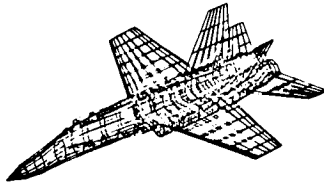


SENSITIVITY ANALYSIS IN COMPUTATIONAL AERODYNAMICS

Dean R. Bristow  
McDonnell Aircraft Company  
St. Louis, MO

SUBSONIC INVISCID PANEL METHOD



MCAERO

INPUT: GEOMETRY

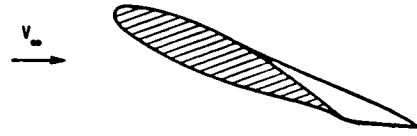
OUTPUT: PRESSURE

DMCAERO

INPUT: GEOMETRY

OUTPUT:  $\frac{\partial \text{PRESSURE}}{\partial \text{GEOMETRY}}$

INTERACTING VISCOUS AND INVISCID THEORIES FOR HIGH  $\alpha$



INVISCID THEORY  $\rightarrow \frac{\partial \text{PRESSURE}}{\partial \text{GEOMETRY}}$

VISCOUS THEORY  $\rightarrow \frac{\partial \text{GEOMETRY}}{\partial \text{PRESSURE}}$

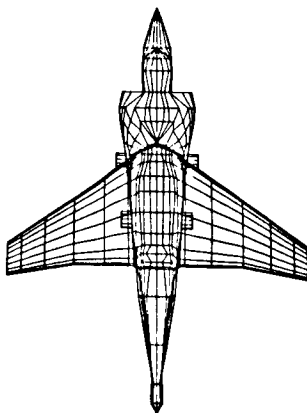


$$\left[ P + \frac{\partial P}{\partial G} \Delta G \right]_{\text{INVISCID}} = \left[ P + \left( \frac{\partial G}{\partial P} \right)^{-1} \Delta G \right]_{\text{VISCIOUS}}$$

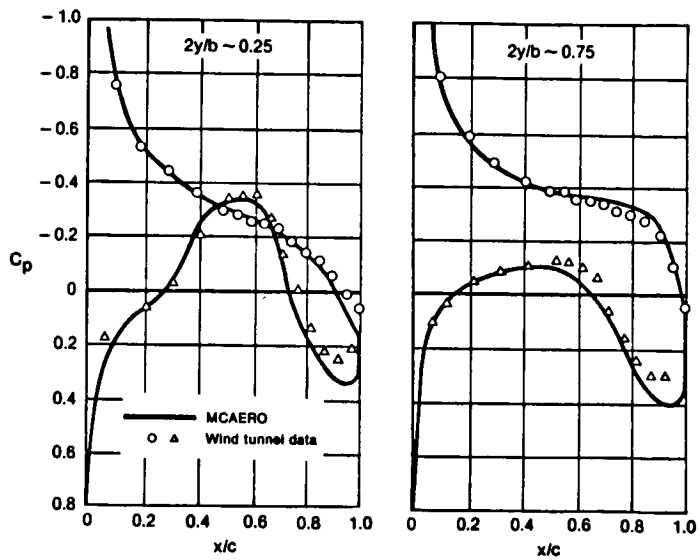
**PREDICTION ACCURACY OF MCAERO**

CONVENTIONAL ANALYSIS MODE

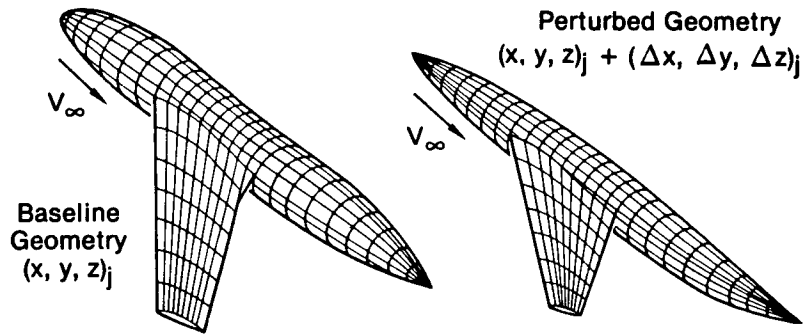
6.4° ANGLE-OF-ATTACK MACH 0.50



**YAV-6B Surface Panel Modeling**



# PERTURBATION ANALYSIS METHOD



## Objective

- Subsonic Inviscid Analysis of Multiple Geometry Perturbations at Small Additional Cost

## Approach

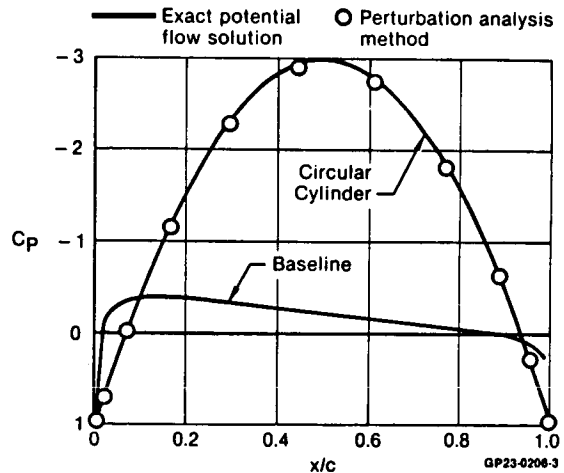
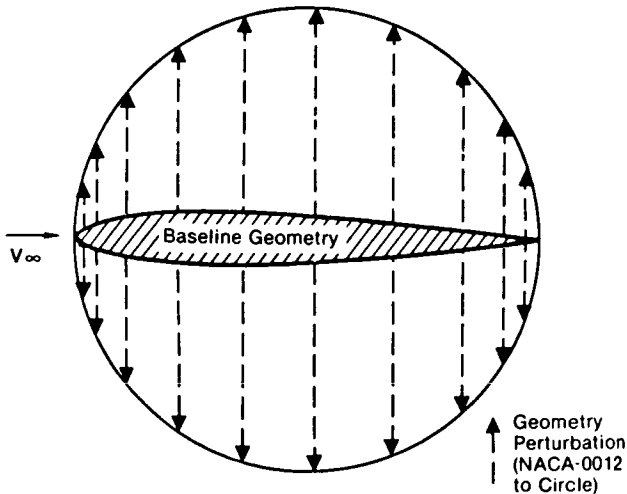
- Precalculated Baseline Matrix of Potential

$$\text{Derivatives } \left\{ \frac{\partial \phi_i}{\partial x_j}, \frac{\partial \phi_i}{\partial y_j}, \frac{\partial \phi_i}{\partial z_j} \right\}$$

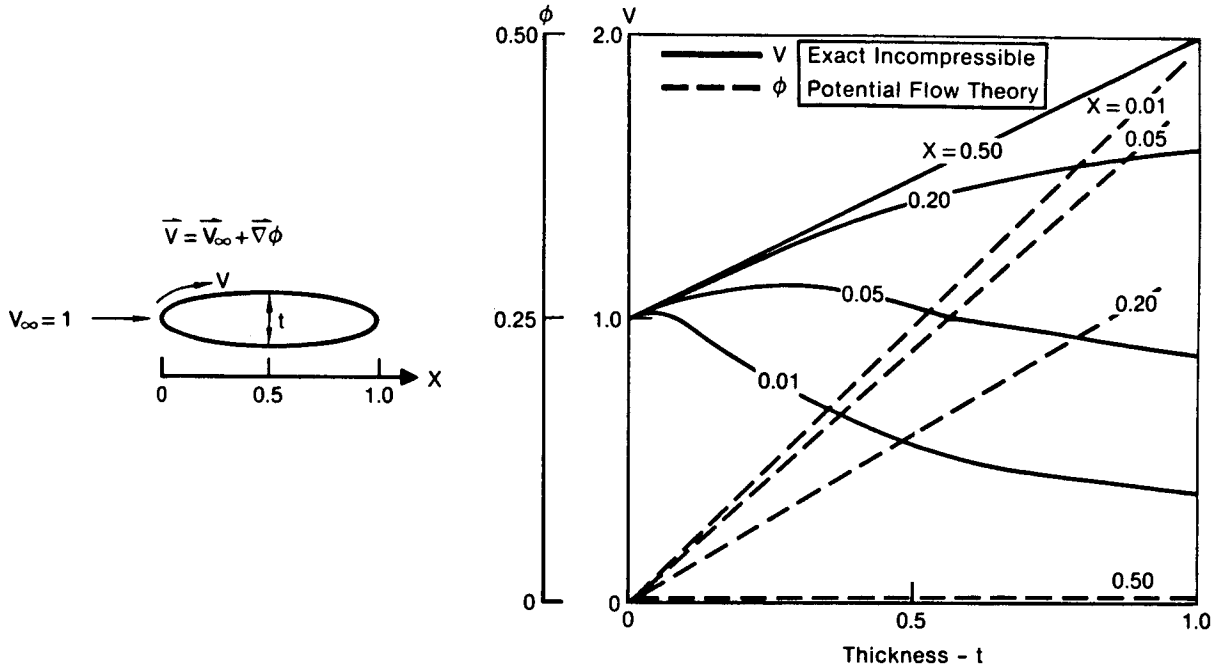
- Linear Extrapolation

$$(\phi_i + \Delta \phi_i) = \phi_i + \sum_j \left\{ \frac{\partial \phi_i}{\partial x_j} \Delta x_j + \frac{\partial \phi_i}{\partial y_j} \Delta y_j + \frac{\partial \phi_i}{\partial z_j} \Delta z_j \right\}$$

## SIMPLE DEMONSTRATION OF PERTURBATION ANALYSIS METHOD 2-D INCOMPRESSIBLE FLOW



## VARIATION OF FLOW PROPERTIES WITH THICKNESS ELLIPTICAL CYLINDER AT 0° INCIDENCE



## Procedure for Calculating Perturbation Matrix

### 1. Conventional Panel Method Calculations

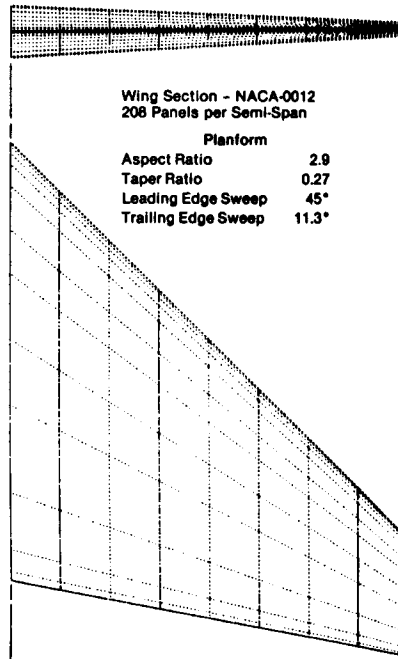
$$[A_{ij}] \cdot \phi_j = BC_i \rightarrow \phi_i = [A_{ij}]^{-1} \cdot BC_j$$

### 2. First-Order Expansion

$$[A_{ij}] \cdot \frac{\partial \phi_j}{\partial z_k} + \phi_j \cdot \left[ \frac{\partial A_{ij}}{\partial z_k} \right] = \frac{\partial BC_i}{\partial z_k}$$

$$\frac{\partial \phi_i}{\partial z_k} = [A_{ij}]^{-1} \cdot \left\{ \frac{\partial BC_j}{\partial z_k} - \phi_l \cdot \left[ \frac{\partial A_{jl}}{\partial z_k} \right] \right\}$$

**BASELINE WING PANELING  
FOR PERTURBATION  
ANALYSIS TEST CASES**

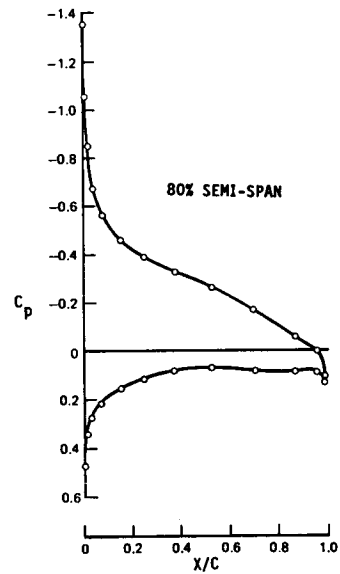
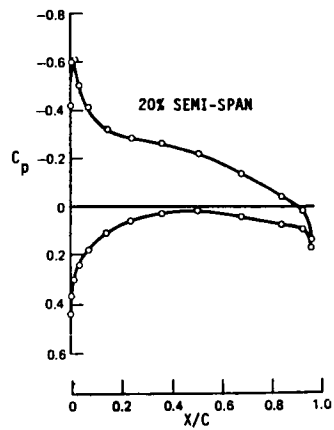


**FIGHTER WING  
AT 5° ANGLE OF ATTACK**

BASELINE	PERTURBED
ROOT	ROOT
TIP	TIP

COMPUTING TIME

— CONVENTIONAL "EXACT" ANALYSIS	245 SECS
○ PERTURBATION ANALYSIS METHOD	7SECS



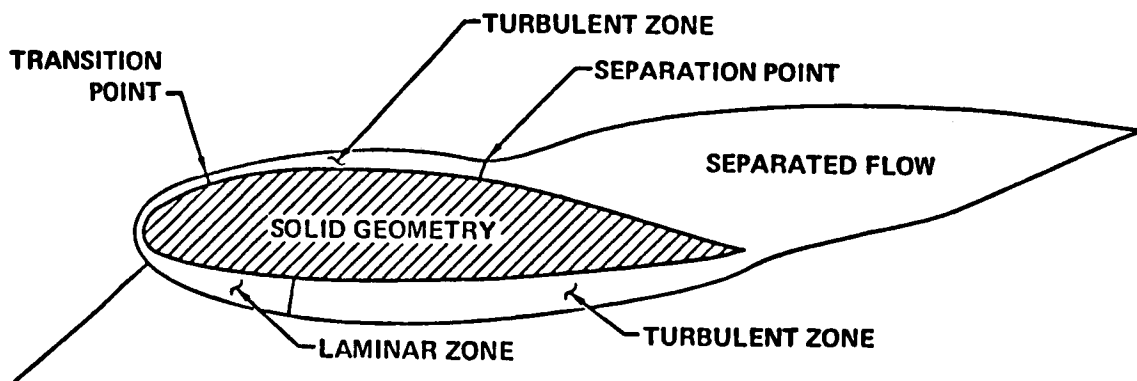
## APPLICATIONS OF INVISCID SENSITIVITY MATRIX

- EFFICIENT ANALYSIS OF MULTIPLE GEOMETRY PERTURBATIONS
- PRESCRIBED PRESSURE WING DESIGN
- UNSTEADY AERODYNAMICS
- AERODYNAMIC-STRUCTURAL DESIGN OPTIMIZATION
- STRONG VISCOUS-INVISCID INTERACTIONS

## TWO-DIMENSIONAL AIRFOIL VISCOUS AERODYNAMICS (LOW SPEED)

### GIVEN

- SOLID GEOMETRY
- REYNOLDS NUMBER



### SOLUTION - GENERAL CASE

- PRESSURE DISTRIBUTION
- LIFT, DRAG, AND PITCHING MOMENT
- SEPARATION POINT
- TRANSITION POINT



# MATCHING PROCEDURE FOR VISCOUS - INVISCID INTERACTION

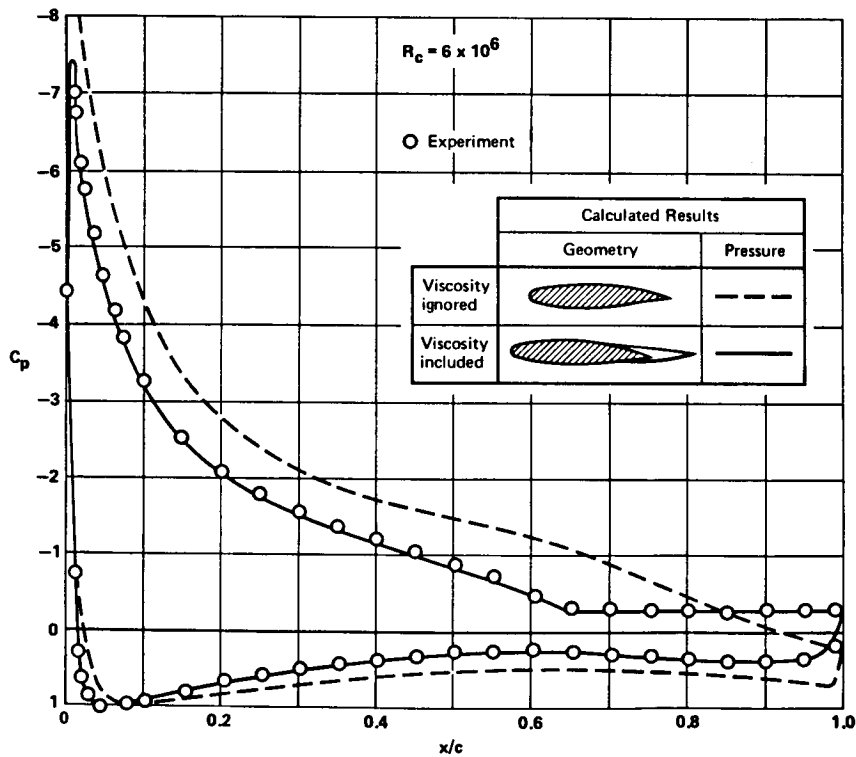
← INVISCID THEORY →

← VISCOUS THEORY →

$$Y_i + \Delta Y_i = \delta_i^* + \sum_j \left( \frac{\partial \delta_i^*}{\partial C_{P_j}} \cdot \Delta C_{P_j} \right)$$

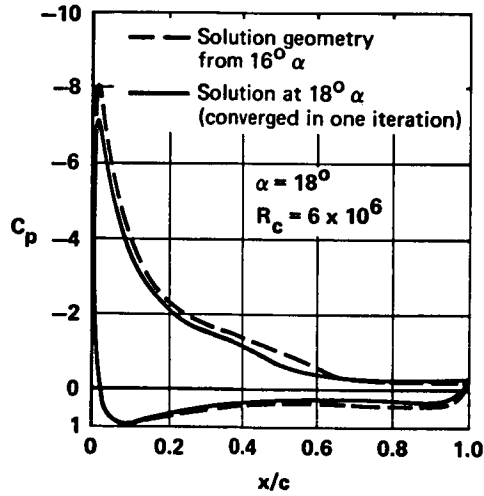
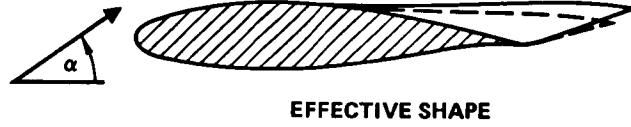
$$C_{P_i} + \sum_j \left( \frac{\partial C_{P_i}}{\partial Y_j} \cdot \Delta Y_j \right) = C_{P_i} + \Delta C_{P_i}$$

## PREDICTION ACCURACY AT HIGH ANGLE-OF-ATTACK (16°α) GA(W)-1 AIRFOIL

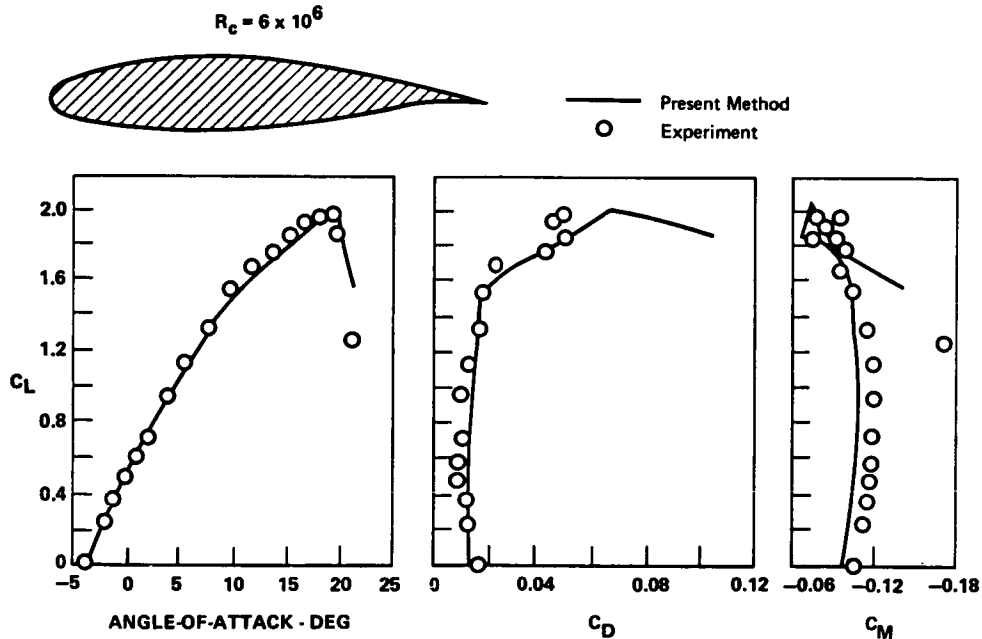




# TYPICAL CHANGE BETWEEN SUCCESSIVE ANGLES OF ATTACK



# ACCURACY OF FORCE AND MOMENT PREDICTIONS NASA GA(W)-1



# EFFECT OF REYNOLDS NUMBER ON AIRFOIL PERFORMANCE



MS<sub>1</sub>- 0313 AIRFOIL

STALLED AIRFOIL  
ANALYSIS PROGRAM

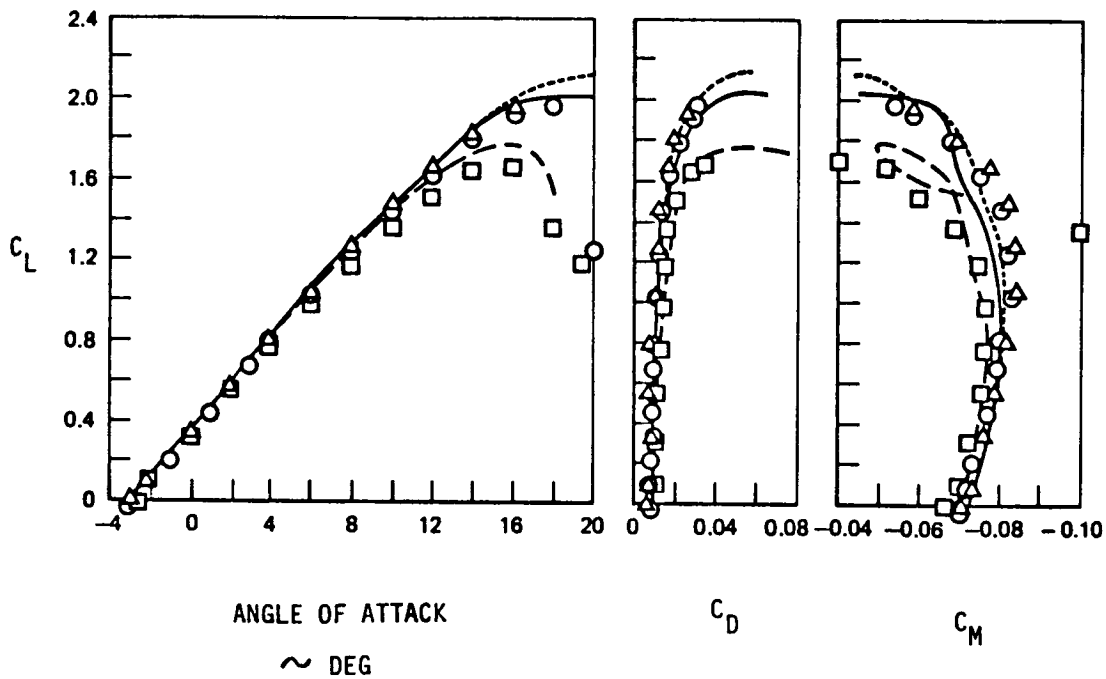
EXPERIMENT

REYNOLDS NUMBER

---  
—  
.....

□  
○  
△

$2 \times 10^6$   
 $6 \times 10^6$   
 $9 \times 10^6$



## CONCLUSIONS REGARDING SENSITIVITY ANALYSIS APPROACH

- POWERFUL EXTRAPOLATION TOOL
- APPROPRIATE FOR STRONG INTERACTION BETWEEN DISTINCT THEORIES