OPTIMIZATION OF CASCADE BLADE MISTUNING UNDER FLUTTER AND FORCED RESPONSE CONSTRAINTS

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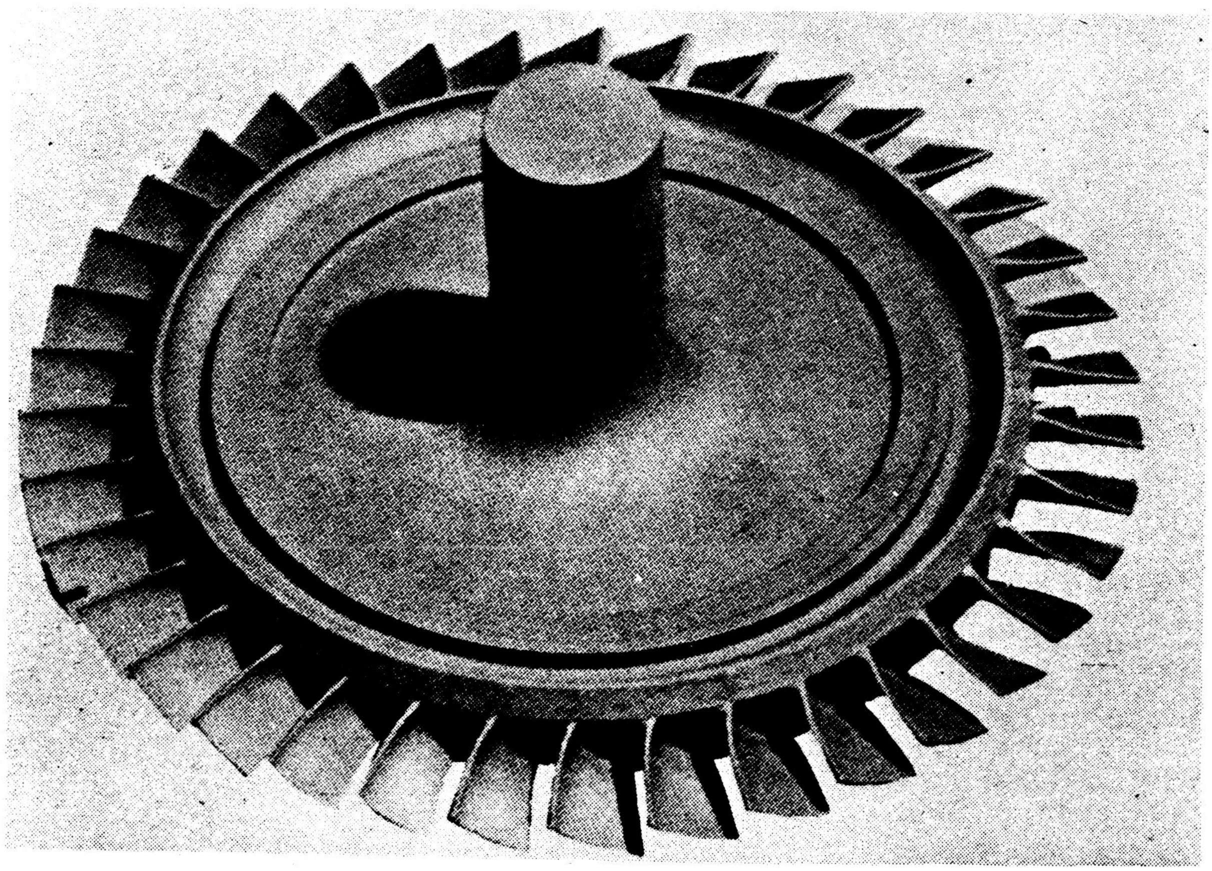
INTRODUCTION

In the development of modern turbomachinery, problems of flutter instabilities and excessive forced response of a cascade of blades that were encountered have often turned out to be extremely difficult to eliminate (refs. 1,2). The study of these instabilities and the forced response is complicated by the presence of mistuning; that is, small differences among the individual blades.

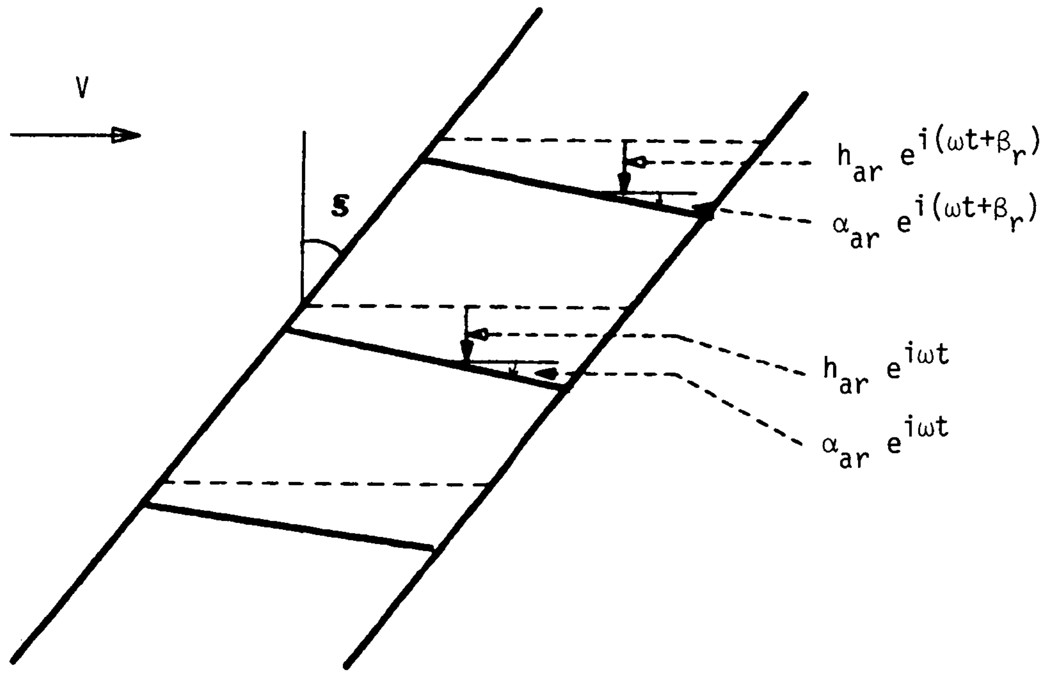
The theory of mistuned cascade behavior (refs. 3-8) shows that mistuning can have a beneficial effect on the stability of the rotor. This beneficial effect is produced by the coupling between the more stable and less stable flutter modes introduced by mistuning (ref. 9). The effect of mistuning on the forced response can be either beneficial or adverse. Kaza and Kielb (refs. 5-8) have studied the effects of two types of mistuning on the flutter and forced response: alternate mistuning where alternate blades are identical and random mistuning.

The objective of the present paper is to investigate other patterns of mistuning which maximize the beneficial effects on the flutter and forced response of the cascade. Numerical optimization techniques are employed to obtain optimal mistuning patterns. The optimization program seeks to minimize the amount of mistuning required to satisfy constraints on flutter speed and forced response.



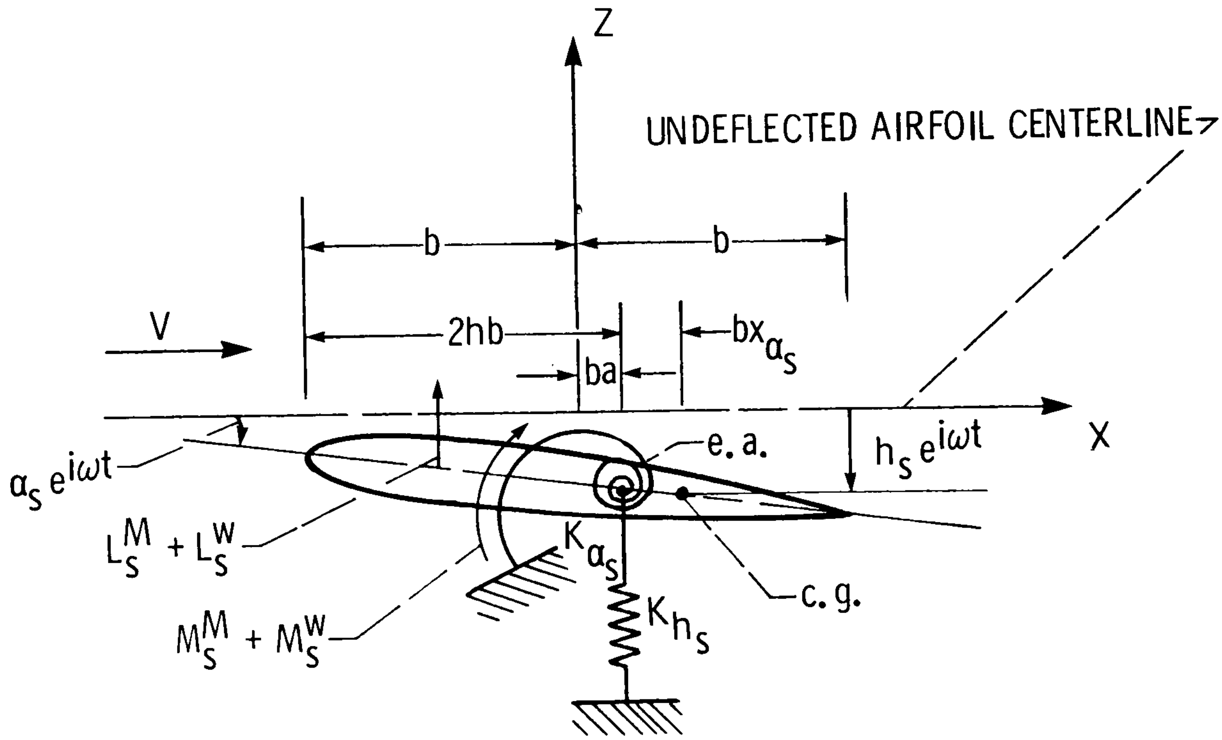
GEOMETRY OF A TUNED CASCADE

As shown in the figure, the blades are modeled as an infinite cascade of airfoils in a uniform upstream flow with a velocity V where ξ is the stagger angle. Only two degrees of freedom (bending and torsion) are considered for each blade. For the tuned cascade, the blades are assumed to be in harmonic motion with h_{ar} being the bending amplitude, α_{ar} the torsional amplitude and β_r the phase angle between adjacent blades. For an N-blade cascade, that phase angle can take only N discrete values $\beta_r = 2\pi r/N$.



The figure illustrates the structural model of the s-th blade. All the length quantities are non-dimensional with respect to the semichord, b. The blade bending and torsional stiffnesses are modeled through the springs K_{h_s} and K_{α_s} respectively. The effects of centrifugal stiffening due to rotation of the disk are included in the spring constants. The elastic coupling between bending and torsion due to pre-twist, shrouds and rotation is modeled through the offset distance (x_{α_s}) of the center of gravity from elastic axis which is located at a distance ba from mid-chord. The chordwise motion of the airfoil is neglected.

The aerodynamic loads on the blade are the lift and moment per unit span L_s^M and M_s^M due to motion and the lift and moment per unit span L_s^W and M_s^W due to wakes. These aerodynamic loads are calculated using Whitehead's extension of Theodorsen's isolated airfoil theory in the incompressible unsteady flow to account for cascade effects (ref. 10).



EQUATIONS OF MOTION

While the motion of a tuned cascade is simple harmonic with a fixed interblade phase angle, the motion of a mistuned cascade is assumed to be a linear combination of the tuned cascade modes. The equation of motion may be written as Eq. (1) where {Y} is a vector of complex amplitudes of the tuned cascade modes, [E] is the modal matrix containing all the possible inter-blade phase angle modes, {Q} is a forcing vector that depends on the aerodynamic wake forces and ω_o is a reference frequency.

When no external loads are applied the eigenvalues of the matrix [P] are calculated for a range of reduced frequencies and the flutter speed is found from the condition that the real part of the eigenvalue is zero. For the forced response calculation, the frequency and mode of the excitation has to be assumed. In the present work an entire range of frequencies is scanned for the most critical forced response. The mode of excitation has its only non-zero component in the (N-1)th harmonic.

$$\Big[[P] - [I]\,\gamma\Big]\{Y\} = -[E]^{-1}\{Q\} \qquad (1)$$

$$\gamma = \left(\frac{\omega_o}{\omega}\right)^2$$

Stability

$$\Big[[P] - [I]\,\gamma\Big]\{Y\} = \{0\} \qquad (2)$$

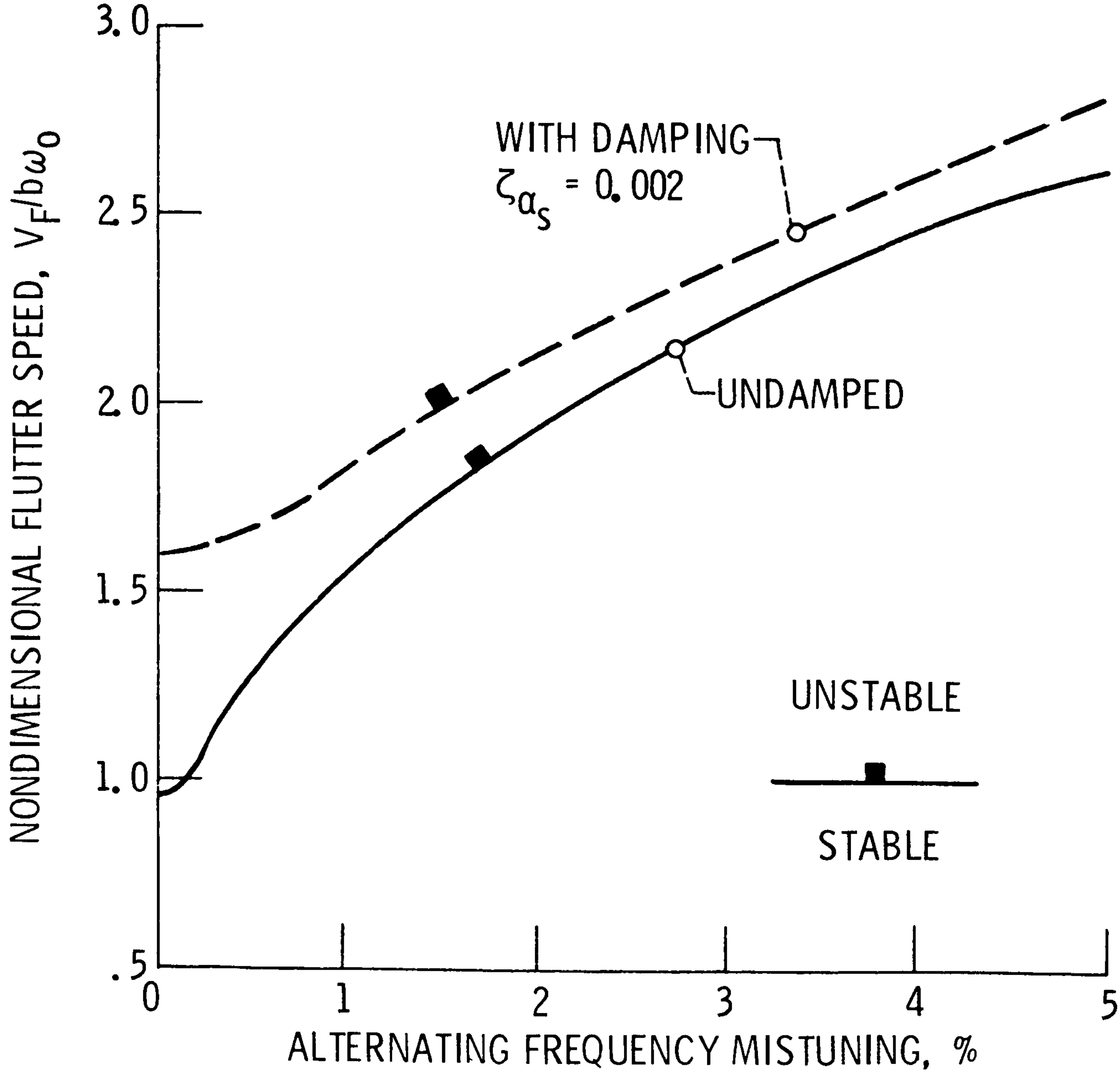
-2N x 2N Eigenvalue problem

$$\frac{i\omega_j}{\omega_o} = \frac{i}{\sqrt{\gamma_j}} = \mu_j \pm i\nu_j$$

Forced Response

$$\{X\} = -[E]\Big[[P] - \gamma_j\,[I]\Big]^{-1}\{Q\} \qquad (3)$$

The figure illustrates the effect of alternate blade mistuning on the flutter speed. The flutter speed increases monotonically with an increase in alternate blade mistuning level. Alternate blade mistuning can have either a beneficial or adverse effect on forced response, depending on the harmonic of excitation.



DESIGN FORMULATION

The objective of the present study is to minimize the amount of mistuning required to satisfy given constraints on the stability and forced response of a cascade. The design variables are the amounts of mistuning in the individual blades ε_i, and the objective function is the sum of the individual mistunings raised to some integer power p. A high value of p corresponds to minimizing the maximal blade mistuning while p=2 corresponds to minimizing the root mean square mistuning. The flutter constraint is based on the result of ref. 9 which showed that maximum stability is obtained when all eigenvalues have the same real part μ. Therefore, the flutter constraint is a limit on the amount of spread of the real part of the eigenvalues about their average value, μ_{av}, as well as a requirement that all real parts are stable.

Design Variables:

$$\varepsilon_i = \gamma_{\alpha i} - (\gamma_\alpha)_{av} \qquad i = 0, 1, \ldots, N-1$$

Objective Function:

$$F(\{\varepsilon\}) = \sum_{i=0}^{n-1} \varepsilon_i^p$$

Flutter Constraint:

Spread Constraint: $$\frac{\mu_{cr}}{\mu_{av}} - a_o \geq 0$$

Stability Constraint: $$\frac{\mu_{cr}}{\mu_o} \geq 0$$

Forced Response Constraint:

$$1 - \frac{r_i(\omega_j)}{r_{max}} \geq 0, \qquad \omega_j \in \Omega_c$$

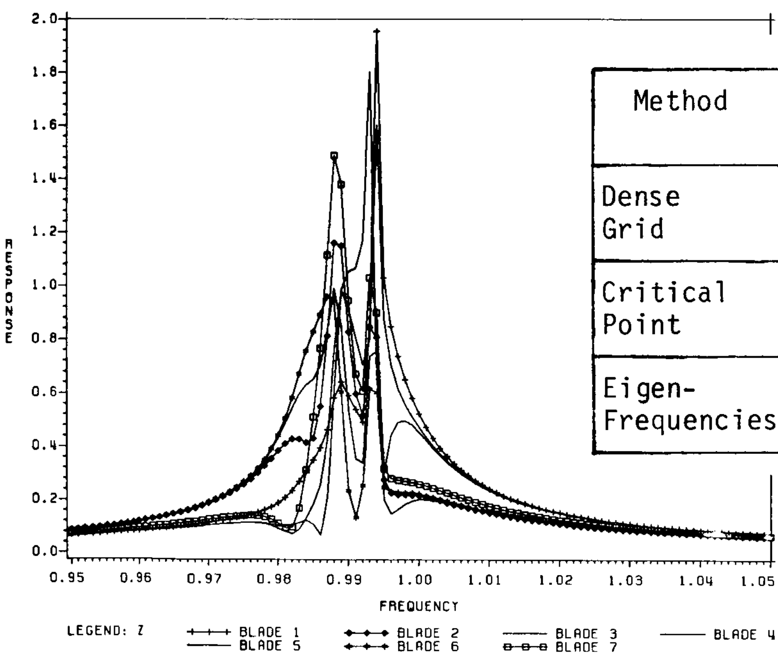
$$i = 0, 1, \ldots N-1$$

EFFICIENT FORCED RESPONSE CONSTRAINT

The constraint on the forced response (Eq. (4) in the figure) requires the calculation of the forced response for a range of frequencies. One way of checking whether any violation occurs in the required range is to evaluate the response at a grid of frequencies dense enough to preclude the possibility of substantial constraint violations between grid points. From the standpoint of the optimization procedure this is very costly because a constraint on the response has to be applied at each grid frequency. For the cases reported here a grid of 101 frequencies had to be used in the range $0.95 \leq \omega/\omega_o \leq 1.05$.

Two alternative techniques were used to reduce the cost of calculating the response constraints and their derivatives. The first is identifying local peaks of the response (as a function of ω) and enforcing the constraints only at these peaks. The main savings of this technique is in terms of derivative calculations. The second technique is based on the assumption that the response is most critical at the eigenfrequencies of the stability problem. This technique results in even larger savings. The results in terms of number of constraints and computer time for a full optimization are shown in the table for a seven-blade cascade.

RESPONSE AT 0.5 PERCENT ALTERNATE DESIGN (K=0.66)



Method	No. of Constraints	CPU Time IBM 3081
Dense Grid	707	30.8 min.
Critical Point	18-21	8.25 min.
Eigen-Frequencies	14	3.67 min.

Forced Response Constraint

$$1 - \frac{r_i(\omega_j)}{r_{max}} \geq 0$$

NEWSUMT OPTIMIZER

The optimization program used to obtain numerical results is the NEWSUMT program (ref. 11). It uses the sequence of Unconstrained Minimization Technique (SUMT) with an extended interior penalty function (ref. 12) to represent the constraints. Each unconstrained minimization is performed by using Newton's method with approximate derivatives (ref. 12). The optimization procedure is particularly efficient when the complexity of a problem is in the constraint and the objective function is fairly simple. For this reason the amount of mistuning is optimized subject to a constraint on the response, rather than optimizing the response subject to a constraint on the amount of mistuning.

Minimize $F(\vec{X})$

subject to $g_j(\vec{X}) \geq 0, \quad j = 1, 2, \ldots, m$

- SUMT - Sequential Unconstrained Minimization Technique
- Extended Interior Penalty Function
- Newton's Method with Approximate Second Derivatives

OPTIMIZATION SUBJECT TO FLUTTER CONSTRAINT

The first set of results were obtained for a twelve blade cascade subject to a flutter constraint of the form

$$\frac{\mu_{cr}}{\mu_{av}} - 0.584 \geq 0$$

where μ_{cr} is the real part of the least stable eigenvalue and μ_{av} the average real part. The results are summarized in the table. They show that the optimized mistuning pattern is about 78.52% better than the alternate mistuning design which satisfies the same constraint. The maximum individual blade mistuning is 0.91% versus 1.4% for the alternate mistuning, and the optimized pattern is still alternating in sign.

TABLE 1

Results of Optimization with Flutter Constraint (k=0.66)

	Alternate Mistuning pattern	Optimized Mistuning pattern
Objective Function	23.52×10^{-4}	5.053×10^{-4}
Max. mistuning ε_{max} (percent)	1.4000	0.9097
Least stable eigenvalue	-0.002525	-0.002526
Mistuning (percentage)		
ε_1	1.4000	0.7628
ε_2	-1.4000	-0.4768
ε_3	1.4000	0.9018
ε_4	-1.4000	-0.6401
ε_5	1.4000	0.9097
ε_6	-1.4000	-0.4683
ε_7	1.4000	0.6619
ε_8	-1.4000	-0.6708
ε_9	1.4000	0.2620
ε_{10}	-1.4000	-0.7796
ε_{11}	1.4000	0.1575
ε_{12}	-1.4000	-0.6201

EFFECT OF OBJECTIVE FUNCTION FORM

The use of the sum of the squares of the individual blade mistunings as the objective function is equivalent to minimizing the root mean square of the mistuning pattern. Another possible objective function is the maximum individual blade mistuning. This objective function has the disadvantage of having discontinuous derivatives with respect to the design variables, ε_i. To avoid this problem the maximum-individual-blade objective function can be approximated by the sum of a high power of the individual mistunings. To check whether the optimized design is sensitive to the objective function the optimization was repeated with the sum of the sixth powers of the ε_i being the objective function. The results are summarized in the table and show the effect to be minimal for this case.

TABLE 2

Effects of change in Objective Function (k=0.66)

	Optimum I (Exponent = 6)	Optimum II (Exponent = 2)
Objective Function	1.698×10^{-4}	5.053×10^{-4}
Max. mistuning ε_{max} (percent)	0.8993	0.9097
Least stable eigenvalue	-0.002525	-0.002526
Mistuning (percentage)		
ε_1	0.7639	0.7628
ε_2	-0.5662	-0.4768
ε_3	0.8927	0.9018
ε_4	-0.5910	-0.6401
ε_5	0.8993	0.9097
ε_6	-0.5352	-0.4683
ε_7	0.6839	0.6619
ε_8	-0.7201	-0.6708
ε_9	0.2476	0.2620
ε_{10}	-0.6621	-0.7796
ε_{11}	0.1709	0.1575
ε_{12}	-0.5837	-0.6201

OPTIMIZATION SUBJECT TO FORCED RESPONSE CONSTRAINT

The optimization was repeated with a forced response constraint. The constraint stipulated that the maximum response amplitude of the optimized design does not exceed the forced response of a 0.5% mistuning alternate-mistuning design. The two designs are compared in the table. It is shown that the objective function was reduced by 70% which corresponds to 45% reduction in the root mean square of the mistuning.

An attempt to obtain an optimal design subject to both flutter and forced response constraint indicated that the alternate mistuning design cannot be improved upon in both categories. That is, improvements in stability resulted in deterioration in forced response, and vice versa.

TABLE 3

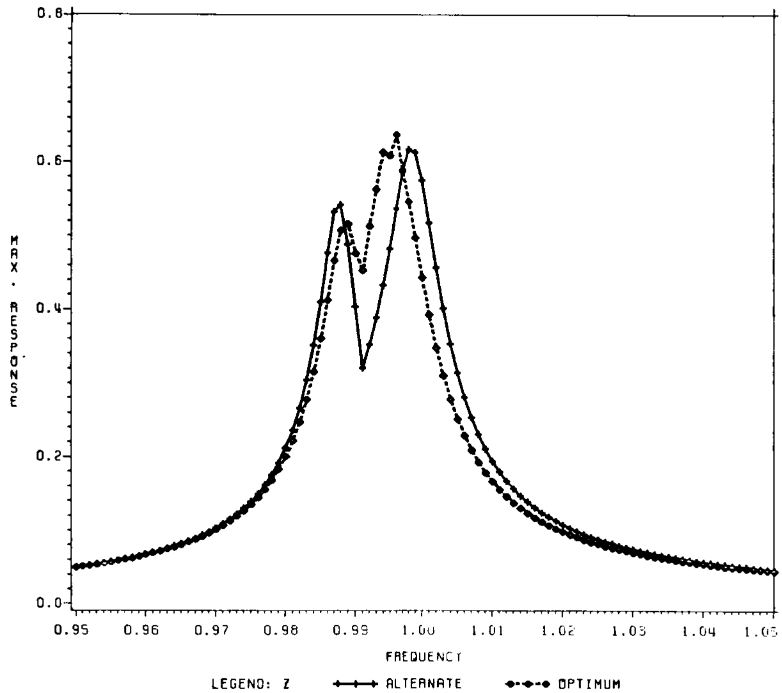
Results of Optimization with Forced Response Constraint
(k=0.8)

	Initial Design	Optimized Design
Objective Function	3.0×10^{-4}	9.144×10^{-5}
Max. mistuning ε_{max}(percent)	0.5	0.4804
Least stable eigenvalue	-0.00138	-0.00003
Mistuning (percentage)		
ε_1	0.5	0.2457
ε_2	-0.5	-0.4804
ε_3	0.5	0.2677
ε_4	-0.5	-0.3049
ε_5	0.5	0.2299
ε_6	-0.5	-0.2620
ε_7	0.5	0.2463
ε_8	-0.5	-0.2024
ε_9	0.5	0.2419
ε_{10}	-0.5	-0.1170
ε_{11}	0.5	0.2474
ε_{12}	-0.5	-0.1123

FORCED RESPONSE COMPARISON

The maximal blade response of the alternate and optimized designs is compared in the figure.

RESPONSE AT ALTERNATE AND OPTIMUM DESIGNS (K=0.80)



CONCLUDING REMARKS

An optimization procedure for finding optimal mistuning patterns for cascades subject to flutter and forced response constraints has been developed. The procedure is based on an extended interior penalty function algorithm and seeks to minimize the amount of mistuning required to satisfy the constraints. An efficient form of the forced response constraint which reduces computation costs by an order of magnitude has also been developed.

The optimization procedure has been applied to the design of a 12-blade cascade and the resulting designs compared to alternate mistuning designs. It was found that mistuning amplitudes could be substantially reduced without hurting either the flutter or the forced response characteristics. However, it was not possible for the example problem to improve on the alternate design subject to both constraints.

The designs obtained by the optimization procedure are not practical because they require many different blades. Work is under way to obtain similar results with only 3 or 4 different blade types.

- Optimization Procedure for Design Under Flutter & Forced Response Constraints Developed
- Efficient Forced Response Constraint Resulted in Large Computer Time Savings
- Optimized Designs Superior to Alternate Designs if only Flutter or only Forced Response is Critical
- Number of Different Blades Should be Reduced

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