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REDISTRIBUTION IN ASTROPHYSICALLY IMPORTANT HYDROGEN LINES

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Abstract

Under typical solar chromospheric conditions (electron density $n_e = 10^{11}$ cm^{-3} and temperature T = 6500 K) for hydrogen radiators, strong collisions due to both electrons and ions are well separated in time, so that a binary collision theory for collisional redistribution is applicable. However, a simple impact approximation may not be used, but rather a "unified" type theory is required in which frequency dependent line shape parameters are used to describe both impact and quasi-static regions of the spectrum. In addition, correlated terms which describe absorption and emission during a collision are important, and, in fact, without correlated terms describing both transfer of excitation and emission during the same collision unphysical predictions (such as negative intensities!) would be obtained.

In this paper theory is specifically developed for the coupled Ly-α, Ly-β, H-α system, and equations of statistical equilibrium and absorption and emission coefficients are given. All correlated events are examined and emission during a collision is found to be important in the line wings. Stimulated emission and absorption is also included within a broadband approximation. The major approximation, adopted for convenience, is to ignore lower state interaction. (For H-α estimated errors in the redistribution formulae from this approximation are the order of 20% in the line center and the order of 40% in the wings.)

It is found that for Ly-β Raman-coupling with H-α occurs and the overall scattering of radiation in the line wings is mostly coherent. In contrast, for H-α, incoherent redistribution due to lower state radiative decay (which occurs even in the absence of collisions) is found to dominate the coherent scattering. Finally, in the Lyman series the dominant incoherent contribution is associated with cascade transitions and inelastic collisions between different principal quantum states.

Preliminary calculations of hydrogen line formation in the solar chromosphere show improved fits to the observations. The most significant improvement is found for Ly-β where the Raman coupling with H-α increases the computed line wing intensity by typically a factor of two or even more, while the cascade contributions give an increase of about 20%. However, significant discrepancies still remain, and, since the redistribution theory is now on firm foundations, they must be attributed to the inadequacy of the solar models.

1. Introduction

The collisional redistribution of radiation by hydrogen is an important problem in astrophysics. In the sun the first two Lyman lines (Ly-α and Ly-β) and the Balmer-α line (H-α) are prominent spectral features. An understanding of their formation is crucial for solving the radiative transfer problem and accurately modeling the chromosphere. In particular, model atmospheres are sensitive to the redistribution in the line wings (outside the Doppler core).

In recent years an essentially exact formulation of collisional redistribution, valid in the binary collision regime, has become available [1,4-5], however, to date relatively little work has been done on the complicated hydrogenic system. Yelnik et al. [1] have considered redistribution of Ly-α under solar conditions. Mindful of the complexity of model atmosphere calculations, we present a simplified formulation, which should enable quantitative comparisons for Ly-α, Ly-β and H-α. So far, most astrophysical studies have used the redistribution function of Omont, Smith and Cooper (OSC) [9], which is however applicable only within the impact approximation and for isolated lines, and is thus inapplicable for hydrogenic lines. For Ly-β, large discrepancies now exist between observations and calculations [12].

Under chromospheric conditions (T=6500-8000°K) the dominant pressure broadening mechanism for hydrogen is provided by electrons and ions (mainly protons) each having a density of order 10^{11} cm^{-3} [1]. Resonance broadening effects should also be included [13] in particular in deeper and denser layers of the chromosphere. Under these conditions the binary collision approximation is valid and the effects of electrons and ions (and neutral hydrogen atoms) are thus additive. We will find, due to the Doppler effect and the broadband nature of the radiation fields, that the redistribution is insensitive to the values of the collisional broadening coefficients close to line center and hence it is a reasonable approximation to neglect fine structure splitting. [For Lyman-α the width of the profile is roughly three Doppler widths, which corresponds to about four times the maximum fine-structure splitting.] We will thus characterize the hydrogen levels by the principal quantum number n and the angular momentum ℓ. Figure 1 shows estimates of the line broadening parameters for Ly-α under typical chromospheric conditions [7]. Notice that the plasma frequencies (ω_{pi} and ω_{pe} for ions and electrons respectively) are small compared to their respective Weisskopf frequencies,

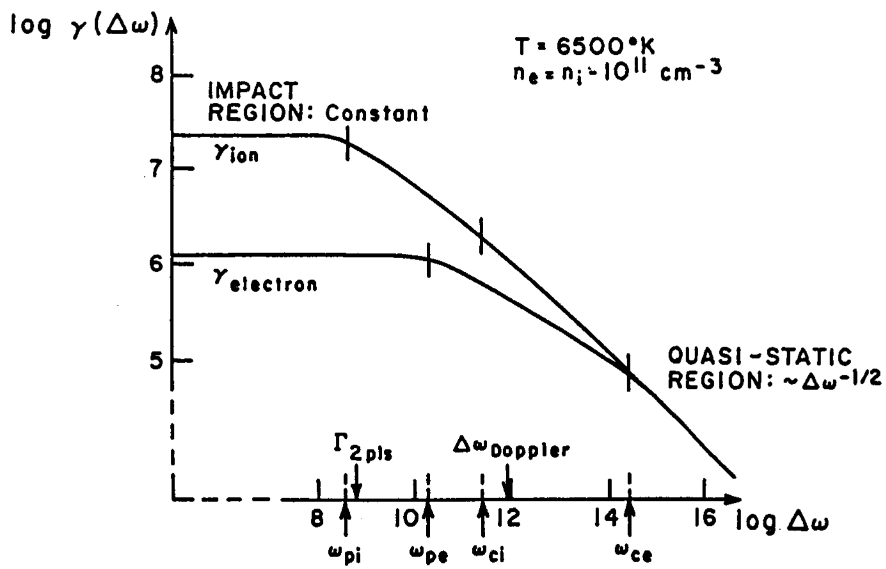

Fig. 1. Collision broadening parameters for Ly-α.

ω_{ci} and ω_{ce} as well as to the Doppler width, $\Delta\omega_D$. The inverse of the Weisskopf frequency is a typical strong collision duration, τ_c, whereas the inverse of ω_p is essentially a weak collision duration, τ_w. Strong collisions dominate in the quasi-static region ($> \omega_c$) and weak collisions dominate (by roughly a factor of 10) in the impact region ($< \omega_p$). The collisional widths are small compared to their respective Weisskopf (or even plasma) frequencies hence the binary collision approximation is valid.

In the present formulation we have carefully examined the correlated terms which describe absorption and emission events which occur <u>during</u> a collision. For the statistical equilibrium equations, which describe the populations of the various levels, since the dominant absorption and stimulated emission occurs within the impact region close to line center these correlated effects are unimportant. However, in the line wings for (e.g.) the emission coefficient, correlated terms which represent collisional mixing of ℓ-degenerate levels during an emission process are of crucial importance. This "destruction" term was designated $D^{(0)}(\Delta\omega)$ by Burnett et al. [5]. However, the coefficients [5] $D^{(i)}$ $D^{(ii)}$ and $D^{(iii)}$ describing both emission and absorption during the same collision are again negligible due to the fact that the dominant absorption is close to line center (and can occur both during and outside of a collision).

The major approximation of this work, adopted for convenience, is the assumption of no lower state interaction (NLSI). This is, of course, no problem for Ly-α and Ly-β since in the dipole approximation the 1s state is unperturbed. For H-α this is reasonable since the n=3 collisional matrix elements are typically ~9/4 times larger than the corresponding n=2 matrix elements. This approximation leads to several important simplifications. It means that the line broadening coefficients are diagonal in ℓ and that there is no collisional transfer of optical coherences. These transfer of coherence rates are equivalent to the "interference term" [8,10] of hydrogen line broadening theory. In the line center, transfer of coherence effects are negligible, and corresponding errors in using NLSI are the order of 20% (i.e. $(4/9)^2$), whereas in the wings the transfer of coherence is more important giving errors of order 40% (i.e. 4/9). (Detailed calculations for H-α give these percentages as approximately 14% and 43% respectively [14]). However, in the line wings the actual collisional terms are much smaller ($\lesssim$ 5%) than their line center value and contribute less significantly.

In Sec. 2 we formulate the redistribution problem and obtain expressions for the equations of statistical equilibrium and the emission and absorption coefficients.

In Sec. 3 we show that without the $D^{(0)}(\Delta\omega)$ correlated terms unphysical negative intensities would be obtained. The wings of Ly-α and Ly-β are found to be dominated by coherent (Rayleigh and Raman) scattering with the only incoherent redistribution being due to cascade terms, but H-α is dominated by an incoherent redistribution due to lower state (Γ_{2p1s}) radiative decay, which occurs even in the absence of collisions. (Redistribution in the absence of collisions has been observed by Lombardi et al. [15]). Finally we present some preliminary calculations using model atmospheres to predict the solar Ly-α and Ly-β profiles.

2. Formulation

We are concerned with hydrogen atoms interacting with Lyman-α, Lyman-β, and Balmer-α radiation in the presence of perturbers (both ions and electrons) (see Figure 2). The atom, with unperturbed Hamiltonian H_0, interacts with the perturbers through the collisional interaction $V_N(t)$ and the radiative interaction $V_E(t)$. Here

$$V_N(t) = \sum_i V_i(t) \qquad (2.1)$$

where $V_i(t)$ indicates the interaction of the i^{th} perturber with the hydrogen atom and

$$V_E(t) = -\vec{d} \cdot \vec{E}(t) \qquad (2.2)$$

where $\vec{d}$ is the dipole moment of the hydrogen atom and

$$\vec{E}(t) = \sum_j \mathcal{E}_j(t)\, e^{-i\omega_j t} + c.c \qquad (2.3)$$

with j indicating the fields due to Ly-α, Ly-β and H-α. The density operator $\rho(t)$ of the combined atom-radiation field-perturber system is then given by [1]

$$\frac{d\rho(t)}{dt} = [\mathcal{L}_0 + \tilde{V}_N(t) + \mathcal{L}_E(t) + \tilde{\Gamma}]\rho(t) \quad . \qquad (2.4)$$

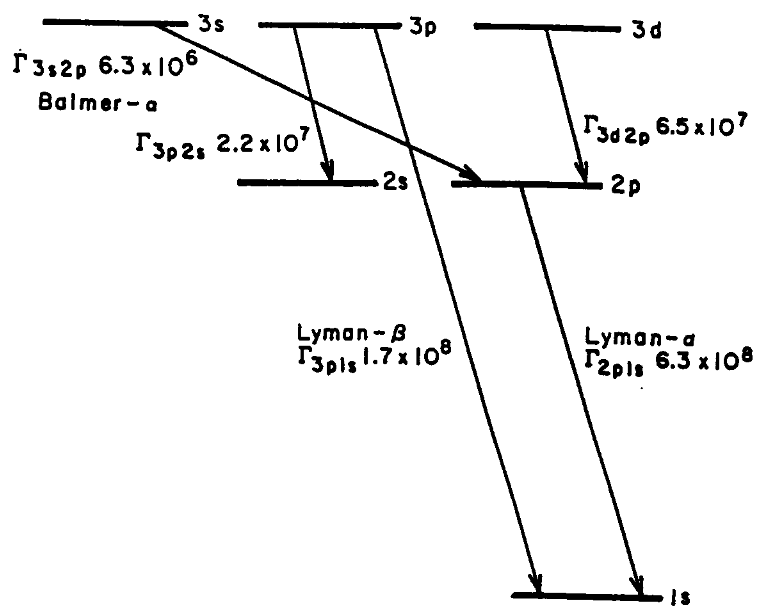

Fig. 2. Radiative decay rates for the Ly-α, Ly-β, H-α system.

The Liouville (tetradic) operators are defined for an arbitrary Hilbert-space operator $\hat{O}$ as follows

$$\mathcal{L}_0\hat{O} = \frac{1}{i\hbar}[H_0,\hat{O}] \qquad (2.5)$$

$$\mathcal{L}_E(t)\,\hat{O} = \frac{1}{i\hbar}[V_E(t),\hat{O}] \qquad (2.6)$$

$$\tilde{V}_N(t)\,\hat{O} = \frac{1}{i\hbar}[V_N(t),\hat{O}] \quad . \qquad (2.7)$$

$\tilde{\Gamma}$ is the (time independent) spontaneous decay damping operator and acts only within the radiator subspace.

2.1 Statistical equilibrium equations

To obtain equations for the populations (i.e. diagonal elements of the density operator) in the various states, we want to derive an equation of motion for the reduced density operator of the radiator $\sigma(t) = Tr_p\{\rho(t)\}$ where $Tr_p\{\ldots\}$ denotes a trace over all perturber coordinates.

As was indicated in the Introduction, strong collisions are separated in time, hence we shall make the binary collision approximation (BCA) [1], which allows complicated projection operators to be dropped. Then

$$\frac{d\sigma(t)}{dt} = [\tilde{L}_0 + \tilde{L}_E(t) + \tilde{\Gamma}]\,\sigma(t) + \int_0^t dt'\, M(t,t')\sigma(t') \quad . \qquad (2.8)$$

This equation is valid as long as $t >> \tau_c$, because effects due to initial correlations of the radiator-perturber system, which decay on a time scale of order τ_c, may then be neglected. We shall see, however, when calculating the dipole-autocorrelation functions from which the emission and absorption coefficients are obtained, that then initial correlations are of crucial importance and cannot be neglected.

Within the BCA the memory kernel $M(t,t')$ is given by

$$M(t,t') = N\, Tr_1[\tilde{V}_1(t)\, \tilde{G}_1(t,t')\, \tilde{V}_1(t')\rho_{p_1}] \qquad (2.9)$$

where $\tilde{V}_1(t)$ represents the tetradic interaction potential of one perturber, $Tr_1\{\ldots\rho_{p_1}\}$ represents a single perturber average and $\tilde{G}_1(t,t')$ satisfies the following equation

$$\frac{d\tilde{G}_1(t,t')}{dt} = [\tilde{L}_0 + \tilde{V}_1(t) + \tilde{L}_E(t) + \tilde{\Gamma}]\,\tilde{G}_1(t,t') \qquad (2.10)$$

with

$$\tilde{G}_1(t',t') = 1 \quad . \qquad (2.11)$$

In evaluating $Tr_1\{\ldots\rho_{p_1}\}$ we assume a spherically symmetric collisional environment, which, by the use of rotational invariance properties, considerably simplifies the analysis. It is customary [1,2] to expand equation (2.10), so that $M(t,t')$ in equation (2.9) may be written in powers of $\tilde{L}_E(t)$. This expansion is valid since the probability of absorption or stimulated emission during a collision is small.

The zeroth order term in $\tilde{L}_E(t)$, which we will designate as $M_0(t-t')$, gives rise to a collisional damping operator. The first order terms in $\tilde{L}_E(t)$, have been designed as C^1 terms [1,4-5], whereas the second order in $\tilde{L}_E(t)$ terms have been designated C^2 or $D^{(i)(ii)(iii)}$ according to whether the two photons involved are respectively the same or different (such as for absorption and emission or two-photon absorption within the same collision).

In order to solve the equations specified by equation (2.8) it is necessary to take matrix elements. The diagonal elements of σ are called "populations." The off-diagonal elements of σ are called "coherences" and those linking different principal quantum numbers are "optical coherences." For the statistical equilibrium equations, we are interested in populations. However, populations and optical coherences are linked by the radiation fields $\mathcal{E}_j(t)$. The rotating wave approximation is invoked [1] but even then we must take into account the fact that the fields are not monochromatic. Thus $\mathcal{E}_j(t)$ fluctuates in time and equation (2.8) represents a stochastic differential equation. Fortunately, in our case the bandwidth of the radiation b (greater than the Doppler width) is large compared with both the collisional and radiative relaxation rates, consequently the correlation time for the fields, depending on $\langle\mathcal{E}_j(t+\tau)\,\mathcal{E}_j^*(t)\rangle$, is so short that a decorrelation procedure may be adopted.

The mixing of the fields may be handled in the following manner (an extension of the method used by CBBH [7]). Consider, for example, a representative coupling for the 3s population:

$$\frac{d\sigma(3s,3s)}{dt} = i d_{2p3s}\, \mathcal{E}_H(t)\sigma(3s,2p) + \ldots \qquad (2.12)$$

$$\frac{d\sigma(3s,2p)}{dt} = i d_{3s2p}\, \mathcal{E}_H^*(t)\, [\sigma(3s,3s) - \sigma(2p,2p)] + i d_{1s2p}\sigma(3s,1s)\mathcal{E}_\alpha(t) + \cdots \qquad (2.13)$$

Formally integrating equation (2.13) and substituting into equation (2.12) leads to terms like

$$\int_0^t \mathcal{E}_H(t)\,\mathcal{E}_H^*(t')\,[\sigma(3s,3s;t') - \sigma(2p,2p;t')]dt' + \int_0^t \mathcal{E}_H(t)\,\mathcal{E}_\alpha(t')\sigma(3s,1s;t')dt' \; .$$

Averaging over the field fluctuations and performing a decorrelation gives

$$\int_0^t \langle \mathcal{E}_H(t)\, \mathcal{E}_H^*(t')\, [\sigma(3s,3s;t') - \sigma(2p,2p;t')]\rangle\, dt'$$

$$= \int_0^t \langle \mathcal{E}_H(t)\, \mathcal{E}_H^*(t')\rangle \langle [\sigma(3s,3s;t') - \sigma(2p,2p;t')]\rangle dt'$$

$$= \int_0^t f(t-t')\, [\sigma(3s,3s;t') - \sigma(2p,2p;t')]\, dt' \qquad (2.14)$$

We also put the term $\langle \mathcal{E}_H(t)\mathcal{E}_\alpha(t')\rangle$ which involves different photons equal to zero. This $\sigma(3s,1s)$ coherence is representation of two-photon (Ly-α and H-α) absorption. Detailed evaluation shows that it is of the order of γ/b where γ is a representative relaxation rate and hence is negligible for broadband radiation. Similarly $\langle \mathcal{E}_j \mathcal{E}_j\rangle = \langle \mathcal{E}_j^* \mathcal{E}_j^*\rangle = 0$.

Since $\langle \mathcal{E}_H(t)\, \mathcal{E}_H^*(t')\rangle$ in equation (2.14) just depends on $(t-t')$, the equations are most conveniently solved by Laplace transforms. Often it is convenient to assume the field represent a Lorentzian profile of width b_j

$$\text{i.e. } \langle \mathcal{E}_j^*(t)\, \mathcal{E}_j(t')\rangle = \Omega_j^2\, e^{-b_j(t-t')} \quad \text{for } t>t' \qquad (2.15)$$

In general, the intensity is just the transform of $\langle \mathcal{E}_j^*(t)\mathcal{E}_j(t')\rangle$ [6].

If z is the Laplace transform variable, steady state solutions can be obtained in the limit $z \to 0$.

Due to stationarity [3], i.e. $\langle \tilde{V}_1(t)\, \tilde{V}_1(t')\rangle = \langle \tilde{V}_1(t-t')\tilde{V}_1(0)\rangle$, the M terms are also easy to Laplace transform.

The actual values of the collisional quantities depend on the matrix elements involved. Examples of the terms associated with $\hat{M}_0(z)$ are:

$$(1) \qquad \frac{d\sigma(3p,3p)}{dt} = \langle\langle 3p,3p|\hat{M}_0(z=0)|3p,3p\rangle\rangle\, \sigma(3p,3p) + \ldots$$

$$= -\kappa_{3p}\, \sigma(3p,3p) + \ldots \qquad (2.16)$$

κ_{3p} is the total inelastic rate from the 3p state.

$$(2) \qquad \frac{d\sigma(3p,3p)}{dt} = \langle\langle 3p,3p|\hat{M}_0(z=0)|3s,3s\rangle\rangle\, \sigma(3s,3s) + \ldots$$

$$= \kappa_{3s3p}\, \sigma(3s,3s) + \ldots \qquad (2.17)$$

κ_{3s3p} is the inelastic rate from the 3s to 3p state.

$$(3) \qquad \frac{d\sigma(3p,2s)}{dt} = \langle\langle 3p,2s|\hat{M}_0(z = -i(\omega_H - \omega_{32}))|3p,2s\rangle\rangle\, \sigma(3p,2s) + \ldots$$

$$= -\gamma_{3p2s}(\Delta\omega_H)\, \sigma(3p,2s) + \ldots \qquad (2.18)$$

Ignoring an imaginary part (i.e. a line shift), $\gamma_{3p2s}(\Delta\omega_H)$ is the destruction rate of the $3p \to 2s$ optical coherence and corresponds to the collisional line broadening rate for this transition. As in the "unified theory" [3], these rates are frequency dependent. For $\Delta\omega_H$ small it gives the impact approximation results, whereas $\Delta\omega_H$ larges gives rise to quasi-static theory results

$$(4) \qquad \frac{d\sigma(3p,2s)}{dt} = \langle\langle 3p,2s|\hat{M}_0(z=-i(\omega_H-\omega_{32}))|3s,2p\rangle\rangle\, \sigma(3s,2p) + \ldots$$

$$= \langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,2p\rangle\rangle\, \sigma(3s,2p) + \ldots \qquad (2.19)$$

$\langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,2p\rangle\rangle$ corresponds to a collisional transfer of coherence rate. It is zero for no lower state interaction (NLSI). Examining the matrix elements in detail (compare OSC [9], appendix A) shows that it is exactly equivalent is the so called "interference term" of hydrogen line broadening [1,10]. Although not negligible in the wings for the equations for the absorption and emission coefficients, in the statistical equilibrium equations the transfer of coherence introduces negligible terms, again of order γ/b.

Due to the presence of $\bar{\Gamma}$ in equation (2.10) the collisional rates are modified from the usual line broadening rates by the effect of spontaneous decay during collisions. This effect may be easily estimated for weak collisions. Denoting K(z) by

$$K(z) = N \int_0^\infty d\tau\, e^{z\tau} \langle \tilde{V}_1(\tau)\, \tilde{V}_1(0)\rangle \qquad (2.20)$$

due to the introduction of $e^{-\Gamma\tau}$ type factors, the correction depends on $\Gamma\left[\frac{K(0)-K(-\Gamma)}{\Gamma}\right]$. Using the method of Yelnik et al. [7] (their appendix) this correction to the collisional rates is of order $\Gamma/[\omega_p \log(\omega_p \tau_c)]$. For Lyman-α this is about 7% under typical conditions and will henceforth be ignored.

The corrections due to $\hat{M}_1(z)$ and $\hat{M}_2(z)$ have also been subjected to detailed estimates [2,4-5]. The C^1 terms connect populations with optical coherences, and the C^2 terms connect populations with populations (or coherences with coherences). Often, these terms give zero contribution due to a sum over states [4]. For examples, in absorption of a photon, the final state is of no interest and is summed over to give a zero contribution. On the other hand, for stimulated emission the C^1 or C^2 terms can describe processes in which, for example, the system starting in either the 2s or 2p states could be stimulated to emit <u>during</u> a collision. If the initial states are populated according to their statistical weights, a sum over initial states again gives zero contribution. Invoking NLSI considerably reduces the number of non-zero correlation terms.

Examples of various correlation terms are given in Figure 3. Since we are dealing with tetradic operators two initial and two final states are involved. This figure includes the $D^{(ii)}$ term corresponding to two-photon absorption. For our purposes, in the statistical equilibrium equations, estimates based on weak collisions suffice. Most of the terms are at most of order γ/b. There is a C^2 term which corresponds to stimulated emission during collisions which modifies the rates in an analogous manner to the spontaneous emission during a collision discussed above. Its correction is again of order 7% for Ly-α under typical conditions.

There are thus no significant corrections for the statistical equilibrium equations. For convenience, to avoid off-diagonal elements of the density matrix due to optical pumping, we perform an angle average by assuming the radiation field is isotropic. Also coherences such as $\sigma(3s,3d)$ now vanish. Then denoting $\sigma(1s,1s) = n(1s)$ etc.

$$\frac{dn(1s)}{dt} = -\kappa_{1s}n(1s) + \Gamma_{2p1s}\, n(2p) + \Gamma_{3p1s}\, n(3p) - R(1s,2p) - R(1s,3p) + s(1s) \tag{2.21}$$

$$\frac{dn(2p)}{dt} = \Gamma_{3s2p}\, n(3s) + \Gamma_{3d2p}\, n(3d) - (\Gamma_{2p1s}+\kappa_{2p})\, n(2p) + \kappa_{2s2p}n(2s) + R(1s,2p) - R(2p,3s) - R(2p,3d) + s(2p) \tag{2.22}$$

$$\frac{dn(2s)}{dt} = \Gamma_{3p2s}\, n(3p) - \kappa_{2s}\, n(2s) + \kappa_{2p2s}n(2p) - R(2s,3p) + s(2s) \tag{2.23}$$

$$\frac{dn(3s)}{dt} = -(\Gamma_{3s2p}+\kappa_{3s})\, n(3s) + \kappa_{3p3s}\, n(3p) + \kappa_{3d3s}n(3d) + R(2p,3s) + s(3s) \tag{2.24}$$

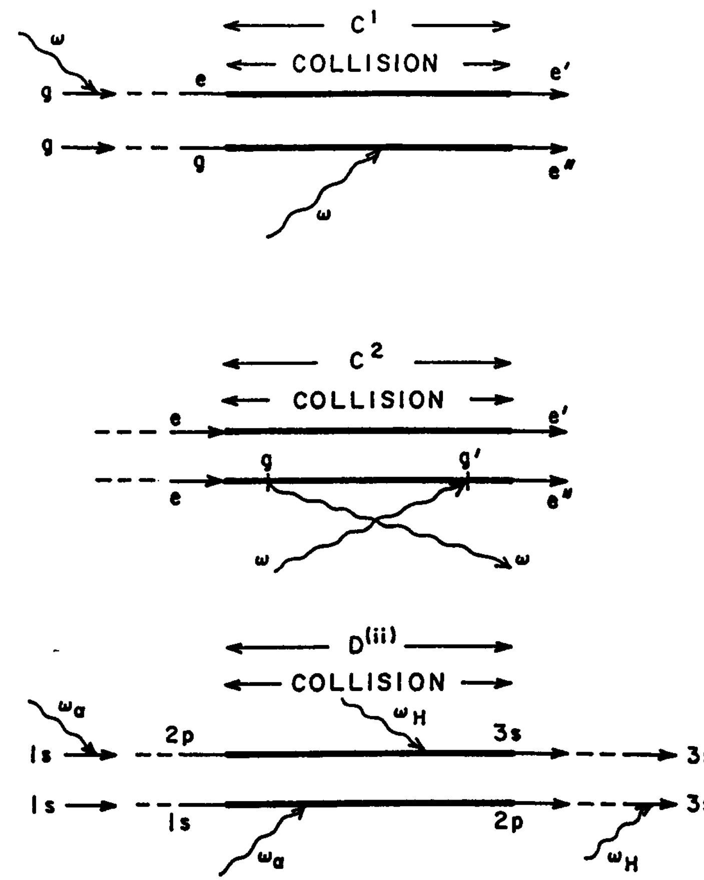

Fig. 3. Examples of correlation terms for populations.

$$\frac{dn(3p)}{dt} = -(\Gamma_{3p1s}+\Gamma_{3p2s}+\kappa_{3p})n(3p)+\kappa_{3d3p}n(3d)+\kappa_{3s3p}n(3s)+R(1s,3p)+R(2s,3p)+s(3p) \quad (2.25)$$

$$\frac{dn(3d)}{dt} = -(\Gamma_{3d2p}+\kappa_{3d})n(3d) + \kappa_{3p3d}n(3p) + \kappa_{3s3d}n(3s) + R(2p,3d) + s(3d) \quad . \quad (2.26)$$

Here quantities like s(2s) represent inelastic rates into 2s from other principal quantum numbers, as well as possible other cascade contributions. The quantities R represent absorption and stimulated emission, and are given by (for example)

$$R(1s,3p) = B_{1s3p} \int d\omega'_\beta \, J_\beta(\Delta\omega'_\beta) \, f_{1s3p}(\Delta\omega'_\beta) \, [n(1s) - 1/3 \, n(3p)] \quad (2.27)$$

$$R(2p,3d) = B_{2p3d} \int d\omega'_H \, J_H(\Delta\omega'_H) \, f_{2p3d}(\Delta\omega'_H) \, [n(2p) - 3/5 \, n(3d)] \quad (2.28)$$

etc. Here the $J(\Delta\omega)$ are angle-averaged intensities. The corresponding absorption (or emission) profiles are

$$f_{1s2p}(\Delta\omega_\alpha) = \frac{\Gamma_{2p}(\Delta\omega_\alpha)/\pi}{\Delta\omega_\alpha^2 + \Gamma_{2p}(\Delta\omega_\alpha)^2} \quad (2.29)$$

$$f_{1s3p}(\Delta\omega_\beta) = \frac{\Gamma_{3p}(\Delta\omega_\beta)/\pi}{\Delta\omega_\beta^2 + \Gamma_{3p}(\Delta\omega_\beta)^2} \quad (2.30)$$

$$f_{2s3p}(\Delta\omega_H) = \frac{\Gamma_{3p}(\Delta\omega_H)/\pi}{\Delta\omega_H^2 + \Gamma_{3p}(\Delta\omega_H)^2} \quad (2.31)$$

$$f_{2p3s}(\Delta\omega_H) = \frac{\Gamma_{3s}(\Delta\omega_H)/\pi}{\Delta\omega_H^2 + \Gamma_{3s}(\Delta\omega_H)^2} \quad (2.32)$$

$$f_{2p3d}(\Delta\omega_H) = \frac{\Gamma_{3d}(\Delta\omega_H)/\pi}{\Delta\omega_H^2 + \Gamma_{3d}(\Delta\omega_H)^2} \quad (2.33)$$

where

$$\Gamma_{2p}(\Delta\omega_\alpha) = \gamma_{2p}(\Delta\omega_\alpha) + \frac{\Gamma_{2p1s}}{2}$$

$$\Gamma_{3p}(\Delta\omega) = \gamma_{3p}(\Delta\omega) + \frac{\Gamma_{3p2s}}{2} + \frac{\Gamma_{3p1s}}{2}$$

$$\Gamma_{3s}(\Delta\omega_H) = \gamma_{3s}(\Delta\omega_H) + \frac{\Gamma_{3s2p}}{2} + \frac{\Gamma_{2p1s}}{2}$$

$$\Gamma_{3d}(\Delta\omega_H) = \gamma_{3d}(\Delta\omega_H) + \frac{\Gamma_{3d2p}}{2} + \frac{\Gamma_{2p1s}}{2}$$

Notice that lower state radiative decay is included in equations (2.32) and (2.33). In order to include the stimulated terms in equation (2.27) etc., (i.e. the - 1/3 n(p) term) we have assumed that the upper state velocity distributions are Maxwellian.

The integrals in equations (2.27) and (2.28) are approximately $J_\beta(0)$ and $J_H(0)$ respectively for broadband radiation ($b \gg \gamma,\Gamma$), consequently, these equations are essentially independent of $\gamma(\Delta\omega)$ and only $\Delta\omega' \sim 0$ is important. The fact that only small $\Delta\omega'$ is important for determining the populations is the reason that the correlation terms (D, C^1 etc.) were negligible.

In addition, it is convenient to assume NLSI. Then the broadening due to ions and electrons is diagonal in ℓ, with the absorption and emission during the collision characterized by the frequency dependence.

Specifically [11],

$$\gamma_{2p}(\Delta\omega_\alpha) = \langle 2p|\gamma(\Delta\omega_\alpha)|2p\rangle \quad (2.34)$$

$$\gamma_{3s}(\Delta\omega_H) = \langle 3s|\gamma(\Delta\omega_H)|3s\rangle \quad (2.35)$$

$$\gamma_{3p}(\Delta\omega_i) = \langle 3p|\gamma(\Delta\omega_i)|3p\rangle \quad ; \quad i = \beta \text{ or } H \quad (2.36)$$

$$\gamma_{3d}(\Delta\omega_H) = \langle 3d|\gamma(\Delta\omega_H)|3d\rangle \quad . \quad (2.37)$$

Each is the sum of ion and electron terms.

A line shift $\Delta(\Delta\omega)$ should also be included in equations (2.29) to (2.33) to allow for exact normalization, but in the binary collision regime, to the accuracy considered in this paper, it may safely be neglected.

2.2 Emission coefficients

The emission coefficient $j(\omega)$ is obtained from the Fourier-Laplace transform of the dipole autocorrelation function, thus [5]

$$J(\omega) \sim 2\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\, \langle \vec{d}^+(t_0)\vec{d}^-(t_0+t)\rangle$$

$$= 2\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\, \mathrm{Tr}_A[g(t)\vec{d}\,] \qquad (2.38)$$

where $\vec{d}^\pm$ are raising and lowering parts of the dipole operator and $\mathrm{Tr}_A[\ldots]$ denotes a trace over atomic states. In the binary collision approximation the function g(t) satisfies [5] with $t_0 = 0$

$$\frac{dg(t)}{dt} = [L_0 + L_E(t) + \tilde{\Gamma}]g(t) + \int_0^t dt'\, M(t,t')g(t')$$

$$+ N\,\mathrm{Tr}_1[\tilde{V}_1(t)\,\tilde{G}_1(t,0)\,[(\int_{-\infty}^0 dt'\,\tilde{G}_1(0,t')\tilde{V}_1(t')\,\sigma(t'))\vec{d}^+] \quad . \qquad (2.39)$$

The emission coefficient is therefore obtained from the Laplace transform $\hat{g}(z)$ of g(t) i.e.

$$J(\omega) \propto 2\,\mathrm{Re}\,\mathrm{Tr}_A[\vec{d}^-\hat{g}(z=-i\omega)] \quad . \qquad (2.40)$$

The equation of motion for g(t) is essentially the same as that for the density matrix σ(t) [eq. (2.8)]. There are two major differences. The first is the third term in equation (2.39) which represents initial correlations due to the fact that g(0) cannot be decorrelated into atom and perturber subspaces at t=0. Note that this term depends on σ(t') for t' ≤ 0. Secondly, g(t) has different initial conditions when compared to σ(t). The spectrum is obtained from the off-diagonal elements of g(t).

In order to illustrate the features involved in the solution of equation (2.39), as an example, we consider $\hat{g}(3p,2s)$ associated with the 3p → 2s transition in H-α. The initial conditions of importance for these transitions are

$$g(3p,2s;0) = \sigma(3p,3p;0)d_{2s3p} \qquad (2.41)$$

$$g(1s,2s;0) = \sigma(1s,3p;0)d_{2s3p} \qquad (2.42)$$

and

$$g(2s,2s;0) = \sigma(2s,3p;0)d_{2s3p} \quad . \qquad (2.43)$$

Equation (2.42) gives rise to Raman coupling between H-α and Ly-β and equation (2.43) leads to Rayleigh scattering in H-α. Before we can Laplace transform equation (2.39) we have to see how averaging over the fluctuating fields lead to convolutions. Thus we have to consider terms like

$$\frac{dg(3p,2s)}{dt} = id_{3p2s}\mathcal{E}_H^*(t)\,[g(3p,3p) - g(2s,2s)] - id_{3p1s}\mathcal{E}_\beta^*(t)g(1s,2s) + \ldots$$
$$(2.44)$$

$$\frac{dg(3p,3p)}{dt} = id_{2s3p}[\mathcal{E}_H(t)\,g(3p,2s) - \mathcal{E}_H^*(t)g(2s,3p)] + \ldots \qquad (2.45)$$

$$\frac{dg(1s,2s)}{dt} = id_{1s3p}\,\mathcal{E}_\beta(t)\,g(3p,2s) + \ldots \qquad (2.46)$$

Formally integrating equation (2.45), substituting into equation (2.44), and decorrelating the field averages with $\langle \mathcal{E}_H^*\mathcal{E}_H^*\rangle = 0$ gives

$$\frac{dg(3p,2s)}{dt} = -|d_{3p2s}|^2 \int_0^t dt'\, \langle \mathcal{E}_H(t)\,\mathcal{E}_H^*(t')\rangle\, g(3p,2s) + \ldots \qquad (2.47)$$

The Laplace transform of the RHS of eq. (2.47) gives [6] $-B_{3p2s}J_H(\Delta\omega_H)\hat{g}(3p,2s)$ where B_{3p2s} is the stimulated emission coefficient from 3p to 2s and $J_H(\Delta\omega_H)$ is the intensity at frequency $\Delta\omega_H = \omega - \omega_{32}$. (A light shift has been ignored.) This term corresponds to power broadening by an amount $B_{3p2s}\,J_H\,(\Delta\omega_H)$ due to the stimulated lifetime. Similarly, integrating equation (2.46) gives a power broadening $B_{3p1s}\,J_\beta(\Delta\omega_H)$ due to the stimulation due to Lyman-β photons (with $\Delta\omega_\beta = \Delta\omega_H$ or $\omega_\beta = \omega + \omega_{21}$). The initial condition for g(1s,2s) also contributes, thus

$$\frac{dg(3p,2s)}{dt} = -\,id_{3p1s}\,\langle \mathcal{E}_\beta^*(t)\,\sigma(1s,3p;0)\rangle\, d_{2s2p} \bullet \ldots$$

$$= d_{2s3p}|d_{3p1s}|^2 \int_{-\infty}^0 dt'\, \langle \mathcal{E}_\beta^*(t)\,\mathcal{E}_\beta(t')\rangle\, e^{-i\omega_\beta' t'}\, [\sigma(1s,1s) - \sigma(3p,3p)] + \ldots$$
$$(2.48)$$

The second term is obtained by integrating the relevant part of the equation for σ(1s,3p) and ignoring damping since the correlation time for the field is short (b >> γ,Γ). Following CBBH [6] the Laplace transform of equation (2.48) is easy to perform, to obtain

$$\frac{1}{2}\,\Gamma_{3p2s}B_{1s3p}(n(1s) - 1/3\; n(3p))\,\frac{f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)} \int d\omega_\beta' J_\beta(\Delta\omega_\beta')\; \{\delta(\omega_\beta' - \omega_{21} - \omega) - f_{1s3p}(\Delta\omega_\beta')\}$$
$$(2.49)$$

with $\Gamma_{3p}(\Delta\omega_H) = \gamma_{3p}(\Delta\omega_H) + \frac{1}{2}\Gamma_{3p2s} + \frac{1}{2}\Gamma_{3p1s}$. The Laplace transform of equation (2.48) gives rise to the terms under the integral in equation (2.49). They correspond to Raman coupling between H-α and Ly-β. In line center, $\Delta\omega_H \sim 0$, the integral is essentially zero (of order γ/b). In the wings, $\Delta\omega_H$ large, the result is practically independent of damping parameters, since the integral is approximately $[J_\beta(\Delta\omega_H) - J_\beta(0)]$ and $f_{2s3p}(\Delta\omega_H)/\Gamma_{3p}(\Delta\omega_H) = 1/(\pi\, \Delta\omega_H^2)$. In a similar manner eliminating the g(2s,2s) coherence gives rise to power broadening, $B_{2s3p}J_H(\Delta\omega_H)$, and the initial condition [eq. (2.43)] gives rise to Rayleigh scattering in a form similar to equation (2.49).

We can now examine the effects due to collisional damping. We need only consider $\hat{M}_0(z)$, since, just as in the statistical equilibrium equations, absorption and stimulated emission for the $\hat{M}_1$ and $\hat{M}_2$ terms (i.e. C^1, C^2 etc.) are negligible. Taking into account the initial condition of equation (2.41)

$$[i\Delta\omega_H+\gamma_{3p2s}(\Delta\omega_H)]\hat{g}(3p,2s) = \sigma(3p,3p;0)d_{2s3p} + \langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,3p\rangle\rangle\hat{g}(3s,2p)+\ldots \tag{2.50}$$

$$[i\Delta\omega_H+\gamma_{3s2p}(\Delta\omega_H)]\,\hat{g}(3s,2p) = \sigma(3s,3s;0)d_{3s2p} + \ldots \tag{2.51}$$

Thus

$$\hat{g}(3p,2s) = \frac{\sigma(3p,3p;0)d_{2s3p}}{i\Delta\omega_H + \gamma_{3p2s}(\Delta\omega_H)} + \frac{\langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,2p\rangle\rangle\ \sigma(3s,3s;0)d_{3s2p}}{[i\Delta\omega_H+\gamma_{3p2s}(\Delta\omega_H)]\ [i\Delta\omega_H+\gamma_{3s2p}(\Delta\omega_H)]} + \ldots \quad . \tag{2.52}$$

To obtain the final spectrum we take the real part and since the observed frequency is Doppler shifted, i.e. $\omega \to \omega - \vec{k}\cdot\vec{v}$, we perform a Doppler convolution. For $\Delta\omega_H \ll \Delta\omega_D$, the effect of the integration is to put $i\Delta\omega_H \sim \Delta\omega_D$ in equation (2.52), and thus the second term is negligible. However, for $\Delta\omega_H \gg \Delta\omega_D$ we have to compare

$$\mathrm{Re}\,\frac{1}{[i\Delta\omega_H + \gamma_{3p2s}(\Delta\omega_H)]} = \frac{\gamma_{3p2s}(\Delta\omega_H)}{\Delta\omega_H^2} \text{ with } -\frac{\langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,2p\rangle\rangle}{\Delta\omega_H^2} \quad ,$$

which are comparable. Thus, as discussed in the Introduction, transfer of coherence effects are important in the line wings. Due to their complicating factors, we adopt NLSI for which $\langle\langle 3p,2s|\gamma(\Delta\omega_H)|3s,2p\rangle\rangle = 0$, etc.

Lastly, we have to consider the so called "destruction" terms -- the last term in equation (2.39). Again we expand in powers of $\tilde{L}_E(t)$. The zeroth order term [5] is designated $D^{(0)}$. This term describes emission during a collision occurring at the initial time t = 0. Since lower state interaction leads to complicated expressions, we will again adopt NLSI.

Again, as an example, we consider the 3p2s transition. The contribution to the angle-averaged emission coefficient due to the $D^{(0)}(\Delta\omega_H)$ term is [5]

$$J_H(\Delta\omega_H) = \frac{1}{\pi}\,\mathrm{Re}\,\left[\frac{1}{\Gamma_{3p}(\Delta\omega_H) - i\Delta\omega_H}\sum_c C^1(0;3p2s,3c3c,\Delta\omega_H)\ n(3c)\ \left(\frac{2\ell_p+1}{2\ell_c+1}\right)^{1/2}\right] + \ldots \tag{2.53}$$

with c = s,p, and d. These terms are illustrated in Figure 4 -- note in particular in cases (a) and (c) that the collisional interaction is necessary to change the states from e.g., 2s to 2p so that radiation can occur.

Simple estimates for the C^1 terms are straightforward. We have already seen that they are negligible for $\Delta\omega_H$ small (then of order $\gamma/[\omega_p\log(\omega_p\tau_c)]$ or of order $\gamma/\Delta\omega_D$ if a Doppler convolution is performed). For large $\Delta\omega_H(>\Delta\omega_c)$ [Ref 7:appendix]

$$\left(\frac{2\ell_p+1}{2\ell_s+1}\right)^{1/2} C^1(0;3p2s,3s3s,\Delta\omega_H) = \left[-\frac{\kappa_{3s3p}}{2} + 3a_{3s3p}\ \gamma_{3p}(\Delta\omega_H)\right]/i\Delta\omega_H \quad . \tag{2.54}$$

The integrals necessary to evaluate C^1 have both an end point (principal value) contribution, [the first term in eq. (2.54)], and a stationary phase contribution [the second term in eq. (2.54)]. The quantity a, of order unity, is easily evaluated by transforming the matrix elements of $\tilde{G}_+(t,t')$ to parabolic states [11]. Terms like $\exp[in(q_1\int V_1(t')dt' - q_2\int V_1(t')dt')]$ occur, which for $q_1 = q_2$ are approximated as $\exp[inq_2V(0)\tau]$ and, for $q_1 \neq q_2$, due to the large phase shift during a collision, are effectively zero. The $\exp[inq_2V(0)\tau]$ is just the form for the time development operator which gives rise to the usual quasi-static profile. In fact, for off-diagonal elements [e.g. c ≠ p in eq. (2.53)] the $a\gamma(\Delta\omega)$ term is the quasi-static emission profile for the atom initially in a <u>specified</u> initial state [the 3s state for eq. (2.54)].

If the initial states n(3c) were populated according to their statistical weights the sum over c in equation (2.54) would give zero. However, deviations of the populations from statistical weighting are of the order of Γ/κ, thus since C^1 is proportional to $\kappa/\Delta\omega$ [from eq. (2.54)] a significant contribution is obtained from $D^{(0)}$ $(\sim \Gamma/\Delta\omega^2)$ even for very large collision rates.

We have found that a sufficient approximation for the $D^{(0)}(\Delta\omega_H)$ term of equation (2.53) is

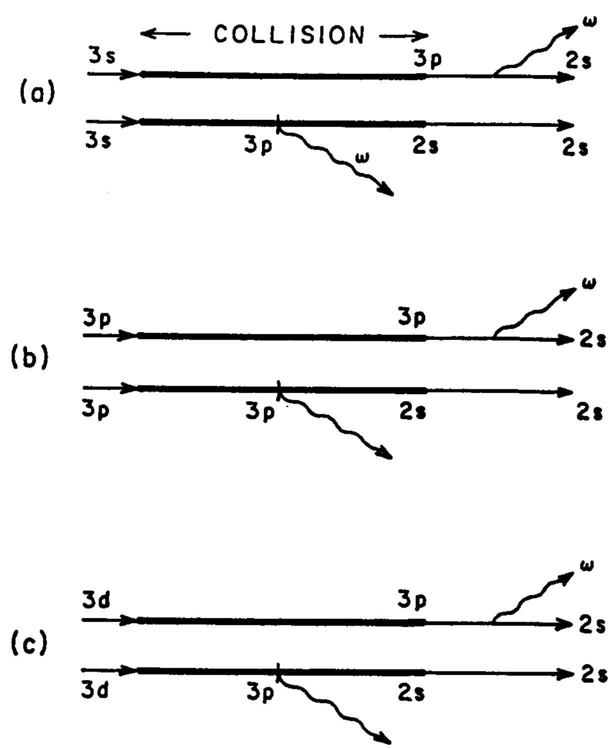

Fig. 4. Contributions from $D^{(0)}$ term.

$$\frac{\hbar\omega_H}{4\pi}\Gamma_{3p2s}\frac{f_{3p2s}(\Delta\omega_H)}{2\Gamma_{3p}(\Delta\omega_H)}\,[\kappa_{3p}n(3p)-\kappa_{3s3p}n(3s)-\kappa_{3d3p}n(3d)$$

$$+2\,\gamma_{3p}(\Delta\omega_H)\,[a_{3s3p}(3n(3s)-n(3p))+a_{3d3p}(3/5\;n(3d)-n(3p))]]G(\Delta\omega_H) \quad (2.55)$$

where the factor $G(\Delta\omega_H)$ has the behavior

$$G(\Delta\omega_H)=0 \quad \text{for} \quad \Delta\omega_H<\omega_p$$

$$G(\Delta\omega_H)=1 \quad \text{for} \quad \Delta\omega_H<\omega_c \quad (2.56)$$

and in the intermediate region a sufficient approximation is:

$$G(\Delta\omega_H)=\frac{\log\,(\Delta\omega_H/\omega_p)}{\log\,(\omega_c/\omega_p)} \quad . \quad (2.57)$$

Finally the destruction terms $D^{(i)}$ $D^{(ii)}$ and $D^{(iii)}$ which here correspond to absorption and emission during the same collision are again negligible (of order γ/b or less) due to the fact that dominant absorption is close to line center. We therefore obtain the following expressions for the emission coefficients, assuming the levels have a Maxwellian velocity distribution, and having performed an angle average over outgoing and incoming (isotropic) radiation fields.

$$J_\alpha(\Delta\omega_\alpha)=\frac{\hbar\omega_\alpha}{4\pi}\Bigg[n(2p)\,\Gamma_{2p1s}\,f^P_{1s2p}(\Delta\omega_\alpha)$$

$$+\frac{\Gamma_{2p1s}}{2}\,B_{1s2p}(n(1s)-1/3\;n(2p))\,\frac{f_{1s2p}(\Delta\omega_\alpha)}{\Gamma_{2p}(\Delta\omega_\alpha)}\int d\omega'_\alpha J_\alpha(\Delta\omega'_\alpha)\;[\delta(\omega_\alpha-\omega'_\alpha)-f_{1s2p}(\Delta\omega'_\alpha)]$$

$$+\frac{\Gamma_{2p1s}}{2}\,\frac{f_{1s2p}(\Delta\omega_\alpha)}{\Gamma_{2p}(\Delta\omega_\alpha)}\,[\kappa_{2p2s}n(2p)-\kappa_{2s2p}n(2s)+2\gamma_{2p}(\Delta\omega_\alpha)a_{2s2p}(3n(2s)-n(2p))]G(\Delta\omega_\alpha)\Bigg] \;. \quad (2.58)$$

Here $a_{2s2p}=1/2$ and $f^P_{1s2p}(\Delta\omega_\alpha)$ has the power broadened rate $\Gamma^P_{2p1s}(\Delta\omega_\alpha)$ where

$$\Gamma^P_{2p1s}(\Delta\omega_\alpha)=\gamma_{2p}(\Delta\omega_\alpha)+\frac{\Gamma_{2p1s}}{2}$$

$$+\frac{1}{2}\,[B_{1s2p}J_\alpha(\Delta\omega_\alpha)+B_{2p1s}J_\alpha(\Delta\omega_\alpha)+B_{1s3p}J_\beta(\Delta\omega_\alpha)+B_{2p3s}J_H(\Delta\omega_\alpha)+B_{2p3d}J_H(\Delta\omega_\alpha)] \;. \quad (2.59)$$

(We could actually add power broadening in the other f's but due to the large bandwidth the terms are insensitive to the damping rates, so such an addition is irrelevant.)

$$J_\beta(\Delta\omega_\beta) = \frac{\hbar\omega_\beta}{4\pi}\Bigg[n(3p)\,\Gamma_{3p1s}\,f^P_{1s3p}(\Delta\omega_\beta)$$

$$+\frac{\Gamma_{3p1s}}{2}B_{1s3p}(n(1s)-1/3\,n(3p))\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}\int d\omega'_\beta J_\beta(\Delta\omega'_\beta)[\delta(\omega_\beta-\omega'_\beta)-f_{1s3p}(\Delta\omega'_\beta)]$$

$$+\frac{\Gamma_{3p1s}}{2}B_{2s3p}(n(2s)-1/3\,n(3p))\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}\int d\omega'_H J_H(\Delta\omega'_H)[\delta(\omega_\beta-\omega_{21}-\omega'_H)-f_{2s3p}(\Delta\omega'_H)]$$

$$+\frac{\Gamma_{3p1s}}{2}\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}[\kappa_{3p}n(3p)-\kappa_{3s3p}n(3s)-\kappa_{3d3p}n(3d)$$

$$+2\gamma_{3p}(\Delta\omega_\beta)\,[a_{3s3p}(3n(3s)-n(3p))+a_{3d3p}(3/5\,n(3d)-n(3p))]]\,G(\Delta\omega_\beta)\Bigg] \tag{2.60}$$

with $a_{3s3p} = 0.195$ and $a_{3d3p} = 0.305$ and

$$\Gamma^P_{3p1s}(\Delta\omega_\beta) = \gamma_{3p}(\Delta\omega_\beta) + \frac{1}{2}(\Gamma_{3p2s}+\Gamma_{3p1s})$$

$$+\frac{1}{2}[B_{1s3p}J_\beta(\Delta\omega_\beta)+B_{3p1s}J_\beta(\Delta\omega_\beta)+B_{3p2s}J_H(\Delta\omega_\beta)+B_{1s2p}J_\alpha(\Delta\omega_\beta)] \tag{2.61}$$

$$J_H(\Delta\omega_H) = \frac{\hbar\omega_H}{4\pi}\Bigg[n(3s)\Gamma_{3s2p}\,f^P_{2p3s}(\Delta\omega_H)$$

$$+\frac{\Gamma_{3s2p}}{2}B_{2p3s}(n(2p)-3n(3s))\frac{f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s}(\Delta\omega_H)}\int d\omega'_H\,J_H(\Delta\omega'_H)\,[\delta(\omega_H-\omega'_H)-f_{2p3s}(\Delta\omega'_H)]$$

$$+\frac{\Gamma_{3s2p}}{2}\frac{f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s}(\Delta\omega_H)}[\kappa_{3s}n(3s)-\kappa_{3p3s}n(3p)-\kappa_{3d3s}n(3d)$$

$$+2\gamma_{3s}(\Delta\omega_H)\,[a_{3p3s}(1/3\,n(3p)-n(3s))+a_{3d3s}(1/5\,n(3d)-n(3s))]]G(\Delta\omega_H)$$

$$+n(3p)\,\Gamma_{3p2s}\,f^P_{2s3p}(\Delta\omega_H)$$

$$+\frac{\Gamma_{3p2s}}{2}B_{2s3p}(n(2s)-1/3\,n(3p))\frac{f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)}\int d\omega'_H J_H(\Delta\omega'_H)\,[\delta(\omega_H-\omega'_H)-f_{2s3p}(\Delta\omega'_H)]$$

$$+\frac{\Gamma_{3p2s}}{2}B_{1s3p}(n(1s)-1/3\,n(3p))\frac{f_{1s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)}\int d\omega'_\beta J_\beta(\Delta\omega'_\beta)[\delta(\omega'_\beta-\omega_{21}-\omega_H)-f_{1s3p}(\Delta\omega'_\beta)]$$

$$+\frac{\Gamma_{3p2s}}{2}\frac{f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)}[\kappa_{3p}n(3p)-\kappa_{3s3p}n(3s)-\kappa_{3d3p}n(3d)$$

$$+2\gamma_{3p}(\Delta\omega_H)\,[a_{3s3p}(3n(3s)-n(3p))+a_{3d3p}(3/5\,n(3d)-n(3p))]]\,G(\Delta\omega_H)$$

$$+n(3d)\,\Gamma_{3d2p}\,f^P_{3p3d}(\Delta\omega_H)$$

$$+\frac{\Gamma_{3d2p}}{2}B_{2p3d}(n(2p)-3/5\,n(3d))\frac{f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d}(\Delta\omega_H)}\int d\omega'_H J_H(\Delta\omega'_H)\,[\delta(\omega_H-\omega'_H)-f_{2p3d}(\Delta\omega'_H)]$$

$$+\frac{\Gamma_{3d2p}}{2}\frac{f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d}(\Delta\omega_H)}[\kappa_{3d}n(3d)-\kappa_{3s3d}n(3s)-\kappa_{3p3d}n(3p)$$

$$+2\gamma_{3d}(\Delta\omega_H)\,[a_{3s3d}(5n(3s)-n(3d))+a_{3p3d}(5/3\,n(3p)-n(3d))]]\,G(\Delta\omega_H)\Bigg] \tag{2.62}$$

where

$a_{3p3s} = 0.500$ $a_{3d3s} = 0.167$
$a_{3s3p} = 0.195$ $a_{3d3p} = 0.305$
$a_{3s3d} = 0.107$ $a_{3p3d} = 0.500$

$$\Gamma^{P}_{3s2p}(\Delta\omega_H) = \gamma_{3s}(\Delta\omega_H) + \frac{1}{2}(\Gamma_{3s2p}+\Gamma_{2p1s})$$

$$+ \frac{1}{2}[B_{3s2p}J_H(\Delta\omega_H) + B_{2p3s}J_H(\Delta\omega_H) + B_{2p3d}J_H(\Delta\omega_H) + B_{2p1s}J_\alpha(\Delta\omega_H)]$$

$$\Gamma^{P}_{3p2s}(\Delta\omega_H) = \gamma_{3p}(\Delta\omega_H) + \frac{1}{2}(\Gamma_{3p2s}+\Gamma_{3p1s})$$

$$+ \frac{1}{2}[B_{3p2s}J_H(\Delta\omega_H) + B_{2s3p}J_H(\Delta\omega_H) + B_{3p1s}J_\beta(\Delta\omega_H)]$$

$$\Gamma^{P}_{3d2p}(\Delta\omega_H) = \gamma_{3d}(\Delta\omega_H) + \frac{1}{2}(\Gamma_{3d2p}+\Gamma_{2p1s})$$

$$+ \frac{1}{2}[B_{3d2p}J_H(\Delta\omega_H) + B_{2p3d}J_H(\Delta\omega_H) + B_{2p3s}J_H(\Delta\omega_H) + B_{2p1s}J_\alpha(\Delta\omega_H)] \quad (2.63)$$

For levels with 2p lower state the δ-function is only a convenient approximation, $\delta(\omega_H-\omega'_H)$ should preferably be replaced by $F(\Delta\omega'_H, \Delta\omega_H) = \dfrac{\Gamma_{2p1s}/\pi}{(\Delta\omega'_H - \Delta\omega_H)^2 + \Gamma^2_{2p1s}}$.

2.3 The absorption coefficients

The absorption coefficients are obtained from the same correlation function equations [i.e. eq. (2.39)] as the emission coefficients [16,6]. The initial conditions for stimulated emission are the same as for emission (Sec. 2.2), hence the stimulated and spontaneous profiles are the same. Specifically, the stimulated emission coefficient $K^s(\Delta\omega)$ is obtained by replacing Γ_{ab} by $B_{ab} = \frac{(2\ell_b+1)}{(2\ell_a+1)} B_{ba}$ in equations (2.58), (2.60) and (2.62).

The initial conditions for absorption are different. For example for the $2p \rightarrow 3s$ transition, we need to consider $\hat{g}(3s,2p)$ for which the following initial conditions are important:

$$g(3s,1s;0) = d_{3s2p}\sigma(2p,1s;0) \qquad (2.64)$$

$$g(3s,2p;0) = d_{3s2p}\sigma(2p,2p;0) \qquad (2.65)$$

$$g(3s,3s;0) = d_{3s2p}\sigma(2p,3s;0) \qquad (2.66)$$

$$g(3s,3d;0) = d_{3s2p}\sigma(2p,3d;0) \quad . \qquad (2.67)$$

The first condition [equation (2.64)] corresponds to a two-photon absorption process from the ground state involving the simultaneous absorption of a Ly-α photon (close to line center) followed by an H-α photon. It is important in the line wings.

The third term [eq. (2.66)] represents the effect on the beam being absorbed due to Rayleigh scattering. Since the net absorption is the difference between absorption and stimulated emission, which also contains a Rayleigh term, their effects must be considered together. Both absorption from the beam and stimulated emission into the beam occur, BUT, contrary to CBBH [6], there is NOT cancellation of the δ-function terms. Since these terms are unimportant at low fields (which is the usual case under solar conditions) although we give explicit expressions for them in the following absorption coefficients we will not pursue their physical implications in detail here. We would remark, however, that their effect is to change appropriate power broadening terms (in this example, those terms for power broadening on the 3s to 2p transition due to H-α) from depending on $J(\Delta\omega)$ to $J(0)$. This is not surprising since the strong field, centered at $\Delta\omega = 0$, is causing the broadening.

The fourth term [eq. (2.67)] is a Raman coupling. It represents the fact that absorption on the $2p \rightarrow 3d$ transition can change the absorption on the $2p \rightarrow 3s$ transition. It changes the power broadening for the 2p level due to the $2p \rightarrow 3d$ transition from $J(\Delta\omega_H)$ to $J(0)$. In addition, there is a stimulated Raman absorption, where transfer from the 3d to 3s state occurs due to stimulated emission on $3d \rightarrow 2p$ followed by absorption on $2p \rightarrow 2s$.

Finally, when we assume NLSI there are no terms equivalent to $D^{(0)}(\Delta\omega)$ for absorption.

The net angle-averaged absorption coefficients (i.e. difference between absorption and stimulated emission) are then

$$K(\Delta\omega) = K^a(\Delta\omega) - K^s(\Delta\omega) \quad . \qquad (2.68)$$

Thus

$$K^a_\alpha(\Delta\omega_\alpha) = \frac{\hbar\omega_\alpha}{4\pi}\Bigg[B_{1s2p}\, n(1s)\, \Gamma^P_{1s2p}(\Delta\omega_\alpha)$$

$$- \frac{1}{2} B^2_{1s2p}\,(n(1s) - 1/3\, n(2p))\, \frac{f_{1s2p}(\Delta\omega_\alpha)}{\Gamma_{2p}(\Delta\omega_\alpha)} \int d\omega'_\alpha J_\alpha(\Delta\omega'_\alpha)[\delta(\omega_\alpha-\omega'_\alpha) - f_{1s2p}(\Delta\omega'_\alpha)]$$

$$- \frac{1}{2} B_{1s2p}B_{1s3p}(n(1s) - 1/3\, n(3p))\, \frac{f_{1s2p}(\Delta\omega_\alpha)}{\Gamma_{2p}(\Delta\omega_\alpha)} \int d\omega'_\beta J_\beta(\Delta\omega'_\beta)[\delta(\omega'_\beta-\omega_{32}-\omega_\alpha) - f_{1s3p}(\Delta\omega'_\beta)]\Bigg] \quad . \qquad (2.69)$$

Only the first term contributes at low intensities.

$$K_\beta^a(\Delta\omega_\beta) = \frac{\hbar\omega_\beta}{4\pi}\Bigg[B_{1s3p}\; n(1s)\; f^P_{1s3p}(\Delta\omega_\beta)$$

$$-\frac{1}{2}B^2_{1s3p}\;(n(1s) - 1/3\; n(3p))\;\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}\int d\omega'_\beta J_\beta(\Delta\omega'_\beta)\;[\delta(\omega_\beta-\omega'_\beta) - f_{1s3p}(\Delta\omega'_\beta)]$$

$$-\frac{B_{1s3p}}{6}B_{2s3p}(n(2s) - 1/3\; n(3p))\;\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}\int d\omega'_H J_H(\Delta\omega'_H)[\delta(\omega_\beta-\omega_{21}-\omega'_H)-f_{2s3p}(\Delta\omega'_H)]$$

$$-\frac{B_{1s3p}B_{1s2p}}{2}\;(n(1s)-1/3\; n(2p))\;\frac{f_{1s3p}(\Delta\omega_\beta)}{\Gamma_{3p}(\Delta\omega_\beta)}\int d\omega'_\alpha J_\alpha(\Delta\omega'_\alpha)[\delta(\omega'_\alpha-\omega_\beta+\omega_{32})-f_{1s2p}(\Delta\omega'_\alpha)]\Bigg] \quad . \tag{2.70}$$

$$K_H^a(\Delta\omega_H) = \frac{\hbar\omega_H}{4\pi}\Bigg[B_{2p3s}\; n(2p)\; f^P_{2p3s}(\Delta\omega_H)+B_{2s3p}\; n(2s)\; f^P_{2s3p}(\Delta\omega_H)$$

$$+ B_{2p3d}\; n(2p)\; f^P_{2p3d}(\Delta\omega_H)$$

$$+\frac{B_{1s2p}B_{2p3s}}{2}\;(n(1s)-1/3\; n(2p))\;\frac{f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s}(\Delta\omega_H)}\int d\omega'_\alpha J_\alpha(\Delta\omega'_\alpha)[\delta(\omega_H+\omega'_\alpha-\omega_{31})-f_{1s2p}(\Delta\omega'_\alpha)]$$

$$+\frac{B_{1s2p}B_{2p3d}}{2}\;(n(1s)-1/3\; n(2p))\;\frac{f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d}(\Delta\omega_H)}\int d\omega'_\alpha J_\alpha(\Delta\omega'_\alpha)[\delta(\omega_H+\omega'_\alpha-\omega_{31})-f_{1s2p}(\Delta\omega'_\alpha)]$$

$$-\frac{1}{2}B^2_{2p3s}\;(n(2p) - 3n(3s))\;\frac{f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s}(\Delta\omega_H)}\int d\omega'_H\; J_H(\Delta\omega'_H)\;[\delta(\omega'_H-\omega_H) - f_{2p3s}(\Delta\omega'_H)]$$

$$-\frac{1}{2}B^2_{2s3p}\;(n(2s) - 1/3\; n(3p))\;\frac{f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)}\int d\omega'_H\; J_H(\Delta\omega'_H)\;[\delta(\omega'_H-\omega_H) - f_{2s3p}(\Delta\omega'_H)]$$

$$-\frac{1}{2}B^2_{2p3d}(n(2p) - 3/5\; n(3d))\;\frac{f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d}(\Delta\omega_H)}\int d\omega'_H\; J_H(\Delta\omega'_H)[\delta(\omega'_H-\omega_H) - f_{2p3d}(\Delta\omega'_H)]$$

$$-\frac{B_{2p3s}B_{2p3d}}{2}\;(n(2p) - 1/5\; n(3d))\;\frac{f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s}(\Delta\omega_H)}\int d\omega'_H J_H(\Delta\omega'_H)\;[\delta(\omega_H-\omega'_H) - f_{2p3d}(\Delta\omega')]$$

$$-\frac{1}{6}B_{2s3p}B_{1s3p}(n(1s)-1/3\; n(3p))\;\frac{f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p}(\Delta\omega_H)}\int d\omega'_\beta J_\beta(\Delta\omega'_\beta)[\delta(\omega'_\beta-\omega_{21}-\omega_H)-f_{1s3p}(\Delta\omega'_\beta)]$$

$$-\frac{B_{2p3d}B_{2p3s}}{2}\;(n(2p) - 3n(3s))\;\frac{f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d}(\Delta\omega_H)}\int d\omega'_H J_H(\Delta\omega'_H)[\delta(\omega'_H-\omega_H) - f_{2p3s}(\Delta\omega'_H)]\Bigg] \quad . \tag{2.71}$$

Only the first five terms are important at low intensities.

3. Discussion

The main results of this work are the statistical equilibrium equations [eqs. (2.4) to (2.26)], the emission and stimulated emission coefficients [eqs. (2.58), (2.60) and (2.62)] and the absorption coefficients [eqs. (2.69), (2.70) and (2.71)]. Angle averages have been performed over the radiation fields to make these equations tractable. The statistical equilibrium equations are relatively straightforward, since due to the fact that absorption occurs close to line center, they do not involve correlated terms. The absorption coefficients are relatively simple at low field intensities, then only the first term in equations (2.68) and (2.69) occur for Ly-α and Ly-β and the first five terms in equation (2.70) for H-α, where two-photon absorption from the ground (1s) state occurs. At high intensities stimulated emission and Raman processes considerably complicate the results.

The emission coefficents are complicated even at low fields. They contain Raman and Rayleigh scattering terms and the "destruction" term which, through $G(\Delta\omega)$, represents emission during an initial collision. (Note: The Yelnik et al. [7] results are contained within the present expressions.)

To examine the effect of this correlation term it is sufficient to examine Ly-β [eq. (2.60)] for weak field in the far line wings (where $\gamma_{3p}(\Delta\omega) \to 0$) and in the limit where H-α emission is also small (i.e. $J_H \to 0$). Then, using equation (2.27)

$$J_\beta(\Delta\omega_\beta) \to \frac{\hbar\omega_\beta}{4\pi}\Bigg[\Gamma_{3p1s}\frac{[(\Gamma_{3p2s}+\Gamma_{3p1s})n(3p)-R(1s,3p)]}{2\pi\Delta\omega_\beta^2}$$

$$+\frac{\Gamma_{3p1s}}{2}B_{1s3p}\; n(1s)\int d\omega'_\beta J_\beta(\Delta\omega'_\beta)\delta(\omega_\beta-\omega'_\beta)$$

$$+\frac{\Gamma_{3p1s}}{2\pi\;\Delta\omega_\beta^2}\;[\kappa_{3p}n(3p) - \kappa_{3s3p}n(3s) - \kappa_{3d3p}n(3d)]G(\Delta\omega_\beta)\Bigg] \quad . \tag{3.1}$$

In this limit, $R(2s,3p) = 0$ and with $s(3p) = 0$, equation (2.25) yields

$$[\kappa_{3p}n(3p) - \kappa_{3d3p}n(3d) - \kappa_{3s3p}n(3s)] = R(1s,3p) - (\Gamma_{3p2s}+\Gamma_{3p1s})n(3p) \quad (3.2)$$

$$= [\frac{\kappa_{3p3s}\Gamma_{3s2p}}{\Gamma_{3s2p}+\kappa_{3s3p}} + \frac{\kappa_{3p3d}\Gamma_{3d2p}}{\Gamma_{3d2p}+\kappa_{3d3p}}]n(3p) \ . \quad (3.3)$$

Equation (3.3) follows from equations (2.24) to (2.26) with $R(2p,3s) = s(3s) = s(3d) = 0$ and (for convenience) $\kappa_{3d3s} = 0$. Since the RHS of eq. (3.3) is positive, if the correlated term were ignored (i.e. $G(\Delta\omega_\beta) = 0$) then the first term in equation (3.1) would predict unphysical negative intensities. However, with $G(\Delta\omega_\beta) = 1$ the first term is exactly cancelled and, as expected, we obtain just coherent scatterings in the far line wings. Including a cascade contribution s(3p) does however lead to a redistributed component.

The effect of the Γ_{2p1s} decay of the lower state in H-α is also of interest. In the absence of collisions we obtain the following redistributed component

$$J_H(\Delta\omega_H)^{Redist}_{no\ collisions} = \frac{\hbar\omega_H}{4\pi}[\frac{\Gamma_{3s2p}f_{2p3s}(\Delta\omega_H)}{\Gamma_{3s2p}+\Gamma_{2p1s}}[n(3s)\Gamma_{2p1s}+s(3s)]$$

$$+ \frac{\Gamma_{3p2s}f_{2s3p}(\Delta\omega_H)}{\Gamma_{3p2s}} s(3p) + \frac{\Gamma_{3d2p}f_{2p3d}(\Delta\omega_H)}{\Gamma_{3d2p}+\Gamma_{2p1s}}[n(3d)\Gamma_{2p1s}+s(3d)]] \ . \quad (3.4)$$

Comparison of this term with the Rayleigh and Raman components for H-α shows that it dominates (by roughly an order of magnitude).

In order to perform model atmosphere calculations, practical expressions for the source functions, in which the Doppler convolution (with $\omega \to \omega - \vec{k}\cdot\vec{v}$) have been performed, are necessary. Such expressions have been obtained by Cooper, Ballagh and Hubeny [17]. These results have been applied to solar model atmosphere calculations. Using expressions adapted from VCS [11,17] for the collisional coefficients. As shown in Figures 5 and 6 improved agreement with observations have been obtained for both Ly-α and Ly-β. However, significant discrepancies still remain. Now that the redistribution is on a firm basis, it is clear that improved information concerning the solar model can be obtained. Obviously the model VAL III-C (Vernazza, Avrett, and Loeser [18]) used in the present computations is inadequate.

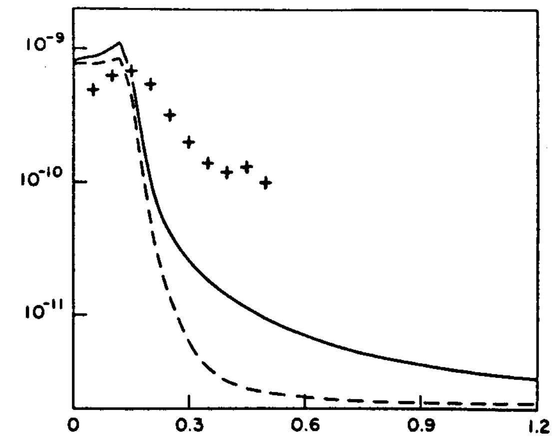

Fig. 5. Intensity versus Δλ(Å) for Lyman-α: ____ Present results, ----- Resonance scattering only (both using VAL III-C model); + Observations [19].

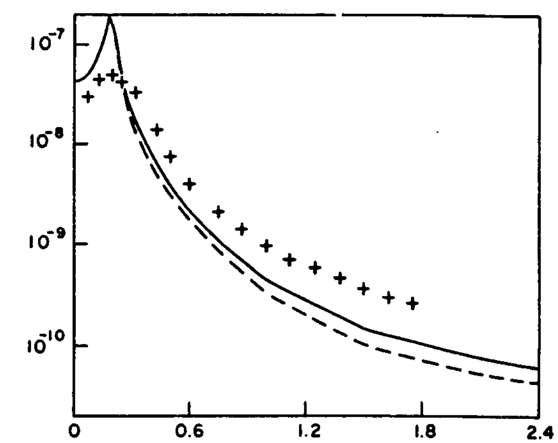

Fig. 6. Intensity versus Δλ(Å) for Lyman-β: ____ Present results, ----- Resonance scattering only (both using VAL III-C model); + Observations [20].

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