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# Calculation of Thermomechanical Fatigue Life Based on Isothermal Behavior

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## CALCULATION OF THERMOMECHANICAL FATIGUE LIFE BASED ON ISOTHERMAL BEHAVIOR

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### ABSTRACT

The isothermal and thermomechanical fatigue (TMF) crack initiation response of a hypothetical material was analyzed. Expected thermomechanical behavior was evaluated numerically based on simple, isothermal, cyclic stress-strain - time characteristics and on strainrange versus cyclic life relations that have been assigned to the material.

The attempt was made to establish basic minimum requirements for the development of a physically accurate TMF life-prediction model. A worthy method must be able to deal with the simplest of conditions: that is, those for which thermal cycling, per se, introduces no damage mechanisms other than those found in isothermal behavior. Under these assumed conditions, the TMF life should be obtained uniquely from known isothermal behavior. The ramifications of making more complex assumptions will be dealt with in future studies.

Although analyses are only in their early stages, considerable insight has been gained in understanding the characteristics of several existing high-temperature life-prediction methods. The present work indicates that the most viable damage parameter is based on the inelastic strainrange.

### INTRODUCTION

Thermomechanical fatigue (TMF) life predictions are often made on the basis of established, isothermal fatigue behavior (1). Strong economic and engineering reasons exist for continuing to rely on the vast quantity of existing, isothermal material characterizations for life-prediction data bases. However, recognition of the inadequacy of using isothermal behavior to predict the TMF life of certain classes of materials has been increasing.

Accuracy of TMF predictions generally is acknowledged to be poorer than that of corresponding isothermal life predictions. The primary deficiency lies in

the simple fact that isothermal material characterization can never capture those micromechanisms which appear only under thermal-cycling conditions.

For example, alloys with strengthening precipitates can experience an additional, localized component of cyclic strain during thermal cycling because of the differential thermal expansion between the precipitate and the matrix metal; such an additional strain would not be present during isothermal cycling. Imposing identical, global mechanical strain during both types of cycling would not produce the same local strain, and hence the test with the highest local strain would likely have the shortest life (i.e., the thermal-cycling test). Numerous other micromechanisms of cyclic deformation may also be present depending on the material and its deformation mechanisms and on the circumstances in a thermal-cycling test.

The overall purpose of this study is to identify and quantify as many of these mechanistic differences as possible and thus develop a more rational and accurate TMF life-prediction methodology.

This report is limited to examining a few of the simpler isothermal life-prediction models and to exploring the logical implications of applying these methods to TMF problems. For the present, the study is analytic and avoids the uncertainties that tend to confuse experimental comparisons of isothermal and TMF behavior. To keep the initial study as basic as possible, only the simplest of possible assumptions are made. It is assumed, for example, that the alloy and its environment are such that no thermal-cycling mechanisms other than those encountered in isothermal cycling are present. In fact, only one isothermal, inelastic damage mechanism (e.g., to-and-fro crystallographic slip) is assumed to be present. To-and-fro crystallographic slip is the simplest of all conditions to consider and, consequently, serves as an excellent base from which to formulate future studies.

Under these assumptions, it is reasonable to expect that the isothermal fatigue model based on this single mechanism would be able to predict thermal-cycling fatigue lives. Hence, it is possible to establish a required condition for a valid, isothermal fatigue life-prediction method. If an isothermal method does not accurately predict thermal-cycling fatigue life under these idealized conditions, then the method is considered to be technically deficient.

While the results of our analyses could be presented in terms of parametric equations and normalized plots, we have chosen to express the results using numerical examples. These give the reader a direct, first-hand impression of the degree of agreement (or disagreement) between isothermal and TMF behavior.

#### SYMBOLS

- E modulus of elasticity
- $N_f$  cycles to fatigue failure (crack initiation)
- T temperature
- $\Delta W_T$  tensile hysteresis energy
- $\Delta \epsilon$  total strainrange
- $\Delta \epsilon_{el}$  elastic strainrange
- $\Delta \epsilon_{in}$  inelastic strainrange
- $\Delta \sigma$  stress range
- $\epsilon$  axial strain
- $\dot{\epsilon}_{in}$  inelastic strain rate
- $\sigma$  axial stress
- $\sigma_T$  peak tensile stress

#### Superscripts:

- $\alpha$  power of cyclic life for elastic strainrange versus cyclic life relations
- $\beta$  power of cyclic life for inelastic strainrange versus cyclic life relations
- $\gamma$  power of cyclic life in Ostergren criterion versus cyclic life relations
- n cyclic strain hardening exponent
- m inelastic-strain-rate hardening coefficient

#### Coefficients:

- A intercept of elastic strainrange versus cyclic life relations
- B intercept of inelastic strainrange versus cyclic life relations
- F intercept of Ostergren criterion versus cyclic life relations
- K strength coefficient; stress range when the inelastic strainrange is unity

#### BACKGROUND

Historically, investigators have sought an equivalence between TMF and isothermal fatigue by postulating the existence of a single, isothermal temperature level for which the low-cycle fatigue resistance is the same as the TMF resistance. Suggested temperature levels have been maximum thermal-cycling temperature (1), minimum temperature, average temperature (2), or more complex choices such as an equivalent temperature (3) or the temperature within the thermal-cycling range for which the isothermal fatigue resistance is minimum (4). A uniquely defined temperature level has not been found thus far that can represent all thermal-cycling conditions for all materials.

The proper basis for comparing isothermal and TMF resistance has also been open to debate. Quoted isothermal and TMF results may differ considerably, depending on the choice of parameter used for comparison. In fact, the relative fatigue resistances can reverse depending on the parameters used. Comparisons have been made on the basis of inelastic strainrange, total strain-range, stress range, peak tensile stress, and a variety of other so-called failure criteria or damage factors. This issue, of course, may never be experimentally resolved because of the host of damage-accumulation mechanisms that may differ from one material to the next and the potential for differing damage mechanisms between isothermal fatigue and TMF.

Numerous examples from the literature have been surveyed recently (5), and a wide range of behaviors between isothermal and TMF have been noted. Thermal fatigue lives are frequently less than isothermal lives when the basis for comparison is constant, inelastic strainrange. However, if the total strainrange is used as the basis of comparison, the order may be reversed, but only if the inelastic strainrange is small compared to the total strainrange.

Another complication arises when dealing with TMF because of the potentially pronounced effect of temperature and strain phasing. Inphase thermal cycling (maximum temperature at maximum strain) produces lives that can be greater than, or less than, out-of-phase cycling (minimum temperature at maximum strain) depending on the material and environment involved. Hence, if isothermal fatigue life-prediction methods are to be able to predict phase effects, they must recognize the different mechanistic aspects of the different phasings. Since in the current study we have limited behavior to a single, operative crack-initiation mechanism, no temperature-strain phase effect is possible. Some isothermal life-prediction methods attempt to distinguish the effects of phasing, others do not. The approach of Ostergren (6), for example, predicts an effect of phasing.

#### CONSTANT MANSON-COFFIN FAILURE CRITERION

Our approach is to assume an idealized, isothermal behavior for a material and then to infer what the likely thermomechanical response would be. History independence of flow behavior is assumed and no additional flow or damage mechanisms are permitted during thermal excursions. Such idealizations are seldom, if ever, realized in engineering materials. However, they do serve a useful purpose in providing a simple, common

basis for directly comparing various life-prediction approaches. If an isothermal life-prediction method properly predicts the idealized TMF behavior, an essential condition has been achieved for assessing the validity of the method. On the other hand, if incorrect TMF behavior is predicted, the prediction method is obviously faulty.

Two distinct material behaviors are examined in this report time-independent, cyclic stress-strain behavior over the temperature range of interest, and time-dependent behavior in a prescribed manner over the temperature range of interest. Initially, the inelastic strainrange versus life relation shown in Eq. (1) and Fig. 1 (i.e., the Manson-Coffin relation) is assumed to be independent of both time and temperature:

$$\Delta \epsilon_{in} = B(N_f)^\beta \quad (1)$$

Because this relation is assumed to be independent of both time and temperature, B and  $\beta$  are constants, and the inelastic strainrange uniquely defines cyclic life.

#### TMF Life Prediction for Time-Independent, Cyclic, Constitutive Behavior

Although the cyclic stress-strain properties are assumed to be independent of time, they are recognized to be temperature dependent. The cyclic stress-strain response is defined by

$$\Delta \sigma = K(\Delta \epsilon_{in})^n \quad (2)$$

The strain-hardening exponent n is assumed to be independent of both time and temperature. The temperature dependency of the strength coefficient K is assumed to be determined by

$$K = 2760 - 2.21(T), \text{ MPa} \quad (3)$$

Figure 2 schematically illustrates the form of the cyclic stress-strain relation used - linear, elastic strain plus a power-law plastic strain response:

$$\Delta \epsilon = \Delta \epsilon_{el} + \Delta \epsilon_{in} = \frac{\Delta \sigma}{E} + \left(\frac{\Delta \sigma}{K}\right)^{1/n} \quad (4)$$

The modulus of elasticity E, also temperature dependent, is defined by

$$E = 207,000 - 69(T), \text{ MPa} \quad (5)$$

The material properties K and E are taken to vary with temperature according to the previous straight-line relations (Eqs. (3) and (5)), which are shown in Fig. 3. While the numerical values are arbitrary, the trends reflect the general behavior of high-temperature alloys. A temperature range of 0 to 1000 °C has been chosen.

Fixing the constants in the cyclic stress-strain relation (Eq. (2)) and in the inelastic strainrange versus life relation (Eq. (1)) fixes the elastic strainrange versus life relation at each temperature:

$$\Delta \epsilon_{el} = A(N_f)^\alpha \quad (6)$$

Since  $\beta$  and n are temperature independent,  $\alpha$  is also temperature independent ( $\alpha = \beta n$ ). The property A varies with temperature because of the corresponding temperature dependency of K and E:

$$A = \frac{K(B)^\beta}{E} \quad (7)$$

The elastic strainrange versus life relation is shown in Fig. 4, and the constants for this relation and for Eq. (1) are shown in Fig. 5 for the temperature range considered.

Since the inelastic strainrange versus life relation is not a function of temperature, and since no new deformation or crack initiation micromechanisms are permitted because of thermal cycling, it can be deduced that a TMF cycle will have exactly the same life as any isothermal cycle with the same inelastic strainrange. Consequently, we can deduce the life of any TMF cycle simply by knowing its inelastic strainrange. Now we are in a position to ask what life predictions other methods (which utilize other criteria for failure) would infer under these circumstances.

Two failure criteria are examined: (1) total strainrange of the cycle at some prescribed temperature and (2) the approach of Ostergren (6) (with no time dependencies), which postulates that the product of the inelastic strainrange and the peak tensile stress is a power function of the cyclic life. The tensile hysteresis energy of the cycle, which is an alternate interpretation of the Ostergren criterion, is also examined.

Total strainrange criterion. The isothermal, total strainrange versus life relation for any temperature is obtained from Eqs. (1) and (6):

$$\Delta \epsilon = A(N_f)^\alpha + B(N_f)^\beta \quad (8)$$

Calculated curves for the maximum, average, and minimum temperatures are presented in Fig. 6. We are now in a position to examine a TMF cycle and predict its life based on the total strainrange versus life relations in Fig. 6.

As an example, an inelastic strainrange of 0.005 is assumed and the TMF hysteresis loop of Fig. 7 is calculated from the assumed constitutive behavior for the temperature range from 0 to 1000 °C. As noted earlier, Eq. (1) is independent of temperature and uniquely defines cyclic life. Thus, for the assumed inelastic strainrange (0.005) and the values of B and  $\beta$  given in Fig. 5, the cyclic life is calculated to be 400 cycles. The loop shown in Fig. 7 does not include the elastic strain. However, the only important elastic strains are those at the extremes of the TMF cycle. In this case, the elastic strainrange of 0.0030 is composed of the 0 °C amplitude of 0.00233 and the 1000 °C amplitude of 0.00067. The total strainrange is 0.0080.

If we enter the isothermal total strainrange life curve for the maximum temperature of the TMF cycle, we will predict a life of 233 cycles to failure. This value, however, is in disagreement with what we know to be the correct life (400 cycles). Similar calculations for other isotherms reveal the predictions shown in Table I.

None of the three predictions agree with the known life of 400 cycles. Since the known life is between the extreme predictions, it is obvious that a temperature level (equal-life temperature) exists for which the prediction would be equal to the known life. However, that temperature can be shown to be a function of

the values of the constants in the cyclic stress-strain curve and in the failure curves themselves. Calculations show that the equal-life temperature is also a function of the temperature range and peak temperature. Since the equal-life temperature is dependent on all the variables involved, it has no special significance. Blind use of a total strainrange failure criterion for a preselected, isothermal temperature fails to produce correct life predictions.

Ostergren approach. The Ostergren failure criterion for time-independent behavior is the product of the inelastic strainrange  $\Delta\epsilon_{in}$  and the peak tensile stress  $\sigma_T$  of the cycle. This quantity is directly related to the tensile hysteresis energy  $\Delta W_T$  for isothermal fatigue cycles. For the assumed cyclic stress-strain behavior and  $n = 0.20$ , the product and energy terms are linearly related by the factor  $(1 - n)/(1 + n) = 2/3$ . For isothermal life prediction there is little need to distinguish between the two. However, when TMF cycles are involved the energy and product terms become nonlinearly related. The reasons for this behavior will become apparent later. We will examine the following product form first:

$$\Delta\epsilon_{in}\sigma_T = F(N_f)^\gamma \quad (9)$$

Equation (9) is shown in Fig. 8 for several temperature levels. The constants are determined by using Eqs. (1) and (2) where  $\gamma = \alpha + \beta$  and

$$F = \frac{K}{2} (B)^{1+n} \quad (10)$$

The isothermal curves for the tensile hysteresis-energy failure criterion would lie a factor of 2/3 below the curves in Fig. 8.

Reexamining the inphase TMF loop in Fig. 7, we can determine the product  $\Delta\epsilon_{in}\sigma_T$ . In this case  $\sigma_T = 212$  MPa at point B and  $\Delta\epsilon_{in} = 0.005$ . The product of these two terms is 1.06 MPa. Following the same procedure as used in examining the total strainrange failure criterion, we examine the minimum, average, and maximum temperature. For an inphase cycle, a fourth temperature might also be selected - that is the temperature at which the peak tensile stress is encountered (point B, Fig. 7). For an out-of-phase TMF cycle, the peak tensile stress of 478 MPa occurs at maximum strain and minimum temperature. Table II(a) compares the TMF life predictions at the four isotherms with the previously determined life known to exist (400 cycles) for the inelastic strainrange of 0.005. Both inphase and out-of-phase TMF cycles are examined.

Again, it is readily apparent that none of the inphase predictions are in agreement with the known life of 400 cycles. In addition, an equal-life temperature is so highly dependent on all of the input variables that the idea of treating the parameter  $\Delta\epsilon_{in}\sigma_T$  and an equal-life temperature as failure criteria is not appealing.

For the out-of-phase cycle, agreement is achieved by using the minimum temperature of the cycle. This agreement, however, is circumstantial and is far from being a general conclusion. For example, if the point of view is taken that the tensile hysteresis energy is a more appropriate term, then the previously noted successful prediction is no longer correct. By integrating the tensile hysteresis energy for the loop of Fig. 7 (or the other half of the loop in the case of an out-of-phase TMF cycle) and entering isothermal failure

curves based on tensile hysteresis energy, we obtain the predictions in Table II(b). For this case, there is no agreement between predictions and the known life of 400 cycles.

#### TMF Life Prediction for Time-Dependent, Cyclic, Constitutive Behavior

In addition to the previously adopted temperature dependence of the cyclic stress-strain behavior, we now examine the consequences of assuming a time dependency. The assumed time dependency begins below an arbitrarily set, inelastic strain rate (1.0/sec in the present case). The degree of time dependency increases as the temperature increases. As stated earlier, the failure behavior, which is taken to be inelastic strainrange versus life, is not assumed to be time dependent, nor is it temperature dependent. The time-dependent, cyclic stress-strain curve is given in the following equation and is illustrated in Fig. 9:

$$\Delta\sigma = K(\dot{\epsilon}_{in})^m (\Delta\epsilon_{in})^n \quad (11)$$

The material properties  $K$  and  $n$  retain their originally assigned values. The inelastic strain-rate hardening exponent  $m$  is assumed to vary with temperature in an exponential manner as shown in Eq. (12) and Fig. 10:

$$m = 0.001 \exp[0.00461(T)] \quad (12)$$

Based on these assumptions, it follows that the elastic strainrange versus life relation is both time and temperature dependent (Fig. 11). Thus, the total strainrange versus life relation is also time and temperature dependent as indicated in Fig. 12.

Since the inelastic strainrange versus life relation is not a function of temperature or time, and since no new deformation or crack initiation micromechanism is permitted, it is logical to deduce that a TMF cycle will again have exactly the same life as any isothermal cycle with the same inelastic strainrange. Thus, we can deduce the life of any TMF cycle simply by knowing its inelastic strainrange. The fact that the stress response is time dependent dictates that the total strainrange and the Ostergren models exhibit time-dependent failure curves.

As an example, we select a single, slow, inelastic strain rate of 0.0001/sec to evaluate the TMF and the isothermal fatigue lives. An inelastic strainrange of 0.005 is again assumed. Figure 13 shows the TMF hysteresis loop for an inelastic strain rate of 0.0001/sec and temperature extremes of 0 and 1000 °C.

Total strainrange criterion. The isothermal total strainrange versus life relation for the slow strain rate of 0.0001/sec is shown in Fig. 14 for the minimum, average, and maximum temperatures. The effect of the slow strain rate on the 1000 °C curve is quite apparent when compared with Fig. 6. The total strainrange is now only 0.00756 (the 1000 °C tensile, elastic strain amplitude is only 0.00025), and the TMF life is 400 cycles to failure as determined earlier. Life predictions based on the three isothermal total strainrange fatigue curves of Fig. 14 are tabulated in Table III. Life predictions at each temperature are the same for inphase and out-of-phase TMF cycles. As noted for the time-independent analysis (Table I), the predicted lives are not in agreement with the known life.

Ostergren approach. With the introduction of the time dependency of the stress response, the Ostergren

approach can no longer be taken in its time-independent form. Instead, the equation must be written as

$$\Delta \epsilon_{in} \sigma_T = F (\dot{\epsilon}_{in})^m (N_f)^\gamma \quad (13)$$

where  $F$  is given by Eq. (10).

Since the isothermal and TMF inelastic strain rates are identical, no interpolation or extrapolation is required with respect to time-dependent effects. Furthermore, the tensile and compressive inelastic strain rates are also identical, thus eliminating the need to consider the time-dependent waveform effects that are handled in the Ostergren approach by a special interpretation of tensile and compressive frequencies.

For an inelastic strain rate of 0.0001/sec and an inelastic strainrange of 0.005, we can calculate a life for each of the temperatures of interest in Fig. 15. These represent isothermal life predictions of the TMF cycle. For an inphase TMF cycle, a fourth isothermal temperature may have significance - that is, the temperature at the maximum tensile stress (point B in Fig. 13). Table IV(a) summarizes the life calculations.

Again, there is disagreement between the established life of 400 cycles and the isothermally predicted lives. The one point of agreement is for the out-of-phase cycle for which the minimum isothermal temperature exists. This is somewhat circumstantial and it would be unwise to generalize the agreement for other conditions. To illustrate the point, we need only to look at the tensile hysteresis energy representation of the time-dependent Ostergren approach. Predictions for the same TMF cycle are tabulated in Table IV(b). Here it is seen that the previously obtained agreement for the minimum-temperature prediction of the out-of-phase cycle no longer exists.

Life-prediction calculations have also been made for a broader range of the variables of interest. These results are contained in Figs. 16 to 19 for the range of temperatures considered and for the inelastic strainranges of 0.0002 and 0.005. The cycle parameters for these calculations are shown in Table V. Figure 16 displays the TMF fatigue life predictions based on the isothermal, total strainrange criterion for the two strainranges and for both time-dependent and time-independent behavior. Predictions are identical for the inphase and out-of-phase TMF cycles. The known TMF lives are also independent of phasing for the present case. Note that there are two total strainrange values for each inelastic strainrange. This is because of the different, elastic strainrange contributions to the total strainrange resulting from the time-independent and time-dependent TMF loops.

Similar results are displayed in Figs. 17(a) and (b) and 18(a) and (b) for the two formulations of the Ostergren approach. The same general trends as found for the 0.005 inelastic strainrange apply for the smaller strainrange (0.0002).

The previous sections dealt with a temperature-independent, inelastic strainrange versus cyclic-life failure criterion. Other temperature-independent criteria are now examined - namely, total strainrange and the Ostergren parameter.

#### CONSTANT TOTAL STRAINRANGE FAILURE CRITERION

Considering the total strainrange criterion first, it quickly becomes apparent that the only way to make

this criterion independent of temperature is to have two conditions fulfilled. The first condition is to have the cyclic strength coefficient decrease with increasing temperature at exactly the same rate as the decrease of the modulus of elasticity with temperature; that is, the elastic strainrange versus life relation must be independent of temperature (and also must be independent of time or strain rate effects). The second condition is that the inelastic strainrange versus cyclic life relation must be temperature independent. However, this is exactly the failure criterion that was examined earlier in this report. Hence, there is no merit in further pursuit of the constant total strainrange criterion.

#### CONSTANT OSTERGREN FAILURE CRITERION

We will now examine the Ostergren approach as a failure criterion. When selecting the time-independent product form (Eq. (9)), we see that the constants  $F$  and  $\gamma$  must be independent of temperature in order that the failure criterion be independent of temperature. Hence,

$$F = (K/2)(B)^{1+n} = \text{constant}$$

$$\gamma = \alpha + \beta = \text{constant}$$

In all previous analyses in this study, we assumed  $n (= 0.2)$ ,  $\alpha (= -0.1)$ , and  $\beta (= -0.5)$  to be constant with respect to temperature. Therefore,  $\gamma = -0.60$ , and  $(K/2)(B)^{1.2} = \text{constant}$ . According to this constraint it is thus necessary for  $K$  and  $B$  to vary with temperature. Previously, we had considered  $B$  to be temperature independent. To do so now would also require that the strength coefficient  $K$  in the cyclic stress-strain curve be a constant. Such a condition is unrealistic. Hence, we select  $K$  to vary with temperature as assumed earlier (Eq. (3)). Solving for how  $B$  must vary with temperature yields

$$B = \left[ \frac{\text{constant}}{1380 - 1.10(T)} \right]^{0.833} \quad (14)$$

For the sake of numerical example, the constant in this expression is determined at the arbitrary reference condition of 500 °C with  $B = 0.10$ . For this condition, the constant is 52.2. Thus,

$$B = [26.4 - 0.0211(T)]^{-0.833} \quad (15)$$

where  $B$  varies from 0.065 at 0 °C to 0.250 at 1000 °C. Note that it is necessary for  $B$  to increase as temperature is increased. While this is not normal engineering behavior, it is physically viable. Such a trend is noted in Ref. 7 for some nickel-based superalloys at high, homologous temperatures (>0.7).

Before pursuing the implications of Eq. (15) for TMF life predictions, we will examine the case of time-dependent, constitutive response and its effect on the Ostergren approach. For the reasons noted previously, we find that the exponent,  $\gamma$ , and the intercept,  $F(\dot{\epsilon}_{in})^m$ , in Eq. (13) must be constants. Solving for  $B$  gives

$$B = \left\{ \frac{\text{constant}}{[\dot{\epsilon}_{in}]^m [1380 - 1.10(T)]} \right\}^{0.833} \quad (16)$$

For the reference condition ( $T = 500$  °C,  $B = 0.10$ ) and a strain rate of 0.0001/sec, the constant in Eq. (16) is 47.6. Thus, Eq. (16) becomes

$$B = \{ [0.0001]^m [29.0 - 0.0232(T)] \}^{-0.833} \quad (17)$$

Hence, B must vary with both temperature and inelastic strain rate in a highly constrained, physically improbable manner. For example, as the strain rate decreases, the value of B (an indicator of ductility) must increase. Such a response is extremely uncommon except within the realm of superplastic behavior at extremely high, homologous temperatures ( $>0.9$ ). For an inelastic strain rate of unity or greater, the time-dependent, constitutive response degenerates to the time-independent case. Obviously, if the Ostergren failure criterion is independent of temperature, then the prediction of TMF life using the Ostergren approach is trivial; that is the TMF life is identical to the isothermal life. The TMF cycles of interest have already been discussed, and the cycle parameters are shown in Table V. Time-independent values ( $\dot{\epsilon} > 1.0/\text{sec}$ ) are shown first, followed by time-dependent values ( $\dot{\epsilon}_{in} = 0.0001/\text{sec}$ ).

For time-independent conditions, an inphase value of 1.06 MPa for the Ostergren product results in an isothermal life of 660 cycles to failure and thus a TMF life also of 660 cycles to failure. For the time-dependent case, the life is 625 cycles. Similarly, the out-of-phase, time-independent and time-dependent lives are 169 and 147 cycles to failure, respectively. These values of life are given in Tables II(a) and IV(b). Note that lives are lower at the slow strain rate than they are at the fast strain rate. TMF lives calculated using the Ostergren tensile hysteresis-energy parameter were numerically different but exhibited identical trends.

#### Manson-Coffin Failure Criterion

Evaluation of the Manson-Coffin equation (with B determined by Eq. (15) for time-independent behavior and Eq. (17) for time-dependent behavior ( $\dot{\epsilon}_{in} = 0.0001/\text{sec}$ ) at an inelastic strainrange of 0.005) gives the results in Table VI(a). None of these lives are in agreement with what is being forced to be the correct life (i.e., 660 cycles). There is no influence of temperature-strain phasing for this failure criterion. When a constant Ostergren failure criterion, independent of temperature, is assumed, the predicted TMF lives increase as the isothermal temperature used in the prediction is increased. Also, at 1000 °C the life at the slow strain rate is prolonged compared to the fast strain rate. Such behavior is not common, although it has been noted for the nickel-base alloy, IN792 + Hf (8).

#### Total Strainrange Criterion

For a constant Ostergren failure criterion, the curves for isothermal, total strainrange versus life must differ from one another at different temperatures. In the present case, both the inelastic and elastic line must vary considerably. This is particularly true for the time-dependent case.

Corresponding to the total strainrange values in Table V are the isothermally predicted TMF lives in Table VI(b). Calculated lives are independent of the temperature-strain phasing of the TMF cycle. Again, the lives listed in Table VI(b) increase as either temperature increases or strain rate decreases.

These results from Table VI(b) are displayed in Figs. 19(a) and (b) for time-independent and time-dependent material behavior, respectively. In addition, a much smaller strainrange condition is examined

(i.e.,  $\Delta\epsilon_{in} = 0.0002$ ). These figures display the diversity in predicted lives that results from the various failure criteria.

#### RESULTS AND CONCLUDING REMARKS

The short-range objectives of this study have been achieved by examining simple, isothermal fatigue life-prediction models and by exploring the logical implications of applying these models to thermal-cycling conditions. Simple sets of assumptions were made regarding isothermal, cyclic flow and failure behavior. These were made to permit easier interpretation of the thermomechanical-cycling behavior. Cyclic life predictions of selected TMF cycles were based on the various assumed, isothermal fatigue resistances. Of the three basic failure criteria examined, inelastic strainrange, total strainrange, and two versions of the Ostergren approach, it is concluded that using an inelastic strainrange criterion gave the most reasonable interpretation of TMF behavior.

The other criteria were shown to require improbable material flow and/or failure behavior to produce numerically correct TMF life predictions. The reason for their poor performance is that the failure criteria for each is composed of two contributions - one related to failure (inelastic strain) and the other to nonlinear cyclic-flow behavior (stress or elastic strain). A similar set of conclusions is projected for the future when we analyze other phenomenological life-prediction methods that incorporate both flow and failure characteristics into their damage parameters.

For a paper study it is by far the simplest to assume that the failure criterion of interest is independent of temperature and time. By assuming that the total strainrange versus life or the Ostergren parameter versus life relation were constant with respect to temperature (and/or time), we had to infer a very specific, cyclic, constitutive relation and a very specific, inelastic strainrange versus cyclic life relation. For example, when a realistic, cyclic stress-strain equation is used, the Manson-Coffin equation must necessarily show an improvement in low-cycle fatigue resistance as temperature is increased or as strain rate is decreased. Neither of these are commonly experienced material behaviors. Thus, the simplifying assumption that the Ostergren criterion is independent of temperature is not a generally viable one.

Procedures for handling nonconstant failure criteria have yet to be established. Assumptions would have to be made as to how to incrementally add damage when going through each TMF cycle. For example, the cycle could be divided into equal increments of inelastic strain. Each increment would have its damage linked to the rate of damage at the temperature of that increment. The linear summation of the increments of damage over a cycle would be straightforward. While this type of procedure could be applied to the inelastic strain criterion, it is unclear as to how to incrementally sum damage due to a total strain criterion or to a product of the peak tensile stress and plastic strain. Obviously, further study is required.

A key feature of the present analytic study is the assumption that no new deformation or failure mechanisms were introduced by thermal cycling. While this is not particularly realistic for many materials, it is not an unreasonable one for others. For example, a relatively pure, single-phase material subjected to

fatigue loading in an inert atmosphere over a relatively narrow, low-temperature regime could readily fall into this category.

By ruling out additional mechanisms due to TMF, one can potentially achieve a TMF life prediction based solely on isothermal behavior. Consequently, one can determine which isothermal life prediction methods can meet this necessary (although not sufficient) condition of validity. The next steps in the analytic study are to examine more complex cycles and more realistic failure behavior as a function of temperature and time, and to examine additional phenomenological life-prediction methods.

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TABLE I. - LIFE PREDICTIONS  
 BASED ON TOTAL STRAIN-  
 RANGE APPROACH FOR  
 TIME-INDEPENDENT  
 BEHAVIOR  
 [ $\Delta\epsilon = 0.0080.$ ]

Isothermal temperature, °C	Predicted life, cycles
Minimum 0	740
Average 500	560
Maximum 1000	233



TABLE II. - LIFE PREDICTIONS BASED ON OSTERGREN  
APPROACH FOR TIME-INDEPENDENT BEHAVIOR

[ $\Delta\epsilon_{in} = 0.005.$ ]

(a) Product form ( $\Delta\epsilon$ )( $\sigma_T$ )

Isothermal temperature, °C	Predicted life, cycles	
	Inphase ( $\Delta\epsilon_{in}\sigma_T = 1.06$ MPa)	Out-of-phase ( $\Delta\epsilon_{in}\sigma_T = 2.39$ MPa)
Minimum 0	1547	400
Point B 410	797	---
Average 500	660	169
Maximum 1000	106	27

(b) Tensile hysteresis energy ( $\Delta W_T$ )

Isothermal temperature, °C	Predicted life, cycles	
	Inphase ( $\Delta W_T = 0.807$ MPa)	Out-of-phase ( $\Delta W_T = 1.12$ MPa)
Minimum 0	1249	717
Point B 410	644	---
Average 500	533	306
Maximum 1000	85	49

TABLE IV. - LIFE PREDICTIONS BASED ON  
OSTERGREN APPROACH FOR TIME-  
DEPENDENT BEHAVIOR

[ $\epsilon_{in} = 0.0001/\text{sec.};$

$\Delta\epsilon_{in} = 0.005.$ ]

(a) Product form ( $\Delta\epsilon$ )( $\sigma_T$ )

Isothermal temperature, °C	Predicted life, cycles	
	Inphase $\Delta\epsilon_{in}\sigma_T = 1.00$ MPa	Out-of-phase $\Delta\epsilon_{in}\sigma_T = 2.37$ MPa
Minimum 0	1681	400
Point B 370	874	---
Average 500	625	147
Maximum 1000	25	6

(b) Tensile hysteresis energy ( $\Delta W_T$ )

Isothermal temperature, °C	Predicted life, cycles	
	Inphase $\Delta W_T = 0.681$ MPa	Out-of-phase $\Delta W_T = 1.05$ MPa
Minimum 0	1621	794
Point B 370	842	---
Average 500	602	295
Maximum 1000	24	12

TABLE III. - LIFE PREDICTIONS  
BASED ON TOTAL STRAINRANGE

CRITERION FOR TIME-  
DEPENDENT BEHAVIOR

[ $\epsilon_{in} = 0.0001/\text{sec.};$

$\Delta\epsilon = 0.00754.$ ]

Isothermal temperature, °C	Predicted life, cycles
Minimum 0	910
Average 500	474
Maximum 1000	205

TABLE V. - THERMOMECHANICAL CYCLE PARAMETERS  
 [Temperature, 0 to 1000 °C.]

(a)  $\Delta\epsilon_{1n} = 0.0002$

Parameter	Parameter values, time independent (time dependent)	
	Inphase	Out-of-phase
$\Delta\epsilon$	0.00177 (0.00177)	0.00156 (0.00156)
$\Delta\epsilon_{1n}\sigma_T$ , MPa	.0223 (.0210)	.0506 (.0502)
$\Delta W_T$ , MPa	.0172 (.0145)	.0234 (.0221)

(b)  $\Delta\epsilon_{1n} = 0.005$

Parameter	Parameter values, time independent (time dependent)	
	Inphase	Out-of-phase
$\Delta\epsilon$	0.008 (0.00754)	0.008 (0.00754)
$\Delta\epsilon_{1n}\sigma_T$ , MPa	1.06 (1.00)	2.39 (2.37)
$\Delta W_T$ , MPa	.807 (0.681)	1.12 (1.05)

TABLE VI. - LIFE PREDICTIONS BASED ON INELASTIC  
 LINE INTERCEPT B DETERMINED BY CONSTANT  
 OSTERGREN FAILURE CRITERION

(a) Manson-Coffin criterion;  $\Delta\epsilon_{1n} = 0.005$

Isothermal temperature, °C	Predicted life, cycles	
	Time independent	Time dependent
Minimum 0	169	149
Average 500	400	400
Maximum 1000	2500	9920

(b) Total strainrange criterion

Isothermal temperature, °C	Predicted life, cycles	
	Time independent $\Delta\epsilon = 0.008$	Time dependent $\Delta\epsilon = 0.00754$
Minimum 0	320	330
Average 500	450	484
Maximum 1000	1450	5110

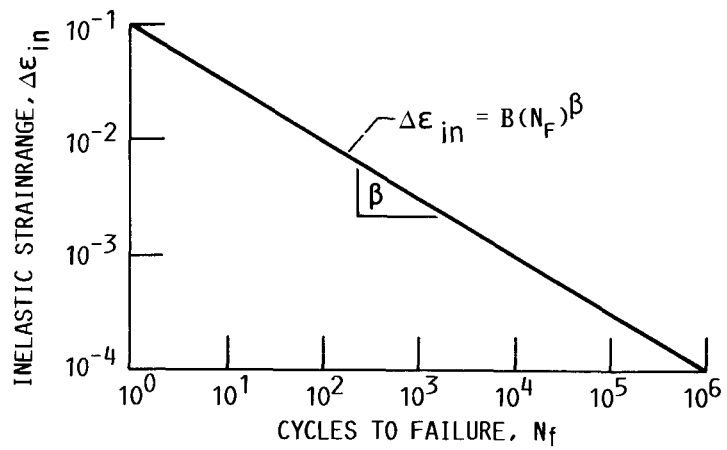


FIGURE 1.- INELASTIC STRAINRANGE VERSUS CYCLIC LIFE RELATION FOR HYPOTHETICAL MATERIAL WITH TIME- AND TEMPERATURE-INDEPENDENT BEHAVIOR.

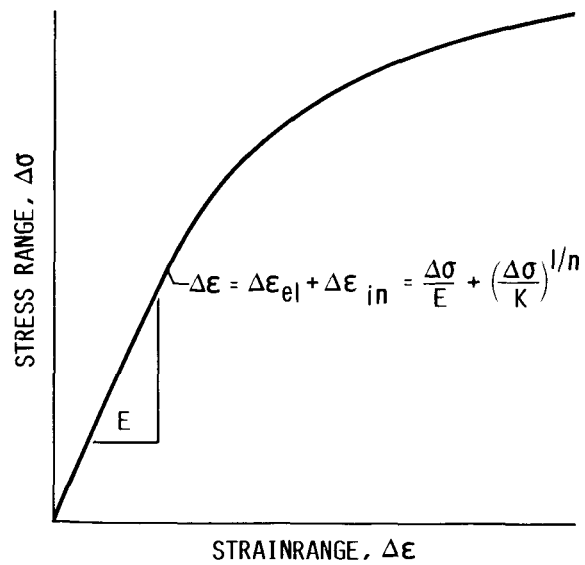


FIGURE 2.- CYCLIC STRESS-STRAIN RELATION FOR HYPOTHETICAL MATERIAL WITH TIME-INDEPENDENT BEHAVIOR.

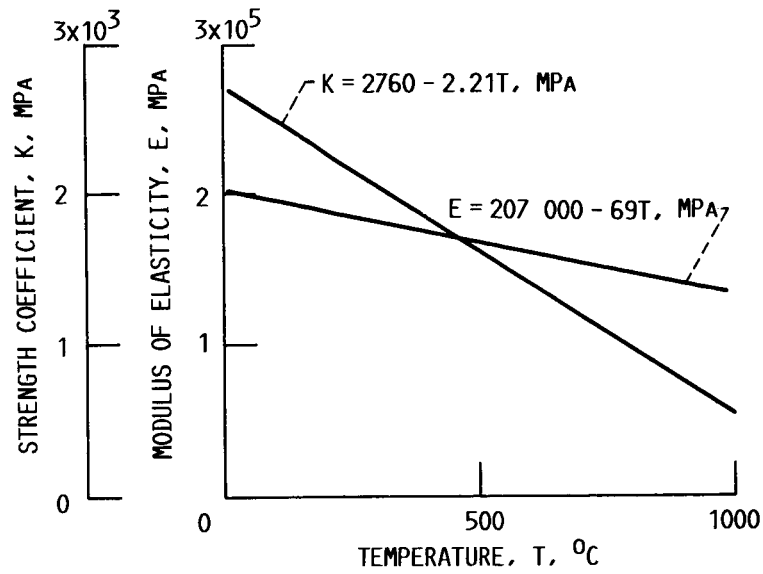


FIGURE 3.- TEMPERATURE DEPENDENCE OF CYCLIC STRESS-STRAIN CONSTANTS E AND K ( $n = 0.2$ ).

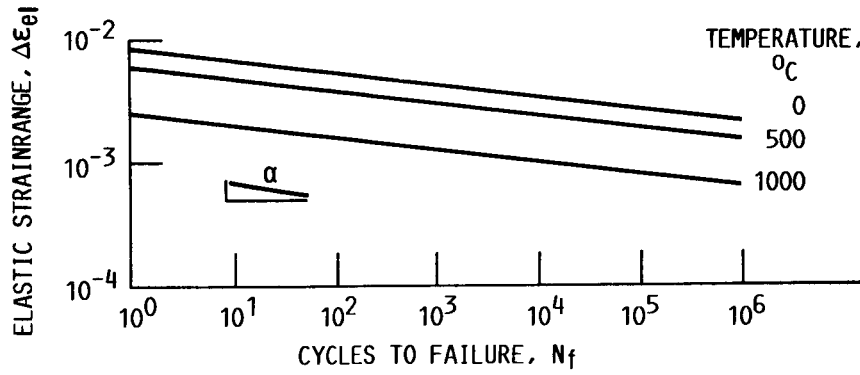


FIGURE 4. - ELASTIC STRAINRANGE VERSUS CYCLIC LIFE RELATION FOR HYPOTHETICAL MATERIAL WITH TIME-INDEPENDENT BEHAVIOR;  $\Delta\epsilon_{eI} = A(N_f)^\alpha$ ;  $A = KB^n/E$ .

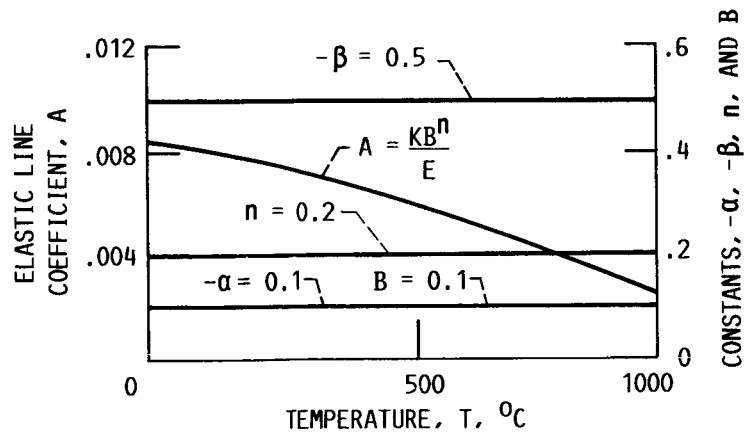


FIGURE 5.- TEMPERATURE DEPENDENCE OF TIME-INDEPENDENT FATIGUE CONSTANTS.

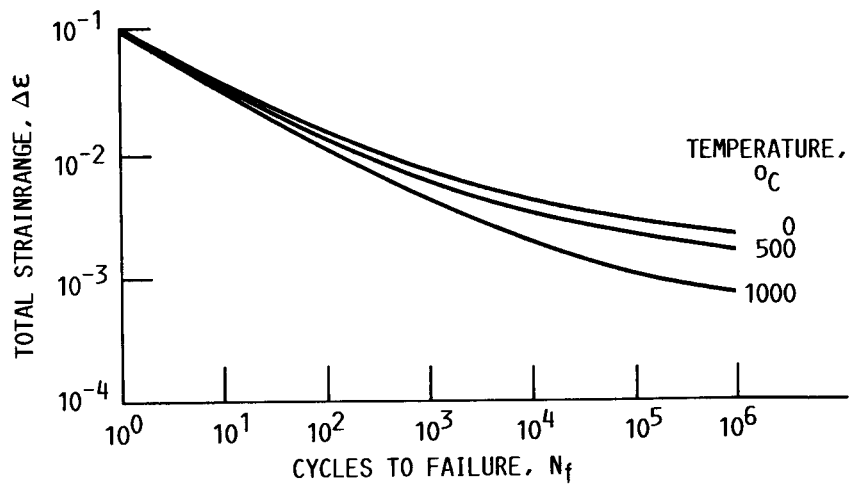


FIGURE 6.- TOTAL STRAINRANGE VERSUS CYCLIC LIFE RELATION FOR HYPOTHETICAL MATERIAL WITH TIME-INDEPENDENT BEHAVIOR. CURVES FOR MINIMUM, AVERAGE, AND MAXIMUM TEMPERATURES OF TMF CYCLE;  
 $\Delta\epsilon = A(N_f)^\alpha + B(N_f)^\beta$ .

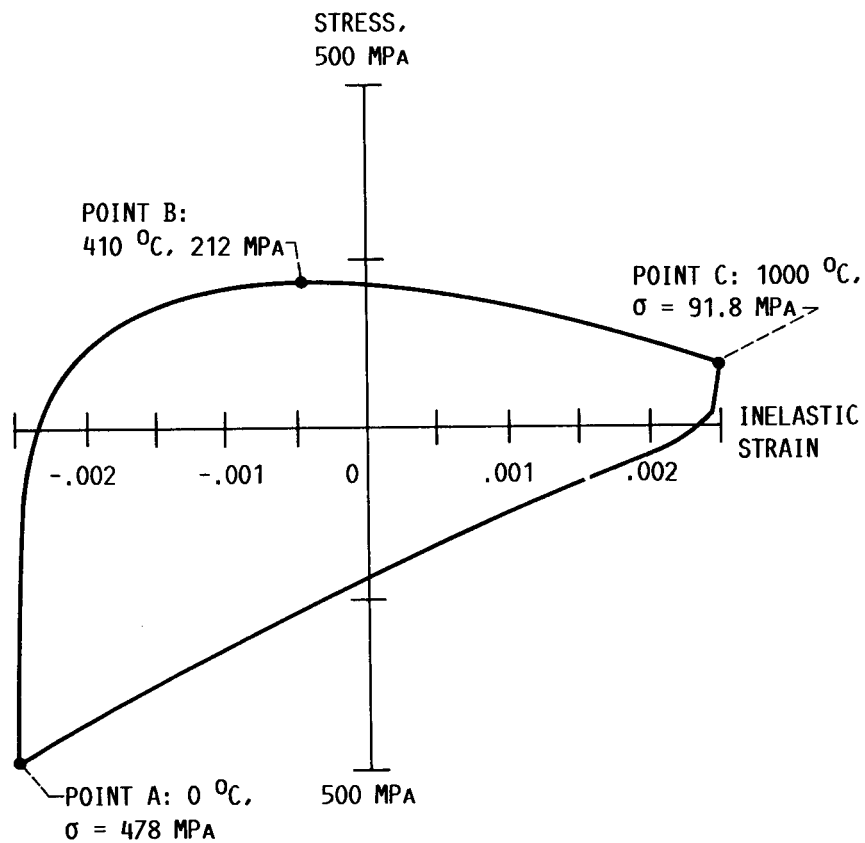


FIGURE 7.- TMF HYSTERESIS LOOP. INPHASE, INELASTIC STRAINRANGE, 0.005; MAXIMUM TEMPERATURE, 1000 °C; MINIMUM TEMPERATURE, 0 °C;  
 $\dot{\epsilon}_{in} \geq 1.0/\text{SEC}$ .

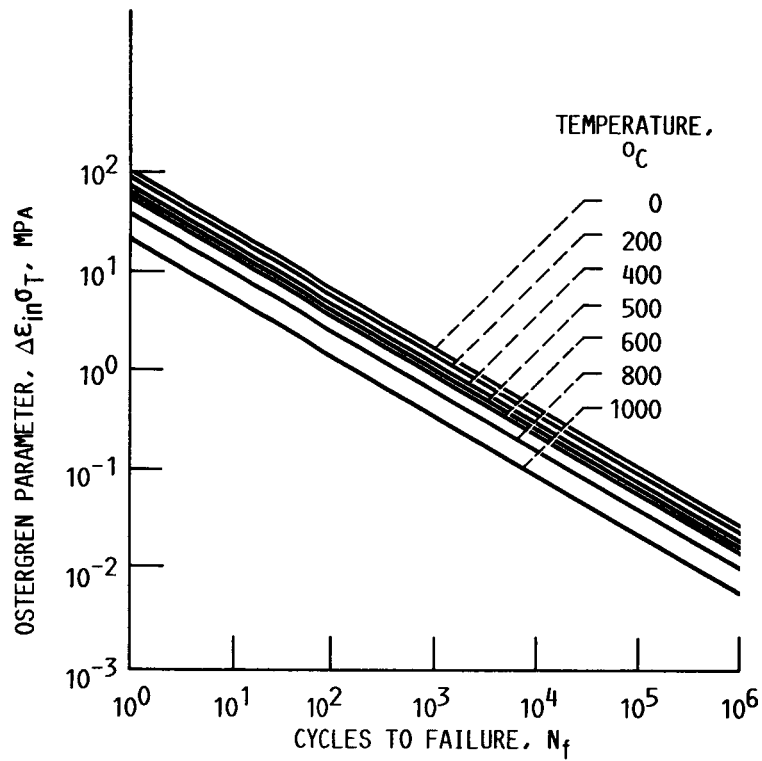


FIGURE 8.- ISOTHERMAL FAILURE CRITERION USING OSTERGREN'S TIME-INDEPENDENT STRESS-INELASTIC STRAINRANGE PRODUCT FORM.

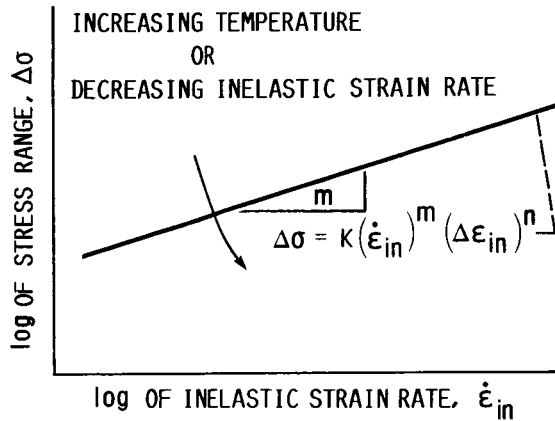


FIGURE 9.- INELASTIC STRAIN-RATE DEPENDENCY OF CYCLIC STRESS-STRAIN CURVE FOR CONSTANT STRAINRANGE AND TEMPERATURE.

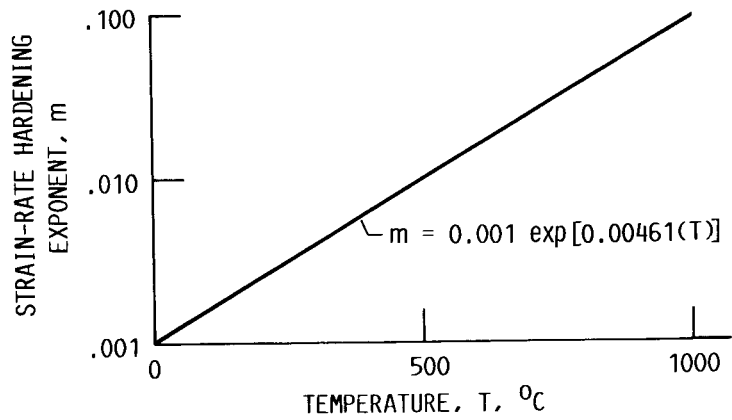


FIGURE 10.- ASSUMED TEMPERATURE DEPENDENCE OF STRAIN-RATE HARDENING EXPONENT  $m$ .

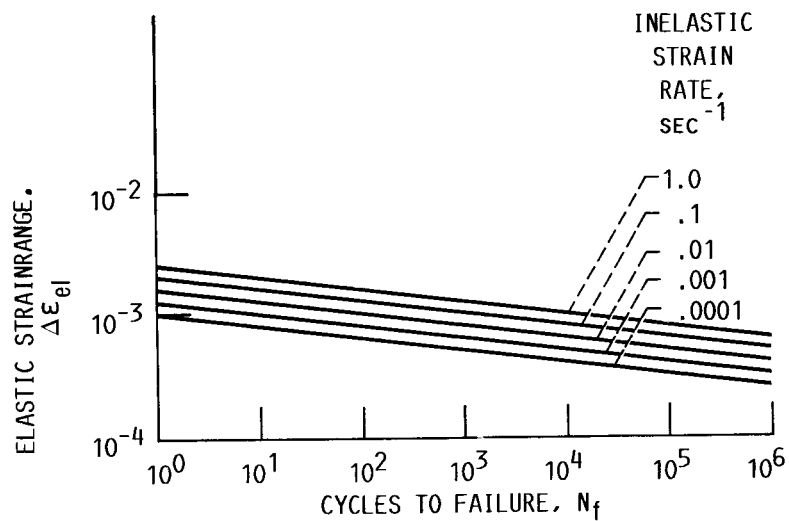


FIGURE 11.- TIME-DEPENDENT ELASTIC STRAINRANGE VERSUS CYCLIC LIFE AS FUNCTION OF INELASTIC STRAIN RATE. TEMPERATURE, 1000 °C;  $\Delta \epsilon_{el} = A(N_f)^a$ ;  $A = (KB^n/E)(\dot{\epsilon}_{in})^m$ .

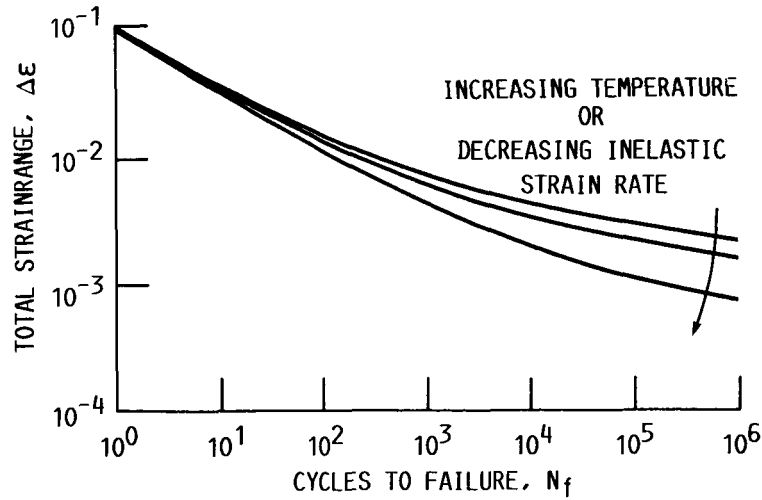


FIGURE 12. - TOTAL STRAINRANGE VERSUS CYCLIC LIFE RELATIONS FOR HYPOTHETICAL MATERIAL FOR CONSTANT, INELASTIC STRAIN RATE AND TEMPERATURE. ELASTIC CONTRIBUTION IS TIME AND TEMPERATURE DEPENDENT.

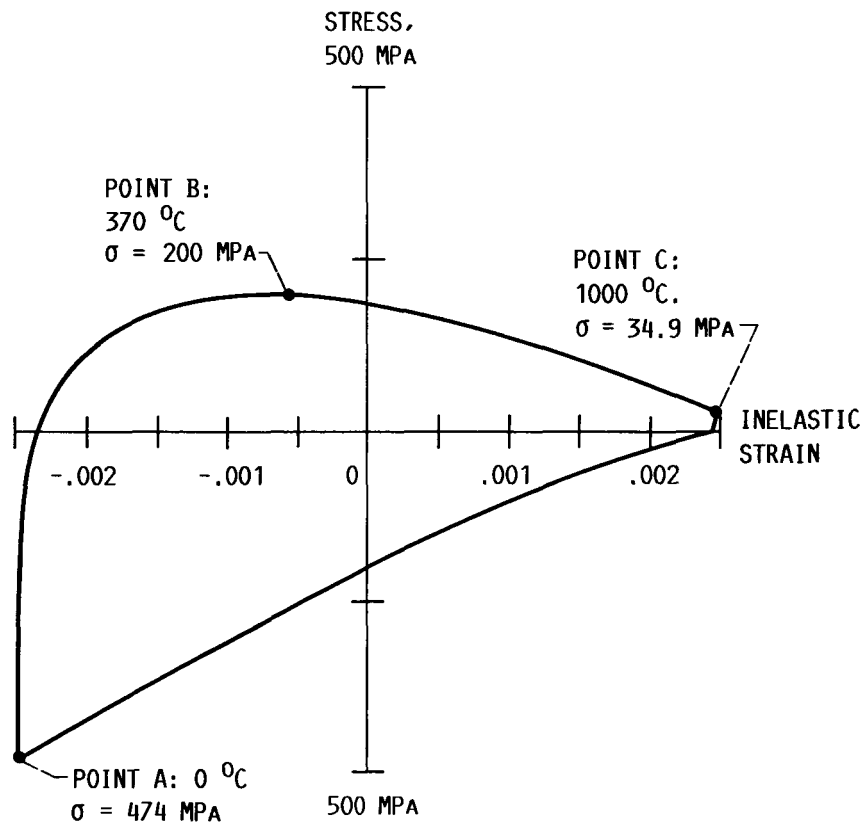


FIGURE 13.- TMF HYSTERESIS LOOP. INPHASE, INELASTIC STRAINRANGE, 0.005; MAXIMUM TEMPERATURE, 1000 °C; MINIMUM TEMPERATURE, 0 °C; INELASTIC STRAIN RATE, 0.0001/SEC.



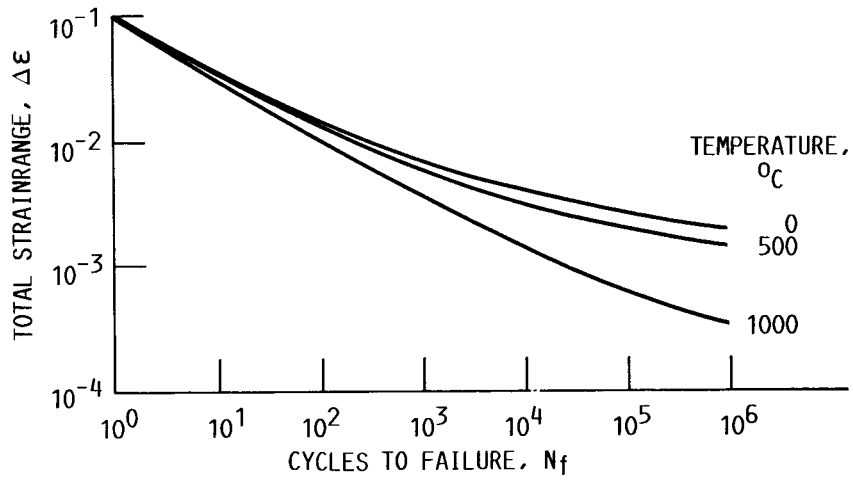


FIGURE 14.- TOTAL STRAINRANGE VERSUS CYCLIC LIFE CURVES FOR MINIMUM, AVERAGE, AND MAXIMUM TEMPERATURES OF TMF CYCLE. INELASTIC STRAIN RATE, 0.0001/SEC.

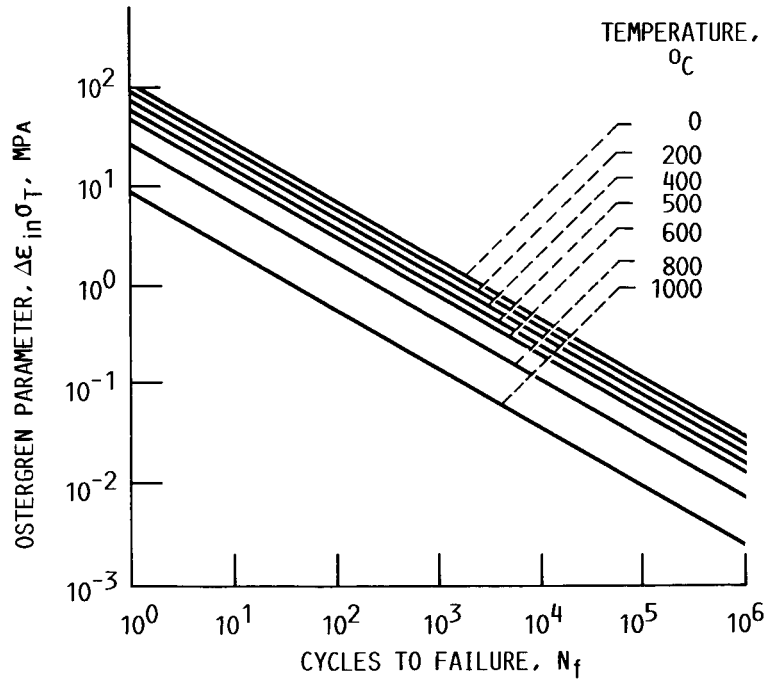


FIGURE 15.- ISOTHERMAL FAILURE CRITERION USING OSTERGREN'S TIME-DEPENDENT PRODUCT FORM. INELASTIC STRAIN RATE, 0.0001/SEC.

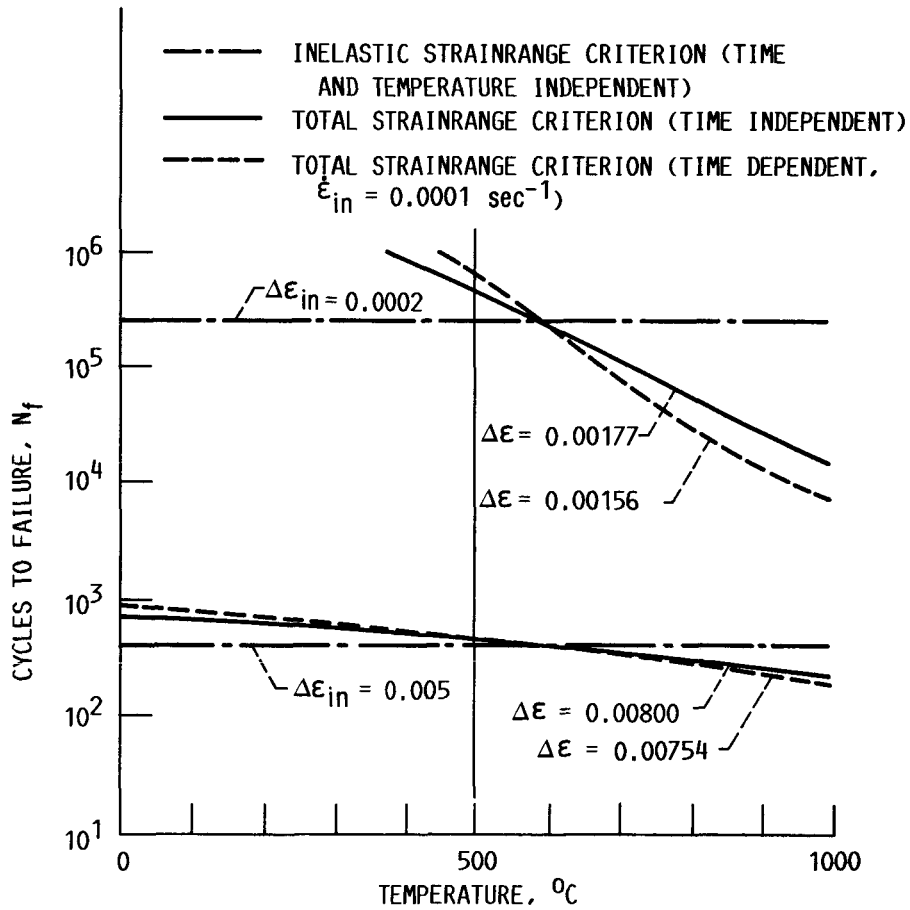


FIGURE 16.- PREDICTIONS OF TMF CYCLIC LIFETIME BASED ON ISOTHERMAL, INELASTIC STRAINRANGE AND TOTAL STRAINRANGE CRITERIA. PREDICTIONS SHOWN AS FUNCTION OF TEMPERATURE FOR 0.0002 AND 0.005 INELASTIC RANGES. CORRESPONDING TOTAL STRAINRANGES OF TMF CYCLE ARE INDICATED.

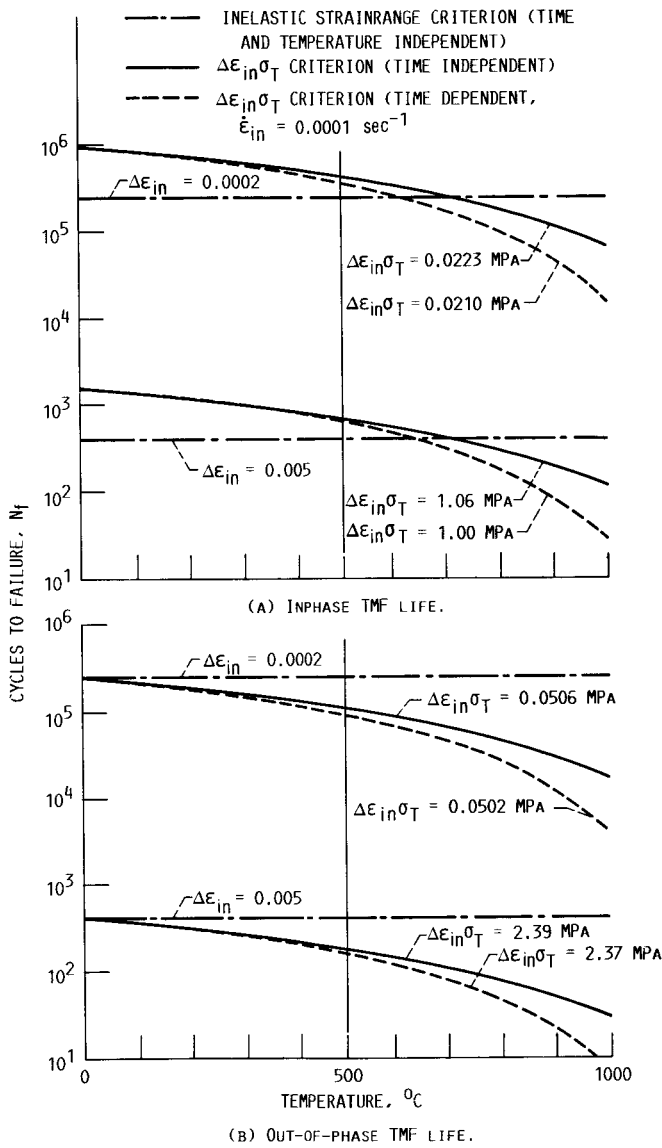


FIGURE 17. - PREDICTIONS OF THERMOMECHANICAL FATIGUE (TMF) LIFE BASED ON ISOTHERMAL FAILURE CRITERION OF OSTERGREN ( $\Delta\epsilon_{in}\sigma_T$ ). PREDICTIONS SHOWN AS FUNCTION OF TEMPERATURE FOR 0.0002 AND 0.005 INELASTIC RANGES.

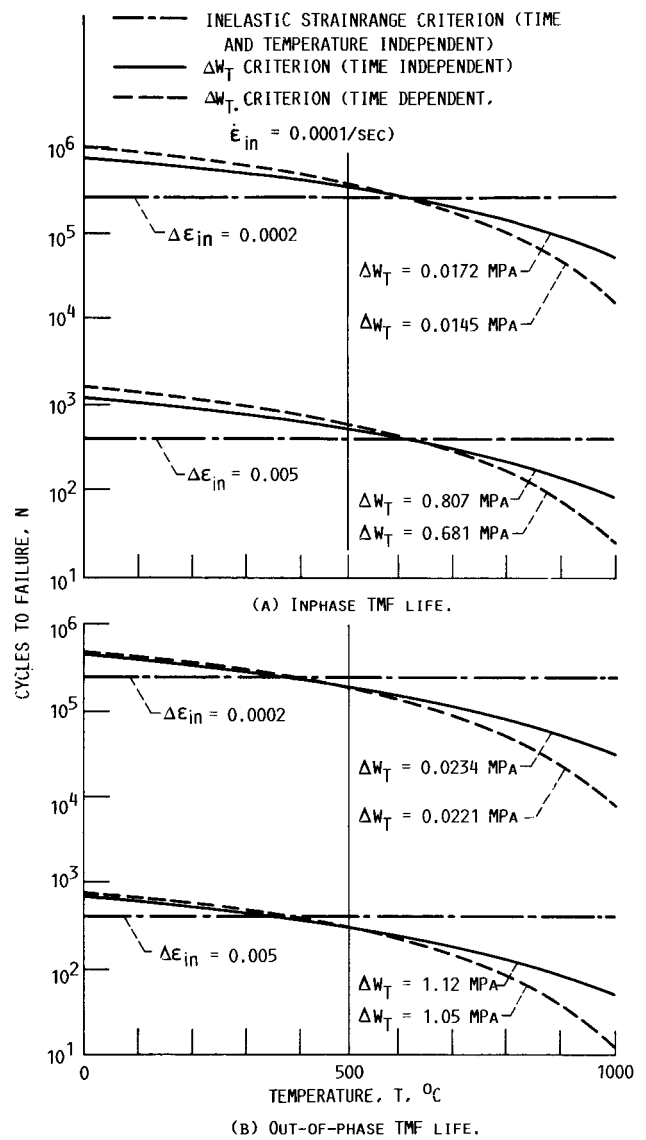


FIGURE 18. - PREDICTIONS OF THERMOMECHANICAL FATIGUE (TMF) LIFE BASED ON ISOTHERMAL FAILURE CRITERION OF OSTERGREN ( $\Delta W_T$ ). PREDICTIONS SHOWN AS FUNCTION OF TEMPERATURE FOR 0.0002 AND 0.005 INELASTIC RANGES.

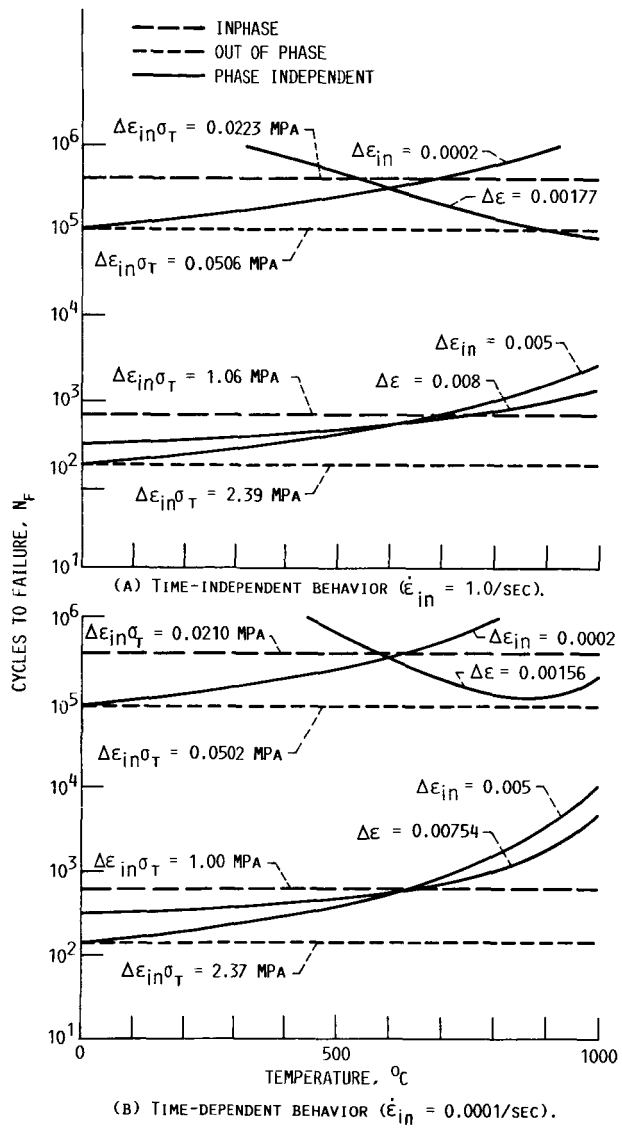


FIGURE 19. - PREDICTIONS OF THERMOMECHANICAL FATIGUE (TMF) LIFE BASED ON ISOTHERMAL FAILURE CRITERIA OF MANSON-COFFIN, TOTAL STRAINRANGE, AND ÖSTERGREN ( $\Delta\epsilon_{in}\sigma_T$ ). ÖSTERGREN CRITERION ASSUMED TO BE TEMPERATURE INDEPENDENT. PREDICTIONS MADE FOR 0.0002 AND 0.005 INELASTIC STRAINRANGES.

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16. Abstract <p>The isothermal and thermomechanical fatigue (TMF) crack initiation response of a hypothetical material was analyzed. Expected thermomechanical behavior was evaluated numerically based on simple, isothermal, cyclic stress-strain - time characteristics and on strainrange versus cyclic life relations that have been assigned to the material. The attempt was made to establish basic minimum requirements for the development of a physically accurate TMF life-prediction model. A worthy method must be able to deal with the simplest of conditions: that is, those for which thermal cycling, per se, introduces no damage mechanisms other than those found in isothermal behavior. Under these assumed conditions, the TMF life should be obtained uniquely from known isothermal behavior. The ramifications of making more complex assumptions will be dealt with in future studies. Although analyses are only in their early stages, considerable insight has been gained in understanding the characteristics of several existing high-temperature life-prediction methods. The present work indicates that the most viable damage parameter is based on the inelastic strainrange.</p>					
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