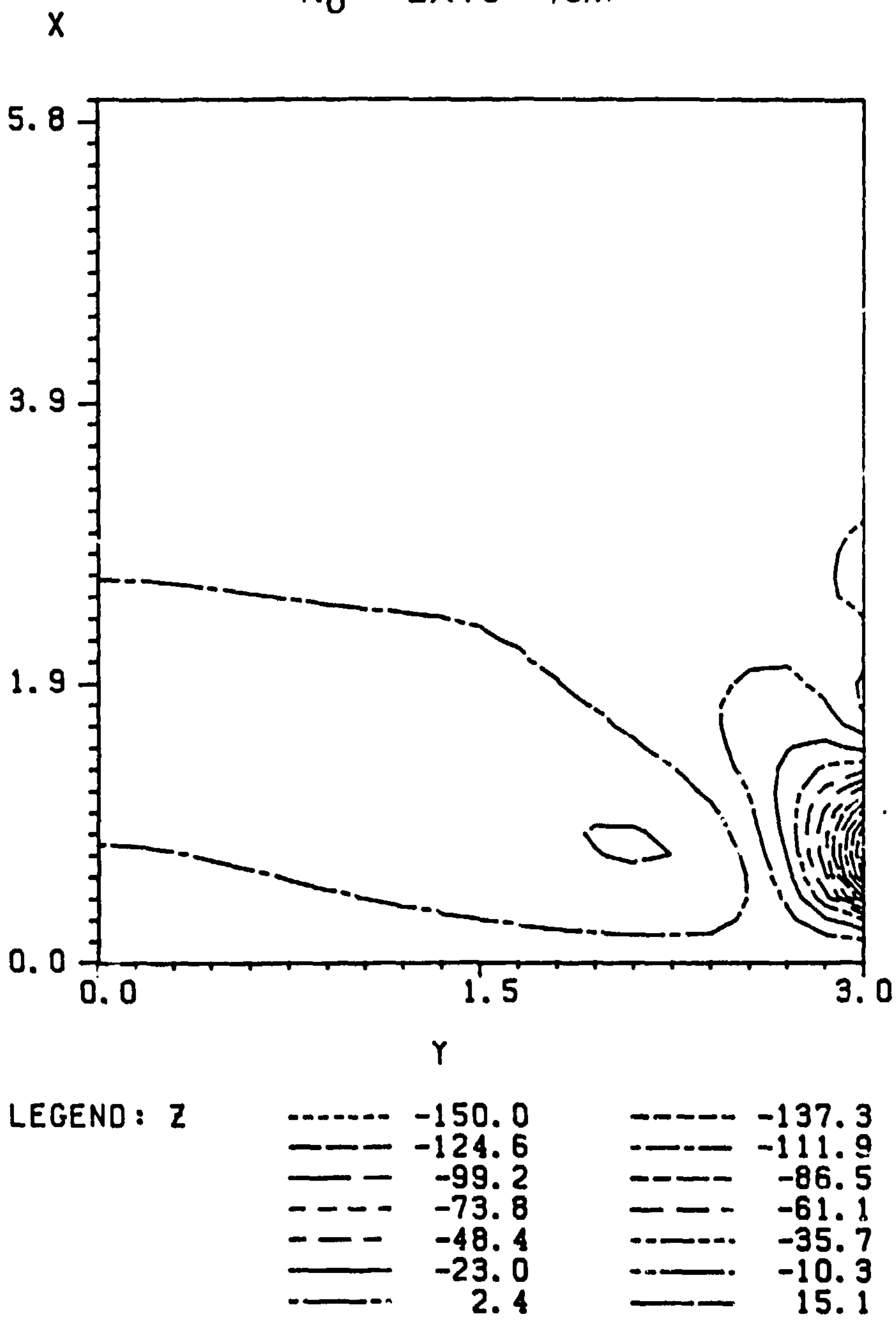
The Normal Stress YY Contour Plot
for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

$N_o = 2X10^{-7}/cm^2$

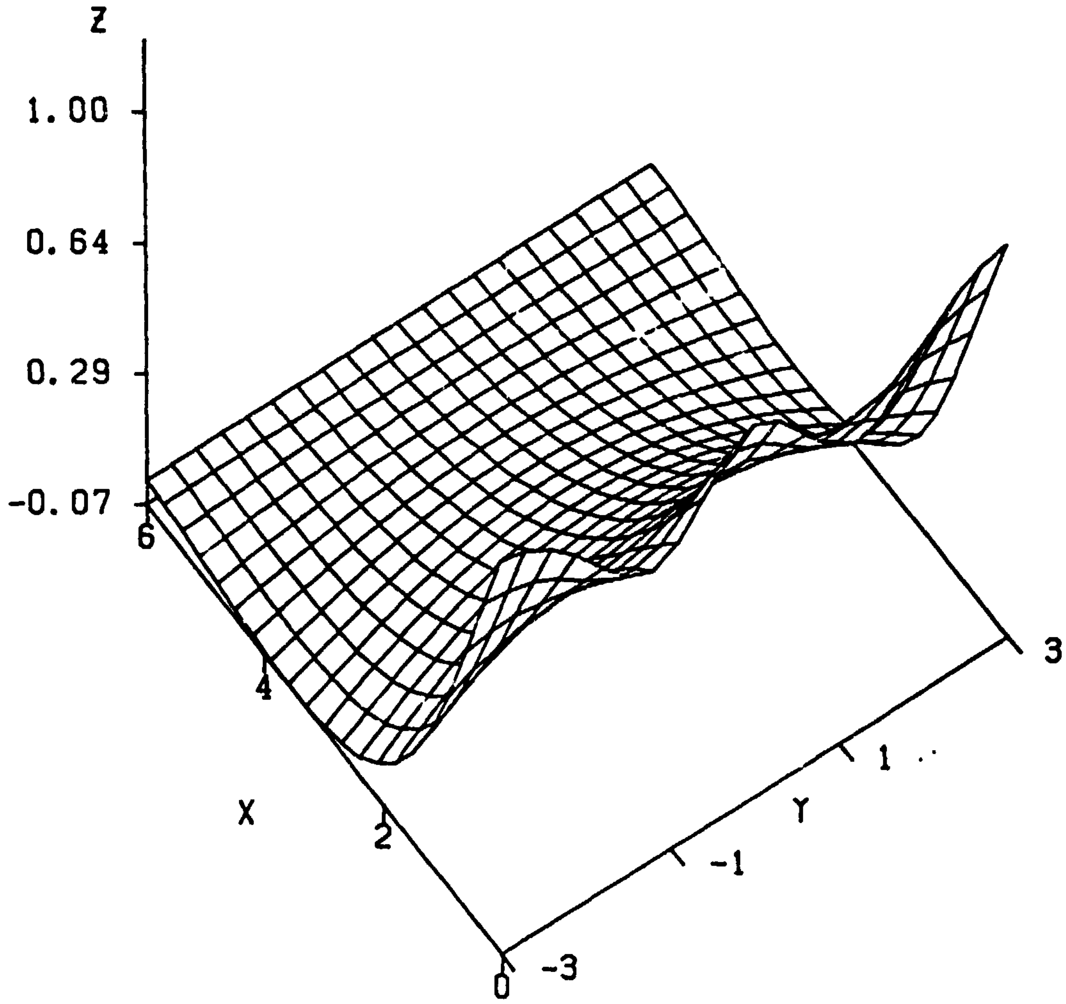


The Normal Stress XX Contour Plot
for Modified EFG Profile
Unit of X and Y = CM, Z = MPA

$N_o = 2X10^{-7}/cm^2$



The 13th Buckling Mode
for Modified EFG Profile
Critical Thickness = 0, 027877 MM
Unit of X and Y = CM



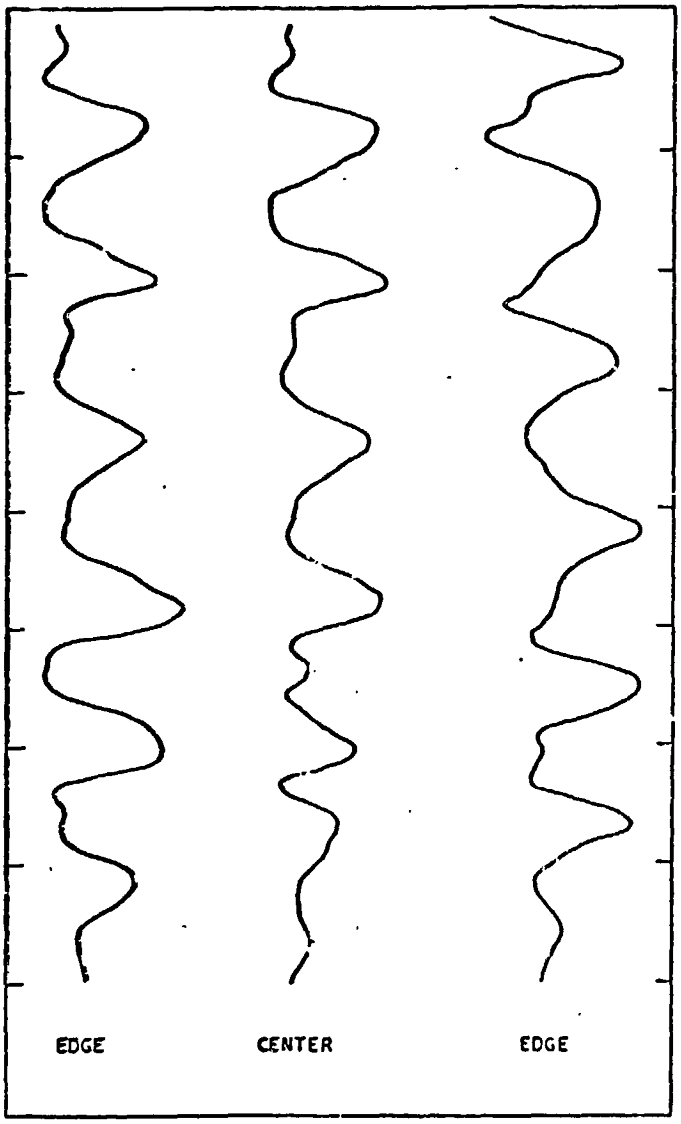


Fig. 1. Surface profile traces illustrating typical edge buckling for ribbon no. 18-102-2 grown at a speed of 3.0 cm/min. Traces are taken along the growth direction, with respect to the width dimension as marked.

MAR 1979

for the case of a 6 cm x 6 cm ribbon pulled at v_0 = .0005 m/sec. The results are shown in Table III.

M /cm	N_o cm/2	N_{f_2} /cm^2	N_{tel} /cm^2	$\sigma_{yy_{max}}$ MPa	Tw mm
1.75	.5	Diverge	4.65×10^8	-151.1*	.4698 *
1.0	.3	Diverge	3.175×10^8	-101.7*	
.25	.3	Diverge	$.5941 \times 10^4$	- 17.4*	
.240625	.3	1.06×10^4	$.3137 \times 10^4$	- 17.8	
.2375	.3	1984	$.2527 \times 10^4$	- 16.8	.21364
.225	.3	963	1049	- 15.0	
.200	.3	173	174	- 12.1	.29581

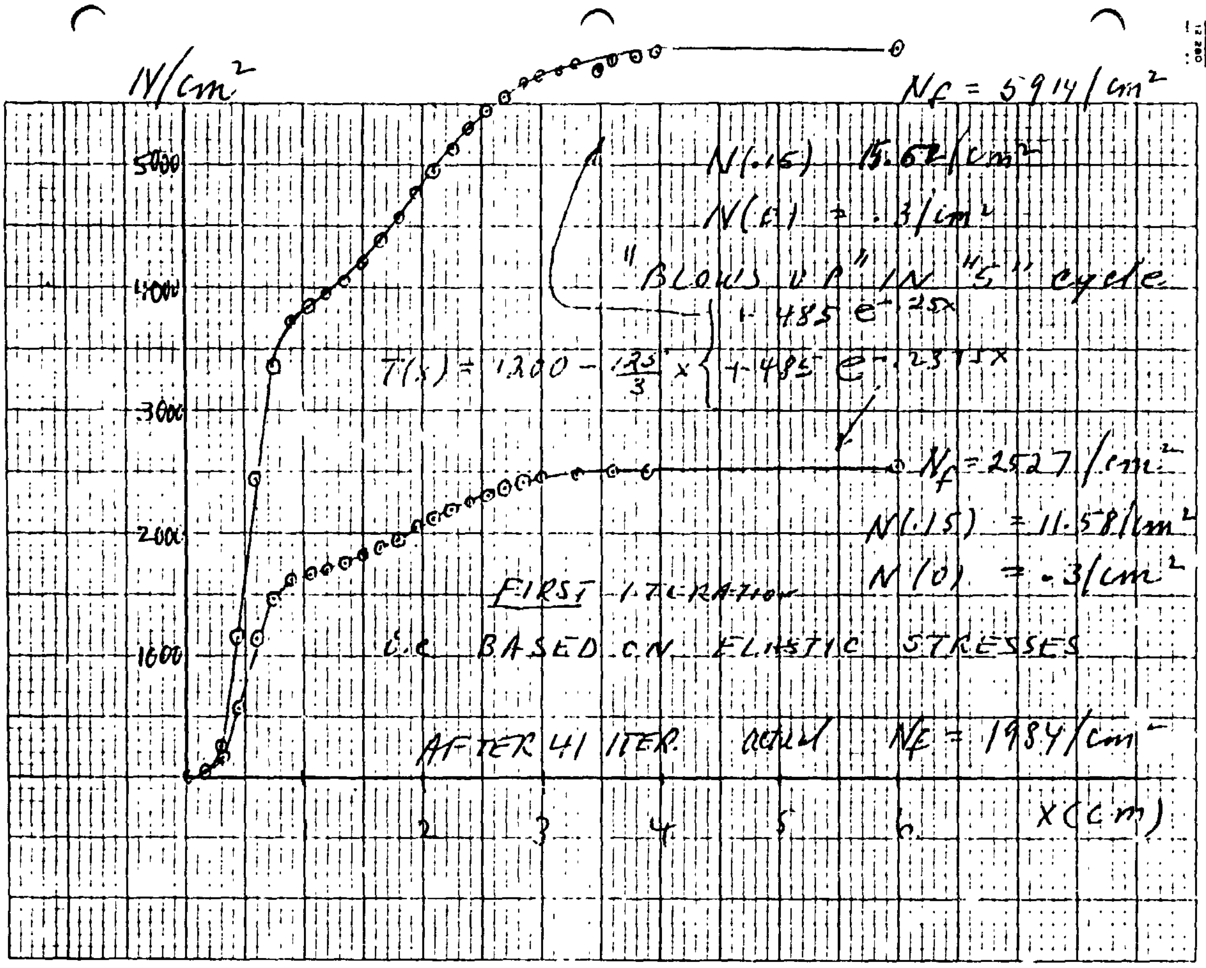
Table III

The * in the last column indicates these are the elastic stresses, because plastic ones are not obtained.

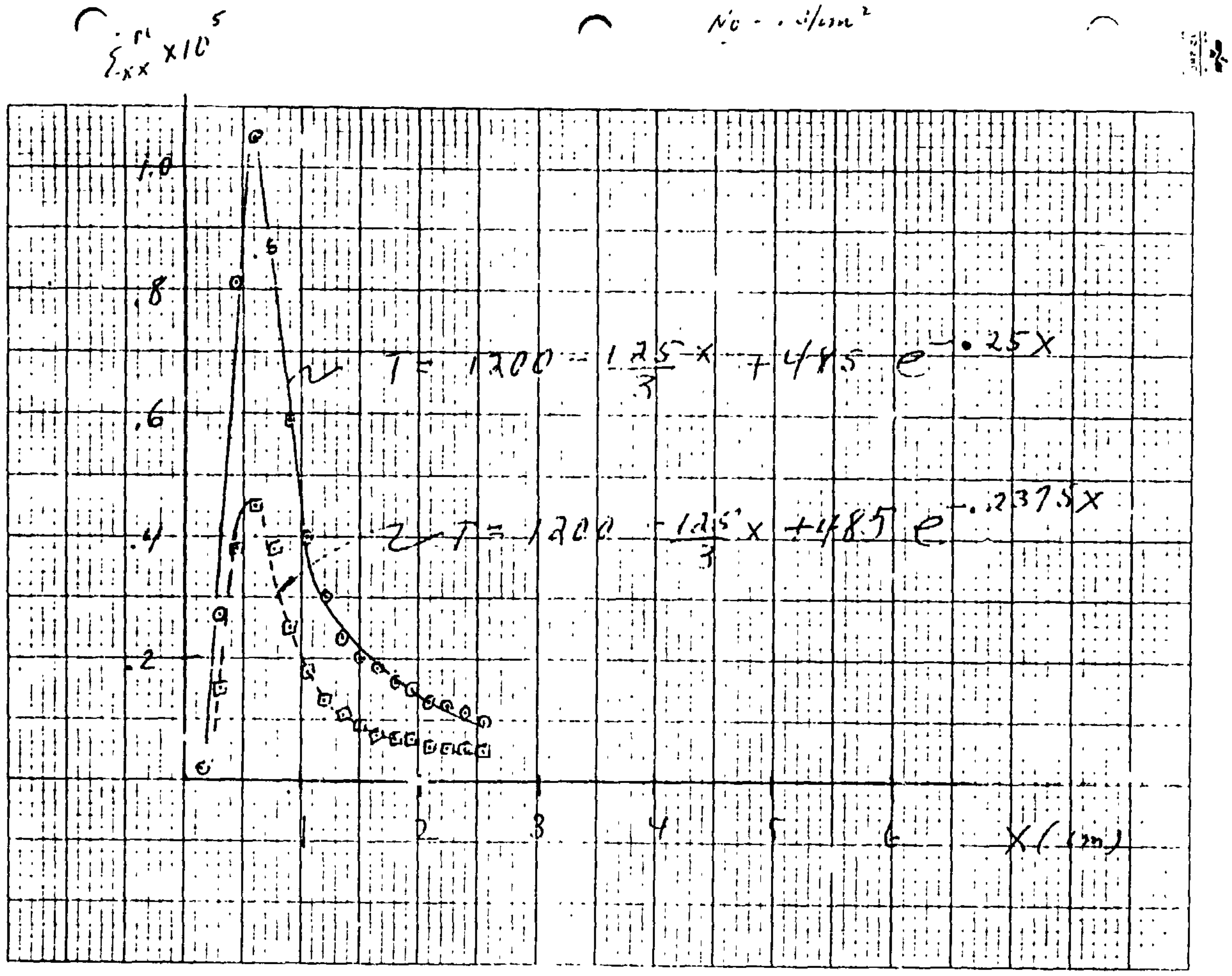
$$T_{NEFG} = 1200 - \frac{125x}{3} + 485e^{-1.75x}$$

This led to divergent solutions under conditions when the Westinghouse profile did not. With this situation in mind we considered the family of thermal profiles, defined by

$$T(x) = 1200 - \frac{125x}{3} + 485\, e^{-Mx}$$


N/cm²
5000
4000
3000
2000
1600
Nf = 5914/cm²
"BLOWS UP" IN "5" cycle
Nf = 2527/cm²
N(.15) = 11.58/cm²
N(0) = .3/cm²
FIRST ITERATION
BASED ON ELASTIC STRESSES
AFTER 41 ITER.
Nf = 1984/cm²
2
3
4
5
6
X(cm)

OF POOR QUALITY



Really New Science

Dislocations as part of the stress analysis.

That is N $\neq$ constant!

New

Creep buckling (lowest mode does not dominate!)

Practical

Elastic very useful

Plastic - residuai stress

$\tau_{cr} \doteq \tau_{cr}$ (elastic)

Keep N small

Very sensitive

(ala melting)