3.3.5 CAN STOCHASTIC, DISSIPATIVE WAVE FIELDS BE TREATED AS RANDOM WALK GENERATORS?

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A suggestion by MEEK et al. (1985) that the gravity wave field be viewed as stochastic, with significant nonlinearities, is applied to calculate diffusivities.

In a talk given in Boulder, REID (personal communication, 1985) described the mesospheric wave field to be predominantly stochastic in character. In fact, a recent paper by MEEK et al. (1985) quoted the BRISCOE (1975) description of the ocean wave field as "...an intermittent stochastic process with significant... nonlinearities", and suggested that this view should be adopted in atmospheric studies. Others have noted the random character of the gravity wave field as well (e.g., Vincent, Balsley).

If the Meek et al. point of view is adopted -- and account is taken that the gravity wave field is often dissipative -- then one might be able to apply the stochastic methods of turbulence transport theory to the gravity wave field. That is, the wave field may be viewed as causing a random walk (of air parcels) in the manner of turbulence: an irreversible process. However, since the waves are not as dissipative or random as turbulence, the random walk can only be an approximation -- an approximation that improves with increasing dissipation and randomness.

Aside from this uncertainty, there is the obvious difference that the wave field "eddies" (which we picture as dissipating gravity waves) are strongly influenced by stratification whereas the neutral turbulence eddies are not. This difference can be accounted for in expressions of turbulent diffusion by replacement of turbulent eddies with gravity wave Fourier components. The question that remains is whether or not such a stochastic wave model is significant for diffusion in the mesosphere.

The purpose of our article is to calculate the diffusivity for that stochastic model and compare with previous diffusivity estimates. We do this for an idealized case in which the wind velocity changes but slowly, and for which saturation is the principal mechanism by which wave energy is lost. A related calculation was given in a very brief way (WEINSTOCK, 1976), but the approximations were not fully justified, nor were the physical pre-suppositions clearly explained. The observations of MEEK et al. (1985) have clarified the pre-suppositions for us, and provided a rationalization and improvement of the approximations employed.

The derivation begins with the diffusivity tensor $\underset{\sim}{D}$ of a stochastic, dissipating velocity field given by TAYLOR (1921) as

$$\underset{\sim}{D} = \lim_{t\to\infty} < (\underset{\sim}{x}_t - \underset{\sim}{x})^2 > /2t$$

$$\underset{\sim}{D} = \int_0^\infty dt <\underset{\sim}{v}'(\underset{\sim}{x},\ t')\underset{\sim}{v}'(\underset{\sim}{x},+\underset{\sim}{x}_t,\ t'+t)> \qquad (1)$$

$$\equiv \int_0^\infty dt\ \{\frac{1}{L^2}\int_0^L dx_1 \int_0^L dx_2 \frac{1}{T}\int_0^T dt'\ \underset{\sim}{v}'(\underset{\sim}{x},\ t')\underset{\sim}{v}'(\underset{\sim}{x}_t,\ t+t')\}$$

where $\underset{\sim}{v}'(\underset{\sim}{x},\ t')$ is the velocity fluctuation at point $\underset{\sim}{x}$ at time t', $\underset{\sim}{x}_t$ is the position a particle will be at time $t + t'$ given that the particle was previously at point $\underset{\sim}{x}$ at time t' (the orbit of a particle in the combined velocity field of mean flow and waves), the angular brackets denote an average over time t' and over horizontal spatial coordinates x_1 and x_2, L is the length scale of the spatial average and T is the time scale of the time average. The time scale T is taken to be much larger than largest gravity wave period $2\Pi/\omega (T >> 2\Pi/\omega)$, and L is much larger than the largest wavelength under consideration. Equation (1) also occurs in the theory of Brownian motion.

For a spectrum of gravity waves, the (stochastic) velocity fluctuation can be represented by

$$
\begin{aligned}
\underset{\sim}{v}'(\underset{\sim}{x}, t') &= \sum_{\underset{\sim}{k}_1} \underset{\sim}{v}_{\underset{\sim}{k}_1} \exp(i\underset{\sim}{k}_1 \cdot \underset{\sim}{x} + i\omega_1 t') \\
\underset{\sim}{v}'(\underset{\sim}{x}_t, t+t') &= \sum_{\underset{\sim}{k}_2} \underset{\sim}{v}_{\underset{\sim}{k}_2} \exp[i\underset{\sim}{k}_2 \cdot \underset{\sim}{x}_t + i\omega_2(t + t')],
\end{aligned}
\qquad (2)
$$

where $\underset{\sim}{k}_1$ is the wave vector of a wave fluctuation, $\underset{\sim}{v}_{\underset{\sim}{k}_1}$ its amplitude, and ω_1 its frequency. Note that $\langle(\underset{\sim}{v}')^2\rangle = \Sigma \underset{\sim}{v}_{k_1} \underset{\sim}{v}_{k_1}^*$, where the asterisk denotes the complex conjugate, and we use $\underset{\sim}{v}(-\underset{\sim}{k}_1) = \underset{\sim}{v}^*_{\underset{\sim}{k}_1}$ to ensure that (2) is real.

Substitution of (2) and (1), it is found in a detailed derivation that D_{zz}, the vertical diffusivity is given by

$$
D_{zz} \approx \sum_{\underset{\sim}{k}} \frac{\langle w_{\underset{\sim}{k}} w_{\underset{\sim}{k}}^* \rangle}{\gamma k_z H\omega} \qquad (3)
$$

where w_k is the vertical component of wave velocity $\underset{\sim}{v}_{\underset{\sim}{k}}$, and ω is the frequency of wave $\underset{\sim}{k}$. This equation was derived for a saturated wave field, and dissipation was required for its derivation.

To generalize (3) to the case of a not completely saturated wave field, it can be shown that H need be replaced by h_o in (3)

$$
D_{zz} \approx \sum_{\underset{\sim}{k}} \frac{\langle w_{\underset{\sim}{k}} w_{\underset{\sim}{k}}^* \rangle}{\gamma k_z h_o \omega}, \qquad (4)
$$

where h_o is the "dissipation length" of the wave field, i.e., the length over which the wave energy decay e-folds owing to saturation.

Whether or not this "stochastic wave" model of diffusion is useful for the atmosphere we are not sure. Perhaps it may be useful as an upper bound -- the more dissipative the waves, the more justified its application. Numerically, the model gives values of D_{zz} in conformity with chemical model estimates (e.g., VINCENT, 1984), but whether this is more than a coincidence, we do not know. It is a straightforward way in which to apply the suggestion of Meek et al. to the problem of diffusivity.

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