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STRUCTURAL SENSITIVITY ANALYSIS: METHODS, APPLICATIONS, AND NEEDS

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INTRODUCTION

The field of sensitivity derivative analysis is emerging as one of the more fruitful areas of engineering research. The reason for this is the recognition of the many practical uses for sensitivity derivatives. Beyond the historical use of derivatives in connection with formal mathematical optimization techniques, recent work has been reported in using sensitivity derivatives in approximate analysis, assessing design trends, analytical model improvement, and determining effects of parameter uncertainties (refs. 1 through 7).

Work supported by the NASA Langley Research Center, under a grant in sensitivity analysis, has been focused on derivatives of thermal response of structures (refs. 8 and 9). Most recently, in-house implementations of generalized structural sensitivity capability in the SPAR and EAL computer programs (refs. 10 and 11) have been completed. Work in the sensitivity area is being expanded, and recent developments both in and outside the structures area have been surveyed to guide the future effort. This paper reviews some innovative techniques applicable to sensitivity analysis of discretized structural systems. These techniques include a finite-difference step-size selection algorithm, a method for derivatives of iterative solutions, a Green's function technique for derivatives of transient response, a simultaneous calculation of temperatures and their derivatives, derivatives with respect to shape, and derivatives of optimum designs with respect to problem parameters. Computerized implementations of sensitivity analysis and applications of sensitivity derivatives are also discussed. Finally, some of the critical needs in the structural sensitivity area are indicated along with Langley plans for dealing with some of these needs.

DISCIPLINES CONTRIBUTING TO SENSITIVITY ANALYSIS DEVELOPMENT

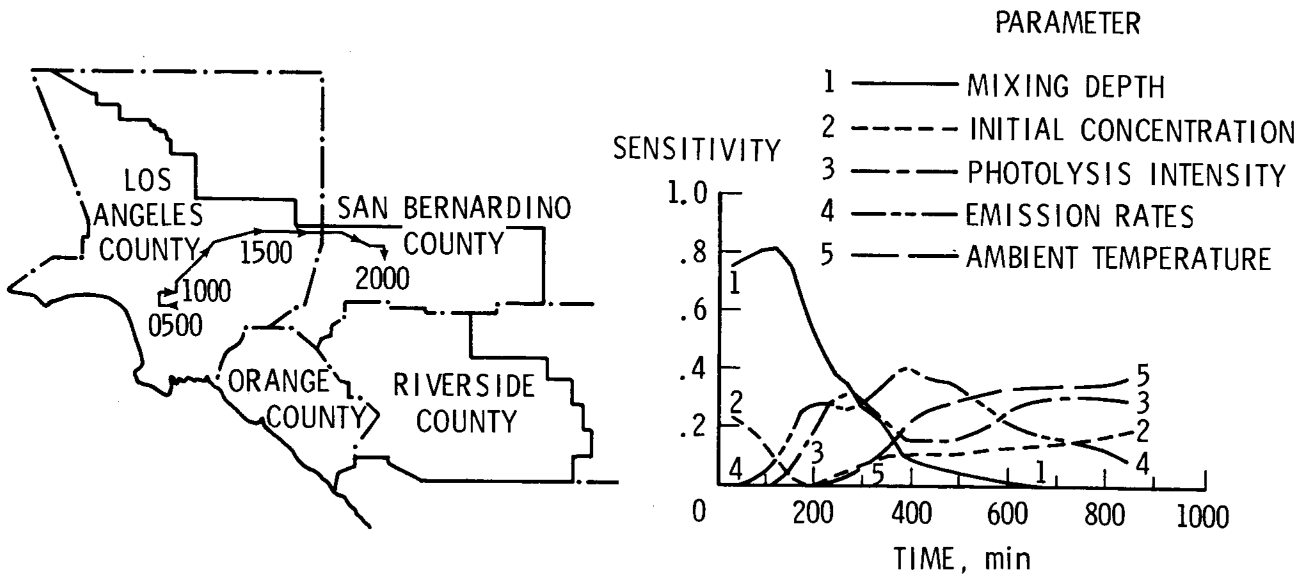
Sensitivity methodology has been and continues to be an important research area for many disciplines. Appreciation for the uses of sensitivity analysis by a broad spectrum of researchers outside the structures area is very evident. Some of those disciplines are indicated in figure 1. For the most part, the motivation in these other disciplines is the need to quantify the effect of uncertainties in parameters of a system model on the predictions of the model. Examples from physical chemistry are described in references 7 and 12 through 18. A specific use is given in figure 2. Electronics and control theory represented the origin of this type of sensitivity work (refs. 19 and 20) in addition to the use of derivatives to synthesize systems. Recent work in physiology with both human and bacteriological system models is described in reference 21. In the thermodynamics area, reference 22 describes the calculations and use of derivatives of the chemical composition with respect to thermodynamic properties in the mathematical modeling of a coal gasification process. Finally, analytical techniques are beginning to emerge to calculate derivatives of aerodynamic quantities with respect to flow parameters (refs. 23 and 24) as described by Bristow (ref. 25). This paper focuses on contributions to sensitivity methodology originating in or applicable to the structural analysis field.

- CHEMICAL KINETICS
- ELECTRONICS AND CONTROL
- PHYSIOLOGY
- THERMODYNAMICS
- AERODYNAMICS
- STRUCTURAL ANALYSIS

Figure 1

APPLICATION OF SENSITIVITY DERIVATIVES TO ATMOSPHERIC POLLUTION MODEL

Sensitivity analysis has been used to assess the effects of uncertainties in emission and meteorological parameters on the predictions from a mathematical model for photochemical air pollution (ref. 7). The atmospheric diffusion equation which governs the degree of pollution of a volume of air (an air parcel) contains several parameters: (1) mixing depth - the vertical height of the air parcel containing pollutants; (2) initial concentration of pollutant; (3) photolysis intensity - the rate of photochemical activity; (4) emission rate - the rate at which the pollutant is emitted into the air parcel; and (5) ambient temperature of the air. The calculation of derivatives of concentrations of various pollutants with respect to the aforementioned parameters is described in reference 7. The derivatives were used to rank the importance of the parameters. The calculations were carried out for the example of an "oxidant episode" which occurred in Southern California in 1974. The mathematical simulation of the event began in downtown Los Angeles at 5 a.m. and terminated in San Bernardino County at 8 p.m. The graph in figure 2 shows the sensitivity of the concentration of the pollutant nitric oxide (NO) with respect to each parameter as a function of time. Results indicate that early in the episode the initial concentration and mixing depth are the most important parameters. Midway through, emission rate and ambient temperature were most important, and late in the calculation, ambient temperature and photolysis intensity were most critical. These types of data indicated the need for more exact measurements of the key parameters to improve the air quality mathematical model.



EXAMPLE PROBLEM —
POLLUTION DUE TO OXIDANT EPISODE

SENSITIVITY OF NO CONCENTRATION

- STUDY RESULTS LED TO RESEARCH FOR MORE PRECISE MEASUREMENTS OF KEY PARAMETERS

(REF.7)

Figure 2

OPTIMUM STEP SIZE FOR FINITE-DIFFERENCE DERIVATIVES

The most straightforward method of calculating derivatives is to use a finite-difference approximation. One of the most serious shortcomings of the finite-difference method is the uncertainty in the choice of a perturbation step size. If the step size is too large, truncation errors may occur. These can be thought of as errors due to retention of only the lowest order terms of a Taylor series representation of a perturbed function. If the step size is too small, condition errors may occur (ref. 26). These errors are due to subtraction of nearly equal numbers. In a recent paper (ref. 27), an algorithm was developed to determine the optimum finite-difference step-size, i.e., one that balances the truncation and condition errors. The algorithm is based on approximating the truncation error as a linear function of step size h and the condition error as a linear function of 1/h. The optimum step size is obtained by equating the condition and truncation errors (fig. 3). This technique has been tested on functions which could be differentiated analytically (ref. 27) and was found to be very effective. A logical extension of this work would be to apply it to matrix equations.

- WANT BEST ESTIMATE OF $\frac{\partial f}{\partial v} \approx \frac{1}{h}\left(f(v+h) - f(v)\right)$

THE PROBLEM
- IF h TOO LARGE — TRUNCATION ERROR $\equiv T(h)$
- IF h TOO SMALL — CONDITION ERROR $\equiv C(h)$

THE SOLUTION
- EXPRESS $T(h)$ AND $C(h)$ AS SIMPLE COMPUTABLE FUNCTIONS
- CHOOSE "OPTIMUM" STEP SIZE $\hat{h}$ SO THAT

$$C(\hat{h}) = T(\hat{h})$$

- RESULT — FORMULA FOR $\hat{h}$
- FORMULA VERIFIED BY TESTS ON ANALYTICAL FUNCTIONS
- NEED TO IMPLEMENT FOR MATRIX EQUATIONS

(REF. 26)

Figure 3

DERIVATIVES OF ITERATIVE SOLUTIONS

In many structural design problems, the response U is the solution of an algebraic system $f(U, v) = 0$, where v is a design parameter (fig. 4). When the system is solved iteratively, the iterative process is terminated when the solution error is reduced below a certain tolerance. To obtain the derivative of U with respect to a design parameter by finite differences, the parameter is perturbed and the solution process is repeated to obtain U_h. The derivative is then approximated by a finite-difference ratio. The error inherent in this process is due to the termination of the iterative solution process before an exact solution is obtained. Thus $\bar{U}$ and $\bar{U}_h$, obtained by iteration, are only approximations to the corresponding exact solutions U and U_h, respectively. Because of noise in the solution process, the difference between $\bar{U}$ and $\bar{U}_h$ can be finite, even for very small values of the perturbation h. In fact, the error is most severe when small values of h are required to avoid large truncation errors in the derivative. A remedy, which is being developed by the second author of this paper, is to define a modified perturbed solution U_h^* which satisfies a modified equation whose right-hand side is not zero but is the residual of the approximate unperturbed equation. By this construction, U_h^* approaches $\bar{U}$ as h approaches zero. Then U_h^* replaces $\bar{U}_h$ in the derivative formula. Finally, $\bar{U}$ serves as the first approximation in the iteration process for U_h^*.

- $\bar{U}$ IS SOLUTION TO $f(U,v) = 0$
- $\bar{U}_h$ IS SOLUTION TO $f\left(U_h, v+h\right) = 0$

$\Big\}$ APPROXIMATE $\bar{U}$ AND $\bar{U}_h$ OBTAINED BY ITERATION

$$\partial U/\partial v = 1/h\left(\bar{U}_h - \bar{U}\right)$$

- ERROR MAY BE LARGE DESPITE SMALL h
- SOLUTION — DEFINE U_h^* SUCH THAT

$$f\left(U_h^*, v+h\right) = \underbrace{f(\bar{U}, v)}_{\text{RESIDUAL}} \qquad \text{THEN } \partial U/\partial v = 1/h\left(U_h^* - \bar{U}\right)$$

- WHY IT WORKS — INITIAL GUESS FOR U_h^* IS $\bar{U}$

  U_h^* APPROACHES $\bar{U}$ FOR SMALL h

Figure 4

GREEN'S FUNCTION METHOD FOR DERIVATIVES OF TRANSIENT RESPONSE

This method, which is well known in applications to solutions of nonhomogeneous differential equations, has been used extensively by physical chemistry researchers (refs. 12 through 18) for calculation of derivatives of response quantities governed by systems of first-order nonlinear ordinary differential equations such as equation (1) shown in figure 5. Numerous applications have been performed for chemical kinetics problems related to air pollution studies. As indicated in this figure, the derivative of the response vector Y with respect to a parameter α satisfies equation (2). The derivative may be represented by an integral expression (eq. 3) involving a kernel K which is the Green's function. The Green's function is the solution to the initial value problem given by equation (4). Comparison of the effort needed to solve equation (2) versus (4) indicates that the Green's function technique is advantageous if the number of design variables m exceeds the number of equations n in the system. One approach to obtaining K is to solve equation (4) directly using an implicit numerical integration technique (ref. 12). An alternate solution for the Green's function is to use the Magnus method (ref. 17) whereby K is expressed as an exponential function of a matrix which is the time integral of the Jacobian matrix J. Because the equation of transient heat transfer is a special case of equation (1), the Green's function method is directly applicable to sensitivity of transient temperatures. It is planned to pursue this line of research at Langley as part of our sensitivity development.

- IMPLEMENTED BY PHYSICAL CHEMISTRY RESEARCHERS

- GENERAL PROBLEM

$$\frac{dY}{dt} = f(Y,\alpha,t) \qquad n \text{ EQUATIONS} \tag{1}$$

$$\frac{dY_\alpha}{dt} = JY_\alpha + f_\alpha \qquad mn \text{ EQUATIONS} \tag{2}$$

$$J \equiv \partial f/\partial Y \qquad Y_\alpha \equiv \partial Y/\partial\alpha$$

- SOLUTION BY GREEN'S FUNCTION

$$Y_\alpha = \int_0^t K(t,\tau) f_\alpha \, d\tau \tag{3}$$

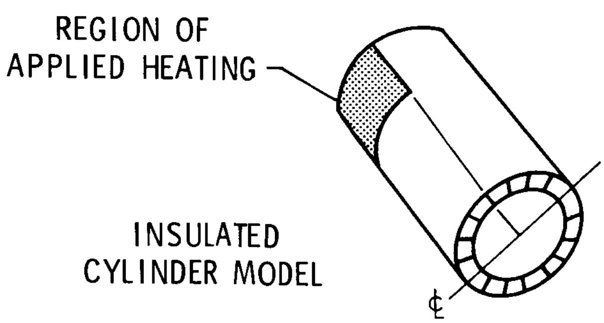
$$\frac{dK}{dt}(t,\tau) - JK(t,\tau) = 0 \qquad n^2 \text{ EQUATIONS} \tag{4}$$

- APPLICABLE TO TRANSIENT HEAT TRANSFER

Figure 5

Derivatives of transient response such as structural displacements and temperatures have been computed following the calculation of the response itself using analytical techniques (refs. 9, 28, 29) and by the Green's function technique (ref. 12). Recently an algorithm for concurrent calculation of transient temperatures and their finite-difference derivatives has been developed (ref. 9). Figure 6 depicts an application to the transient thermal response of an insulated cylindrical shell. Temperatures throughout the shell are computed using an implicit numerical integration technique. Along with temperatures at each time step, finite-difference sensitivity derivatives are calculated with respect to design variables representing insulation thicknesses at 10 locations on the shell surface. The key to the success of the method is that during each time step, when a nonlinear algebraic equation is solved by iteration for the current temperature and the perturbed temperature, the same time step is used for both solutions. Further, the unperturbed temperature serves as the initial guess in the iteration for the perturbed temperature. The numbers in the table are solution times in seconds. The results indicate that the timesaving from the concurrent calculation is substantial, and nearly a factor of 4 advantage is obtained.

ERROR TOLERANCE ON TEMPERATURE	SEQUENTIAL		CONCURRENT	
	CPU TIME FOR TEMPERATURE CALCULATION	CPU TIME FOR EACH DERIVATIVE	CPU TIME FOR TEMPERATURE PLUS TEN DERIVATIVES	CPU TIME FOR EACH DERIVATIVE
.001	113	113	500	39
.003	78	78	296	22
.010	46	46	168	12



TYPICAL DERIVATIVE

$$\frac{\partial T}{\partial t_{ins}}$$

T = TEMPERATURE

t_{ins} = INSULATION THICKNESS

Figure 6

C-5

SENSITIVITY DERIVATIVES FOR SHAPE DESIGN VARIABLES

A relatively new topic in structural sensitivity analysis is the calculation of derivatives with respect to shape design variables. Examples are derivatives of displacements or stresses with respect to a beam length or a membrane area (fig. 7). Two approaches have been used. One approach is to differentiate the discretized equations resulting from a finite-element representation. A drawback to this technique is that when shape design variables change, the finite-element mesh is modified. The resulting mesh distortion changes the discretization error and leads to inaccurate derivatives. The second approach, which avoids mesh distortion errors, is to reverse the order of differentiation and discretization (refs. 30 through 32). The procedure is to differentiate the continuum equations of equilibrium and discretize the resulting integral equations. This method uses the concept of a material derivative from continuum mechanics which is composed of two parts: a derivative corresponding to a fixed shape, and a contribution from the change of the boundary. The preferred choice between the two methods is not yet clear. The second approach avoids mesh distortion by its formulation but does not permit shape differentiation of a discretized set of equations. The first method, although suffering from mesh distortion errors, could benefit from a built-in adaptive mesh generation capability which would reduce the mesh distortion.

- EXAMPLES — DERIVATIVE OF

• DISPLACEMENTS	WITH RESPECT TO	LENGTH OF BEAM
• STRESSES		AREA OF MEMBRANE

- FIRST METHOD — DISCRETIZE FIRST THEN DIFFERENTIATE
    - NUMERICAL ERRORS DUE TO MESH DISTORTION
    - REDUCE ERRORS BY ADAPTIVE MESH GENERATION

- SECOND METHOD — BASED ON MATERIAL DERIVATIVE
    - DIFFERENTIATE CONTINUUM EQUATIONS THEN DISCRETIZE
    - AVOIDS MESH DISTORTION ERRORS

Figure 7

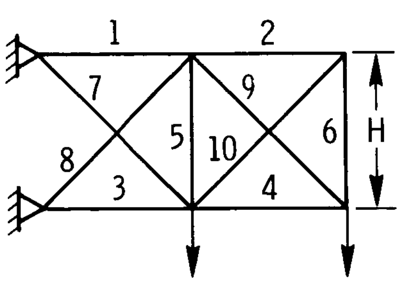
The problem addressed by this technique (refs. 33, 34) is to obtain derivatives of an objective function F and design variables V from an optimized solution with respect to parameters P which were held constant during the optimization. The most obvious and thus most useful application of the technique is extrapolation of an optimum design for variations of a problem parameter. For example, the effect of varying the height H of the truss in figure 8 is assessed by using optimum sensitivity derivatives. Extrapolated values of the mass F and one of the cross-sectional areas A_1, based on derivative with respect to H, are compared with those obtained by reoptimization with different values of H. As shown in the lower right portion of figure 8, the results agree very closely for up to a 20-percent change in H. Other applications of these types of derivatives include optimization for multiple-objective functions, assessing the effects of adding or deleting constraints, and most recently using the derivatives as links between subsystems during multilevel optimization (ref. 35).

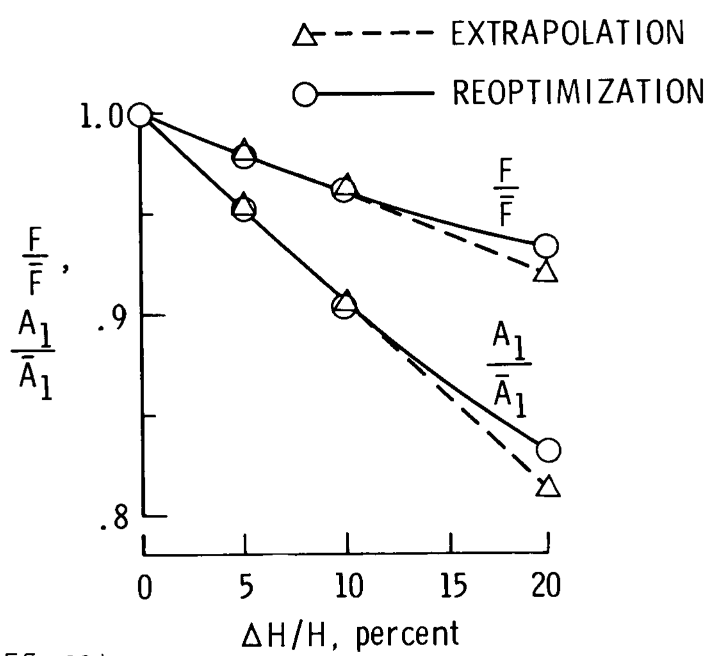
GIVEN AN OPTIMUM DESIGN:

F = OBJECTIVE FUNCTION   V = DESIGN VARIABLES

P = PROBLEM PARAMETERS

ANALYSIS GIVES $\frac{\partial F}{\partial P}$ AND $\frac{\partial V}{\partial P}$





● EXAMPLE — EFFECT OF TRUSS HEIGHT ON OPTIMUM DESIGN

(REF. 33)

Figure 8

COMPUTER IMPLEMENTATION

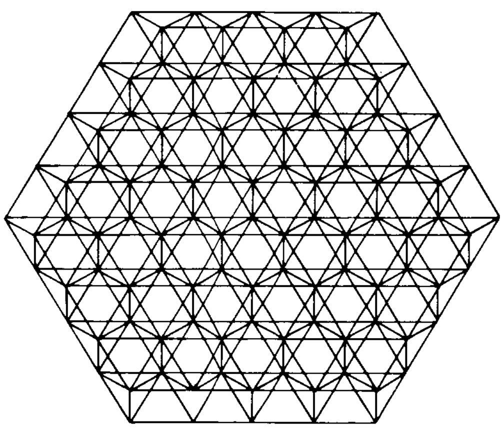
Some progress has occurred in providing general-purpose software for sensitivity analysis (fig. 9). The Green's function technique described earlier has been implemented in a computer program denoted AIM (ref. 18). Use of this program requires supplying subroutines to define the system of equations - specifically, the vector f and the matrix J in figure 5. The capability for computing derivatives of static displacements, stresses, and vibration and buckling eigenvalues and eigenvectors has been implemented in the SPAR finite-element program (ref. 10) and EAL (refs. 11, 36). The EAL (Engineering Analysis Language) system contains the SPAR finite-element modules, but additionally EAL provides FORTRAN-like commands which permit branching, testing data, looping, and calling the SPAR modules (similar to calling FORTRAN subroutines). A recent level of a proprietary version of the NASTRAN computer program also has capability for static displacement, stress, and eigenvalue derivatives (ref. 37).

- AIM (GREEN'S FUNCTION TECHNIQUE)
  - GENERAL FIRST-ORDER EQUATIONS
  - DERIVATIVES WRT PARAMETERS IN EQUATIONS
- SPAR (COSMIC) AND EAL (PROPRIETARY)
  - DERIVATIVES OF —
    - DISPLACEMENTS
    - STRESSES
    - EIGENVALUES
    - EIGENVECTORS
- NASTRAN (PROPRIETARY)
  - DERIVATIVES OF —
    - DISPLACEMENTS
    - STRESSES
    - EIGENVALUES

Figure 9

APPLICATION OF STRUCTURAL SENSITIVITY ANALYSIS TO SPACE ANTENNA

An application of sensitivity analysis to reveal structural design trends is illustrated in figure 10. The structure is an Earth-orbiting antenna reflector subjected to nonuniform heating leading to thermal distortions which can degrade antenna performance (ref. 11). The structure is modeled using only rod elements. There are three design variables representing, respectively, the cross-sectional areas of the elements in the upper surface (A_1), the elements joining the upper and lower surfaces (A_2), and the elements in the lower surface (A_3). Derivatives of the center deflection with respect to each design variable were calculated and are shown in the figure. A positive derivative indicates that increasing the design variable increases the response. A negative derivative indicates that increasing the design variable decreases the response. The seemingly contradictory result that increasing a design variable can increase a response stems from the fact that the thermal loads are proportional to the rod cross-sectional areas. From the table in figure 10, we see that increasing A_1 has the largest effect on reducing deflection but at the cost of a weight increase. On the other hand, decreasing either A_2 or A_3 would reduce the deflection and at the same time reduce weight. It is at the discretion of the designer as to which of the alternatives is a better choice. The sensitivity derivatives provide the data for that judgment.



ANTENNA MODEL

w_0 = CENTER DEFLECTION

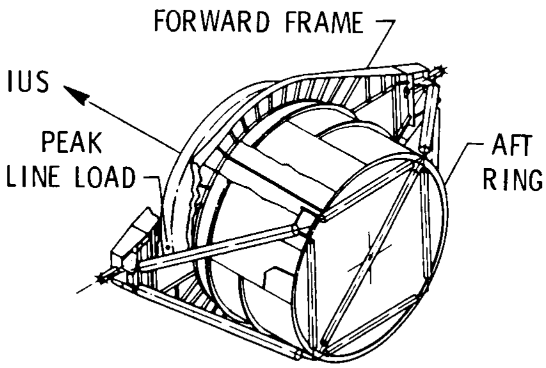
DESIGN VARIABLE — ELEMENT AREA, A_i	$\partial w_0/\partial A_i$
UPPER SURFACE	-2.4×10^{-4}
DIAGONALS	8.3×10^{-5}
LOWER SURFACE	1.8×10^{-4}

DERIVATIVES OF CENTER DEFLECTION

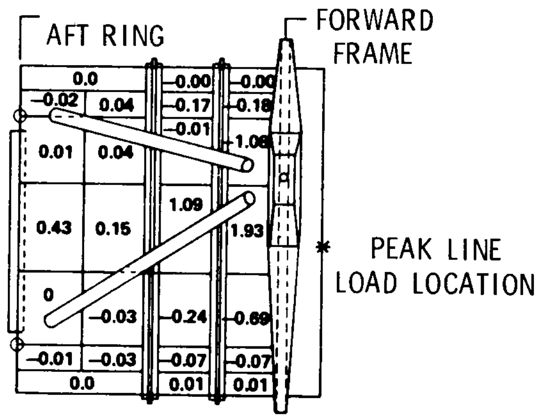
Figure 10

APPLICATION OF SENSITIVITY ANALYSIS TO SHUTTLE PAYLOAD

Sensitivity analysis has been used to redesign the Airborne Support Equipment (ASE) assembly on the Space Shuttle orbiter (ref. 5). The ASE supports the inertial upper stage (IUS) vehicle in the payload bay. The purpose of the IUS vehicle is to transport payloads further into space once the orbiter has established low-Earth orbit. The shell of the ASE is subjected to large launch loads and is designed for an ultimate load of 3000 lb/in. An initial sculptured skin design met the design load but was too heavy. A sensitivity analysis was performed to determine which skin gages had the largest effect on loads and to determine which type of modified construction would give the largest weight reduction (among isogrid, waffle, and stiffened skin). The structure was modeled and divided into zones as shown on the right side of figure 11. Derivatives of compressive loads and weight with respect to longitudinal, circumferential, and shear stiffening were computed for each zone. As an example, consider derivatives with respect to the longitudinal stiffness design variable t_a. The numbers in the zones are ratios of derivatives of load to derivatives of weight for the sculptured skin design. Negative values indicate that increasing a design variable decreases the load, and positive values indicate that increasing a design variable increases the load. The analysis revealed that derivatives with respect to longitudinal and shear stiffness were the largest, derivatives with respect to circumferential stiffness were negligible, and derivatives with respect to shear stiffness were nearly all positive. Based on these results, the shell was redesigned as a longitudinally stiffened machined skin. (The isogrid was rejected because of high shear stiffness; the waffle construction was rejected due to unneeded high circumferential stiffness.) The resulting design satisfies the ultimate load constraint with a large margin of safety and an acceptably low weight.



BASELINE ASE CONFIGURATION



DISTRIBUTION OF DERIVATIVE RATIOS

$$\frac{\partial N/\partial t_a}{\partial W/\partial t_a}$$

t_a = LONGITUDINAL STIFFNESS

- STUDY RESULTS
  - LARGE DERIVATIVES WRT LONGITUDINAL AND SHEAR STIFFNESS
  - SMALL DERIVATIVES WRT CIRCUMFERENTIAL STIFFNESS
- OUTCOME — REDESIGNED SHELL AS LONGITUDINALLY STIFFENED MACHINED PANEL SATISFIED DESIGN REQUIREMENTS WITH LOW WEIGHT

(REF. 5)

Figure 11

STRUCTURAL SENSITIVITY ANALYSIS NEEDS

As a result of surveying methods applicable to computing structural sensitivity derivatives, a list of needs has emerged (fig. 12). First, continued development of methods for derivatives of transient response and derivatives with respect to shape design variables and material properties should have high priority. Further, techniques developed for sensitivity derivatives in nonstructural disciplines such as physical chemistry have much to offer and should be evaluated for their adaptability to structural areas. It appears that structural designers have made insufficient use of the power and utility of sensitivity derivatives to guide design modifications and to assess uncertainties in their models. Their use can be accelerated by demonstrations of practical applications of sensitivity analysis and careful documentation (by optimization and sensitivity specialists) to guide structural analysts and designers not experienced in formal optimization and sensitivity analysis. Finally, sensitivity analysis needs to be routinely included as a standard feature in general-purpose structural analysis software packages. Near-term plans at Langley include evaluation of the Green's function method for derivatives of transient thermal response, methods for derivatives of spacecraft thermal response with respect to material properties, and implementation of the optimum finite-difference step-size technique for finite-element sensitivity analysis. Concurrent with this effort, demonstration problems will be selected and solved.

- TRANSIENT RESPONSE
- MATERIAL PROPERTIES AND SHAPE
- PRACTICAL APPLICATIONS
- ROUTINE INCLUSION IN GENERAL-PURPOSE COMPUTER PROGRAMS

LANGLEY PLANS

- EVALUATE GREEN'S FUNCTION METHOD FOR TRANSIENT TEMPERATURES
- DERIVATIVES WRT MATERIAL PROPERTIES
- IMPLEMENT FINITE-DIFFERENCE STEP-SIZE ALGORITHM
- SENSITIVITY DEMONSTRATION PROBLEMS

Figure 12

SUMMARY

This paper was based on a recently conducted survey of methods for sensitivity analysis of structural response (fig. 13). The survey was not limited to research in the structural area alone and revealed that a broad range of disciplines are using sensitivity analysis and contributing to the methodology. In almost every instance, methods from the nonstructural disciplines are directly applicable to the structures area. An example application from chemical kinetics was described in which sensitivity analysis was used to assess the impact of parameter uncertainties in a mathematical model used in air pollution studies.

The bulk of the paper has focused on a selected set of innovative methods applicable to sensitivity analysis of structural systems. The analysis techniques include a finite-difference step-size selection algorithm, a method for derivatives of iterative solutions, a Green's function technique for derivatives of transient response, concurrent calculation of temperatures and their derivatives, derivatives with respect to shape, and derivatives of optimum designs with respect to problem parameters. Two applications were described wherein derivatives were used to guide structural design changes to improve an engineering design without recourse to formal mathematical optimization. Plans at Langley for contributing to identified critical needs were cited. Among the needs were implementation of methods for derivatives of transient response, derivatives with respect to shape and material properties, solution and documentation of sensitivity analysis demonstration problems, and routine inclusion of sensitivity analysis as a feature in general-purpose structural analysis computer programs. Langley near-term plans in the sensitivity area include evaluating the Green's function method for derivatives of transient thermal response, developing methods for derivatives with respect to material properties, implementation of the finite-difference step-size algorithm, and solution of sensitivity demonstration problems.

- LANGLEY CONDUCTING SURVEY OF METHODS FOR SENSITIVITY DERIVATIVES
- PAPER REVIEWED RECENTLY DEVELOPED TECHNIQUES
  - OPTIMUM FINITE-DIFFERENCE STEP-SIZE ALGORITHM
  - METHOD FOR DERIVATIVES OF ITERATIVE SOLUTIONS
  - GREEN'S FUNCTION METHOD FOR TRANSIENT RESPONSE
  - SIMULTANEOUS CALCULATION OF TEMPERATURES AND DERIVATIVES
  - DERIVATIVES WITH RESPECT TO SHAPE
  - DERIVATIVES OF OPTIMUM DESIGNS WITH RESPECT TO PROBLEM PARAMETERS
- REVIEWED USES OF DERIVATIVES AND CITED APPLICATIONS
- CITED NEEDS AND OUTLINED LANGLEY PLANS

Figure 13

REFERENCES

1. Schmit, L. A., Jr.; and Farshi, B.: Some Approximation Concepts for Structural Synthesis. AIAA J., vol. 12, no. 5, May 1974, pp. 692-699.

2. Storaasli, O.; and Sobieszczanski, J.: On the Accuracy of the Taylor Approximation for Structure Resizing. AIAA J., vol. 12, no. 2, Feb. 1974, pp. 231-233.

3. Noor, K.; and Lowder, E.: Structural Reanalysis Via a Mixed Method. Comput. and Struct., vol. 5, no. 1, Apr. 1975, pp. 9-12.

4. Robinson, J. C.: Application of a Systematic Finite-Element Model Modification Technique to Dynamic Analysis of Structures. NASA TM-83292, 1983.

5. Musgrove, M. D.; Reed, J. M.; and Hauser, C. C.: Optimization Using Sensitivity Analysis. J. Spacecraft & Rockets, vol. 20, no. 1, Jan.-Feb. 1983, pp. 3-4.

6. Cukier, R. I.; Fortuin, C. M.; Shuler, K. E.; Petschek, A. G.; and Schaibly, J. H.: Study of the Sensitivity of Coupled Reaction Systems to Uncertainties in Rate Coefficients. I. Theory, J. Chem. Phys., vol. 59, 1973, pp. 3873-3878.

7. Tilden, J. W.; and Seinfeld, J. H.: Sensitivity Analysis of a Mathematical Model for Photochemical Air Pollution. Atmospheric Environment, vol. 16, no. 6, 1982, pp. 1357-1364.

8. Haftka, R. T.: Techniques for Thermal Sensitivity Analysis. Int. J., Numerical Meth. in Engineering, vol. 17, 1981, pp. 71-80.

9. Haftka, R. T.; and Malkus, D. S.: Calculation of Sensitivity Derivatives in Thermal Problems by Finite Differences. Int. J. Numerical Meth. in Engineering, vol. 17, 1981, pp. 1811-1821.

10. Giles, G. L.; and Rogers, J. L., Jr.: Implementation of Structural Response Sensitivity Calculations in a Large-Scale Finite-Element Analysis System. Proc. 23rd Structures, Structural Dynamics and Materials Conference, May 10-12, 1982, New Orleans, LA., pp. 348-359.

11. Camarda, C. J.; and Adelman, H. M.: Implementation of Static and Dynamic Structural Sensitivity Derivative Calculations in the Finite-Element-Based Engineering Analysis Language System (EAL). NASA TM-85743, 1984.

12. Hwang, J. T.; Dougherty, E. P.; Rabitz, S.; and Rabitz, H.: The Green's Function Method of Sensitivity Analysis in Chemical Kinetics. J. Chem. Phys., vol. 69, Dec. 1978, pp. 5180-5191.

13. Dougherty, E. P.; and Rabitz, H.: Computational Kinetics and Sensitivity Analysis of Hydrogen-Oxygen Combustion. J. Chem. Phys., vol. 72, no. 12, June 15, 1980, pp. 6571-6586.

14. Eslava, L. A.; Eno, L.; and Rabitz, H.: Further Developments and Applications of Sensitivity Analysis to Collisional Energy Transfer. J. Chem. Phys., vol. 73, no. 10, Nov. 1980, pp. 4998-5012.

15. Demiralp, M.; and Rabitz, H.: Chemical Kinetic Functional Sensitivity Analysis: Derived Sensitivities and General Applications. J. Chem. Phys., vol. 75, no. 4, Aug. 1981, pp. 1810-1819.

16. Edelson, D.: Computer Simulation in Chemical Kinetics. Science, vol. 214, no. 27, Nov. 1981, pp. 981-986.

17. Kramer, M. A.; and Calo, J. M.: An Improved Computational Method for Sensitivity Analysis: Green's Function Method with "AIM". Appl. Math. Modeling, vol. 5, Dec. 1981, pp. 432-441.

18. Kramer, M. A.; Calo, J. M.; Rabitz, H.; and Kee, R. J.: AIM: The Analytically Integrated Magnus Method for Linear and Second-Order Sensitivity Coefficients. Sandia National Laboratories Report SAND-82-8231, Aug. 1982.

19. Goddard, P. J.; Vilalaz, P. A.; and Spence, R.: Method for the Efficient Computation of the Large Change Sensitivity of Linear Nonreciprocal Networks. Electronics Letters, vol. 7, Feb. 1971, pp. 112-113..

20. Radanovic, L., ed.: Sensitivity Methods in Control Theory. Pergamon Press Ltd., 1966.

21. Leonard, J. I.: The Application of Sensitivity Analysis to Models of Large Scale Physiological Systems. NASA CR-160228, Oct. 1974.

22. Irwin, C. L.; and O'Brien, T. J.: Sensitivity Analysis of Thermodynamic Calculations. U.S. Dept. of Energy Report DOE/METC/82-53, 1982.

23. Dwyer, H. A.; Peterson, T.; and Brewer, J.: Sensitivity Analysis Applied to Boundary Layer Flow. Proc. 5th International Conference on Numerical Methods in Fluid Dynamics, A. I. van de Vooren and P. J. Zandbergen, eds., Springer-Verlag (Berlin), 1976, pp. 179-184.

24. Bristow, D. R.; and Hawk, J. D.: Subsonic Panel Method for Designing Wing Surfaces from Pressure Distributions. NASA CR-3713, July 1983.

25. Bristow, D. R.: Sensitivity Analysis in Computational Aerodynamics. Recent Experiences in Multidisciplinary Analysis and Optimization, NASA CP-2327, Part 1, 1984, pp. 385-394.

26. Gill, P. E.; Murray, W.; Saunders, M. A.; and Wright, M. H.: Computing the Finite-Difference Approximations to Derivatives for Numerical Optimization. U.S. Army Research Office, Report No. DAAG29-79-C-0110, May 1980.

27. Gill, P. E.; Murray, W.; Saunders, M. A.; and Wright, M. H.: Computing Forward Difference Intervals for Numerical Optimization. SIAM J. Sci. Stat. Comput., vol. 4, June 1983, pp. 310-321.

28. Haug, E. J.; Arora, J. S.; and Feng, T. T.: Sensitivity Analysis and Optimization of Structures for Dynamic Response. J. Mech. Des., vol. 100, Apr. 1978, pp. 311-318.

29. Haug, E. J.; and Feng, T. T.: Optimal Design of Dynamically Loaded Continuous Structures. Int. J. Numerical Meth. in Engineering, vol. 12, 1978, pp. 299-317.

30. Rousselet, B.; and Haug, E. J.: Design Sensitivity Analysis in Structural Mechanics. III. Effects of Shape Variation. J. Struct. Mech., vol. 10, no. 3, 1983, pp. 273-310.

31. Dems, K.; and Mroz, Z.: Variational Approach By Means of Adjoint Systems to Structural Optimization and Sensitivity Analysis - II: Structure Shape Variation. Int. J. Solids and Structures, vol. 20, no. 6, 1984, pp. 527-552.

32. Choi, K. K.; and Haug, E. J.: Shape Design Sensitivity Analysis of Elastic Structures. J. Struct. Mech., vol. 11, 1983, pp. 231-269.

33. Sobieszczanski-Sobieski, J.; Barthelemy, J. F.; and Riley, K. M.: Sensitivity of Optimum Solutions to Problem Parameters. AIAA J., vol. 20, Sept. 1982, pp. 1291-1299.

34. Barthelemy, J. F. M.; and Sobieszczanski-Sobieski, J.: Optimum Sensitivity Derivatives of Objective Functions in Nonlinear Programing. AIAA J., vol. 21, no. 6, pp. 913-915.

35. Sobieszczanski-Sobieski, J.: Structural Optimization: Challenges and Opportunities, NASA TM-85741, 1984.

36. Prasad, B.; and Emerson, J. F.: A General Capability of Design Sensitivity for Finite-Element Systems. Proc. 23rd Structures, Structural Dynamics and Materials Conference, May 10-17, 1982, pp. 175-186.

37. Joseph, J. A., ed: MSC/NASTRAN Application Manual, Version 63A, MacNeal-Schwendler Corp., April 1983.