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RAPID FITTING OF PARTICLE CASCADE DEVELOPMENT FROM X-RAY FILM DENSITOMETRY MEASUREMENTS

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16. ABSTRACT

This report describes a semiautomatic method of fitting transition curves to x-ray film optical density measurements of electromagnetic particle cascades. Several hundred singly and multiply interacting cosmic ray events from the JACEE 8 balloon flights were analyzed using this procedure. In addition to greatly increased speed compared to the previous manual method, the semiautomatic method offers increased accuracy through maximum likelihood fitting.

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INTRODUCTION

In balloon-borne emulsion chambers for cosmic ray composition and spectra measurements, a "calorimetry" section at the bottom of the chamber is used to provide the energy measurement (1). The calorimetry section is usually a multi-layered stack of lead plates interspersed with sheets of nuclear track emulsion and high sensitivity x-ray film, as shown in Figure 1.

When a galactic cosmic ray with an energy of $\sim 10^{12}$ eV strikes a target nucleus in the apparatus, the result is often a spectacular collision in which the primary loses a significant fraction of its energy ($\sim 50\%$). Such a collision may also produce large numbers of mesons (predominately charged and neutral pions), concentrated in a narrow cone a few milliradians wide. A heavy primary, such as iron, may be completely reduced to smaller nuclear fragments, α-particles, neutrons, and protons. These secondary particles and the charged pions may themselves interact again before leaving the bottom of the chamber, adding to the number of particles in the "shower." The heavy fragments have a short interaction mean free path and are especially likely to interact again, contributing strongly to the secondary development of the cascade.

The neutral pions from the collision quickly decay ($\sim 10^{-16}$ s), producing γ-rays. Upon entering the calorimeter the γ-rays produce $e^- e^+$ pairs. The electrons, in turn, produce γ-rays in the lead through the bremsstrahlung process. These γ-rays can then pair-produce, increasing the number of electrons. This process, known as an "electromagnetic cascade," quickly multiplies the number of charged particles in the calorimeter. The characteristic length for doubling the number of electrons is the "radiation length," about 0.57 cm in lead. The number of particles in the electromagnetic cascade increases until the average electron or γ-ray no longer produces a significant number of secondary particles. At this point, the number of particles in cascade begins to decrease and

EMULSION CHAMBERS

PRIMARY COSMIC RAY

$Z \geq 1$

$E \geq 10^{12} eV$

DOUBLE COATED
EMULSION PLATES

CR39 ETCHABLE
TRACK DETECTORS

EMULSIONS
PLASTIC PLATES
AND/OR METAL PLATES

HONEYCOMB

LEAD PLATES

EMULSIONS
X-RAY FILM

CHARGE
DETECTOR

MEASURE Z BY
δ RAY COUNTING IN
EMULSIONS AND
ETCH PIT SIZE IN
CR39

TARGET

INTERACTION PROBABILITY
~0.2 FOR PROTONS
~0.9 FOR IRON NUCLEI

SPACER

PRIMARY FRAGMENTS, PRODUCED $\pi\pm$,
GAMMA RAYS FROM π° DECAY SPREAD
OUT SO THEY MAY BE MEASURED
INDIVIDUALLY

θ (TYPICAL)
$\approx 2 \times 10^{-3}$ RADIANS
AT $10^{12} eV/AMU$

CALORIMETER

GAMMA RAYS PRODUCE
COPIOUS $e\pm$, γ's IN LEAD.
NO. OF $e\pm$ COUNTED IN
EMULSIONS OR X RAY FILM
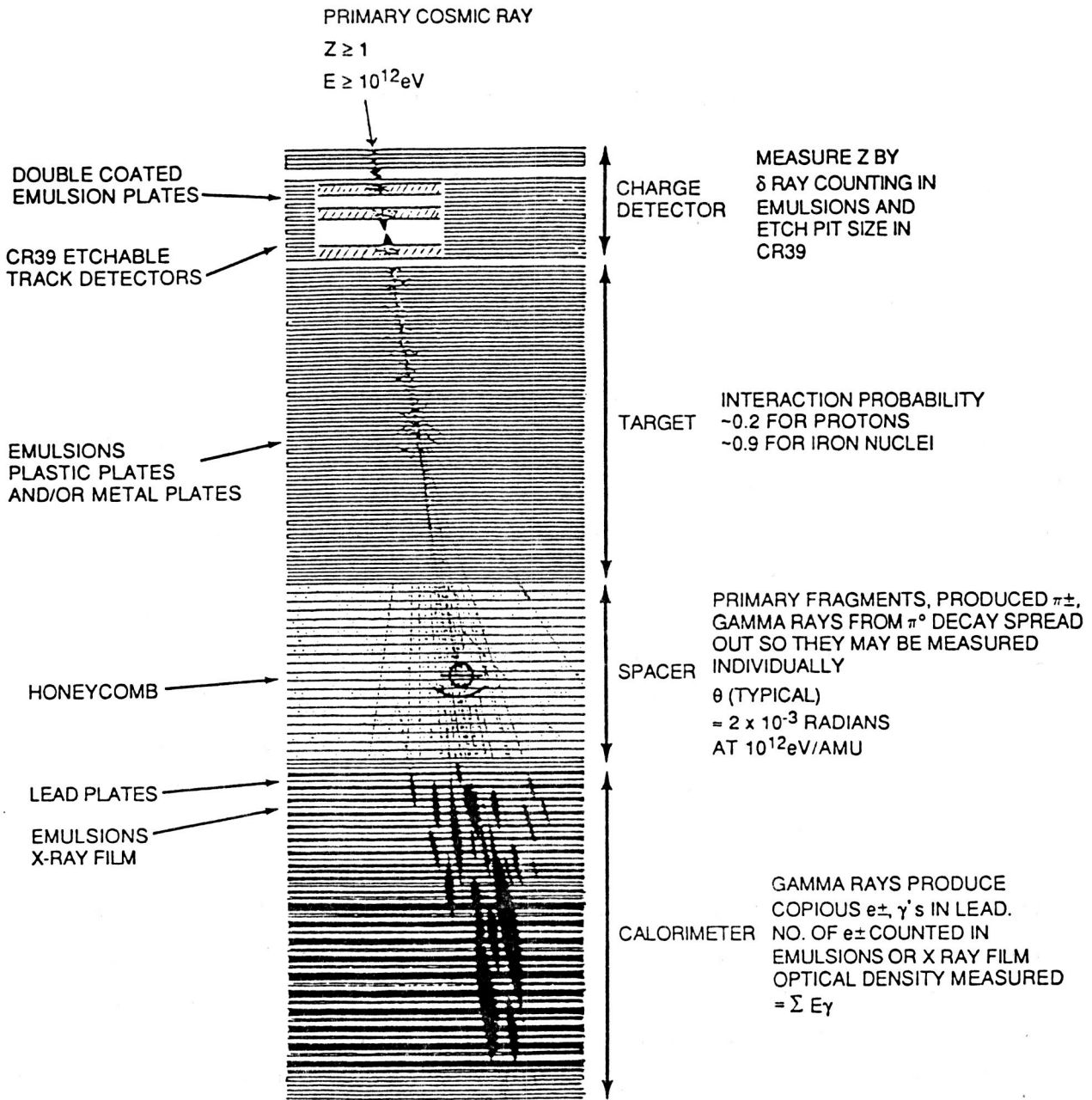OPTICAL DENSITY MEASURED
$= \Sigma E\gamma$

Figure 1.   Schematic diagram of a typical emulsion chamber showing a cosmic ray
event which interacts in the target section section of the chamber. The
upper part of the cascade consists primarily of nuclear fragments,
nucleons, and pions. The electromagnetic portion of the cascade
predominates in the calorimeter section. Produced particle angles are
exaggerated for clarity.

the shower "dies off." A plot of the number of particles in the cascade as a function of depth in the calorimeter is known as a "transition curve" (Figure 2). In order to observe the maximum development of the cascade, the emulsion chambers used in the JACEE experiments are designed to be about 10 radiation lengths thick for cosmic rays with moderately inclined trajectories (45°). Nucleons, pions, α-particles, and a few heavy fragments also pass through the calorimeter, but this "hadronic shower" is vastly overshadowed by the electrons in the electromagnetic cascade.

If the γ-rays produced by the primary cosmic ray have a total energy of about 1 TeV or greater, then the electrons subsequently produced in the apparatus will leave a visible dark spot on the x-ray film carried in the calorimeter. With sufficient energy release, successive x-ray film layers will have spots that can be mapped to locate cascade trajectories. Typically, six successive spots through about three radiation lengths of lead are required to further analyze an event.

The traditional method for estimating the energy of the primary cosmic ray is based on counting the number of electrons produced in the cascade. The major drawback of this procedure is that it is very time-consuming. An alternate method is to determine the optical density of the x-ray film spots as a function of depth. A commercial optical densitometer (Joyce-Loebl or PDS) fitted with a 250-μ aperture is used to measure the optical density of an event for six or more layers around the maximum of the cascade. A "transition curve" of optical densities is plotted for an event (Figure 3). By fitting a calculated transition curve to the data as described below, it is possible to find the maximum optical density (D_{max}) for the event. This fitted value of the maximum optical density usually differs from the maximum experimental value due to measurement errors. Also, the depth at which the fitted maximum occurs may not exactly coincide with a sheet of x-ray film (so that direct experimental measurements cannot be made). At some initial stage in the analysis, electron counts are compared to optical densities for one or more sample events to establish a calibration curve. Thus, D_{max} may be related to the number of electrons.

This report describes a semiautomatic method of fitting optical density transition curves. Basically, this procedure generates transition curves using a set of parameterized formulas, taking into account the angle of incidence of the cascade. It also allows significant secondary cascades to be recognized and fitted
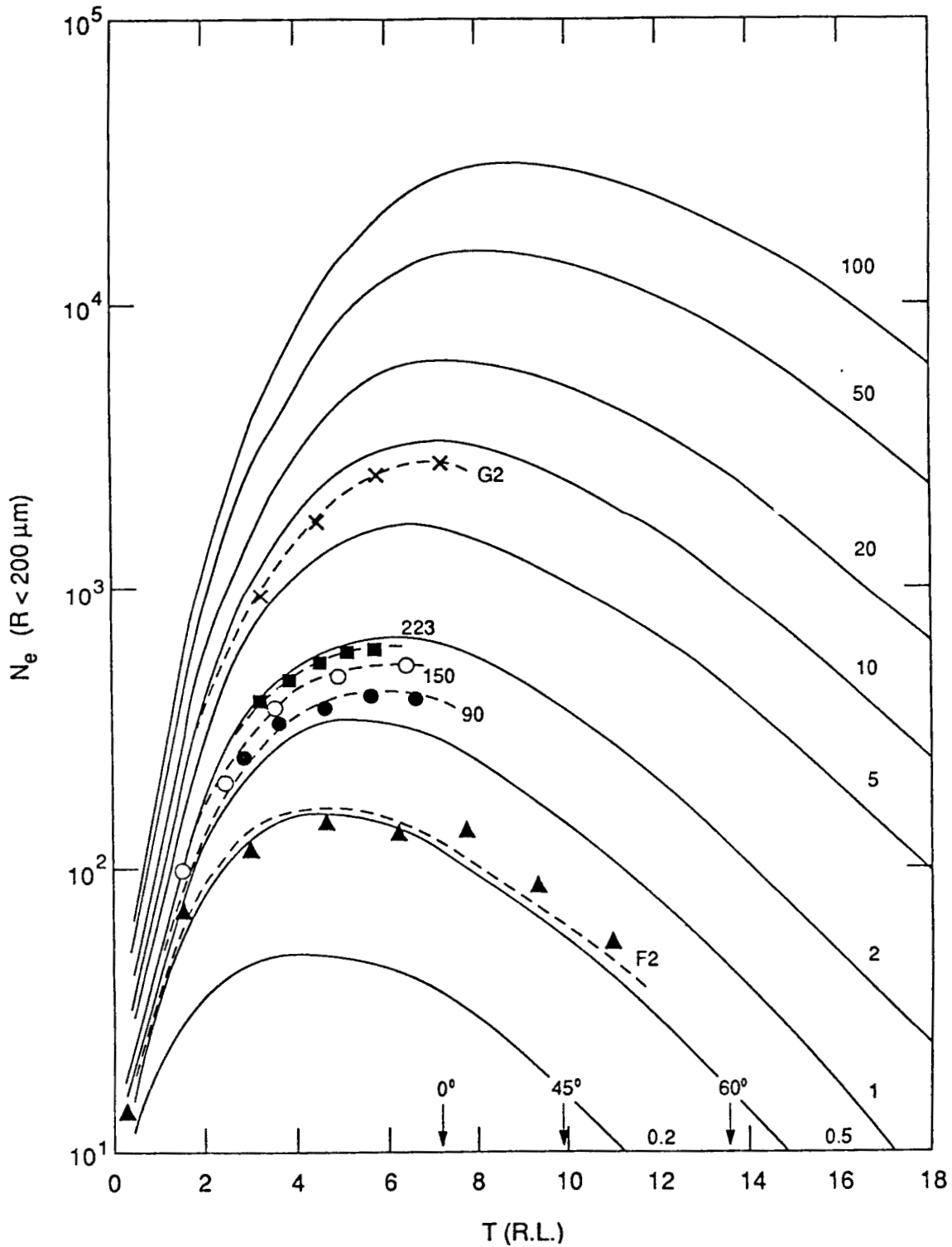
Figure 2. Transition curves for proton-initiated "Pb jet" cascades showing the development in electron number as a function of depth in the calorimeter. The data points are from typical cascades, with the dashed lines indicating fitted transition curves. Adapted from Reference (1), Figure 7.
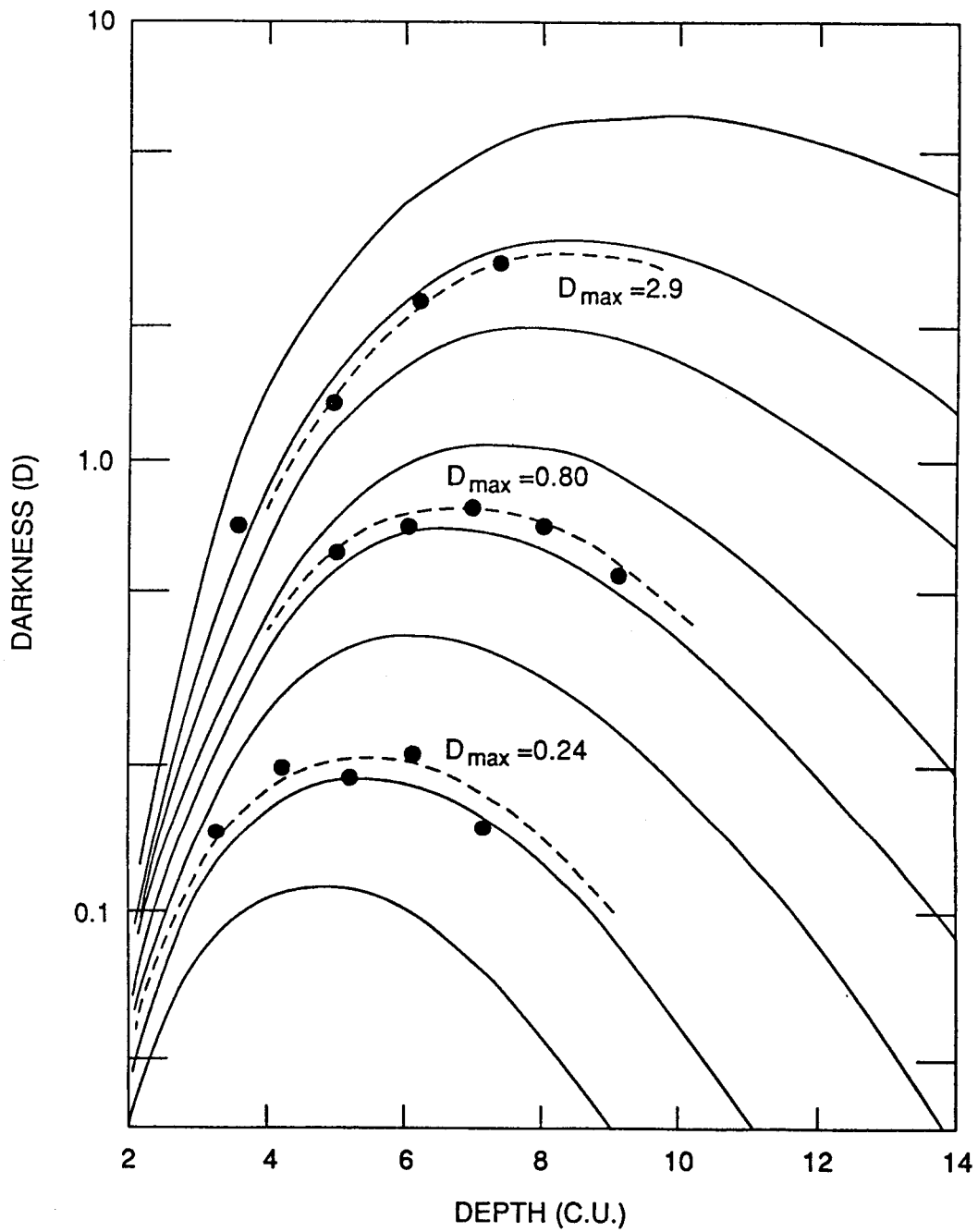
4

Figure 3. Longitudinal development of x-ray film spot optical density for proton-initiated cascades. Data from some typical events shown as plotted points. Adapted from Reference (1), Figure 6.

separately. Moveover, it speeds up the process of curve fitting to between 1 and 2 orders of magnitude over doing the work by hand.

Some general information on densitometry and the use of x-ray films in cosmic ray research is given in References (2)–(12).

METHODS

Originally, the maximum optical density (D_{max}) of a data set was determined by graphically "fitting" the data points to reference curves of optical density versus depth. The procedure is as follows :

(1) Select correct group of reference curves based on the inclination $m = \tan\theta$ of the event.

(2) Find the reference curve of the group that gives the best visual fit to the data points. The D_{max} for this curve is known.

(3) "Subtract" the selected reference curve from the data points to see if another interaction occurs further down in the apparatus.

This work was done by hand on a light-table. It seemed clear that using a computer to fit the reference curves to the data would improve the speed and accuracy of the process. However, a simple formula for the several groups of reference curves was previously unavailable.

Since the film darkness is closely related to the number of electrons produced in the particle cascade, it was suspected that an equation for optical density versus depth might be similar to known formulas relating electron number to depth. A simple approximation for electron number N_e is

$$N_e = \frac{0.31}{\beta^{1/2}} \exp(t(1 - \frac{3}{2}\ln s)) \tag{1}$$

$$s = \frac{3t}{t + 2\beta} \tag{2}$$

where t is the depth in cascade units and β is a constant. It was found that the film darkness D may be described by a similar set of equations :

$$\log D = \log D_{\mathrm{max}} + \frac{1}{\ln 10}(t(1 - \frac{3}{2}\ln s) - t_{\mathrm{max}}) \tag{3}$$

6

$$s = \frac{3t}{t + 2t_{max}} \tag{4}$$

Here, t_{max} is the depth in cascade units at which the maximum darkness D_{max} is attained.

Note that Equations (3) and (4) involve two parameters, namely D_{max} and t_{max}. How are they related? A key observation was that the maxima in each group of the reference curves (optical density versus depth) could be connected by a straight line. That is, for a particular value of $m = \tan\theta$, the quantities $\log D_{max}$ and t_{max} are linearly related. Specifically,

$$\log D_{max} = (a/\cos\theta)t_{max} + b \tag{5}$$

where θ is the zenith angle of the incoming cosmic ray. Though a and b vary slightly with m, they are approximately equal to 0.295 and -3.10, respectively. (For more accurate work, tables of a and b values may be used.)

Equations (3), (4), and (5) give an iterative procedure for fitting a curve to a set of data points :

(1) Interpolate over $m = \tan\theta$ to calculate a and b for the event.

(2) Estimate the maximum optical density D_M for the experimental data. Record the depth T_C in cascade units at which this maximum occurs.

(3) Set $D_{max} = D_M$ in Equation (5) and calculate t_{max} :

$$t_{max} = (\log D_{max} - b)/a \tag{6}$$

(4) With $D_{max} = D_M$ and t_{max} as calculated above, construct a reference curve of optical density D versus depth t using Equations (2).

(5) Compare the experimental data points D_i and the estimated values $D(t_i)$ to check the fit :

$$S = \sum (D_i - D(t_i))^2 \tag{7}$$

(6) Vary D_{max} until the best fit is obtained. That is, repeat steps (3)–(5) until S is minimized.

Several complications enter into this procedure. First, it is important to note that t_{max} is in general not equal to T_C, because the cosmic ray interaction producing the data occurs at some arbitrary point in the apparatus. Hence, the reference curve must be shifted over so that t_{max} and T_C coincide. Second,

the depth t in Equations (3) and (4) must be replaced by the "effective depth" $t/\cos(\theta)$ for particles with an inclined trajectory. Finally, many of the plots of film darkness versus depth exhibit several peaks. The first maximum is associated with the interaction of the original cosmic ray with a target: the subsequent peaks correspond to secondary interactions by fragments left over from the primary interaction. In this case, it is necessary to fit a first curve to the first peak, "subtract out" the calculated values from the data points, and then fit a curve to the next interaction.

RESULTS

The original program developed to do the above curve-fitting read an estimate for D_{max} and the depth T_C of the maximum by reading a cursor position. A curve could be fitted to any subset of the data points for an event, and curves for up to five interactions could be fit to one event. Some sample results from this program are shown in Figures 4 and 5. The curve-fitting calculations were also checked by applying this program to data taken from the original reference curves.

This program was later re-written in BASIC by one of the authors (C. B.) for use on Hewlett-Packard Series 200/300/500 computers. Several features make this version substantially faster than the original version :

(1) The points used for the curve-fit are selected using a digitizer. The user simply touches the digitizer to a point on the screen, instead of typing in the point number.

(2) A "grid search" over D_{max} and t_{max} is made in the vicinity of the maximum of a curve. This replaces moving a cursor around in the original program.

(3) Small changes in the reference curve (darkness D versus depth t) are made by scaling D instead of recalculating the curve. This basically amounts to shifting the reference curve up or down by a small amount.

Additional details on this program are provided in the Appendix.

Using this method, it was possible to analyze approximately 520 events from the FF block of JACEE 8 in less than a day. This represents a very substantial
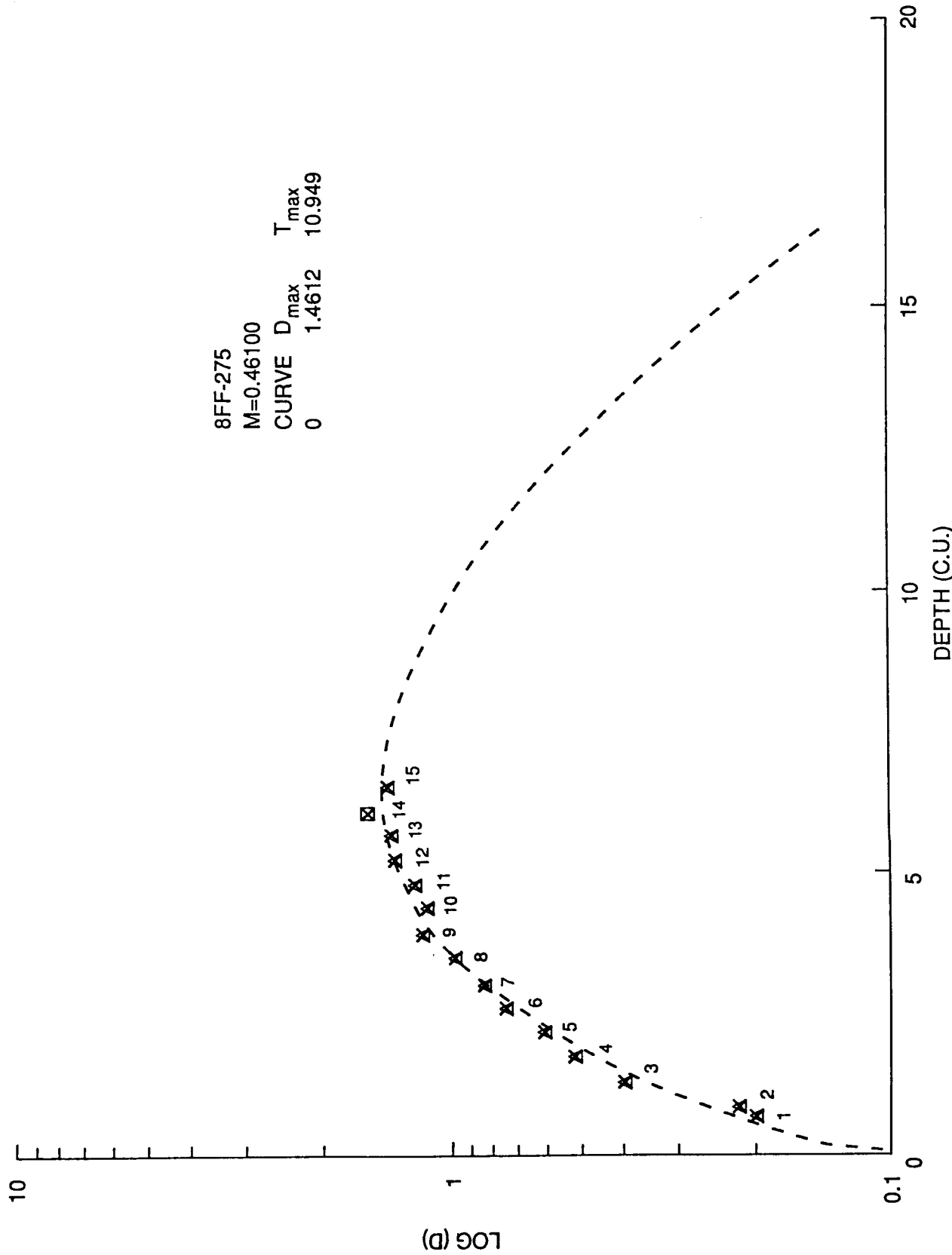
LOG (D)

10

1

0.1

0   5   10   15   20

DEPTH (C.U.)

8FF-275
M=0.46100

CURVE	D_{max}	T_{max}
0	1.4612	10.949

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Figure 4.   Sample output from the curve-fitting program. The primary of this event has only one interaction. Note T_{max} is the parameter t_{max} in Equations (3) and (4) and not the depth at which the maximum occurs in the data.

9

8EE-24
M=3.33

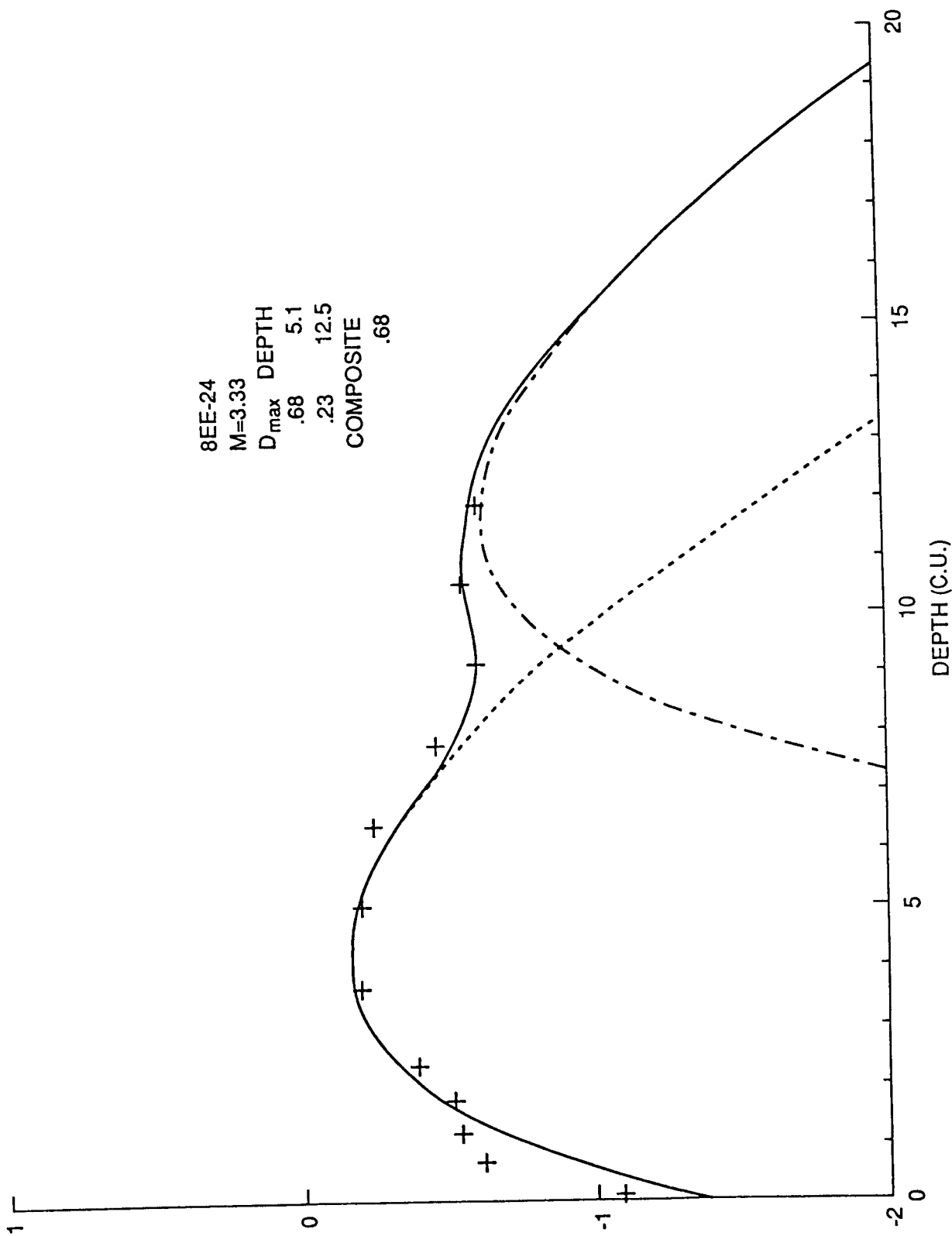| D_{max}	DEPTH
.68	5.1
.23	12.5
COMPOSITE	
.68	

DEPTH (C.U.)

Figure 5.   Sample output from the curve-fitting program showing two separate interactions for this event.

10

savings in time, since it generally requires 20–30 minutes to do one event by hand.

CONCLUSIONS

The procedure described in this report provides a rapid, maximum likelihood method for fitting optical density transition curves to x-ray film densitometry data. With this technique, it was possible to obtain D_{max} values for several hundred cosmic ray events from JACEE 8, reducing a month-long task to a job of a few hours. Many of these events included secondary interactions, which could themselves be analyzed. These results will ultimately be used to estimate the energy of the cosmic ray events observed.

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APPENDIX

JC8DATAFIT: A Brief User's Guide

(For Hewlett-Packard Series 200/300/500 Computers)

(1) Power on computer, disk drive, and graphics tablet.

(2) Load program "JC8DATAFIT" from disk.

(3) Press RUN.

(4) Type in identifier for data block (e.g., "DD" for block DD). This loads the specified data block into memory.

(5) The program automatically selects the first event that was recorded on at least three sheets of x-ray film. If this this event is not suitable or another event is desired, click the digitizing pen on the yellow region of the display screen to select another data set.

(6) For selected event,
    (i) Exclude a "bad" point from the fitting process by setting the digitizing stylus on the point and clicking the pen.
    (ii) Repeat (i) for each point to be excluded.
    (iii) Click pen on left side of y-axis to continue.
    (iv) Move the digitizing stylus to the estimated maximum in the curve. Click the digitizing pen to input this position.
    (v) Click pen on left side of y-axis to fit a curve to this data.

If the data contain several maxima (corresponding to several interactions), separate curves can be fit to each interaction by entering two or three positions in step (iv). If three positions are entered, step (v) is omitted.

(7) Output for data set:
    (i) D_{max} value for each fitted curve.
    (ii) Goodness-of-fit estimate for each curve.
    (iii) Composite curve (when two or move curves fit).

(8) Use digitizing stylus to select option from menu:
    (i) Save data, proceed to next event.
    (ii) Make hard copy of output, return to menu.
    (iii) Re-do event.
    (iv) Save all data to disk, exit program.

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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Director
Space Science Laboratory