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1.1.2 A THEORY FOR THE RETRIEVAL OF VIRTUAL TEMPERATURE FROM WINDS, RADIANCES AND THE EQUATIONS OF FLUID DYNAMICS

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ABSTRACT

A technique to deduce the virtual temperature from the combined use of the equations of fluid dynamics, observed wind and observed radiances is described. The wind information could come e.g., from ground-based sensitive very high frequency (VHF) Doppler radars and/or from space-borne Doppler lidars. The radiometers are also assumed to be either space-borne and/or ground-based. From traditional radiometric techniques the vertical structure of the temperature can be estimated only crudely. While it has been known for quite some time (GAL-CHEN, 1978; HANE and SCOTT, 1978) that the virtual temperature could be deduced from wind information only, such techniques had to assume the infallibility of certain diagnostic relations. The proposed technique is an extension of the Gal-Chen technique. It is assumed that due to modeling uncertainties the equations of fluid dynamics are satisfied only in the least square sense. The retrieved temperature, however, is constrained to reproduce the observed radiances. It is shown that the combined use of the three sources of information (wind, radiances and fluid dynamical equations) can result in a unique determination of the vertical temperature structure with spatial and temporal resolution comparable to that of the observed wind.

1. INTRODUCTION

A few years ago, GAL-CHEN (1978) and HANE and SCOTT (1978) noted that if sufficiently accurate measurements of the wind and its time history could be obtained from Doppler radars, this information would, in principle, define also the thermodynamic structure. In essence, this is done by requiring that the data will also satisfy the momentum equations in the least square sense. This has reduced the problem to a classical calculus of variation problem (COURANT and HILBERT, 1953). The form of the momentum equations assumed in these studies is quite general and is, in principle, applicable to small meso- and large-scale atmospheric motions. While not immediately obvious, when the approximations appropriate to large-scale atmospheric flows are employed, the above variational formulation is reduced to solving a classical balance equation (HALTINER and WILLIAMS, 1980) of obtaining the geopotential from the wind.

GAL-CHEN and KROPFLI (1984), ROUX et al. (1984) and HANE and RAY (1985) have tested the practical utility of the above-mentioned variational formulation on a variety of observed small-scale phenomena; planetary boundary layer (PBL) convection in the Gal-Chen and Kropfli case; severe storms for the Hane and Ray case, and a tropical squall line for the Roux et al. case. In all, the three case studies of temperature and pressure are deduced from observed Doppler radars wind. Satisfactory agreement with in situ thermodynamic observations is reported in all three cases.

As GAGE and BALSLEY (1978) point out, sensitive Doppler radars can be used to obtain mesoscale wind profiles under all weather conditions. The vertical resolution is up to 100 m. The time resolution is about 1 hour and the horizontal resolution is determined by the average distance between the profilers. Comparable resolution is not obtainable from radiometric measurements of the atmosphere either from the ground or from satellites. The purpose of this paper is to extend and modify the GAL-CHEN (1978) technique to satisfy the following requirements:

(a) The horizontal momentum equations are satisfied in the least square sense (to be defined further below).
(b) The hydrostatic constraint is satisfied exactly.
(c) The thermodynamic equation is satisfied in the least square sense.
(d) The radiative transfer equation at various frequencies is satisfied exactly.
(e) Given wind and radiances as input, virtual temperatures should be obtained as an output. The retrieved temperature should have horizontal and vertical resolution compared to that of the observed wind.

In this paper only the theory is developed. The practical utility remains to be checked. This should be done first by simulation studies and then by examining real data. The task is vast and difficult and I hope that the theory developed here will stimulate other researchers to check its practical utility and to seek even better ways to estimate the virtual temperature.

The technique discussed in this paper has some similarities to the techniques considered by KUO and ANTHES (1985) and by BRUMMER et al. (1984). However, it also has some potentially important differences. These include _inter alia_:

(a) Lateral boundary conditions for the temperature are obtained directly from the wind data rather than prescribed from a 12-hour forecast as in Kuo and Anthes or, as in the Brummer et al. case, prescribed from a vertically smoothed temperature profile obtained by pure radiometric techniques.
(b) In both the Kuo and Anthes and Brummer et al. techniques, the horizontal divergence equation is used; as a weak constraint in the Brummer case and as a diagnostic equation for the geopotential in the Kuo and Anthes case. In our case, attempt is made to satisfy, albeit in the least square sense, all the prognostic equations relevant to describing mesoscale motions.
(c) The Kuo and Anthes approach does not utilize the information contained in the radiances. It is assumed that in nature the divergence equation is satisfied exactly. This is not true even if the wind measurements are error free. In the Brummer technique, a temperature profile is sought that will, on the one hand, satisfy the divergence equation as close as possible, and on the other hand, is also not too far from the smooth temperature profile retrieved from radiometric data. Our technique, however, demands that the retrieved temperature satisfy the radiative transfer equation, augmented by additional dynamical constraints. Unlike the pure radiometric techniques, the above set is mathematically well posed and no _a priori_ smoothing or statistical constraints need to be imposed on the retrieved temperature.

2. MODELING ASSUMPTIONS

Governing equations. The governing hydrostatic primitive equations in Cartesian x, y, z coordinates may be written as:

Continuity equation,

$$D\rho/Dt + \rho\nabla\cdot\underset{\sim}{u} = 0 \qquad (1)$$

Horizontal momentum equations,

$$Du/Dt = -(1/\rho)\partial p/\partial x + F_1 + fv \qquad (2)$$

$$Dv/Dt = -(1/\rho)\partial p/\partial y + F_2 - fu \qquad (3)$$

Here D/Dt is a symbol for total derivative

$$D/Dt = \partial/\partial t + u\partial/\partial x + v\partial/\partial y + w\partial/\partial z \qquad (4)$$

f is the Coriolis parameter ($f = 2\Omega\sin\phi$; Ω is the earth angular velocity and ϕ is the latitude). For convenience we are displaying the equations using Cartesian coordinates. Nevertheless, the extension of our ideas to spherical coordinates is obvious and all our subsequent discussions (conclusions) are valid for spherical geometry. The hydrostatic equation is given vis.,

$$\partial p/\partial z = -\rho g \qquad (5)$$

An approximate form of the thermodynamic equation neglecting the contribution of moisture to the density ρ and to the heat capacity (under constant pressure) C_p is

$$C_p\, DT/Dt - (1/\rho)Dp/Dt = S_h \qquad (6)$$

The heat capacity under constant pressure is given vis. $C_p = (7/2)R$ with R the gas constant for dry air. The equation for conservation of water vapor is

$$Dq/Dt = S_v \qquad (7)$$

The equation of state is

$$p = \rho R T_v \qquad (8)$$

Here $\underset{\sim}{u}$ is the three-dimensional wind vector $\underset{\sim}{u} = (u, v, w)$. u is the horizontal velocity in the x direction; v is the velocity in the y direction and w is the velocity in the vertical direction, z. The density of dry air is denoted by ρ; p is the pressure; g is the acceleration of gravity; F_1 and F_2 are symbols for turbulent friction forces (of dimension Newton/kg) which in this study we assume that they can be either measured directly or parameterized based on wind observations. T is a symbol for temperature; p is the pressure. T_v is the virtual temperature defined vis.

$$T_v - T = 0.61\, qT \qquad (9)$$

Here q is the water vapor mixing ratio (expressed in 10^{-3}g/(kg of dry air)). The symbol S_h is for sources (or sinks) of heat energy. Since in this study we are limiting ourselves to relatively short time scales (0-12 hours), radiative processes are presumed to be of secondary importance (SMAGORINSKY, 1974) and the major source of heating in the free atmosphere is due to precipitation. The major heating source in the planetary boundary layer (PBL) is assumed to be fluxes of sensible heat. S_v is a symbol for sources or sinks of water vapor and in accordance with our previous presumption that the major contributor to S_v in the free atmosphere is the removal of vapor by precipitation. In the PBL, the major source is evaporation from the ground.

The nature of the data and/or the parameterizations. Our major assumptions about the nature of the observed data or the parameterizations employed are as follows:

(a) Horizontal motions can be measured by means of powerful Doppler radars (frequency range is 50-900 MHz). The measurements have accuracy of ± 1 ms^{-1}; are such that all motions with time scales with less than 1 hour have been filtered and are possible under all weather conditions (LITTLE, 1982).

(b) Vertical motions with scales described in (a) can be either deduced from the horizontal motions (using the mass continuity equation) or else can be measured directly by Doppler radars (NASTROM et al., 1985). To be useful for predictions of synoptic scale motions, the accuracy of the deduced (or measured) vertical motions must be of the order of $\pm$ 1 cms^{-1} (HALTINER and WILLIAMS, 1980).

(c) Remote sensing of temperature and moisture profiles using ground-based and/or space platforms renders some useful information under almost all weather conditions (WESTWATER et al., 1985). (This is true only if the infrared channels are augmented by additional channels from the microwave. Otherwise, contamination from clouds may be severe. Furthermore, microwave measurements are contaminated under the presence of heavy rain.) The temperature and moisture retrieved from these measurements typically have poor vertical resolution. As a result, the retrieved profiles have an accuracy of no better than $\pm$ 3°C for temperature and $\pm$ 5 g/kg for the moisture.

(d) At the minimum, it is assumed that the measurements described in (a)-(c) are available in at least three spatial locations to be able to define a triangle. The satisfaction of this requirement would enable calculations of horizontal gradients. It must be borne in mind that the distance between the stations also determine the smallest scales that can be resolved by such a network. Thus, even though the horizontal wind measurements described e.g., in (a) may contain spatial scales of motions smaller than the distance between the stations, the computed horizontal gradients cannot properly resolve this information.

(e) As is customary in numerical weather prediction (NWP) models (HALTINER and WILLIAMS, 1980), we assume that all motions and processes with spatial and temporal scales that cannot be resolved by the network can either be "parameterized" in terms of what is observed or measured directly. For instance, F_1 and F_2 (in (2) and (3)) which are turbulent friction terms may be estimated from single Doppler radar data (KROPFLI, 1984). Alternatively, one may attempt to parameterize it in terms of the larger scale winds. (The simplest parameterization is to set $F_1 = F_2 = 0$.) Another example is the precipitation rate and the vertical distribution of latent heat release which may be evaluated using conventional radars (DOVIAK, 1981) or from satellite data (ATLAS and THIELE, 1981) or parameterized (e.g., ignored).

3. ALGORITHM DEVELOPMENTS

<u>Deduction of horizontal virtual temperature gradients</u>. Taking into account our assumption (a) and (b) in the previous section, we may write the horizontal momentum equations (2) and (3) as:

$$(1/\rho)\nabla_H P = \underset{\sim}{G} \qquad (10)$$

Here, $\underset{\sim}{G} \equiv (G_1, G_2)$ is a given two-dimensional vector function ($G_1 = -Du/Dt + fv + F_1$; $G_2 = -Dv/Dt - fu + F_2$) which, in principle, can be computed from the observed wind; ∇_H is the two-dimensional gradient operator. Differentiating (10) with respect to z and using hydrostatic (5), and the equation of state (8) one gets

$$(1/\rho)\nabla_H P \partial \ln T_v/\partial z + g\nabla_H \ln T_v = \partial \underset{\sim}{G}/\partial z.$$

Taking it into account (10)

$$\underset{\sim}{G}\partial \ln T_v/\partial z + g\nabla_H \ln T_V = \partial\underset{\sim}{G}/\partial z \qquad (11)$$

(11) can be considered as a generalization of the thermal wind relation in z coordinates. In fact, for $\underset{\sim}{G} = (fv;-fu)$ the thermal wind is reproduced.

Equation (11) expresses horizontal and vertical temperature gradients in terms of observed quantities (i.e., winds and its derivatives). Together with other relations to be used further below, it can be used to infer the vertical structure. Nevertheless, it is also useful to consider several approximations of (11). First consider the ratio (denoted by R_a) of $g\nabla_H \ln T_v$ to $\underset{\sim}{G}\partial \ln T_v/\partial z$. Traditional scale analysis considers (e.g., PEDLOSKY, 1979, pp. 1-10) dictates that the order of magnitude of the above-mentioned ratio is given by

$$R_a = \frac{g\delta^h T/L}{O(\underset{\sim}{G})\Gamma}$$

Here, $\delta^h T$ is a typical horizontal temperature difference over a typical length scale L and Γ is the lapse rate ($\Gamma \equiv -\partial T_v/\partial z$). We shall now try to obtain for baroclinic weather systems a lower bound of R_a. We know that Γ can hardly exceed the dry adiabatic lapse (g/C_p). Furthermore,

$$\text{Max}|\underset{\sim}{G}| = \text{Max}(U/\tau,\ U^2/L,\ fU)$$

Here, U is a typical velocity associated with the scale L, f is the Coriolis parameter, and τ is a typical time scale. Thus, overall

$$\text{Min}(R_a) = \frac{(\delta^h T/L)C_p}{\max(U/\tau, U^2/L, fU)}$$

For large-scale flows in the middle latitudes, $L \sim 10^6$ m, $U \sim 10\ ms^{-1}$, $f \sim 10^{-4}\ s^{-1}$, $O(\underset{\sim}{G}) \sim fU$. Also, a modest estimate of the large scale temperature gradient in a baroclinic flow is 3 deg/1000 km; in addition, $C_p = 1004\ Jdeg^{-1}kg^{-1}$, thus $\text{Min}(R_a) = O(3)$. This means that in the l.h.s. of (11), the contribution of the terms associated with the horizontal temperature gradient typically dominate that associated with the vertical temperature gradient. The net result is

$$g\nabla_H \ln T_v \simeq \partial\underset{\sim}{G}/\partial z \qquad (11)'$$

For the geostrophic case, $G = (fv,-fu)$ and (11)' is recognized as an approximate form of the thermal wind relation (e.g., HESS, 1959, p. 191). As long as significant baroclinicity exists, the approximation (11)' continues to be valid for mesoscale flows with $L \sim 10^5$ m, $U \sim 10\ ms^{-1}$, $|\underset{\sim}{G}| \sim U^2/L$ and $\delta^h T/L$ = 0.3 deg/100 km.

Another useful form of (11) can be utilized if one recognizes that the l.h.s. of (11) is actually $g\nabla_H \ln T_v)_p$. Here the operator $\nabla_H)_p$ is the horizontal gradient in x, y, p, t space (p is held constant). To see why this is so, note (HESS, 1959, pp. 260-264) that

$$\nabla_H T)_z = \nabla_H T)_p + (\partial T/\partial P)\nabla_H P$$

Using hydrostatic (5) and the chain rule, we obtain

$$\partial T/\partial P = -(1/\rho g)\partial T/\partial z$$

We also know from (10) that, $(1/\rho)\nabla_H P = \underset{\sim}{G}$ thus, overall,

$$g\nabla_H T_v)_z = g\nabla_H T_v)_p - \underset{\sim}{G}\partial T_v/\partial z$$

The net result is

$$g\nabla_H \ln T)_p = \partial \underset{\sim}{G}/\partial z \qquad (11)''$$

(11)" is an exact expression to calculate horizontal temperature gradients; nevertheless, to utilize it, its r.h.s. must be known at selected p levels. This requires knowledge of the pressure as a function of z. Typically, in the absence of rawinsonde, this is accomplished by utilizing a crude first guess of temperature from the radiometers, together with hydrostatic (5) and the equation of state (8). This results in a crude first guess of the pressure (typically $\pm$ 10 mb). Equipped with this information, one can interpolate $\partial \underset{\sim}{G}/\partial z$ which is observed in x, y, z, t space to an x, y, p, t coordinate. In practice, this is accomplished by interpolating to those z which correspond to constant p levels. Setting aside for the purpose of this discussion the standard errors associated with interpolations, there is an error associated with the fact that the pressure is inaccurately known; consequently, the z's associated with the constant p levels are inaccurately known.

We will now proceed to evaluate the above-mentioned errors. From hydrostatic we know that

$$\partial z/\partial \ln p = -RT_v/g$$

Integrating from sea level to some specified height (assuming for convenience p (sea level) $\sim$ 1000 mb) we get

$$z = (R\overline{T}_v/g)\ln(p/1000)$$

Here $\overline{T}_v$ is some vertically averaged temperature in the interval (o,z) and p is the pressure at level z. In the lower troposphere $\ln(p/1000) = O(1)$ and $R\overline{T}_v/g = O(8\text{ km})$ (see e.g., HESS, 1959, pp. 75-77). Assuming a worst case scenario that the errors in estimating $\overline{T}_v$ and T_v strictly from radiometric data are the same, we obtain for δz (the error in z) that,

$$\delta z = (R\,\delta\overline{T}_v/g)O(1)$$

Taking $\delta T_v \sim 3°K$ we obtain $\delta z \sim 100$ m. Now from (11)" and Taylor expansion

$$\delta(\partial \underset{\sim}{G}/\partial z) \sim \delta z\, \partial^2 \underset{\sim}{G}/\partial z^2 \qquad (12)$$

Furthermore, for Rossby number not too large from unity (essentially corresponding to large and mesoscale motion) $\underset{\sim}{G} = O(fU)$ thus,

$$\partial^2 G/\partial z^2 = O(fU/D^2) \qquad (13)$$

Here D is a typical vertical scale over which significant variation of $\partial^2 u/\partial z^2$ are occurring. For the troposphere, $D \sim 5$ km, $f \sim 10^{-4}\ s^{-1}$, $U \sim 10\ ms^{-1}$, $T \sim 273°K$. Also as discussed above, $\delta z \sim 100$ m. Substituting the above results in (12) and (13) and also taking into account (11)" we obtain

$$\delta\nabla_H T)_p \sim O(10^{-2}\text{deg}/100\text{ km}) \qquad (14)$$

$\delta\nabla_H T)_p$ in (14) is an estimate of the error in the evaluation of the horizontal temperature gradient in p coordinates due to interpolation errors from x, y, z, t space to x, y, p, t space. As discussed above, these errors are the result of our inaccurate knowledge of p. The error appears to be quite acceptable. Nevertheless, it should be remembered that our error estimate is quite sensitive to the choice of D, the vertical scale. At any rate, the algorithm to be described further utilizes (11) which is the exact form in x, y, z, t space rather than the approximate form (11)' or the form (11)" which is exact but requires interpolation to x, y, p, t space.

Deduction of vertical virtual temperature gradients. So far we have shown how to find horizontal virtual temperature gradients. To find the vertical temperature gradients we will have to use the thermodynamic equation (6). A difficulty arises because while $T_v \simeq T$ to within 1% (HESS, 1959, p. 44) the contribution of the moisture to the horizontal virtual temperature gradient at the lower troposphere could be comparable to that of the horizontal temperature gradients. To overcome this difficulty we will now derive an alternative form of (6) containing only gradients of the virtual temperature. We start by noting that the continuity equation (1) and the equation of state (8) imply that

$$-(1/\rho)Dp/Dt = -RDT_v/Dt + RT_v \mathrm{div}\cdot \underset{\sim}{u}$$

This enables us to rewrite (6) as

$$C_p DT/Dt - RDT_v/Dt + RT_v \mathrm{div}\cdot \underset{\sim}{u} = S_h \tag{15}$$

From the definition of virtual temperature (9) we obtain

$$DT/Dt = DT_v/Dt - 0.61\, qDT/Dt - 0.61\,(Dq/Dt)T \tag{16}$$

Now, under all meteorological conditions $q \sim 10^{-2}$ and less. Thus, the second term ot the r.h.s. of (16) is always negligible compared to the first term. Under conditions of strong moisture gradient (e.g., dry lines) the third term may be important and is therefore retained. However, in the third term, we may substitute T_v for T. Utilizing the above approximations we may substitute (16) in (15) taking also into account the moisture equation (7) and the fact that $C_p - C_v = R$ to obtain

$$C_v DT_v/Dt - RT_v \mathrm{div}\cdot \underset{\sim}{u} - 0.61\, C_p S_v T_v = S_h \tag{17}$$

We next substitute (11) in (17) replacing the horizontal temperature gradient in (17) by $(T_v/g)\partial \underset{\sim}{G}/\partial z - (\underset{\sim}{G}/g)\partial T_v/\partial z$. The net result is

$$C_v(\partial \ln T_v/\partial t + \tilde{w}\partial \ln T_v/\partial z) = F - S_h/T_v \tag{18}$$

Here F is a symbol for presumably observed quantities, i.e.,

$$F = 1/g(\underset{\sim}{u}_H\cdot\partial \underset{\sim}{G}/\partial z) + R\mathrm{div}\cdot \underset{\sim}{u} - 0.61\, C_p S_v \tag{19}$$

$\tilde{w}$ is the modified vertical velocity given by

$$\tilde{w} = w + (\underset{\sim}{u}_H\cdot \underset{\sim}{G})/g \tag{20}$$

The horizontal velocity vector is denoted by $\underset{\sim}{u}_H$ $[\underset{\sim}{u}_H = (u,v)]$.

The next step is to obtain explicit expressions for $\partial \ln T_v/\partial z$ which do not contain temperature tendencies. This is accomplished by applying the vector operator $g\nabla_H + \underset{\sim}{G}\partial/\partial z$ on both sides of (18). The result using (11) and calculus rules of the sort $f'g = (gf)' - g'f$ are terms like

$$\partial/\partial t(g\nabla_H \ln T_v) + \underset{\sim}{G}\,\partial/\partial t(\partial \ln T_v/\partial z) = (g\nabla_H + \underset{\sim}{G}\,\partial/\partial z)\partial \ln T_v/\partial t$$

$$g\tilde{w}\partial/\partial z(\nabla_H \ln T_v) + g(\nabla_H\tilde{w})\partial\ln T_v/\partial z = g\nabla_H(\tilde{w}\partial\ln T_v)/\partial z$$

$$\underset{\sim}{G}\tilde{w}\partial/\partial z(\partial\ln T_v/\partial z) + \underset{\sim}{G}(\partial\tilde{w}/\partial z)\partial\ln T_v/\partial z = \underset{\sim}{G}\partial/\partial z(\tilde{w}\partial\ln T_v/\partial z)$$

Furthermore, from calculus

$$\underset{\sim}{G}\partial/\partial t(\partial\ln T_v/\partial z) = \partial/\partial t(\underset{\sim}{G}\partial\ln T_v/\partial z) - (\partial G/\partial t)(\partial\ln T_v/\partial z)$$

Thus,

$$(g\nabla_H = \underset{\sim}{G}\partial/\partial z)\partial\ln T_v/\partial t = \partial/\partial t(g\nabla_H + \underset{\sim}{G}\partial/\partial z)\ln T_v - (\partial\underset{\sim}{G}/\partial t)\partial\ln T_v/\partial z$$

Using (11),

$$(g\nabla_H + \underset{\sim}{G}\partial/\partial z)\partial\ln T_v/\partial t = \partial^2\underset{\sim}{G}/(\partial z\partial t) - (\partial\underset{\sim}{G}/\partial t)\partial\ln T_v/\partial z$$

Similarly,

$$(g\nabla_H+\underset{\sim}{G}\frac{\partial}{\partial z})\tilde{w}\cdot\frac{\partial\ln T_v}{\partial z} = g(\nabla_H\tilde{w})\frac{\partial\ln T_v}{\partial z} + \underset{\sim}{G}\frac{\partial\tilde{w}}{\partial z}\cdot\frac{\partial\ln T_v}{\partial z} + \tilde{w}\frac{\partial^2 G}{\partial z^2} - \tilde{w}(\partial\underset{\sim}{G}/\partial z)\frac{\partial\ln T_v}{\partial z}$$

Also utilizing (11)

$$(g\nabla_H+\underset{\sim}{G}\partial/\partial z)\frac{S_h}{T_v} = \frac{1}{T_v}(g\nabla_H+\underset{\sim}{G}\partial/\partial z)S_h - (S_H/T_v)\partial\underset{\sim}{G}/\partial z$$

Overall, the net results are two separate estimates for the vertical temperature gradient, namely

$$C_v(g\nabla_H\tilde{w}+\underset{\sim}{G}\partial\tilde{w}/\partial z - \partial\underset{\sim}{G}/\partial t - \tilde{w}(\partial\underset{\sim}{G}/\partial z)\frac{\partial\ln T_v}{\partial z} = \underset{\sim}{H} \tag{21}$$

where $\underset{\sim}{H}$ is given by

$$\underset{\sim}{H} = (g\nabla_H+\underset{\sim}{G}\partial/\partial z)F - (S_h/T_v)\partial\underset{\sim}{G}/\partial z + (1/T_v)(g\nabla_H+\underset{\sim}{G}\partial/\partial z)S_h \tag{22}$$

Here F is given by (19). Since we have assumed that a first guess of T_v is available from the radiometers and is accurate to within $\pm$ 3°K it is permissible to substitute this first guess in the r.h.s. of (22). The net result is that $\underset{\sim}{H}$ is an observed vector function.

It is now useful to put together the forms of the horizontal and vertical virtual temperature gradients and their dependence on the observed winds. They are:

The generalized thermal wind relation (11) rewritten here as

$$(g\nabla_H+\underset{\sim}{G}\partial/\partial z)\ln T_v = \partial\underset{\sim}{G}/\partial z \tag{23}$$

Equations for vertical temperature gradient (21) rewritten as

$$\underset{\sim}{A}\partial\ln T_v/\partial z = \underset{\sim}{H} \tag{24}$$

Here $\underset{\sim}{G}$ are the horizontal accelerations (with a minus sign), namely

$$\underset{\sim}{G} = (-Du/Dt+fv+F_1;\ -Dv/Dt-fu+F_2 \tag{25}$$

The term H is given by (22) and from (21)

$$\underset{\sim}{A} = C_v(g\nabla_H\tilde{w}+\underset{\sim}{G}\partial\tilde{w}/\partial z-\partial\underset{\sim}{G}/\partial t-\tilde{w}(\partial\underset{\sim}{G}/\partial z) \tag{26}$$

We close this section by noting that considerable simplifications of the expressions for $\underset{\sim}{A}$, $\underset{\sim}{H}$, and F may be realized if either the approximation (11)' is used or pressure coordinates are utilized. In that latter case (11)" may be employed. While we have shown that the above approximations are for the most part reasonable, we prefer the use of the more exact forms because the numerical solution of (23) and (24) does not become easier when (23) is replaced by (11)' or (11)" and approximate forms for $\underset{\sim}{A}$ and $\underset{\sim}{H}$ are utilized.

Retrieval of the virtual temperature from the wind and radiances. Relations (23) and (24) contain information about spatial gradients of the virtual temperature. From this, as is discussed e.g., in GAL-CHEN (1978), a second-order three-dimensional Poisson-like partial differential equation for T_v may be obtained. To be solved in a limited domain, the boundary conditions (BC) need to be prescribed. GAL-CHEN (1978) has shown that Neumann-type boundary conditions (i.e., conditions on the virtual temperature gradient in the direction of the normal to the boundary) may be obtained from the observed wind (essentially from the components of $\underset{\sim}{H}$ and $\underset{\sim}{G}$ in the direction of the normal). Such a procedure appears to be better than using Dirichlet-type BC which require the specification of the virtual temperature itself on the boundaries. In the absence of radiosonde information, Dirichlet-type BC are usually known very crudely (either from radiometric data or from a guess from a larger scale model).

Regardless of what type of BC are used, the use of (23) and (24) may not be optimal because it does not utilize the radiances from the infrared and microwave channels. Also, retrieval techniques based solely on (23) and (24) tacitly assume that the formulation of the dynamical equation (1)-(7) are infallible, i.e., that the retrieval errors of the virtual temperature would be attributed solely to observational uncertainties about the wind.

We shall now proceed to develop a formulation which incorporates the observed radiances into the retrieval procedure. We start by noting that the radiative transfer equation may be reduced often to a Fredholm intergral equation of the first kind (e.g., WESTWATER and STRAND, 1972), namely,

$$\int_0^\infty B_\nu(T)K(\nu,z)dz = \tilde{I}_\nu \tag{27}$$

Here, ν is the frequency, $B_\nu(T)$ is the flanck function, K are the weights and $\tilde{I}_\nu$ are observed radiances. The surface temperature contributions are included in the r.h.s. of (27). These contributions can be determined from the "window channel" measurements for space-borne radiometers and from the "big bang" cosmic background of 2.9°K for ground-based observations. For a well mixed gas, the function $K(\nu,z)$ is known except perhaps for a small temperature dependence. Traditional methods of determination of vertical temperature profile rely on solving (27) for various channels (frequencies) having different weights $K(\nu,z)$. Thus, the contributions from different height layers can be varied and a degree of height resolution can be achieved. Extensive research (e.g., CHESTERS et al., 1982) have demonstrated the limitations of such inversion techniques. In essence, the kernel $K(\nu,z)$ acts as a vertical smoother (low pass filter). As a result, the retrieved temperature profile has a poor vertical resolution (at least in the troposphere). However, if (27) is combined with (23) and (24) the problem of vertical resolution is eliminated. In essence, the large vertical scales may be determined from (27) and the smaller vertical scales, which cannot be resolved by (27) would be determined from (23) and (24).

Before we proceed with further mathematical developments of the idea outlined above, we note that (27) has been formulated for temperatures while (23) and (24) are valid for virtual temperature gradients. Furthermore, as has been noted before, the moisture contributions to the gradient may be important. To express (27) in terms of the virtual temperature, observe that a Taylor expansion of $B_\nu(T_v)$ around T_ν taking into account (9) and the smallness of the virtual temperature correction would result in

$$B_\nu(T_v) = B_\nu(T) + (\partial B_\nu/\partial T)0.61\ qT \tag{28}$$

Now, the second term in the r.h.s. of (28) while small compared to the first, may not be neglected if we desire at least $\pm$ 1°K accuracy for temperature retrievals; nevertheless, we may substitute the radiometers first guess about the moisture and temperature in the second term. To justify this approximation, let us denote by $\delta()$ the first guess retrieval errors. We may recall that $\delta q = \pm 6\ gkg^{-1}$ and $\delta T = \pm 3$°K. We also know that, $T \sim 300$°K and (for the lower troposphere) $q \sim 10\ gkg^{-1}$. Therefore, substituting the radiometers first guess in the second term of the r.h.s. of (28) would result in virtual temperature error δT_v of the order

$$\delta T_v = 0.61\ q\delta T + 061\ T\delta q$$

Taking into account the order of magnitude of the various terms, the error is at most $\pm$ 1°K. Furthermore, the contribution of this error to the radiances [$\tilde{I}_\nu$ in (27)] is further reduced due to the averaging implied. Overall, we may substitute (28) in (27) approximating the second term in the r.h.s. of (28) by the radiometers first guess with the net result

$$\int_0^\infty B_\nu(T_v)K(\nu,z) = I_\nu \tag{29}$$

where the moisture correction to the radiances $\tilde{I}_\nu$ have been absorbed in the term I_ν.

The general retrieval algorithm may now be formulated as follows: Find a T_v such that

$$\iiint[(g\nabla_H + \underset{\sim}{G}\partial/\partial z)\ln T_v - \partial\underset{\sim}{G}/\partial z]^2 + (\underset{\sim}{A}\partial\ln T_v/\partial z - \underset{\sim}{H})^2 = \text{Min} \tag{30a}$$

subject to the constraint that

$$\int_0^\infty B_\nu(T_v)K(\nu,z)dz = I_\nu \tag{30b}$$

This is a familiar calculus of variation problem (COURANT and HILBERT, 1953, Vol. 1, pp. 164-274) whose solution will not be discussed here. We note, however, that (30-a,b) attempts to satisfy the dynamical equations in the least square sense while enforcing the retrieved virtual temperature to satisfy everywhere the radiative transfer equation.

A potential weakness of the retrieval algorithm is that the terms $\underset{\sim}{A}$ and $\underset{\sim}{H}$ involves calculating higher order derivative terms in both space and time. The estimate of such terms from the observed wind and its time history may be "noisy". To alleviate this problem one may use the Kalman filter approach (GHIL et al., 1980) where observations at more than two (or three) time levels are used to improve the estimate obtained from the solution of (30-a,b). Detailed examinations of the terms involved in (30-a,b) reveal that for the most part only two time levels are required. The calculation of $\underset{\sim}{A}$ (equation 26) requires knowledge of $\partial\underset{\sim}{G}/\partial t$. Since $\underset{\sim}{G}$ is acceleration, this requires knowledge of the wind at three time levels. Nevertheless, $\partial\underset{\sim}{G}/\partial t$ would be dropped out if we utilize pressure coordinates and relation (11)" or use the approximate form (11)'. As noted earlier (11)" is exact but the use of pressure as a vertical coordinate requires some <u>a'priori</u> knowledge of the pressure distribution.

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