

THE USEFULNESS OF KLETT'S INVERSION ALGORITHMS TO SIMULATED  
SATELLITE LIDAR RETURNS

M. Kästner, H. Quenzel  
University of Munich, Meteorological Institute  
Munich, Fed. Rep. of Germany

1. Introduction

The lidar equation is a special form of the radiative transport equation in single scattering approximation and describes the return signal of a lidar. Based on this lidar equation satellite backscatter lidar returns  $P$  have been simulated<sup>1</sup> using realistic optical parameters of the atmosphere ( $\beta$  = total backscatter coefficient,  $\sigma$  = total extinction coefficient). The lidar equation reads

$$P(r) = C_G \cdot \frac{1}{r^2} \cdot \beta(r) \cdot \exp\left(-2 \int_{r_0}^r \sigma(r) dr\right) \quad (1)$$

where  $r$  is the range from the lidar to the scattering volume, and  $C_G$  is the instrumental constant including the pulse energy. Because of the viewing mode vertically down to the earth surface the range  $r_0$  is at that range where the return signal is detectable and the maximum range  $r_m$  is the satellite altitude.

The simulated lidar signal shows the bounds of atmospheric layers with different optical density with a height resolution of about 100 m. It is even possible to detect the upper bound of the planetary boundary layer (PBL) through a thin cirrus cloud. 'Subvisible' clouds not detectable with passive remote sensing techniques will be recognized with satellite lidar.

The inversion of satellite lidar data to the profile of the extinction coefficient and so to the optical depth of each atmospheric layer and the transmission of the atmosphere contributes e.g. to studies on the radiation flux and the radiation budget, from which heating/cooling rates can be derived.

2. Sensitivity study on Klett's algorithm

Klett's<sup>2</sup> algorithm for retrieving the total extinction coefficient profile has been developed for application to ground-based lidar returns by a backward integration from the far end  $r_m$  to the near end  $r_0$ , the range where the incident and backscattered pulse overlap totally. Our study results in an assessment of the applicability of Klett's algorithm to satellite backscatter lidar returns. We used the simulated data<sup>1</sup> of a 1 J Alexandrite laser operated at about 0.7  $\mu\text{m}$  and at a satellite flight level of 840 km. Klett assumes a power law for the  $\beta$ - $\sigma$ -relation ( $\beta = c \cdot \sigma^k$ ), the exponent  $k$  has been chosen to be 0.7 according to Fenn<sup>3</sup>. Klett's solution with backward integration mode reads

$$\sigma(r) = \frac{\exp((S(r) - S(r_m))/k)}{(\sigma(r_m))^{-1} + \frac{2}{k} \int_r^{r_m} \exp((S(r) - S(r_m))/k) dr} \quad (2)$$

with  $S(r) = \ln(P(r) \cdot r^2)$

while the forward integration mode reads

$$\sigma(r) = \frac{\exp((S(r) - S(r_0))/k)}{(\sigma(r_0))^{-1} - \frac{2}{k} \int_{r_0}^r \exp((S(\tau) - S(r_0))/k) d\tau} \quad (3)$$

where  $\sigma(r_m)$  is the total extinction coefficient at the far end bound (just above the earth surface) and  $\sigma(r_0)$  is the near end bound value (top of the atmosphere) for a satellite lidar system.

Variations of these boundary values result in a parallel shift of the retrieved extinction profile (lin-log-scale) to greater values for greater  $\sigma(r_m)$  or  $\sigma(r_0)$ . Higher values of the exponent  $k$  makes the retrieved profile steeper. The stability of both solutions is moderate, because in the forward integration mode the nominator increases and the denominator decreases, while the reverse is true in the backward integration mode.

### 3. Results

In Fig. 1 the vertical profiles of the total extinction coefficient  $\sigma$  (log-scale) versus the height (lin-scale) are shown resulting from Eqs.(2) and (3) with correct boundary values of  $\sigma(r_m)$  and  $\sigma(r_0)$ , respectively. The solid lines are the known input  $\sigma_{in}$ -profiles of two atmospheric models. In an atmosphere with low turbidity (Fig. 1a) the forward integration mode leads to a better  $\sigma_{out}$ -profile (dashed) than the backward integration mode (dashed-dotted  $\sigma_{out}$ -profile). On the other hand in an atmosphere with high turbidity (Fig. 1b) the backward integration leads to better results. But these two examples give exceptionally good inversion results.

Both modes have been applied to a large number of simulated satellite signals of multiply layered atmospheres, which differ by the aerosol type or the optical depth of each layer. Typical results of the inversions are given in Fig. 2. Fig. 2a) shows the  $\sigma_{in}$ - and two  $\sigma_{out}$ -profiles for a clear atmosphere with a cirrus layer (optical depth = 0.3) between 7.5 and 9.5 km height, Fig. 2b) shows the set of extinction profiles for an atmosphere with low turbidity in the stratosphere and troposphere but very high turbidity in the PBL (desert dust), and Fig. 2c) shows the set of profiles for a turbid stratosphere (volcanic aerosol) and a clear troposphere. All examples demonstrate that both inversion algorithms are unsatisfying for satellite lidar signals, because the inverted profiles are incorrect to about one order of magnitude. A reason for these unsatisfying results is that the values of the pair  $(c,k)$  of the power law are different for different aerosol types and that  $c$  is assumed to be constant in the derivation of Eqs.(2) and (3) and only one value of  $k$  is used in the inversion algorithm.

Recently Klett<sup>4</sup> proposed an improved algorithm distinguishing between air molecules (subscript R) and aerosol particles (subscript M). The lidar equation for these two scatterers reads

$$P(r) = C_G \cdot \frac{1}{r^2} \cdot (\beta_R(r) + \beta_M(r)) \cdot \exp\left(-2 \int_{r_0}^r (\sigma_R(\tau) + \sigma_M(\tau)) d\tau\right) \quad (4)$$

A linear law for the  $\beta$ - $\sigma$ -relation, the aerosol lidar ratio  $S_M(r) = \sigma_M(r)/\beta_M(r)$  is assumed in the solution to the aerosol extinction coeffi-

cient. The lidar ratio may change from layer to layer according to the aerosol type. The aerosol extinction coefficient profiles

$$\sigma_M(r) = -A(r) + \frac{X(r) \cdot \exp\left(\int_T^{r_m} (A(\tau) - \sigma_R(\tau)) d\tau\right)}{\frac{X(r_m)}{A(r_m) + \sigma_M(r_m)} + 2 \cdot \int_T^{r_m} X(\tau) \cdot \exp\left(\int_T^{\tau} (A(\nu) - \sigma_R(\nu)) d\nu\right) d\tau} \quad (5)$$

with  $A(r) = S_M(r) \cdot \beta_R(r)$  and  $X(r) = S_M(r) \cdot P(r) \cdot r^2$  and

$$\sigma_M(r) = -A(r) + \frac{X(r) \cdot \exp\left(-\int_r^{r_0} (A(\tau) - \sigma_R(\tau)) d\tau\right)}{\frac{X(r_0)}{A(r_0) + \sigma_M(r_0)} - 2 \cdot \int_r^{r_0} X(\tau) \cdot \exp\left(-\int_{\tau_0}^{\tau} (A(\nu) - \sigma_R(\nu)) d\nu\right) d\tau} \quad (6)$$

are the solutions of the backward and forward integration modes of the lidar equation (4).

Above the PBL the values of  $A(r)$  and the second term (called  $B(r)$ ) are at least one order of magnitude greater than the value of  $\sigma_M(r)$ . Slight errors in both terms lead to great errors in  $\sigma_M$ . If  $B(r)$  is less than  $A(r)$ , negative values of  $\sigma_M(r)$  are gained, which is more likely with the backward integration mode. Fig. 3 gives the  $\sigma_M$ -profiles according to Eq.(5) (dashed-dotted) and according to Eq.(6) (dashed) and the correct  $\sigma_M$  input profile (solid), the simulated  $P(r)$  profile is without noise, the boundary values are the correct ones, the lidar ratio  $S_M(r)$  used is by a factor 0.5 too little. The figure shows a relative deviation of the retrieved profile from the input profile to about 0.43 at the surface for the forward and to about 6.7 at the top of the atmosphere for the backward integration mode. The problem how to get the correct values of the lidar ratio has not been solved up to now; perhaps it can be estimated from the lidar signal itself, where the increased information of a multiple wavelength lidar would certainly help.

It is not always advantageous, as Fernald<sup>5</sup> states, to apply the last calculated  $\sigma_M(r)$  as a new boundary value  $\sigma_M(r_m)$  or  $\sigma_M(r_0)$  because, if this  $\sigma_M$ -value is wrong, this error will propagate through the whole inversion.

In fact, the problem of inverting the lidar returns may be even more complex because there is some evidence<sup>6</sup> that even in optically rather thin atmospheres the second order scattering cannot be ignored, so that an extended version of the lidar equation must be used. Then there is certainly no chance anymore to get an analytical solution of the inversion problem.

So we try to solve the lidar equation to the aerosol extinction coefficient with an iterative method, which seems to be more stable with respect to inaccuracies in the input data.

- 
- 1 Quenzel, H., E. Thomalla and K. Nodop: Performance Simulations of Backscatter Lidar. In: Vol. 1 of M. Endemann: Orbiting Lidars for Atmospheric Sounding, Battelle-Institut e.V., Frankfurt a.M., December 1984, 107-197.
  - 2 Klett, J.D., 1981, Appl. Opt. 20, 211-220.
  - 3 Fenn, R.W., 1966, Appl. Opt. 5, 293-295.
  - 4 Fernald, F.G., 1984, Appl. Opt. 23, 652-653.
  - 5 Klett, J.D., 1985, Appl. Opt. 24, 1638-1643.
  - 6 Ricklefs, U., 1985, Doctorate-Thesis, Univ. Karlsruhe, 153 p.

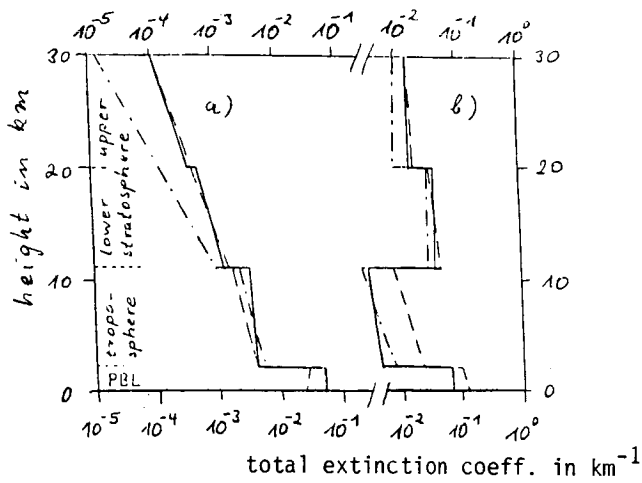


Figure 1

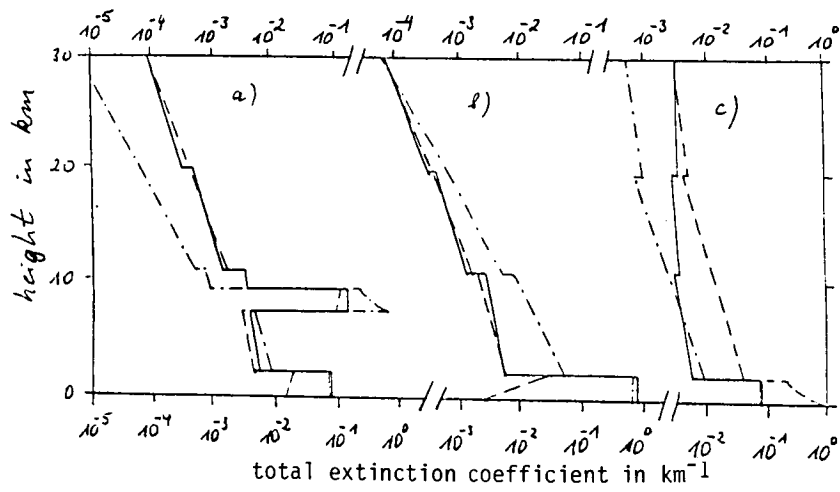


Figure 2

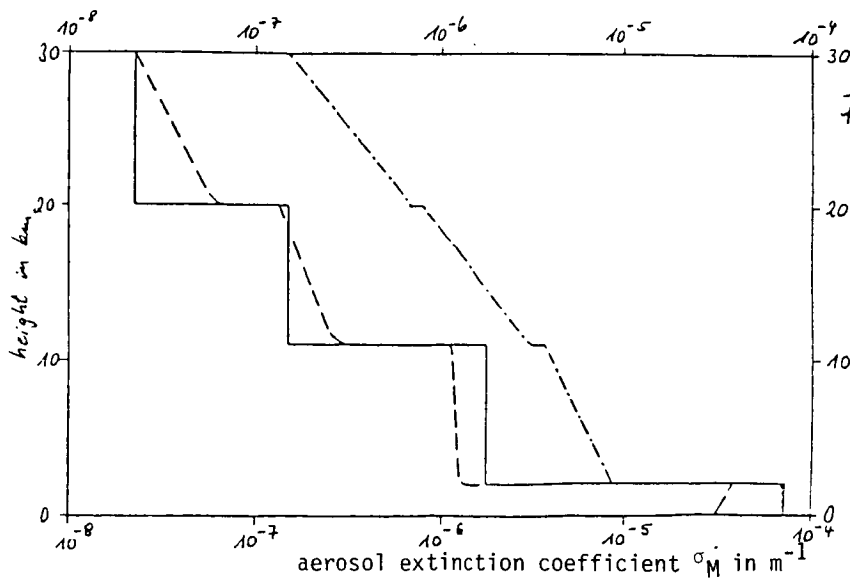


Figure 3