3.3.2 DETERMINATION OF THE BRUNT-VAISALA FREQUENCY FROM VERTICAL VELOCITY SPECTRA

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Recent work on the spectra of vertical velocity oscillations due to gravity waves in the troposphere, stratosphere and the mesosphere has revealed a typical feature which we call the "Brunt-Vaisala cutoff". Several observers (RASTOGI, 1975; ROTTGER, 1980a,b; ECKLUND et al., 1985) noticed a spectral peak near the Brunt-Vaisala frequency. This peak often is characterized by a very steep slope at the high frequency part, but a fairly shallow slope towards lower frequencies. Some example spectra of stratosphere observations are shown in Figure 1. This distinct spectral shape (most clear at the upper height 22.5 km) can be explained by the fact that the vertical velocity amplitudes of atmospheric gravity waves increase with frequency up to their natural cutoff at the Brunt-Vaisala frequency. ROTTGER and IERKIC (1985) showed that this peak around the 4-6 min period is very definitely due to gravity waves. VANZANDT (1982) suggested that the total spectra of vertical velocity variations is a manifestation of a universal spectrum of gravity waves.

RASTOGI (1975) found that the upper cutoff frequency of his mesospheric observations is consistent with the Brunt-Vaisala frequency deduced from model temperature profiles. ROTTGER (1980 a,b) compared the cutoff directly with the profiles of the Brunt-Vaisala frequency deduced from radiosonde temperature profiles of the troposphere and stratosphere. The spectrum-contour plot of Figure 2 shows a typical example, indicating the consistency of the cutoff with the Brunt-Vaisala frequency.

The observed spectral shape (Figure 1) almost exactly resembles the model spectra (Figure 3, vertical = 0°) of SCHEFFLER and LIU (1985), when one disregards here the low-frequency cutoff at the inertial frequency in the model. It was pointed out by FRITTS (1984) and VANZANDT, LIU and GAGE (personal communications, 1985) that Doppler shifts can substantially distort the spectra. LIU and SCHEFFLER (personal communication, 1985) recently did some model computations and showed that the spectral energy is redistributed through the spectrum due to a Doppler shift. Although Liu and Scheffler used the Boussinesq approximation in their simplified calculations to determine this effect of the Doppler shift, it is reasonably evident that it will also be revealed in the full wave solutions. The Doppler shift is most pronounced for just those waves with frequencies very close to the Brunt-Vaisala frequency.

We assume that the distribution of gravity-wave phase velocities is isotropic in azimuth with respect to the wind velocity (within a suitable observation period). Then about one quarter of the waves are shifted to higher frequencies, one quarter to lower frequencies and two quarters are not very little shifted because their phase velocities are (exact or almost) perpendicular to the wind velocity. The effect is that the spectrum is well smeared out, but the peak at the Brunt-Vaisala frequency still remains unshifted (due to the perpendicular waves) although it becomes less distinguishable from the spread-out background spectrum. Another effect, wave steepening due to amplitude growth of gravity waves can also have an influence on the spectral shape. It was pointed out by WEINSTOCK (1985) that the wave

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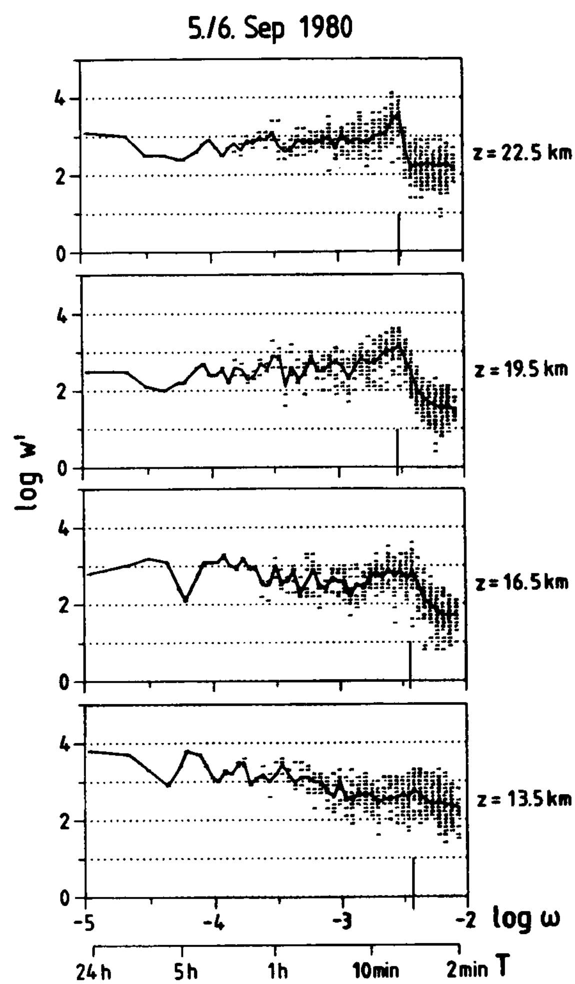

Figure 1. Power spectrum of vertical velocity, measured with the SOUSY VHF Radar (see ROTTGER, 1981, for details). The large tickmarks on the log scale indicate the position of the spectral peak, which is supposed to be at the Brunt-Vaisala frequency.

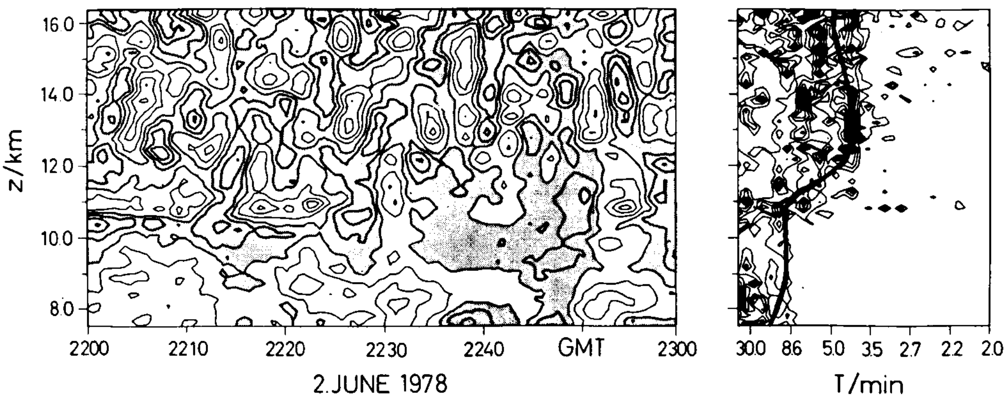

Figure 2. Contour plot of vertical velocity and spectral density (right-hand side). The continuous curve indicates the profile of the Brunt-Vaisala frequency deduced from radiosonde temperature profiles (from ROTTGER, 1980a).

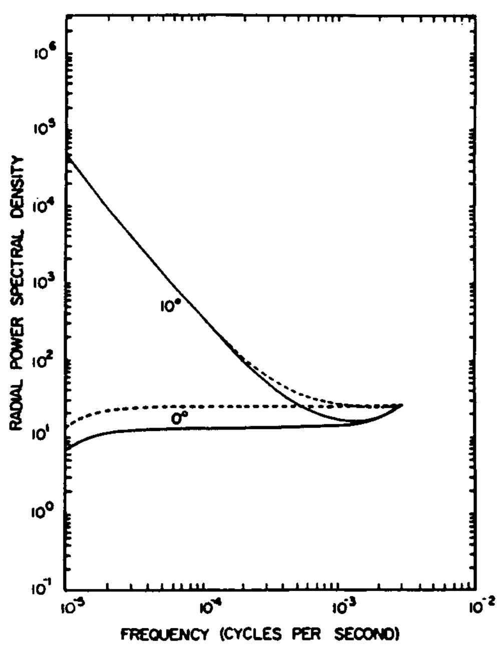

Figure 3. Frequency spectra of gravity wave velocities for 0° (vertical) and 10° zenith angle. The dashed curves are for the Boussinesq solution and the continuous curves are for the full gravity wave solution (from SCHEFFLER and LIU, 1985).

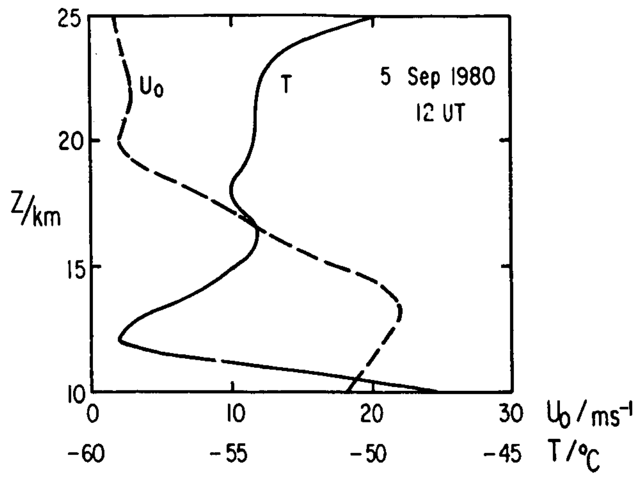

Figure 4. Radiosonde profiles of wind speed U_o and temperature T measured at a time in the interval 5/6 Sept. 1980, when the spectra of Figure 1 were obtained.

velocity steepens before it breaks. The steepening is because of harmonics which also tend the spectrum to be shifted towards higher frequencies. A cut-off above the Brunt-Vaisala frequency, however, is still apparent.

This effect may have influenced the velocity spectra observed by ECKLUND et al. (1985) during disturbed conditions (high wind), but their spectral power increased conspicuously, which may be attributed to spill-over from the horizontal velocities through a wide antenna beam width or sidelobes. The spread-out expected without the power increase can be noticed in the spectra of heights 13.5 km and 16.5 km in Figure 1, where a substantial amount of spectral energy if found at periods of a few minutes, i.e., at frequencies higher than the Brunt-Vaisala frequency. (This is particularly evident when one compares the spectra of different heights.)

Figure 4 shows the profiles of the mean wind U_o and temperature T. We notice a wind maximum of 22 ms^{-1} at 13 km, and a gradual decay of the wind velocity down to a few ms^{-1} at 20 km. It was found by ROTTGER and IERKIC (1985) that the gravity waves at the 4-6 min period have phase velocities of about 40 ms^{-1} and horizontal wavelengths of about 10 km. A wind velocity of 20 ms^{-1} can consequently yield a Doppler shift towards higher or lower frequencies by up to almost a factor of two. The low wind velocity of a few ms^{-1} above 18 km has only a negligible influence on the spectral shape. This effect of cutoff-steepening during low-wind velocities is clearly noticed when comparing Figures 1 and 4. The peak near the Brunt-Vaisala frequency is rather seen at z = 13.5 km, but it gets more salient with height. It is also shifted towards lower frequencies with height. Since a Doppler shift only widens but does not shift the peak, this frequency shift can be attributed only to a change of the Brunt-Vaisala frequency itself, which is equivalent to a change of the vertical temperature gradient. This is quite apparent in Figure 4, where we notice a highly stable temperature profile, i.e., a large Brunt-Vaisala frequency, between 12 km and 16 km, and a lower-stability profile, i.e., lower Brunt-Vaisala frequency, above 16 km.

We thus regard the measurement of the frequency of the peak in a vertical velocity spectrum to yield most directly the Brunt-Vaisala frequency from MST-radar measurements. Knowing the Brunt-Vaisala-frequency profile, one can deduce the potential temperature profile, if one has a calibration temperature at one height. However, even the uncalibrated profile will be quite useful, e.g., to determine fronts (defined by temperature inversions) and the tropopause height. It has to be noted, however, that this method fails for superadiabatic lapse rates when the Brunt-Vaisala frequency is imaginary. Examples can be found in the spectral plots published by ROTTGER (1980b). The application of this method will also be difficult when the wind velocity is too high, causing the Doppler effect to smear out the total spectrum and blur the Brunt-Vaisala cutoff. A similar deficiency will also appear if the gravity-wave distribution has a maximum in wind direction.

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