

3.7.5 DOPPLER EFFECTS ON VELOCITY SPECTRA
OBSERVED BY MST RADARS

A. O. Scheffler and C. H. Liu

Department of Electrical and Computer Engineering
University of Illinois
Urbana, IL 61801

IB647432

1. INTRODUCTION

Recently, wind data from MST radars have been used to study the spectra of gravity waves in the atmosphere (SCHEFFLER and LIU, 1985; VANZANDT et al., 1985). Since MST radar measures the line-of-sight Doppler velocities, it senses the components of the wave-associated velocities along its beam directions. These components are related through the polarization relations which depend on the frequency and wave number of the wave. Therefore, the radar-observed velocity spectrum will be different from the original gravity-wave spectrum. Their relationship depends on the frequency and wave number of the wave as well as the propagation geometry. This relation can be used to interpret the observed data. It can also be used to test the assumption of gravity-wave spectrum (SCHEFFLER and LIU, 1985).

In deriving this relation, the background atmosphere has been assumed to be motionless. Obviously, the Doppler shift due to the background wind will change the shape of the gravity-wave power spectrum as well as its relation with the radar-observed spectrum. In this paper, we attempt to investigate these changes.

2. DOPPLER-SHIFTED FREQUENCY SPECTRUM

Let us first assume that the background wind is constant, blowing in the x-direction. In the rest frame coordinate of the atmosphere, the frequency Ω is related to the frequency ω of the laboratory frame through the relation

$$\Omega = \omega - k_h v_0 \cos \phi \tag{1}$$

where k_h is the horizontal wave number, v_0 is the wind speed and ϕ is the angle the horizontal wave vector \vec{k}_h makes with the x-axis. Following the derivation in SCHEFFLER and LIU (1985), the frequency spectrum of the observed velocity fluctuations along a radar beam pointed at zenith angle θ_B can be written as

$$E_{0b}(\omega) = \int_0^\infty \int_{-\infty}^\infty Q(\phi, \Omega) E(\vec{k}, \Omega) \delta(\Omega - \omega + k_h v_0 \cos \phi) d\Omega d\vec{k} \tag{2}$$

where

$$Q(\phi, \Omega) = \frac{\Omega^2 - \omega_1^2}{\omega_b^2 - \omega_1^2} \cos^2 \theta_B + \frac{\omega_b^2 - \Omega^2}{\omega_b^2 - \omega_1^2} [1 - (1 - \frac{\omega_1^2}{\Omega^2}) \sin^2(\phi - \phi_B)] \sin^2 \theta_B \tag{3}$$

$E(\vec{k}, \Omega)$ is the power spectrum of the gravity wave in the rest frame (Lagrange frame). We further assume that the wave spectrum has the same form as that in the laboratory frame such that

$$E(\vec{k}, \Omega) = \frac{E_0}{2\pi k_h} A(k_z) B(\Omega) \delta[k_h \pm (\frac{\omega^2 - \omega_1^2}{\omega_b^2 - \omega^2})^{1/2} |k_z|] \tag{4}$$

where

18929

048-46
228
3P

$$A(k_z) = \frac{(t-1)k_z^{*t-1}}{(k_z^{*+}|k_z|^t)^t}$$

$$B(\Omega) = \frac{p-1}{\omega_1^{1-p} - \omega_b^{1-p}} \Omega^{-p} \quad \omega_1 \leq \Omega \leq \omega_b \quad (5)$$

Here, isotropy of the spectrum has been assumed. Equation (2), together with equations (3), (4), and (5), yields the observed frequency spectrum of the velocity fluctuation. We note that because of the two δ -functions, the original 4-fold integration in (2) is reduced to 2-fold which can be integrated numerically.

3. RESULTS

Figure 1 shows an example of numerical computation for the frequency spectrum. This is for the case where the radar beam is pointed vertically. The Doppler effect redistributes the wave energy so that the spectrum spreads out beyond the original limits (ω_1 , ω_b). The level is depressed and a slope is introduced in the high frequency end of the spectrum. These effects become more apparent as the background wind speed v_u is increased. Figure 2 shows the result for an oblique radar beam with $\theta_B = 10^\circ$. Again, the spectrum spreads out beyond ω_1 and ω_b . The redistribution of energy due to the Doppler effect in this case makes the spectrum shallower. In a certain frequency range, the level can be higher than that for the case with no Doppler.

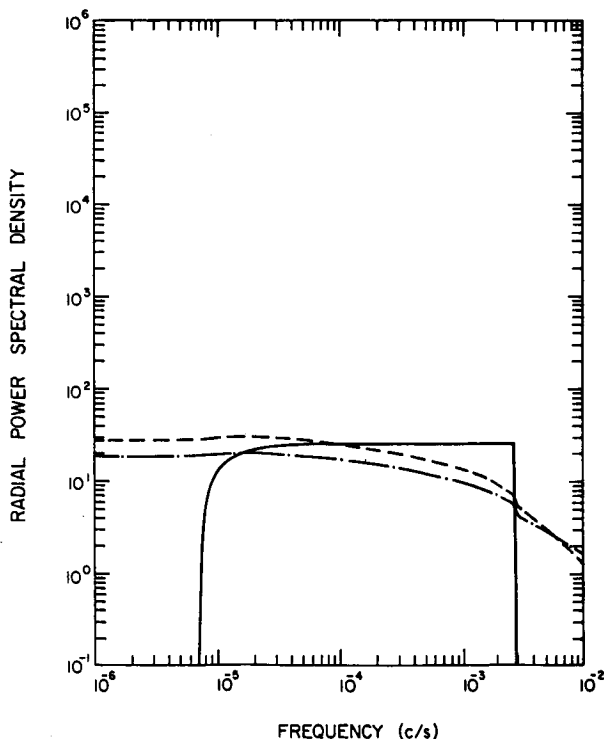


Figure 1. Model vertical frequency spectra ($\theta_B = 0$) for three different values of β , where $\beta = v_u k_z / \omega$. $\beta = 0$ is solid curve, $\beta = 5$ is dashed curve, $\beta = 10$ is dotted and dash-dot. Other parameters are $p=2$, $t=2.25$, $E = 31$ J/kg, $f_1 = 7.2 \times 10^{-6}$ Hz, and $f_b = 3.0 \times 10^{-4}$ Hz. Typical values of kz^* are of the order of $2\pi \times 10^{-4}$ to 10^{-1} .

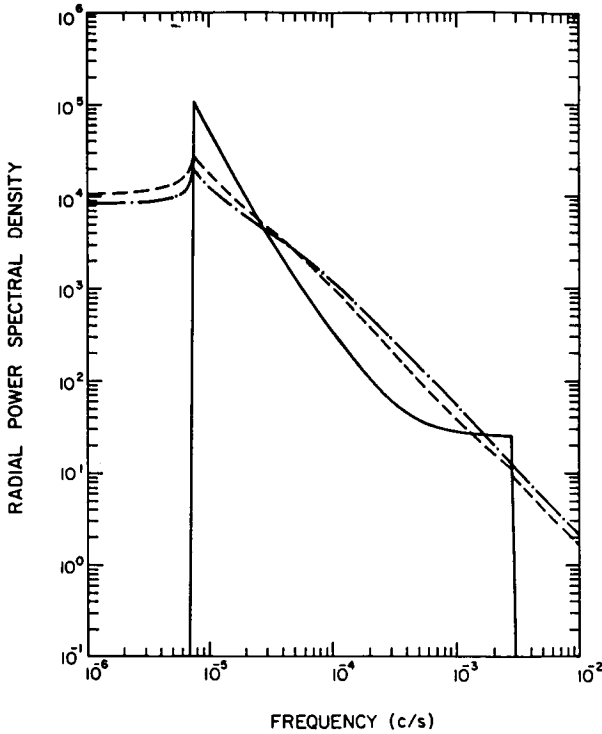


Figure 2. Same as Figure 1 except $\theta_B = 10^\circ$.

4. CONCLUSIONS

The results show that the radar-observed frequency spectrum of velocity fluctuations due to gravity waves will be affected by the Doppler shift due to the background wind. The numerical results are for the case of isotropic gravity-wave spectrum in a constant background wind. The technique could be applied to the more general cases.

REFERENCES

- Scheffler, A. O., and C. H. Liu (1985), On observation of gravity wave spectra in the atmosphere by using MST radar, *Radio Sci.*, 20, 1309-1322.
 VanZandt, T. E. (1985), A model for buoyancy wave spectra observed by Doppler sounding systems, *Radio Sci.*, 20, 1323-1330.