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3.1.7 A PROPOSED EXPERIMENTAL TEST TO DISTINGUISH WAVES FROM 2-D TURBULENCE

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While the companion paper on buoyancy range turbulence given here leads to a unique scale, k_B, that allows one to differentiate between waves and turbulence for the special case of $\theta = 0$ (i.e., horizontally propagating waves), it does not seem to lead to a practical empirical distinction for the general situation. This is due to the fact that, as θ is increased, one has the ever-increasing presence of BRT for longer wavelengths (see Figure 1 below). The fact that the numerical values of ε' are not yet available compounds the difficulty. In addition, it does not appear possible to encompass true 2-D turbulence in the above picture. We are thus driven to a test which circumvents all these difficulties.

Our proposed test is based on the idea shown in Table 1 (of the companion paper) that waves are coherent and propagate while in turbulence we have the opposite situation. In particular, our test is suggested by the following quotation from MULLER (1984), on the nature of such turbulence: "The turbulence in each horizontal plane is independent from the turbulence in the other planes." If this statement were to be taken literally, it would imply that the temporal coherence between horizontal speeds, separated only in altitude, would be zero. Any vertical separation would be enough to destroy coherence. Naturally, in the real world, one would be forced to take into account the effects of viscosity; that is to say, a specific finite vertical separation would be needed to destroy coherence. In order to estimate this distance, L, one can use (see PRANDTL, 1952, P. 107)

$$L = C(\nu/S)^{1/2} \tag{1}$$

where ν is the kinematic viscosity, S is the shear scale, and C is a constant of order unity. Thus, if the coherence were very close to zero for vertical separations somewhat larger than L, then this would constitute strong evidence for two-dimensional turbulence and against other types of fluctuations such as gravity waves or three-dimensional turbulence over that frequency range. Numerically, L is of the order of 10 m in the troposphere and stratosphere. If however, ν in Equation (1) is replaced by turbulent eddy viscosity, then L would be increased by something like an order of magnitude. If C = 5, we would have something like 500 m. Perhaps, then, L = 1 km would be a safe value.

In view of the practical importance we will now present some of the mathematical details of the above test (see BENDAT and PIERSOL, 1971, 335-339). The coherence γ_{xy}^2 between two time series x and y is defined by

$$\gamma_{xy}^{2} \equiv \frac{|\phi_{xy}(f_m)|^2}{\phi_x(f_m)\phi_y(fm)} \tag{2}$$

Here the x and y time series are obtained at N discrete times separated by the sampling time interval Δt. The frequencies f_m are given by

$$f_m = \frac{m}{N\Delta t} \quad , \ (m=0,1\ldots\frac{N}{2}) \tag{3}$$

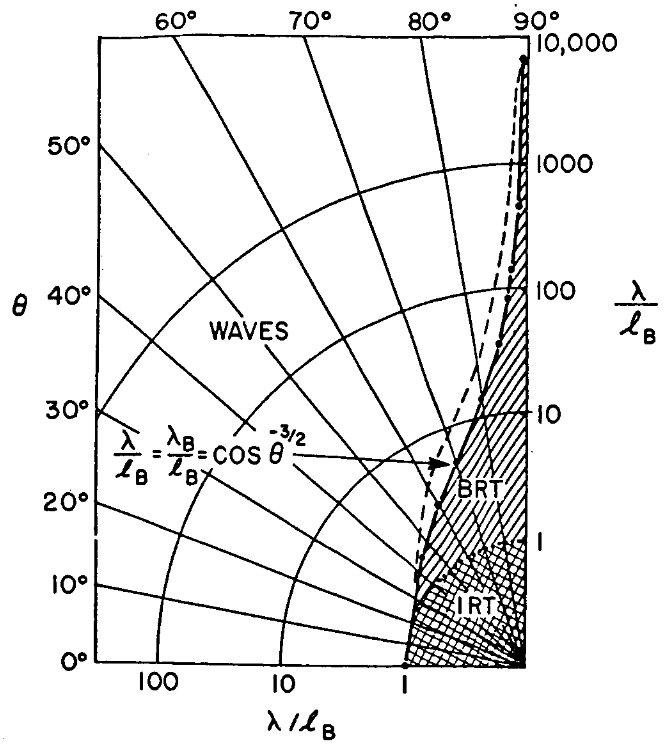

Figure 1. A polar plot of λ/ℓ_B as a function of $\theta \cdot \lambda_B \equiv k_B^{-1}$, $\ell_B \equiv (\frac{\varepsilon}{N_B^3})^{1/2}$. Regions of turbulence and waves are indicated. Note that here we take $\lambda_B = (N_B^3 \cos^3 \theta/\varepsilon)^{1/2}$ but to be strictly correct $\lambda_B = (N_B^3 \cos^3 \theta/\varepsilon')^{1/2}$. ($\varepsilon'$ is unknown). Note that BRT can extend to long wavelengths in the vertical direction when θ approaches 90°.

The cross spectrum, $\phi_{xy}(f_m)$ is defined by

$$\phi_{xy}(f_m) \equiv \frac{2\Delta t}{N}|X^*(f_m)Y(f_m)| \tag{4}$$

where

$$X(f_m) \equiv \sum X_n \exp[-j\,\frac{2\pi nm}{N}] \tag{5}$$

and this would be computed by means of the "fast Fourier transform" and then smoothed by means of standard windowing and averaging methods as described by BENDAT and PIERSOL [1971]. The $Y(f_m)$ is related to y by the same relation. X_m^* is the complex conjugate of X_m. The PSD, $\phi_x(f_m)$, is obtained from (4) by setting y = x and using only the x series. A similar thing is done to obtain ϕ_y. In other words,

$$\phi_x(f_m) = \frac{2\Delta t}{N}|X_m^* X_m| \qquad (6)$$

and similarly for ϕ_y.

At the meeting, we shall discuss the caveats associated with this test, and perhaps we can arrive at that time at a numerical specification of coherence which will satisfy most people in regard to a definitive test between waves and 2-D turbulence.

As a final remark, it should be pointed out that a certain amount of care is needed in order to avoid artifact when calculating the coherence. In particular, a single unsmoothed data set would automatically lead to a value of unity for γ_{xy}^2. In this way, an enthusiast for the gravity-wave interpretation would unwittingly delude himself into thinking that he had proven his case. To circumvent this artifact the procedure is to (a) calculate ϕ_{xy}, ϕ_x, and ϕ_y on a significantly large number of data records, or (b) to use appropriate smoothing. Bendat and Piersol point this out on p. 339 of their book in Section 6.6.

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