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# A Rapid Method for the Computation of Equilibrium Chemical Composition of Air to 15000 K 

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## Summary

A rapid computational method has been developed to determine the equilibrium chemical composition of air to 15000 K . Eleven chemically reacting species- $\mathrm{O}_{2}, \mathrm{~N}_{2}, \mathrm{O}, \mathrm{NO}, \mathrm{N}, \mathrm{NO}^{+}, \mathrm{e}^{-}, \mathrm{N}^{+}$, $\mathrm{O}^{+}, \mathrm{Ar}$, and $\mathrm{Ar}^{+}$-are included. The method involves the algebraic combination of the equilibrium, elemental mass-balance, and charge-neutrality equations and solves the resulting equation by an iterative method. Computational speeds for the determination of the equilibrium chemical composition are significantly faster than those of the often-used free-energy-minimization procedure. Data are also included from which the thermodynamic properties of air can be computed. Additionally, a listing of the computer program and a set of sample results are included.

## Introduction

The application of computational fluid dynamics (CFD) techniques to hypersonic atmospheric flight vehicles requires methods that rapidly compute the composition of air over a wide range of temperatures and densities. Most of the available methods are very general in nature and employ the free-energy-minimization procedure. The generality of any method may be desirable if only a few calculations are to be carried out. But in a CFD code for hypersonic-flow-field computations for which the chemistry code is a subroutine executed a very large number of times, a large fraction of the computational time could be spent in executing the chemistry subroutine alone unless this subroutine is very efficient. For such an application it is evident that it would be beneficial to forego a certain amount of generality in favor of computational speed. The overall gain in the computational time, of course, depends on the particular application.

A method of combining the chemical equilibrium equations and the mass constraint equations algebraically and solving the resulting equation numerically was suggested by Erickson, Kemper, and Allison (ref. 1). Recently, Erickson and Prabhu (ref. 2) applied this method to hydrocarbon combustion products. Smith, Erickson, and Eastwood (ref. 3) also used a similar method for high-temperature equilibrium air. The present method, like that used by Smith et al., divides the temperature-density domain into four regions, each with a limited set of major species and a different computational scheme. Although the present work follows the same general approach used by Smith et al., it differs in that a different set and a smaller number of major species are selected to form the primary iteration equations, and
all the minor species are included in a series of secondary computations. The present approach leads to a more rapid computation and ensures continuity of results between adjacent regions. Eleven species are included, and air is assumed to be a reacting mixture of ideal gases.

The following sections describe the four models, their governing equations, and the solution procedures. A computer program to determine the equilibrium composition of air over a wide range of temperature and density conditions has been written with the present scheme. A listing of this code and a set of results are included. Computational speed for the present program is significantly faster than that of conventional free-energy-minimization routines.

## Symbols

| $a_{i, j}$ | coefficients of the approximating polynomials for thermodynamic functions |
| :---: | :---: |
| $b_{n}$ | coefficients of the polynomials |
| $c_{p, i}$ | specific heat at constant pressure of $i$ th species, J/mol-K |
| $E_{i}^{0}$ | molar internal energy of $i$ th species, $\mathrm{J} / \mathrm{mol}$ |
| $e$ | specific internal energy of the mixture, $\mathrm{J} / \mathrm{kg}$ |
| $G_{i}^{0}$ | molar Gibbs energy of $i$ th species, $\mathrm{J} / \mathrm{mol}$ |
| $H_{i}^{0}$ | molar enthalpy of $i$ th species, $\mathrm{J} / \mathrm{mol}$ |
| $h$ | specific enthalpy of the mixture, $\mathrm{J} / \mathrm{kg}$ |
| $K_{j}$ | equilibrium constants in terms of mole numbers ( $j=1,2, \ldots, 7$ ) |
| $K_{p, j}$ | equilibrium constants in terms of partial pressure ( $j=1,2, \ldots, 7$ ) |
| $N(\varepsilon)$ | number of iterations for an accuracy $\varepsilon$ |
| $p$ | pressure of the mixture, Pa |
| $p^{0}$. | standard-state pressure, 101325 Pa |
| $R$ | universal gas constant, $8.314 \mathrm{~J} / \mathrm{mol}-\mathrm{K}$ |
| $S_{i}^{0}$ | molar entropy of $i$ th species, $\mathrm{J} / \mathrm{mol}-\mathrm{K}$ |
| $s$ | specific entropy of the mixture, J/kg-K |
| $T$ | temperature of the mixture, K |
| $T_{\text {ref }}$ | reference temperature for enthalpy, 0 K |
| $t$ | computational time, $t_{0}+t_{i} N(\varepsilon), \mu \mathrm{s}$ |


| $t_{i}$ | time per iteration, $\mu \mathrm{s}$ |
| :---: | :---: |
| $t_{0}$ | constant time outside the iteration loop, $\mu \mathrm{s}$ |
| $\Delta G^{0}$ | Gibbs energy for a reaction |
| $\Delta H_{f, i}^{0}$ | enthalpy of formation of $i$ th species at $T_{\text {ref }}, \mathrm{J} / \mathrm{mol}$ |
| $\Delta N$ | increase in mole number in a reaction |
| $\varepsilon$ | accuracy in mole fraction |
| $\rho$ | density of the mixture, $\mathrm{kg} / \mathrm{m}^{3}$ |
| $\sigma$ | total mole number of the mixture, $\mathrm{mol} / \mathrm{kg}$ |
| $\sigma_{i}$ | mole number of $i$ th species $(i=1,2, \ldots, 11), \mathrm{mol} / \mathrm{kg}$ |
| $\sigma_{\mathrm{O}}, \sigma_{\mathrm{N}}, \sigma_{\mathrm{A}}$ | equivalent mole numbers of oxygen, nitrogen, and argon, mol $/ \mathrm{kg}$ |

## Analysis

In the present analysis, low-temperature air is assumed to consist of $\mathrm{O}_{2}, \mathrm{~N}_{2}$, and Ar in molar proportions of $20.96: 78.11: 0.93$. Eleven chemically reacting species are assumed to be present in equilibrium in air $-\mathrm{O}_{2}, \mathrm{~N}_{2}, \mathrm{O}, \mathrm{NO}, \mathrm{N}, \mathrm{NO}^{+}, \mathrm{e}^{-}, \mathrm{N}^{+}, \mathrm{O}^{+}$, Ar , and $\mathrm{Ar}^{+}$-and are designated by the respective indices $i=1,2, \ldots, 11$.

Seven independent chemical reactions involving the 11 reacting chemical species are considered. These are as follows:

$$
\begin{align*}
\mathrm{O}_{2} & =2 \mathrm{O}  \tag{1}\\
\mathrm{~N}_{2}+\mathrm{O}_{2} & =2 \mathrm{NO}  \tag{2}\\
\mathrm{~N}_{2} & =2 \mathrm{~N}  \tag{3}\\
\mathrm{NO}^{+}+\mathrm{e}^{-} & =\mathrm{NO}  \tag{4}\\
\mathrm{~N}^{+}+\mathrm{e}^{-} & =\mathrm{N}  \tag{5}\\
\mathrm{O}^{+}+\mathrm{e}^{-} & =\mathrm{O}  \tag{6}\\
\mathrm{Ar}^{+}+\mathrm{e}^{-} & =\mathrm{Ar} \tag{7}
\end{align*}
$$

The corresponding 7 nonlinear equilibrium equations involving the unknown mole numbers of the 11 species and the 7 equilibrium constants are the following:

$$
\begin{align*}
& K_{1}=\sigma_{3}^{2} / \sigma_{1}  \tag{1}\\
& K_{2}=\sigma_{4}^{2} / \sigma_{1} \sigma_{2}  \tag{2}\\
& K_{3}=\sigma_{5}^{2} / \sigma_{2}  \tag{3}\\
& K_{4}=\sigma_{4} / \sigma_{6} \sigma_{7} \tag{4}
\end{align*}
$$

$$
\begin{align*}
& K_{5}=\sigma_{5} / \sigma_{8} \sigma_{7}  \tag{5}\\
& K_{6}=\sigma_{3} / \sigma_{9} \sigma_{7}  \tag{6}\\
& K_{7}=\sigma_{10} / \sigma_{11} \sigma_{7} \tag{7}
\end{align*}
$$

The equilibrium constants $K_{j}$ in terms of mole numbers are related to the equilibrium constants $K_{p, j}$ in terms of partial pressures by the following relation:

$$
K_{j}=K_{p, j}\left(p^{0} / \rho R T\right)^{\Delta N}
$$

where $\Delta N$ is the increase in the number of moles when the reaction proceeds from reactants to products as written for reactions (1) to (7). Note that

$$
\begin{aligned}
\Delta N & =1 \text { for reactions }(1) \text { and }(3) \\
& =-1 \text { for reactions }(4),(5),(6), \text { and }(7) \\
& =0 \text { for reaction }(2)
\end{aligned}
$$

Also, $K_{j}$ has the units of moles per kilogram raised to the $\Delta N$ power, and $K_{p, j}$ has units of atmospheres raised to $\Delta N$ power. The equilibrium constants $K_{1}$ to $K_{7}$ are determined through the use of Gibbs energies of formation for each of the chemical species participating in the chemical reactions.

The mass-balance equations for oxygen, nitrogen, and argon provide three linear equations involving the unknown values of $\sigma_{i}(i=1,2, \ldots, 11)$ and the known values of $\sigma_{\mathrm{N}}, \sigma_{\mathrm{O}}$, and $\sigma_{\mathrm{A}}$.

$$
\begin{gather*}
2 \sigma_{1}+\sigma_{3}+\sigma_{4}+\sigma_{6}+\sigma_{9}=\sigma_{\mathrm{O}}  \tag{8}\\
2 \sigma_{2}+\sigma_{4}+\sigma_{5}+\sigma_{6}+\sigma_{8}=\sigma_{\mathrm{N}}  \tag{9}\\
\sigma_{10}+\sigma_{11}=\sigma_{\mathrm{A}} \tag{10}
\end{gather*}
$$

In addition, the charge-neutrality condition of the mixture provides the following linear equation:

$$
\begin{equation*}
\sigma_{6}+\sigma_{8}+\sigma_{9}+\sigma_{11}=\sigma_{7} \tag{11}
\end{equation*}
$$

In principle, it is possible to combine equations (1) to (11) into a single equation in one unknown and solve by some iteration procedure. The basic approach employed in the present analysis, however, is to identify 4 regions in the temperaturedensity domain such that in each of these regions, only some of the 11 species are found in abundance. These species are referred to as the major species and the rest as the minor species in that region. The nonlinear equations governing the major species and the appropriate linear equations are combined into a single equation, with the assumption that the minor species are adjustable constants. Such governing equations are solved by an iterative procedure updating the minor species at each iteration step. (This approach
is essentially a combination of Newton and fixedpoint iteration schemes.) Although some of the species are treated as the major species and the others as the minor species, these models for the 4 regions compute the correct values of all the 11 reacting chemical species considered. These models also have some overlapping regions of applicability, and the choice of one model over the other for a given condition is entirely based on the criterion of minimum computational time.

The following sections discuss these four models. In each case the governing equations and the solution procedures are described.

## Model I

At very low temperatures, the only major species are $\mathrm{O}_{2}, \mathrm{~N}_{2}$, and Ar. Among the remaining species, those with a neutral charge (i.e., NO, N, and O) are small and are treated as minor species; the ionized species (i.e., $\mathrm{NO}^{+}, \mathrm{e}^{-}, \mathrm{N}^{+}, \mathrm{O}^{+}$, and $\mathrm{Ar}^{+}$) are in very small quantities and hence are denoted trace species. Model I is a very simple model that considers this case. The mole numbers of the minor species and of the trace species are assumed to be zero to start with, and the major species, namely $\mathrm{O}_{2}, \mathrm{~N}_{2}$, and Ar , are computed from the mass-balance equations. The equilibrium equations then determine the minor species, which are used to update the major species. The trace species are computed after this process has converged. The steps involved in this simple iteration scheme are given in the following:

1. Assume $\sigma_{1}^{\prime}=\sigma_{\mathrm{O}} / 2, \sigma_{2}^{\prime}=\sigma_{\mathrm{N}} / 2$, and $\sigma_{10}=\sigma_{\mathrm{A}}$.
2. Compute the minor species:

$$
\begin{aligned}
\sigma_{3} & =\sqrt{K_{1} \sigma_{1}^{\prime}} \\
\sigma_{5} & =\sqrt{K_{3} \sigma_{2}^{\prime}} \\
\sigma_{4} & =K_{e} \sigma_{5} \sigma_{3}
\end{aligned}
$$

3. Update the major species:

$$
\begin{aligned}
& \sigma_{1}=\left(\sigma_{\mathrm{O}}-\sigma_{3}-\sigma_{4}\right) / 2 \\
& \sigma_{2}=\left(\sigma_{\mathrm{N}}-\sigma_{4}-\sigma_{5}\right) / 2
\end{aligned}
$$

4. Check and go to step 5 if the results have converged. If they have not converged, set $\sigma_{1}^{\prime}=\sigma_{1}$ and $\sigma_{2}^{\prime}=\sigma_{2}$ and go to step 2.
5. Compute the trace species:

$$
\begin{aligned}
\sigma_{7} & =\sqrt{\left(\sigma_{4} / K_{4}\right)+\left(\sigma_{5} / K_{5}\right)+\left(\sigma_{3} / K_{6}\right)+\left(\sigma_{10} / K_{7}\right)} \\
\sigma_{6} & =\sigma_{4} /\left(K_{4} \sigma_{7}\right) \\
\sigma_{8} & =\sigma_{5} /\left(K_{5} \sigma_{7}\right) \\
\sigma_{9} & =\sigma_{3} /\left(K_{6} \sigma_{7}\right) \\
\sigma_{11} & =\sigma_{10} /\left(K_{7} \sigma_{7}\right)
\end{aligned}
$$

As one might expect, this iteration scheme converges in a couple of iterations at low temperatures, where the minor species are present in extremely small quantities.

## Model II

This model is applicable for temperatures which are high enough for dissociation but not high enough for significant ionization. Under this condition, air is assumed to be a mixture of $\mathrm{O}_{2}, \mathrm{~N}_{2}, \mathrm{O}, \mathrm{NO}, \mathrm{N}$, and Ar , with the ionized species present only in small quantities. The derivation of the governing equation and the method employed in solving it are as follows. The equations governing the six major species are equations (1) to (3) and (8) to (10). With the mole numbers of the ionized species $\sigma_{6}$ to $\sigma_{9}$ and $\sigma_{11}$ set equal to pseudo constants (equal to zero at the start), these equations are combined into a single equation involving one unknown, namely $\sigma_{3}$. The steps are as follows. Equations (1), (2), and (3) yield

$$
\sigma_{4} / \sigma_{3} \sigma_{5}=\sqrt{K_{2} / K_{1} K_{3}}=K_{e}
$$

Therefore, $\sigma_{4}=K_{e} \sigma_{3} \sigma_{5}$. Equations (1) and (3) give

$$
\begin{aligned}
& \sigma_{1}=\sigma_{3}^{2} / K_{1} \\
& \sigma_{2}=\sigma_{5}^{2} / K_{3}
\end{aligned}
$$

Substituting the above into equation (8) and solving for $\sigma_{5}$ in terms of $\sigma_{3}$ gives

$$
\sigma_{5}=\left[\sigma_{\mathrm{O}}^{\prime}-\left(2 \sigma_{3}^{2} / K_{1}\right)-\sigma_{3}\right] / K_{e} \sigma_{3}
$$

Finally, upon substituting for $\sigma_{4}, \sigma_{2}$, and $\sigma_{5}$ in equation (9) and simplifying, we obtain the following quartic equation with $\sigma_{3}$ as the unknown:

$$
\sum_{n=0}^{4} b_{n} \sigma_{3}^{n}=0
$$

where

$$
\begin{aligned}
b_{4} & =\left(8-2 K_{2}\right) / K_{1} \\
b_{3} & =8-K_{2}-2 K_{3} K_{e} \\
b_{2} & =-8 \sigma_{\mathrm{O}}^{\prime}+2 K_{1}+\left(\sigma_{\mathrm{O}}^{\prime}-\sigma_{\mathrm{N}}^{\prime}\right) K_{2}-K_{1} K_{3} K_{e} \\
b_{1} & =\left(-4+K_{3} K_{e}\right) K_{1} \sigma_{\mathrm{O}}^{\prime} \\
\text { and } & \\
b_{0} & =2 K_{1} \sigma_{\mathrm{O}}^{\prime 2}
\end{aligned}
$$

Note that $\sigma_{\mathrm{O}}^{\prime}=\sigma_{\mathrm{O}}-\sigma_{6}-\sigma_{9}$ and $\sigma_{\mathrm{N}}^{\prime}=\sigma_{\mathrm{N}}-\sigma_{6}-$ $\sigma_{8}$. The equation for $\sigma_{3}$ is solved by the Newton iteration scheme. The initial value for the unknown $\sigma_{3}$ is determined through solution of the following equations:

$$
\begin{aligned}
2 \sigma_{1}+\sigma_{3}+\sigma_{4}^{\prime} & =\sigma_{\mathrm{O}} \\
\sigma_{1} & =\sigma_{3}^{2} / K_{1} \\
\sigma_{4}^{\prime} / \sigma_{3} & =\sqrt{2 K_{1} / \sigma_{\mathrm{N}} K_{2}}
\end{aligned}
$$

where $\sigma_{4}^{\prime}$ is an approximate value of $\sigma_{4}$. These equations imply that all the oxygen is in the species $\mathrm{O}_{2}, \mathrm{O}$, and NO , which is a good approximation for the initial guess in this model.

The following steps are taken in the solution procedure:

1. Preset $\sigma_{6}=\sigma_{8}=\sigma_{9}=0$ and $\sigma_{10}=\sigma_{\mathrm{A}}$.
2. Compute $\sigma_{3}$.
3. Determine $\sigma_{\mathrm{O}}^{\prime}=\sigma_{\mathrm{O}}-\sigma_{6}-\sigma_{9}$
and $\sigma_{\mathrm{N}}^{\prime}=\sigma_{\mathrm{N}}-\sigma_{6}-\sigma_{8}$.
4. Determine $b_{n}(n=0,1, \ldots, 4)$.
5. Compute $\Delta \sigma_{3}=-F / F^{\prime}$, where

$$
\begin{aligned}
F & =\sum_{n=0}^{4} b_{n} \sigma_{3}^{n} \\
F^{\prime} & =\sum_{n=1}^{4} n b_{n} \sigma_{3}^{n-1}
\end{aligned}
$$

6. Update $\sigma_{3}$ and compute the major species:

$$
\begin{aligned}
& \sigma_{3}=\sigma_{3}+\Delta \sigma_{3} \\
& \sigma_{5}=\left[\sigma_{\mathrm{O}}-\left(2 \sigma_{3}^{2} / K_{1}\right)-\sigma_{3}\right] / K_{e} \sigma_{3} \\
& \sigma_{4}=K_{e} \sigma_{3} \sigma_{5} \\
& \sigma_{2}=\left(\sigma_{\mathrm{N}}^{\prime}-\sigma_{4}-\sigma_{5}\right) / 2 \\
& \sigma_{1}=\left(\sigma_{\mathrm{O}}^{\prime}-\sigma_{4}-\sigma_{3}\right) / 2
\end{aligned}
$$

7. Compute the minor species:

$$
\begin{aligned}
\sigma_{7} & =\sqrt{\left(\sigma_{4} / K_{4}\right)+\left(\sigma_{5} / K_{5}\right)+\left(\sigma_{3} / K_{6}\right)+\left(\sigma_{10} / K_{7}\right)} \\
\sigma_{6} & =\sigma_{4} / K_{4} \sigma_{7} \\
\sigma_{8} & =\sigma_{5} / K_{5} \sigma_{7} \\
\sigma_{9} & =\sigma_{3} / K_{6} \sigma_{7} \\
\sigma_{11} & =\sigma_{10} / K_{7} \sigma_{7} \\
\sigma_{10} & =\sigma_{\mathrm{A}}-\sigma_{11}
\end{aligned}
$$

8. Check and stop if the results are converged. If they are not converged, go to step 3.
As one might expect, the range of applicability of model II overlaps that of model I at the lower temperatures (at about 800 K ). Similarly, the applicability of this model at higher temperatures overlaps that of model III, which is discussed in the next section.

## Model III

This model is applicable under certain temperature and density conditions when oxygen is almost completely dissociated and not much ionization has taken place. The major species in this case are $O$, $\mathrm{N}_{2}, \mathrm{~N}$, and Ar , and the remaining seven species are treated as minor species. The equations governing the major species (eqs. (3) and (8) to (10)) are combined into a single equation in one unknown, namely $\sigma_{5}$. This turns out to be a quadratic equation for which the coefficients involve the unknown mole numbers of the minor species. As in the previous case, these are treated as pseudo constants while the quadratic equation is solved. After the quadratic is solved for $\sigma_{5}$, the mole numbers of the minor species are determined and the coefficients of the quadratic equation are updated. This process is continued until a converged solution is obtained. The following steps describe this solution procedure:

1. Assume $\sigma_{3}^{\prime}=\sigma_{\mathrm{O}}$ and $\sigma_{10}=\sigma_{\mathrm{A}}$ and set $\sigma_{4}=\sigma_{6}=\sigma_{8}=0$ initially.
2. Compute $\sigma_{\mathrm{N}}^{\prime}=\sigma_{\mathrm{N}}-\sigma_{4}-\sigma_{6}-\sigma_{8}$.
3. Determine the coefficients $b_{n}(n=0,1$, and 2$)$ :

$$
\begin{aligned}
& b_{0}=-K_{3} \sigma_{\mathrm{N}}^{\prime} / 2 \\
& b_{1}=K_{3} / 2 \\
& b_{2}=1
\end{aligned}
$$

4. Solve the quadratic equation $b_{2} \sigma_{5}^{2}+b_{1} \sigma_{5}+b_{0}=0$ for $\sigma_{5}$.
5. Determine the unknown mole numbers:

$$
\begin{aligned}
\sigma_{2} & =\left(\sigma_{\mathrm{N}}^{\prime}-\sigma_{5}\right) / 2 \\
\sigma_{1} & =\sigma_{3}^{\prime 2} / K_{1} \\
\sigma_{4} & =K_{e} \sigma_{3}^{\prime} \sigma_{5} \\
\sigma_{7} & =\sqrt{\left(\sigma_{3}^{\prime} / K_{6}\right)+\left(\sigma_{4} / K_{4}\right)+\left(\sigma_{5} / K_{5}\right)+\left(\sigma_{10} / K_{7}\right)} \\
\sigma_{6} & =\sigma_{4} / K_{4} \sigma_{7} \\
\sigma_{8} & =\sigma_{5} / K_{5} \sigma_{7} \\
\sigma_{9} & =\sigma_{3}^{\prime} / K_{6} \sigma_{7} \\
\sigma_{11} & =\sigma_{10} / K_{7} \sigma_{7} \\
\sigma_{10} & =\sigma_{\mathrm{A}}-\sigma_{11}
\end{aligned}
$$

Finally,

$$
\sigma_{3}=\sigma_{\mathrm{O}}-2 \sigma_{1}-\sigma_{4}-\sigma_{6}-\sigma_{9} .
$$

6. Check and stop if results are converged. If they are not converged, set $\sigma_{3}^{\prime}=\sigma_{3}$ and go to step 2.

## Model IV

This model considers the very-high-temperature case, which is dominated by atomic and ionic species. The major species in this model are $\mathrm{O}, \mathrm{N}, \mathrm{e}^{-}, \mathrm{N}^{+}$, $\mathrm{O}^{+}$, and Ar , and the others are treated as minor species. When equations (5) to (11) governing the major species are combined, they reduce to a cubic equation involving one unknown, namely $\sigma_{7}$. The minor species mole numbers that appear in the coefficients of this cubic equation are treated as pseudo constants. Starting with a value of zero for the mole numbers of the minor species, we solve the cubic equation. Next the mole numbers of the remaining major and of all the minor species are computed, and the coefficients of the cubic equation are updated. This iteration process is continued until the results are converged. The following steps describe the iteration procedure:

1. Preset $\sigma_{1}=\sigma_{2}=\sigma_{4}=\sigma_{6}=\sigma_{11}=0$.
2. Determine $\sigma_{\mathrm{O}}^{\prime}=\sigma_{\mathrm{O}}-2 \sigma_{1}-\sigma_{4}-\sigma_{6}$ and

$$
\sigma_{\mathrm{N}}^{\prime}=\sigma_{\mathrm{N}}-2 \sigma_{2}-\sigma_{4}-\sigma_{6}
$$

3. Determine $b_{n}(n=0,1, \ldots, 3)$ :

$$
\begin{aligned}
& b_{0}=-\sigma_{\mathrm{N}}^{\prime}-\sigma_{\mathrm{O}}^{\prime}-\sigma_{11}-\sigma_{6} \\
& b_{1}=1-\sigma_{\mathrm{N}}^{\prime} K_{6}-\sigma_{\mathrm{O}}^{\prime} K_{5}-\left(\sigma_{11}+\sigma_{6}\right)\left(K_{5}+K_{6}\right) \\
& b_{2}=K_{5}+K_{6}-\left(\sigma_{11}+\sigma_{6}\right) K_{5} K_{6} \\
& b_{3}=K_{5} K_{6}
\end{aligned}
$$

4. Compute $\Delta \sigma_{7}=-F / F^{\prime}$, where

$$
\begin{aligned}
F & =\sum_{n=0}^{3} b_{n} \sigma_{7}^{n} \\
F^{\prime} & =\sum_{n=1}^{3} n b_{n} \sigma_{7}^{n-1}
\end{aligned}
$$

5. Update $\sigma_{7}$ and compute the major species:

$$
\begin{aligned}
\sigma_{7} & =\sigma_{7}+\Delta \sigma_{7} \\
\sigma_{3} & =\sigma_{\mathrm{O}}^{\prime} \sigma_{7} K_{6} /\left(1+K_{6} \sigma_{7}\right) \\
\sigma_{5} & =\sigma_{\mathrm{N}}^{\prime} \sigma_{7} K_{5} /\left(1+K_{5} \sigma_{7}\right) \\
\sigma_{11} & =\sigma_{\mathrm{A}} /\left(1+K_{7} \sigma_{7}\right) \\
\sigma_{10} & =\sigma_{\mathrm{A}}-\sigma_{11} \\
\sigma_{9} & =\sigma_{3} / K_{6} \sigma_{6} \\
\sigma_{8} & =\sigma_{5} / K_{5} \sigma_{7}
\end{aligned}
$$

6. Compute the minor species:

$$
\begin{aligned}
& \sigma_{1}=\sigma_{3}^{2} / K_{1} \\
& \sigma_{2}=\sigma_{5}^{2} / K_{1} \\
& \sigma_{4}=K_{e} \sigma_{3} \sigma_{5} \\
& \sigma_{6}=\sigma_{4} / K_{4} \sigma_{7}
\end{aligned}
$$

7. Check and stop if results are converged. If they are not converged, go to step 2.

Table I gives a summary of the major and minor species in the four temperature ranges considered here. As pointed out earlier, although these four models are developed assuming a certain set of major species in each, they have overlapping areas of applicability, and the choice of one model over the other for a given temperature and density is based strictly on minimum computational time. Figure 1 shows the regions in the temperature-density domain, where the four models are applicable and give results in minimum time.

## Thermodynamic Data

Computation of thermodynamic properties for each of the 11 species as well as for the mixture has been included in the program. The data for the determination of the thermodynamic properties were obtained from several sources. The data for the molecular species were obtained directly from the tables of reference 4. The heat capacities for the monatomic species were computed by Bonnie J. McBride of NASA Lewis Research Center using the appropriate statistical-thermodynamic relations and
the data from references 5,6 , and 7 . The specific heat data for all the species were processed, also by McBride, using the least-squares method of curve fitting employing the approximating polynomials of the following forms:

$$
\begin{gathered}
c_{p, i} / R=\sum_{j=1}^{5} a_{i, j} T^{j-1} \\
H_{i}^{0} /(R T)=\sum_{j=1}^{5} a_{i, j} T^{j-1} / j+a_{i, 6} / T \\
S_{i}^{0} / R=\sum_{j=2}^{5} a_{i, j} T^{j-1} /(j-1)+a_{i, j} \ln T+a_{i, 7} \\
G_{i}^{0} /(R T)=H_{i}^{0} /(R T)-S_{i}^{0} / R
\end{gathered}
$$

The coefficients $a_{i, 6}$ and $a_{i, 7}(i=1,2, \ldots, 11)$ are the integration constants. The values of $a_{i, 6}$ were determined from the enthalpy of formation (taken from the JANAF Thermochemical Tables published by Dow Chemical Co.) for all the species at the reference temperature of 0 K . The values of $a_{i, 7}$ were determined from the values of entropy at $T=298.15 \mathrm{~K}$.

The coefficients $a_{i, 1}$ to $a_{i, 7}(i=1,2, \ldots, 11)$ are given in table II. The entire temperature range from 200 to 15000 K is divided into five smaller ranges; these ranges are 200 to $800 \mathrm{~K}, 800$ to 3000 K , 3000 to $6000 \mathrm{~K}, 6000$ to 10000 K , and 10000 to 15000 K . The curves for the approximating leastsquares polynomials are forced to pass through common points at the junctions between the five temperature ranges, thereby ensuring continuity in the values at the junction points.

The standard-state Gibbs energy for the 11 reacting species is required for the determination of the equilibrium constants for the 7 reactions. Equilibrium constants $K_{p, j}$ for reactions (1) to (7) are given by

$$
K_{p, j}=\exp \left[-\Delta G_{j}^{0} /(R T)\right](j=1,2, \ldots, 7)
$$

where $\Delta G_{j}^{0}=\sum_{i} \nu_{i} G_{i}^{0}-\sum_{i} \nu_{i}^{\prime} G_{i}^{0}, \nu_{i}$, and $\nu_{i}^{\prime}$ are the stoichiometric coefficients, and $\Delta G_{i}^{0}$ is the standardstate Gibbs energy of formation of the $i$ th species. For example, for reaction (2) $\left(\mathrm{N}_{2}+\mathrm{O}_{2}=2 \mathrm{NO}\right)$,

$$
\Delta G_{2}^{0}=2\left(G^{0}\right)_{\mathrm{NO}}-\left(G^{0}\right)_{\mathrm{N}_{2}}-\left(G^{0}\right)_{\mathrm{O}_{2}}
$$

and

$$
K_{p, 2}=\exp \left(-\Delta G_{2}^{0} / R T\right)
$$

The enthalpy of the mixture in terms of the enthalpies of the constituent species and their mole numbers is given by

$$
h=\sum_{i=1}^{11} H_{i}^{0} \sigma_{i}
$$

Because the reference temperature is 0 K , internal energy of each species is simply given by

$$
E_{i}^{0}=H_{i}^{0}-R T(i=1,2, \ldots, 11)
$$

and the internal energy of the mixture is given by

$$
\begin{aligned}
e & =\sum_{i=1}^{11} E_{i}^{0} \sigma_{i} \\
& =h-R T \sigma
\end{aligned}
$$

The entropy of the mixture is determined from the expression

$$
s=-R \sigma \ln \left(p / p^{0}\right)+\sum_{i=1}^{11} \sigma_{i}\left[S_{i}^{0}-R \ln \left(\sigma_{i}\right)\right]
$$

where $p^{0}$ is the standard-state pressure ( 101325 Pa ) and $p$ is the pressure of the mixture given by

$$
p=\rho R T \sigma
$$

## Results and Discussion

A computer code has been written to implement the computational scheme. The thermodynamic data in the form of coefficients $a_{i, j}(j=1,2, \ldots, 7)$ for the 11 chemical species ( $i=1,2, \ldots, 11$ ) are included in the code. The program has four subroutines, one for each model. This program reads a set of values for temperature, density, and convergence criterion and computes the chemical equilibrium composition and the thermodynamic properties of air. The appendix gives a listing of the computer code and a set of sample results.

The total time $t$ for the computation of chemical equilibrium composition for any iterative method is given by the expression

$$
t=t_{0}+t_{i} N(\varepsilon)
$$

where $t_{0}$ is a constant time required to compute the terms outside the iteration loop and includes the time for computation of equilibrium constants and estimates of initial values to be used for the
iteration, $t_{i}$ is the time required per iteration, and $N(\varepsilon)$ is the number of iterations required for the desired accuracy $\varepsilon$.

A series of cases were run on the CDC CYBER 170/750 to determine the three quantities $t_{0}, t_{i}$, and $N(\varepsilon)$ for the present computational scheme. Sample results are shown in table III. The constant time is essentially the same for all four models. However, the time per iteration and the number of iterations required for convergence are different for the four models. The number of iterations for convergence depends on the desired accuracy. Over the wide range of conditions explored, one or two iterations were required for a mole fraction accuracy of $\varepsilon=0.001$, and in most cases two or three iterations were sufficient for an accuracy of $\varepsilon=0.0001$. Results for points along the boundaries between various models are indicated with an asterisk.

Computations have been carried out with the computer code to obtain equilibrium chemical
composition for air over a temperature range of 200 to 15000 K and for density values of $1 \times 10^{-5}$, $1 \times 10^{-2}$, and $1 \times 10^{2} \mathrm{~kg} / \mathrm{m}^{3}$. These results are presented in figures 2(a), 2(b), and 2(c).

## Concluding Remarks

The computational scheme presented provides a means for rapidly computing the equilibrium chemical composition of air to 15000 K . This scheme was developed for the purpose of providing a relatively rapid computer code for use with computational fluid dynamics programs that deal with high-temperature air. A computer code and a thermodynamic data base are also included from which the thermodynamic properties of air to 15000 K may be computed.

NASA Langley Research Center
Hampton, Virginia 23665-5225
January 7, 1988

## Appendix

## Computer Code Listing and Sample Results

## Computer Code

PROGRAM AIRNEL (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

C

| C | THIS PROGRAM COMPUTES THE EQUILIBRIUM CHEMICAL COMPOSITION OF AIR ASSUMING |
| :---: | :---: |
| C | (1) THE DENSITY IS CONSTANT. |
| C | (2) THE ELEVEN SPECIES: $02, \mathrm{~N}, \mathrm{O}, \mathrm{NO}, \mathrm{N}, \mathrm{NO}, \mathrm{E}, \mathrm{N}, \mathrm{N}, \mathrm{O}, \mathrm{AR}, \mathrm{AND}$ AR+. |
| C | (3) CURUE FITTED DATA RECEIVED FROM BONNIE J. MCBRIDE. |
| C | (4) REFERENCE TEMPERATURE IS ZERO KELUIN. |
| C | ALSO COMPUTES ENTHALPY, INTERNAL ENERGY, AND ENTROPY. |
| C |  |
|  |  |
| REAL AA(11, 7,5$), \mathrm{C}(11), \mathrm{H}(11), \mathrm{EN}(11), \mathrm{S}(11), \mathrm{E}(11)$ |  |
| REAL K1, K2, K3, K4, K5, K6, K7, KE |  |
| COMMON/A/ K1, K2, K3, K4, K5, K6, K7, KE |  |
| COMMON / B/ SN,SO,SR,EPS |  |
| COMMON / C/S |  |
| COMMON / D / NITER |  |
|  |  |
|  | 1 (S6,S(6)),(S7,S(7)),(S8,S(8)),(S9,S(9)),(SA,S(10)), |
|  | 2 (SB,S(11)) |
| C |  |
|  | $G B Y R T(I, K, T)=(\quad A A(I, 1, K) *(1,-A L O G(T))-A A(I, 2, K) * T / 2 .-A A(I, 3, K)$ |
| $1 * T * * 2 / 6 .-A A(I, 4, K) * T * * 3 / 12.0-A A(I, 5, K) * T * * 4 / 20.0+A A(I, 6, K) / T$ |  |
| 1-AA( $1,7, K)$ ) |  |
| C |  |
|  | $\operatorname{HBYRT}(I, K, T)=A A(I, 1, K) \quad+A A(I, 2, K) * T \quad / 2 .+A A(I, 3, K) * T * * 2 / 3$. |
|  | $1+\mathrm{AA}(1,4, K) * T * * 3 / 4 .+A A(I, 5, K) * T * * 4 / 5$. |
|  | $2+\mathrm{AA}(I, 6, K) / T$ |
| C |  |
|  |  |
|  | 1 +AA(I, 5, K)*T**4/4.+AA(I,1,K)*ALOG(T)+AA(I, $7, K)$ |
| c |  |
| C | THE FOLLOWING CONSTANTS ARE OBTAINED FROM B. J. MCBRIDE, |
| C | THE BASIC DATA SOURCES ARE: |
| C | (1) TABULATED DATA FROM: THERMODYNAMIC PROPERTIES OF |
| C | INDIVIDUAL SUBSTANCES, ACADEMY OF SCIENCES, MOSCOW, USSR, |
| C | 1978, VOL.1, PART 2. (FOR MOLECULAR SPECIES, |
| C | AND FORMATION UALUES OF ALL SPECIES.) |
| c | (2) UALUES COMPUTED (BY MCBRIDE) USING STATISTICAL DATA |
| C | FOR MONATOMIC SPECIES. |
| $C$ SOURCE OF BASIC STATISTICAL DATA: |  |
| C (A) NSRDS NBS VOLUME 35, 1971 |  |
| C (B) NBS 3, SECTION 5, 1975 |  |
| C | (C) NBS 3, SECTION 7, 1976 |
| C |  |
|  | NEUTRAL MOLECULAR OXYGEN 02. |
| DATA ( $(A A(1, J, K), J=1,7), \mathrm{K}=1,5)$ ) |  |
| *.37703733E+01, -. 28952206E-02, .95332231E-05, -. 92469913E-08, |  |
| *.30191908E-11,-.18859751E+02, .36933495E+01, |  |
| * . 28969178E+01, .23736545E-02,-.14917097E-05, .46603392E-09, |  |
| *-.53945166E-13, .82240429E+02, . $75019384 \mathrm{E}+01$, |  |
| *. 28421112E+01, .13320561E-02,-.33915853E-06, .44652178E-10, |  |
| *-. 22914824E-14, .58350386E+03, . $86255038 \mathrm{E}+01$, |  |
| * .56821087E+01,-.75300588E-03, .22980075E-06,-.23955920E-10, |  |
|  | * .80048472E-15,-.24700372E+04, $-.98740054 \mathrm{E}+01$, |

```
    *-.27258966E+00, .20115141E-02,-.24547714E-06, .12025200E-10,
    *-.21389859E-15, .76117740E+04, .31631739E+02/
C..... NEUTTRAL MOLECULAR NITROGEN N2
    DATA ((AA(2,J,K),J=1,7),K=1,5)/
    *..34622648E+01. .58202350E-03,-.30525456E-05, .62280057E-08,
    *-.33755957E-11, .87951739E+00, .32192648E+01,
    *.27022403E+01. .19443933E-02,-.89300044E-06, .19739206E-09,
    *-.16967813E-13, .20180222E+03, .72040846E+01,
    * .39143506E+01, .31537108E-03,-.56481043E-07, .36012299E-11,
    * .52359433E-16,-.53551958E+03, .21671998E-01,
    *.12657466E+01, .18269791E-02,-.37583896E-06, .33033716E-10,
    *-.93651108E-15,.31426960E+04, .17943292E+02,
    * .27715943E+02,-.74173486E-02, .82395970E-06,-.35285300E-10,
    * .49671195E-15,-.56942692E+05,-.17402873E+03/
C..... NEUTRAL MONATOMIC OXYGEN O
    DATA ((AA(3,J,K),J=1,7),K=1,5)/
    * . 32167143E+01,-.37822689E-02, .84746788E-05,-.88658246E-08,
    * .35365597E-11, .29640462E+05,.18526415E+01,
    * .26045368E+01,-.17235464E-03,.11574138E-06,-.36417858E-10,
    * .46011536E-14, .29728962E+05, .45865262E+01.
    * .28101683E+01,-.29039910E-03, .90833347E-07,-.99427817E-11,
    * .37704361E-15, .29536612E+05, .32536467E+01,
    * .19209266E+01, .21776554E-03,-.18288405E-07. .49050959E-12,
    * .28507296E-17, .30783423E+05, .92748629E+01,
    * .19209266E+01, .21776554E-03,-.18288405E-07, .49050959E-12,
    * .28507296E-17, .30783423E+05, .92748629E+01/
C..... NEUTRAL NITRIC OXIDE,NO
    DATA ((AA(4,J,K),J=1,7),K=1,5)/
    * .42064360E+01,-.45098354E-02, .10557385E-04,-.85919382E-08,
    * .24047097E-11, .10888965E+05, .23137934E+01,
    * .27543778E+01, .23093284E-02,-.12823359E-05, .34043524E-09,
    *-.34807544E-13, .11134325E+05, .90789671E+01,
    * .38015418E+01, .49857546E-03,-.12531319E-06, .14093893E-10,
    *-.44820194E-15, . 10666566E+05, . 31619390E+01,
    *.49133164E+01, .61755267E-06,-.55222382E-07, .11686489E-10,
    *-.54642492E-15,.88453776E+04,-.45786900E+01,
    * .20456654E+02,-.61498079E-02,.86914275E-06,-.50877512E-10,
    * .10624157E-14,-.22955242E+05,-.11561956E+03/
C..... NEUTRAL MONATOMIC NITROGEN N
    DATA ((AA(5,J,K),J=1,7),K=1,5)/
    * .25000000E+01,0. ,0. 0.
    *0. . .56626706E+05. .41807280E+01,
    * .25075109E+01,-.24797882E-04, .29641517E-07,-.15288108E-10,
    * .28913714E-14, .56624949E+05,. .41431864E+01,
    * .26376051E+01,-.87373323E-05,-.64772651E-07, .23473436E-10,
    *-.17396164E-14, .56452267E+05, .32232104E+01,
    * .33720620E+01,-.88554639E-03, .25293270E-06,-.23187896E-10,
    *.70471420E-15,.56270151E+05,-.10563976E+01,
    *-.10205642E+02, .42931347E-02,-.49310664E-06, .24951067E-10,
    *-.46969943E-15,.84933779E+05,.96403744E+02/
C..... UNIPOSITIUE NITRIC OXIDE, NO+
    DATA ((AA(6,J,K),J=1,7),K=1,5)/
    * .35659323E+01,-.27420307E-03,-.52284273E-06, .30228201E-08,
    *-.19112702E-11, .11841356E+06, .35935691E+01,
    * .27152625E+01, .18998146E-02,-.85345216E-06, .18390288E-09,
```

```
    *-.15328469E-13, .11862129E+06, .79369146E+01,
    * . 39310989E+01, . 30660785E-03,-.56705942E-07, .45573508E-11,
    *-.77762022E-16, .11786296E+06, .70198182E+00,
    * .10656163E+01, .17917961E-02,-.33077778E-06, .25052719E-10,
    *-.54540127E-15, .12214242E+06, . 20328302E+02,
    * . 31376951E+02,-.78170663E-02, .75437124E-06,-.24734142E-10,
    * .15952361E-15, .48124504E+05,-.20218492E+03/
C..... ELECTRGN, E-
    DATA ((AA(7,J,K),J=1,7),K=1,5),
    * .25000000E+01,0. ,0. ,0. 
    *0. ,-.42498957E-01,-.11733917E+02.
    * .25000000E+01,0. ,0. ,0. ,
    *0. .-.42498957E-01,-.11733917E+02,
    * .25000000E+01,0. ,0. ,0. ,
    *0. ,-.42498957E-01,-.11733917E+02.
    * .25000000E+01,0. ,0. ,0. ,
    *0. ,-.42498957E-01,-.11733917E+02,
    * .25000000E+01,0. ,0. ,0. ,
    *0. ,-.42493957E-01,-.11733917E+02/
C..... UNIPOSITIUE MONATOMIC NITROGEN, N+
        DATA ((AA(B,J,K),J=1,7),K=1,5)/
        * .30816977E+01,-.37737512E-02,. .96771466E-05,-.11099460E-07.
        * .47248633E-11, .22532807E+06, .24315936E+01,
        * .25347005E+01,-.55067384E-04, .36440013E-07,-.12313199E-10,
        * .19224650E-14, .22539524E+06, .47895586E+01,
        * . 28005551E+01,-.29566365E-03, .96649663E-07,-.11053590E-10,
        * .44145151E-15, .22518494E+06, . 31305296E+01,
        *.20047667E+01;.19189730E-03,-.15499136E-07, .41165940E-12,
        * .26287640E-17,.22622600E+06, .84634959E+01,
        * .20598363E+01, .18875424E-03,-.18075532E-07, .80284344E-12,
    *-.13089587E-16, .22602766E+06, .80254371E+01/
C..... UNIFOSITIVE MONATOMIC QXYGEN, O+
            DATA ((AA(9,J,K),J=1,7),K=1,5)/
        *.25000000E+01,0. ,0. ,0. ,
    *0. ;.18771321E+06, .43801998E+01,
    * .25013354E+01,-.40431297E-05, .43271450E-08,-.19527702E-11,
    * .31632791E-15, .18771288E+06, .43734240E+01,
    * .22258863E+01, .29650143E-03,-.11561144E-06, .18461192E-10,
    *-.89249152E-15, .18791159E+06, .60576133E+01,
    * .41210991E+01,-.86786175E-03, .15337469E-06,-.92260135E-11,
    *.17842049E-15, .18543702E+06,-.66388864E+01,
    * . 17349474E+01,-.80710732E-04, .59806317E-07,-.45998476E-11,
    * .10295182E-15, .19107440E+06, .10791908E+02/
C..... NEUTRAL ARGON, A.
    DATA ((AA(10,J,K),J=1,7),K=1,5)/
    * .25000000E+01,0. ,0.
    *0. ,-.42498957E-01,.43664979E+01,
    *.25000000E+01,0. ,0. ,0. ,
    *0. . -.42498957E-01, .43654979E+01,
    * .25000000E+01,0. ,0. ,0. ,
    *0. ,-.42498957E-01,.43664979E+01,
    * .26303545E+01,-.77393441E-04, .17219729E-07,-.17028231E-11,
    * .63198847E-16,-.17548010E+03, .34890096E+01,
    *-.91423987E+01, . 38508169E-02,-.46800440E-06, .24506918E-10,
    *-.45646899E-15, .27751915E+05, .89461765E+02/
```

```
C..... UNIPQSITIVE MONATOMIC ARGON, A.
    DATA ((AA(11,N,K),J=1,7),K=1,5)
    * .26818797E+01,-.20720300E-02, .74786002E-05,-.87672013E-08,
    * .34484189E-11, .18287091E+06, .50764556E+01,
    * .24220646E+01, .86346979E-03,-.78459395E-06, .26673998E-09,
    *-.31850590E-13, .18285266E+06, .59236280E+01,
    * .25588416E+01, .89736230E-04,-.50091586E-07, .88298198E-11,
    *-.52371930E-15,.18301380E+06, .55313033E+01,
    * .25014164E+01, .29446558E-04,-.80471729E-08, .77535785E-12,
    *-.25780935E-16,.18325152E+06, .50544022E+01,
    * .25706058E+01,-.20621051E-04, .29786013E-08,-.21101899E-12,
    * .57476758E-17, .18322322E+06, .56165023E+01/
        RR=8.31441
        P0=1.01325E+05
C
C..... ASSUME AIR TO HAVE 02 : N2 : A = 20.96:78.11 : 0.93
C
    S0 = 14.4802
    SN =53.9620
    SR=0.3212
```



```
C
C INPUT DATA:
C
C TKEL = TEMFERATURE (KELUIN)
C RKGMS = DENSITY (KG/M***3)
C EFS = CONUERGENCE CRITERION
C
```



```
    60 WRITE (6,66)
    66 FORMAT(/,2X,*INPUT:- TKEL, RKGM3, EPS*,
        1 (,2X,* ENTER <RETURN〉 TO STOP*)
            READ (5,*) TKEL,RKGM3,EFS
            IF(EOF(5)) 62,5,62
            5 CONTINUE
C
C.....CHOOSE APPROPRIATE CONSTANTS DEPENDING UPON THE TEMPERATURE.
C
    k=1
    IF(TKEL .GT. 800.) K=2
    IF(TKEL .GT. 3000.) K=3
    IF(TKEL .GT. 6000.) K=4
    IF(TKEL .GT. 10000.) K=5
C
C.....COMPUTE DG/RT, H/RT, AND S/R FOR ALL THE ELEVEN SPECIES.
C
            DO B I=1,11
            C(I)=DGBYRT(I,K,TKEL)
            H(I)=(HBYRT (I,K,TKEL)*TKEL)*RR
            EN(I)=SBYR (I,K,TKEL) *RR
C WRITE(6,50) I,K,TKEL,C(I),H(I),EN(I)
    50 FORMAT(2I2,7E12.5)
            8 CONTINUE
```

```
C
```

C..... COMPUTE THE EQUILIERIUM CONSTANTS FOR THE GIUEN TEMPERATURE C.....AND DENSITY CONDITIONS.
C......THE REACTIONS ARE:

C 0
(NOt) $+E-=N O$
(NO+) $+E-=N D$
$(N+)+E-=N$
$(O+)+E-=0$
$(A R+)+E-=A R$
C
Cれk
$K 1=C(3)+C(3)-C(1)$
$K 2=C(4)+C(4)-C(1)-C(2)$
$K 3=C(5)+C(5)-C(2)$
$K 4=C(4)-C(5)-C(7)$
$K 5=C(5)-C(8)-C(7)$
$K 6=C(3)-C(9)-C(7)$
$K 7=C(10)-C(11)-C(7)$
$K 1=E \times X P(-K 1) \quad$ F $\quad K 2=E \times \times P(-K 2) \quad K B=E \times \times P(-K 3)$
$K 4=E \times X P(-K 4) \quad$ क $K 5=E \times X P(-K 5) \quad$ ) $K E=E X X P(-K \Theta)$
$K 7=E \times X P(-K, 7)$
POBYRT=PO/(RR*TKEL*RKGMB)
$K 1=K 1 * P 0 B Y R T$
$K 3=K 3 * P O B Y R T$
$K 4=K 4 /$ POBYRT
$K 5=K 5 / P 0 B Y R T$
$K G=K 6 / P O B Y R T$
$K 7=K 7 / P O B Y R T$
$K E=S Q R T(K 2 /(K 1 * K 3))$
C
C.....CALL APPROPRIATE SUBROUTINE DEPENDING UPON THE TEMPERATURE
C.....TO COMPUTE THE COMPOSITION.

C
IF(TKEL .LE. 1600.0) GO TO 151
RLOG $=$ ALOG10 (RKGM3)
TEMP1 $=1000.0 *(3.5+10.0 *((R L D G+5.0) / 7.0) * * 2.5)$
IF(TKEL .LE. TEMP1) GO TO 154
TEMP $2=1000 . *(5.4+3.2 *($ RLOG 6.$) / 6 .+5.55 *(A B S(R L O G+6) / 6.) * * 3.35$.
IF (TKEL .LE. TEMP2) GO TO 152
153 MODEL $=3$
CALL CUBIC
GO TO 156
152 MODEL $=2$
CALL QUDRATC
GO TO 156
154 MODEL $=4$
CALL QUARTIC

```
        GO TO 156
    151 MODEL = 1
        CALL LOWTEMP
C
C.....TOTAL MOL NUMBER.
C
    156 SUM = S1+54+52+53+55+56+57+58+59+5A+5B
C
C.....cOMPUTE PRESSURE
C
    PBAR = RKGM3*8.314*SUM*TKEL/100000.
    PATM = PBAR/1.01325
C
C.....COMPUTE RESIDUES....
C
    R1 = S3*S3/K1/S1-1.0
    R2 = 54*54/K2/51/52-1.0
    R3 = S5*S5/K3/S2-1.0
    R4 = S4/K4/S6/S7-1.0
    R5 = S5/K5/S8/S7-1.0
    RG = S3/K6/S9/S7-1.0
    R7 = S0-2.0*S1-S3-54-S6-59
    R8 = SN-2.0*S2-S4-S5-56-S8
    R9 = S7-S6-58-S9-SB
    RA = SR - SA -SB
    RB = SA - K7*SB*S7
    IF(SB .EO. 0.0) RB = 0.0
```



```
C
C.....COMPUTE THE ENTHALPY, INT, ENERGY, AND ENTROPY OF THE MIXTURE.
C.....NOTE: ENTHALPY AND INT. ENERGY ARE IN JOULES/KG.
C..... ENTROPY IS IN JOULES/KG.KELUIN
C
```



```
    HO = 0.0
    EO=0.0
    ENTO = -RR*SUM*ALOG(PATM/SUM)
    DO 96 I=1,11
    H0 = H0+S(I)*H(I)
    E(I) = H(I)-RFNTKEL
    EO=EO+S(I)*E(I)
    1F(S(I) .EQ. 0.0) GO TO GG
    ENT0 = ENTO+S(I)*(EN(I)-RR*ALOG(S(I)))
    96 CONTINUE
C
C.....WRITE OUT THE RESULTS.
C
    WRITE(6,101) TKEL,RKGM3,EPS
    WRITE(G,100) SO,SN,SR
    WRITE(6,102) K1,K2,K3,K4,K5,K6,K7,KE
    IF(MODEL .EQ. 1) WRITE(6,304) NITER
    IF(MODEL .EQ. 2) WRITE(6.307) NITER
    IF(MODEL .EQ. 3) WRITE(6,306) NITER
    IF(MODEL .EQ. 4) WRITE(6,305) NITER
    WRITE(6,303) 51,52,53,54,55,56,57,58,59,5A,5B
```

```
    WRITE(6,318) (H(I),I =1,11)
    WRITE(E,316) (E(I),I =1,11)
    WRITE(6,319) (EN(I),I =1,11)
    WRITE(6,317) SUM,HO,ENTO,EO,FBAR
    WRITE(6,204) R1,R2,R3,R4,RS,R6,R7,R8,R9,RA,RB
303 FORMAT(/, 2X,38HMOLE NUMEERS OF THE SPECIES (MOL/KG):-,
    1/, 9X,2HO2, 11X,2HN2,11X,2HO ,11X,2HNO,11X,2HN,10X,3HNO+,
    1/, }3\times,2HE-,11\times,2HN+,11\times,2HO+,11\times,2HAR,11\times,3HAR+,/%
    1 2X,6E13.5,/,2X,5E13.5)
304 FORMAT(/. 2X,43HSOLUTIGN BY THE SIMPLE ITERATION PROCEDURE;,
    1 I3,11H ITERATIONS)
305 FORMAT(/, 2X,33HSOLUITION OF THE QUARTIC EQUAATION:,
    1 IS,11H ITERATIONS)
306 FORMAT(/, 2X,3IHSOLUTION OF THE CUBIC EQUATION:,
    1 I3,11H ITERATIONS)
307 FORMAT(/ , 2X,35HSOLUTION OF THE QUADRATIC EQUATION;,
    I I3,11H ITERATIONS)
100 FORMAT(/, 2X,*OXYGEN = *,F8.4,2X,*NITROGEN = * F F8.4,2X,
    1 *ARGON = *,FG.4,* MOLS/KG.*)
101 FORMAT (/, 2X,*T (KELUIN) = *,FQ.2,2X,*DENSITY (KG.M**3) = *,E12.6
    1 ,2X,*EPS = *,E10.4)
204 FORMAT(%,2X,*RESIDUES:-*,3(%,2X,6E13.5))
102 FORMAT:/, 2X,*EOUILIBRIUM EONSTANTS K(I):-*,
    1 / , 2X,6E13.5,/,2X,6E13.5)
317 FORMAT(
    1.,2X,29HTOTAL MOL NUMBER = ,F12.4,11H MOLS/KG. .
    1/,2\times,29HENTHALPY OF THE MIXTURE = ,E12.6.11H JOULES/KG.,
    1/,2X,29HENTROPY OF THE MIXTURE = ,E12.6,18H JOULES/KG./KELUIN,
    1, ,2X,29HINT. ENERGY OF THE MIXTURE = ,E12.6,11H JOULES/KG.,
    1/,2X,29HPRESSURE = ,E12.6,6H (BAR))
318 FORMAT(/,2X,39HENTHALPY OF EACH SPECIES (JOULES/MOL):-,%,
    1 3(2X,6E13.5,/,))
31G FORMAT(%, 2X,46HINTERNAL ENERGY OF EACH SPECIES (NOULES/MOL):-./.
    1 3(2X,6E13.5,/,))
319 FORMAT(/, 2X,45HENTROPY OF EACH SPECIES (IOULES/MOL/KELUIN):-,/,
    1 3(2X,6E13.5,%,))
        GO TO 50
62 STOP
    END
```


## SUBROUTINE LOWTEMF

C
C......THIS SUBROUTINE COMPUTES THE COMPOSITION OF AIR
c.....AT LOW TEMPERATURES WHERE THERE IS SOME DISSOCIATION
c......BUT VERY LITTLE IONIZATION.
C.....the major species are oz. N2. AND ar.
c.....(MODEL I)
c
 COMMON/A/K1,K2,K3.K4,K5,K6,K7.KE COMMON / B / SN.SO,SR,EPS COMMON / C / S1,52,53,54,55,56,57,58,59,5A,5B COMMON / D / ITER REAL K1,K2,K3,K4,K5,K6,K7,KE
c
C..... initialize and start iterating.
c $51 P=50 / 2.0$ F $52=5 N / 2.0$ F $5 A=5 R$ DO $200 \mathrm{ITER}=1.20$
C
c.....compute the minor species.
c

c
C......update the major species. c $\mathrm{S} 1=(\mathrm{sO}-\mathrm{S} 4-\mathrm{s} 3) / 2.0 \quad \mathrm{~F} \quad \mathrm{~S}=(\mathrm{SN}-\mathrm{S} 4-\mathrm{S} 5) / 2.0$
C
C..... CHECK FOR CONUERGENCE.
c
DS1 = S1 - S1P 1F(ABS(DS1) .LE. EPS) GO TO 201 S1P $=51$
200 CONTINUE
201 CONTINUE
c.
c.....compute the trace sfecies.

C


SUBROUTINE QUARTIC
 C
C.....THIS SUBROUTINE COMPUTES THE COMPOSITION OF AIR
C.....AT MODERATE TEMPERATURES WHERE THERE IS DISSOCIATION
C.....BUT NOT MUCH IONIZATION.
C.....THE MAJOR SPECIES ARE 02, N2, O, NO, AND N.
C.....(MODEL II)

C
C.

COMMON / A/ K1,K2,K3,K4,K5,K6,K7,KE
COMMON / B/SN,SO,SR,EPS
COMMON / C / S1,S2,53,54,55,S6, $57,58,59, S A, S B$
COMMON / D/ ITER
REAL K1,K2,K3,K4,K5,K6,K7,KE
C
C.....INITIALIZE, DETERMINE THE INITIAL GUESS, AND START ITERATION.

## C

$S 6=0.0$ \& $S 8=0.0$ \& $S 9=0.0 \quad$ \& $S A=S R$
$B 4=(8.0-K 2-K 2) / K 1$
$B 3=8.0-\mathrm{K2}-2.0 \times \mathrm{K} 3 * \mathrm{KE}$
$\mathrm{BB}=K 1 *\left(1.0+\operatorname{SQRT}\left(K 2 \star \operatorname{SN} /\left(K_{1}+K_{1}\right)\right)\right)$
$S 3=(-\mathrm{BB}+\mathrm{SQRT}(\mathrm{BB} * \mathrm{BB}+8.0 * S 0 * K 1)) / 4.0$
C1 $=\mathrm{K} 1 * K 3 * K E$
DO 200 ITER $=1,20$
$501=50-56-59$
$\mathrm{SN1}=\mathrm{SN}-\mathrm{S6}-\mathrm{S8}$
$B 2=-8.0 * 501+K 1+K 1+(S 01-S N 1) * K 2-C 1$
$B 1=501 *(-K 1-K 1-K 1-K 1+C 1)$
$\mathrm{BO}=2.0 * 501 * 501 * K 1$
$F=B 0+53 *(B 1+53 *(B 2+53 *(B 3+53 * B 4)))$
$F P=B 1+S 3 *(B 2+B 2+S 3 *(B 3+B 3+B 3+53 k(B 4+B 4+B 4+B 4)))$
$D 3=-F / F P$
$S 3=53+D 3$
C
C.....UNRAVEL THE REST OF THE MAJOR SPECIES

C
$\mathrm{S} 5=(\mathrm{SO1}-53-2.0 * 53 * 53 / K 1) /(K E * 53)$
$54=K E * S 5 * S 3$
$52=(S N 1-54-55) / 2.0$
$S 1=(501-54-53) / 2.0$
C
C..... COMPUTE THE MINOR SPECIES.

C

$C$
C..... CHECK FOR CONVERGENCE

C
IF (ABS(D3) .LE. EPS) GO TO 201
200 CONTINUE
201 CONTINUE

- RETURN

END

SUBROUTINE CUBIC


```
C
C.....THIS SUBROUTINE COMPUTES THE COMPOSITION OF AIR
C.....AT VERY HIGH TEMPERATURES WHERE NOT MANY DIATOMIC PARTICLES ARE FOUND.
C.....THE MAJOR SPECIES ARE O,N, E- ,N+, AND O+.
C.....(MODEL IV)
C
```



```
COMMON/A/A/K1,K2,K3,K4,K5,K6,K7,KE
COMMON / B / SN,SO,SR,EPS
COMMON / C / S1,S2,S3,54,55,S6,57,S8,S9,SA,SB
COMMON / D/ITER
REAL K1,K2,K3,K4,K5,K6,K7,KE
C
C.....INITIALIZE, DETERMINE THE INITIAL GUESS, AND START ITERATION.
C
    S1=0.0 & S2 = 0.0 & S4=0.0 & S6 = 0.0
    SA = SR & SB = 0.0
    ST = (SQRT(1.0 + 4.0*SN*K5)-1.0) / (K5 + K5)
    S9 = S0 / (1.0 + K6*S7)
    SB = SR / (1.0 + K7*S7)
    57 = 57 + 59 + 5B
    B3 = K5*kK5
    C2 = K5 + K6
    DO 200 ITER = 1.20
    501 = 50-51 - 51 - 54 - 56
    SN1 = SN - S2 - S2 - S4 - S6
    S6P = 56 + SB
    B2 = C2 - B3*S6P
    B1 = 1.0 - SN1*K6 - SO1*K5 - 56P*C2
    B0 = - SN1 - S01 - S6P
    F}=\textrm{B0}+57*(B1+57*(B2+57*B3)
    FP}=\textrm{B}1+S7*(B2+B2+S7*3.0*B3
    D7 = - F/FP
    S7 = S7 + D7
E
C.....UNRAUEL THE REST OF THE MAJOR SPECIES
C
S3 = S01*57*K6 / (1.0 + K6*S7)
S5 = SN1*S7*K5 / (1.0 + K5*S7)
SB = SR / (1.0 + K7*S7)
SA = SR - SB
S9 = S3 / (K6*S7)
S8=55 /(K5*S7)
C
C.....COMPUTE THE MINOR SPECIES.
C
s1 = S3*53/k1
s2 = 55*55/K3
S4 = KEkS3*S5
S6 = S4/(K4*S7)
C
C.....CHECK FOR CONUERGENCE
C
IF(ABS(D7) .LE. EPS) GO TO 201
200 CONTINUE
201 CONTINUE
RETURN
END
```

SUBROUTINE QUDRATC
 C
C..... THIS SUBROUTINE COMPUTES THE COMPOSITION OF AIR AT INTERMEDIATE
C.....TEMPERATURES WHERE OXYGEN IS ALMOST FULLY DISSOCIATED,
c.....but not much ionization has taken place.
c.....the major species are n2, 0, AND N. C.....(MODEL IIJ)

C

COMMON / A/ K1,K2,K3,K4,K5,K6,K7,KE
COMMON / B/ES,SO,SR,EPS
COMMON / C / S1,52,53,54,55,56,57,58,59,SA,SB COMMON / D / ITER REAL K1,K2,K3,K4,K5,K6,K7,KE
C
C......INITIALIZE, DETERMINE THE INITIAL GUESS, AND START ITERATION.
c
c
$S 4=0.0$ \$ $S 6=0.0 \quad \$ \quad S 8=0.0 \quad \$ \quad S A=S R$
$B 1=K 3 / 2.0 \quad 末 \quad B 2=K 1 / 2.0$
S5 = ( $-\mathrm{B} 1+\operatorname{SQRT}(\mathrm{B} 1 * \mathrm{~B} 1+4.0 * B 1 * \operatorname{SN})) / 2.0$
$53 P=(-\mathrm{B} 2+\mathrm{SQRT}(B 2 \times B 2+4.0 \times B 2 \times 50)) / 2.0$
DO 200 ITER = 1,20
$\mathrm{SN1}=\mathrm{SN}-\mathrm{S} 4-\mathrm{S} 6-\mathrm{S} 8$
$F=-B 1 * S N 1+55 *(B 1+55)$
$\mathrm{FP}=\mathrm{B} 1+\mathrm{S} 5+\mathrm{S} 5$
$\mathrm{s} 5=\mathrm{s} 5-\mathrm{F} / \mathrm{FP}$
$\mathrm{S} 2=(\mathrm{SN} 1-\mathrm{S} 5) / 2.0$
c
c.....UNRAVEL THE REMAINING SPECIES

C
S1 = S3P*S3P / K1
$54=K E \star S 3 P \star S 5$

$\mathrm{S} 7=\mathrm{SQRT}(\mathrm{C} 3+\mathrm{C} 4+\mathrm{C5}+\mathrm{C} 6)$

$\mathrm{SA}=\mathrm{SR}-\mathrm{SB}$
$\mathrm{s} 3=\mathrm{s} 0-\mathrm{S} 1-\mathrm{S} 1-\mathrm{s} 4-56-59$
c
C......CHECK FOR CONUERGENCE.
c
$D 3=53 P-53$
$53 P=53$
IF(ITER .EQ. 1) GO TO 200
IF ( ABS(D3) .LE. EPS) GO TO 201
200 CONTINUE
201 CONTINLE
RETURN
END
FUNCTION EXXP(X)
EXXP=0.0
IF(X .LT. - 670.0) RETURN
$\operatorname{EXXP}=E \times P(X)$
RETURN
END

## Sample Results

```
/190
    INPUT:- TKEL, RKGMS, EPS
            ENTER 〈RETURN〉 TO STOP
? 10000.,1.e-6,1.e-10
T(KELUIN) = 10000.00 DENSITY (KG/M3) = .100000E-05 EFS = .1000E-09
OXYGEN = 14.4802 NITROGEN = 53.9620 ARGON = .3212 MOLS/KG.
EQUILIBRIUM CONSTANTS K(I):-
        .29508E+11 . 19149E+01 . 23078E+09 .34684E-04 .13169E-02 .20269E-02
        .19413E-02 .53027E-09
SOLUTION OF THE CUBIC EQUATION; 5 ITERATIONS
MOLE NUMBERS OF THE SPECIES (MOL/KG):-
\begin{tabular}{|c|c|c|c|c|c|}
\hline 02 & N 2 & 0 & NO & \(N\) & \(\mathrm{NO}+\) \\
\hline E- & \(\mathrm{N}+\) & O+ & AR & AR+ & \\
\hline . \(91003 \mathrm{E}-10\) & . \(73958 \mathrm{E}-07\) & .16387E+01 & . \(35900 \mathrm{E}-08\) & . \(41314 E+01\) & \(.16440 \mathrm{E}-05\) \\
\hline \(.62958 E+02\) & . \(49831 \mathrm{E}+02\) & .12841E+02 & . \(34983 \mathrm{E}-01\) & . \(28622 \mathrm{E}+00\) & \\
\hline \multicolumn{6}{|l|}{\multirow[t]{3}{*}{HALPY OF EACH SPECIES (JOULES/MOL):-}} \\
\hline & & & & & \\
\hline & & & & & \\
\hline
\end{tabular}
INTERNAL ENERGY GF EACH SPECIES (JOULES/MOL):\(.32776 \mathrm{E}+06 \quad .29702 \mathrm{E}+06 \quad .38303 \mathrm{E}+06 \quad .39817 \mathrm{E}+06 \quad .63313 \mathrm{E}+06 \quad .12792 \mathrm{E}+07\) \(.12472 \mathrm{E}+06 \quad .20103 \mathrm{E}+07\). \(17035 \mathrm{E}+07\). \(12476 \mathrm{E}+06 \quad .16502 \mathrm{E}+07\)
ENTROPY OF EACH SPECIES (JOULES/MOL/KELUIN):\(.33620 E+03 \quad .31386 E+03 \quad .23514 E+03 \quad .33631 E+03 \quad .23135 E+03 \quad .32018 E+03\) \(.93886 \mathrm{E}+02 \mathrm{I} .23460 \mathrm{E}+03 \quad .23013 \mathrm{E}+03 \quad .22776 \mathrm{E}+03 \quad .24261 \mathrm{E}+03\)
```

```
TOTAL MOL NUMBER = 131.7218 MOLS/KG.
```

TOTAL MOL NUMBER = 131.7218 MOLS/KG.
ENTHALPY OF THE MIXTURE = .144573E+09 JOULES/KG.
ENTHALPY OF THE MIXTURE = .144573E+09 JOULES/KG.
ENTROPY OF THE MIXTURE = .332118E+05 JOULES/KG./KELUIN
ENTROPY OF THE MIXTURE = .332118E+05 JOULES/KG./KELUIN
INT. ENERGY OF THE MIXTURE = .133621E+09 JOULES/KG.
INT. ENERGY OF THE MIXTURE = .133621E+09 JOULES/KG.
PRESSURE = .109513E-03 (BAR)
PRESSURE = .109513E-03 (BAR)
RESIDUES:-
0. -.71054E-14 0. 0. 0. 0.
.56843E-13 .22737E-12 .58797E-12 0. -.35527E-13

```

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Table I. Major and Minor Species in the Present Air Model
\begin{tabular}{|c|c|c|c|}
\hline Model I* & Model II & Model III & Model IV \\
\hline \multicolumn{4}{|c|}{Major species} \\
\hline \(\mathrm{O}_{2}\) & \(\mathrm{O}_{2}\) & \(\mathrm{N}_{2}\) & O \\
\hline \(\mathrm{N}_{2}\) & \(\mathrm{N}_{2}\) & O & N \\
\hline Ar & 0 & N & \(\mathrm{e}^{-}\) \\
\hline & NO & Ar & \(\mathrm{N}^{+}\) \\
\hline & N & & \(\mathrm{O}^{+}\) \\
\hline & Ar & & Ar \\
\hline \multicolumn{4}{|c|}{Minor species} \\
\hline \(\bigcirc\) & \(\mathrm{NO}^{+}\) & \(\mathrm{O}_{2}\) & \(\mathrm{O}_{2}\) \\
\hline NO & \(\mathbf{e}^{-}\) & NO & \(\mathrm{N}_{2}\) \\
\hline N & \(\mathrm{N}^{+}\) & \(\mathrm{NO}^{+}\) & NO \\
\hline \(\mathrm{NO}^{+}\) & \(\mathrm{O}^{+}\) & \(\mathrm{e}^{-}\) & \(\mathrm{NO}^{+}\) \\
\hline \(\mathrm{e}^{-}\) & \(\mathrm{Ar}^{+}\) & \(\mathrm{N}^{+}\) & \(\mathrm{Ar}^{+}\) \\
\hline \(\mathrm{N}^{+}\) & & \(\mathrm{O}^{+}\) & \\
\hline \(\mathrm{O}^{+}\) & & \(\mathrm{Ar}^{+}\) & \\
\hline \(\mathrm{Ar}^{+}\) & & & \\
\hline
\end{tabular}
\({ }^{*}\) The species \(\mathrm{NO}^{+}, \mathrm{e}^{-}, \mathrm{N}^{+}\), and \(\mathrm{Ar}^{+}\)are considered as trace species in model I.

Table II. Coefficients of Approximating Polynomials for Thermodynamic Functions


Table II. Concluded


Table III. Computational Time and Number of Iterations for Sample Cases
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Model & \[
\begin{aligned}
& t_{0}, \\
& \mu \mathrm{~S}
\end{aligned}
\] & \[
\begin{aligned}
& t_{i}, \\
& \mu \mathrm{~S}
\end{aligned}
\] & \[
\begin{gathered}
\rho, \\
\mathrm{kg} / \mathrm{m}^{3} \\
\hline
\end{gathered}
\] & \[
\begin{aligned}
& \hline T, \\
& \mathrm{~K} \\
& \hline
\end{aligned}
\] & \[
\begin{gathered}
N(\varepsilon) \\
(\varepsilon=0.001)
\end{gathered}
\] & \[
\begin{gathered}
N(\varepsilon) \\
(\varepsilon=0.0001)
\end{gathered}
\] \\
\hline \multirow[t]{6}{*}{I} & 286 & 20 & \(1 \times 10^{1}\) & 800 & 1 & 1 \\
\hline & & & \(1 \times 10^{-2}\) & 800 & 1 & 1 \\
\hline & & & \(1 \times 10^{-5}\) & 800 & 1 & 1 \\
\hline & & & \(1 \times 10^{1}\) & \(1600^{*}\) & 2 & 2 \\
\hline & & & \(1 \times 10^{-2}\) & \(1600^{*}\) & 2 & 2 \\
\hline & & & \(1 \times 10^{-5}\) & \(1600^{*}\) & 2 & 2 \\
\hline \multirow[t]{9}{*}{II} & 294 & 40 & \(1 \times 10^{1}\) & 1600* & 1 & 1 \\
\hline & & & \(1 \times 10^{-2}\) & 1600* & 1 & 1 \\
\hline & & & \(1 \times 10^{-5}\) & \(1600 *\) & 1 & 1 \\
\hline & & & \(1 \times 10^{1}\) & 7000 & 1 & 2 \\
\hline & & & \(1 \times 10^{-2}\) & 3000 & 1 & 1 \\
\hline & & & \(1 \times 10^{-5}\) & 2000 & 1 & 1 \\
\hline & & & \(1 \times 10^{1}\) & 13500* & 2 & 3 \\
\hline & & & \(1 \times 10^{-2}\) & 5968* & 1 & 1 \\
\hline & & & \(1 \times 10^{-5}\) & \(3577 *\) & 1 & 1 \\
\hline \multirow[t]{9}{*}{III} & 286 & 31 & \(1 \times 10^{1}\) & 13500* & 2 & 4 \\
\hline & & & \(1 \times 10^{-2}\) & \(5968 *\) & 1 & 3 \\
\hline & & & \(1 \times 10^{-5}\) & 3577* & 1 & 3 \\
\hline & & & \(1 \times 10^{1}\) & 13000 & 2 & 3 \\
\hline & & & \(1 \times 10^{-2}\) & 10000 & 1 & 2 \\
\hline & & & \(1 \times 10^{-5}\) & 7000 & 1 & 2 \\
\hline & & & \(1 \times 10^{1}\) & 15000 & 2 & 3 \\
\hline & & & \(1 \times 10^{-2}\) & 8960* & 2 & 2 \\
\hline & & & \(1 \times 10^{-5}\) & 5950* & 1 & 2 \\
\hline \multirow[t]{6}{*}{IV} & 293 & 31 & \(1 \times 10^{-2}\) & 8960* & 2 & 3 \\
\hline & & & \(1 \times 10^{-5}\) & \(5950 *\) & 2 & 3 \\
\hline & & & \(1 \times 10^{-2}\) & 14000 & 1 & 2 \\
\hline & & & \(1 \times 10^{-5}\) & 13000 & 1 & 2 \\
\hline & & & \(1 \times 10^{-2}\) & 15000 & 1 & 2 \\
\hline & & & \(1 \times 10^{-5}\) & 15000 & 1 & 1 \\
\hline
\end{tabular}
*Point on boundary between models.



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