INNOVATIVE DESIGN OF COMPOSITE STRUCTURES: THE USE OF CURVILINEAR FIBER FORMAT IN COMPOSITE STRUCTURE DESIGN

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Abstract

This paper investigates the gains in structural efficiency that can be achieved by aligning the fibers in some or all of the layers in a laminate with the principal stress directions in those layers. The name curvilinear fiber format is given to this idea. The problem studied is a plate with a central circular hole subjected to a uniaxial tensile load. An iteration scheme is used to find the fiber directions at each point in the laminate. Two failure criteria are used to evaluate the tensile load capacity of the plates with a curvilinear format, and for comparison, counterpart plates with a conventional straightline fiber format. The curvilinear designs for improved tensile capacity are then checked for buckling resistance. It is concluded that gains in efficiency can be realized with the curvilinear format.

Introduction

Since their introduction and initial development, advanced composite materials have been used successfully in many military aircraft and both military and civilian spacecraft. Currently, fiber-reinforced materials are being used in business aircraft and, to some extent, commercial aircraft. Despite the increased usage, there is a strong feeling among some researchers that the potential advantages of advanced fiber-reinforced composite materials have not been fully exploited, or even explored. The development and use of advanced composites in structures has followed a rather slow evolutionary course rather than a quantum increment approach. As a result, many of the advantages of composite materials have been eroded by some of their detrimental characteristics. The result is, in some instances, a marginal gain, compared to a metallic, when composite materials are considered. For example, a quasi-isotropic laminate has stiffness characteristics much like aluminum and has similar inplane load capacity. However, unlike aluminum, a quasi-isotropic laminate can delaminate. Aluminum has no such problem. A quasi-isotropic laminate is susceptible to environmental degradation. Aluminum has less serious problems with this. Moreover, the manufacture and repair of composite materials is less fully understood than the manufacture and repair of aluminum. When more weight is added to a composite structure to compensate for these problems, the gains by using composite materials in a conservative fashion begin to decrease. When all issues are considered, a composite material may not be the final choice. This is particularly true when one considers the increases in performance of new aluminum alloys. In the final analysis, with current philosophies regarding the use of advanced composite materials, the gains being discussed are in the 15-20% range, the evolutionary level, as opposed to gains of 50-100%, the revolutionary level. The only way to overcome this problem is to begin to abandon the overly conservative utilization of a material which seems to have considerably more potential. Some of the design philosophies and traditions developed during the last two decades must be reexamined and perhaps even ignored altogether. The work reported on here represents a departure from traditional usage of composite materials and attempts to explore the idea of using composite materials more effectively and perhaps realize step increases in structural performance. Specifically, this study examines the issue of using fiber reinforcing in such a manner that the direction of the fibers, or at least some of the fibers, is a function of spatial position. Herein this is referred to as a curvilinear fiber format. In fact, if one has at their disposal strong stiff fibers for reinforcing a material, it is not altogether clear that aligning the fibers parallel to each other and in straight lines is the best way to utilize the fiber. It is true that in the past this is the format in which fibers have been supplied. The fibers have been impregnated with resin, aligned parallel with each other, and rolled onto a spool for transportation or storage. The user makes a structural component by stacking together multiple layers of fibers, the fibers in each layer being in a straight line. Yet it is possible that the component contains a geometric discontinuity, such as a hole, that interrupts fiber continuity in all of the layers and causes a concentration and realignment of the stresses. If it could be done, it would seem that not breaking fiber continuity, and somehow using the fibers to advantage near the geometric discontinuity, would seem the efficient way to use strong stiff fibers to advantage. In particular, it would seem that the fibers should "flow" continuously around the discontinuity and be oriented in such a way as to transmit the load efficiently around the discontinuity. To be sure, there are many issues that must be studied with this idea, the most important being fabrication. However, the availability of raw fiber, the increased power and flexibility of robotics, and new matrix development do provide promise for fabricating components with something other than a straightline format. These promising fabrication techniques aside, researchers would be remiss if only straightline fiber format were considered.

The particular problem studied here is a prime candidate problem for deviating from the straightline fiber format. The particular problem is a plate loaded in its plane and containing a central circular hole. The problem has been studied hundreds of times by many researchers dealing with isotropic and composite materials. It has been studied to some extent in the context of curvilinear reinforcement (1,2). It has been studied a number of times because it is an important problem. Here it is studied specifically because aircraft structures contain many holes for access and fabrication, in addition to numerous windows, if commercial

transports are being considered. These are all discontinuities in otherwise continuous structures that can lead to inefficiencies in the use of material. In this paper the design of a plate to resist inplane tensile loads is discussed, the plate design being based on the curvilinear format. The basic issue is whether or not the load capacity of the plate can be improved using the curvilinear format. As will be seen, the present work leads to the conclusion that tensile capacity can be improved.

With curvilinear designs established for increased resistance to tensile loads, the influence of these designs on the compressive buckling load is evaluated. This is an important step because increased tensile performance could well come at the expense of compression performance.

This paper begins by describing the geometry and nomenclature of the problem. The design philosophy and the method of analysis are discussed. Attention then turns to the curvilinear designs themselves. Designs which may be unrealistic and strictly academic, as well as more realistic designs are considered. To put the results for the curvilinear designs into context, and for purposes of comparison, several conventional straightline format designs are briefly discussed. The straightline designs studied are considered counterparts of the curvilinear designs. An explanation of the observed differences between the tensile capacities of the curvilinear and the counterpart straightline designs is presented. Buckling of the designs is then examined.

Problem Description

Figure 1 shows a plate with a central circular hole. The plate width is W, its length L, and the hole diameter D. Shown is a uniform load applied at opposite ends of the plate. Two plate geometries, $L/W = 1$ and 2, and two hole diameters, $W/D = 3.33$ and 1.67 are considered. The results to be presented are not a strong function of these parameters and thus only one geometry will be discussed in detail. The basic issue is as follows: Given that a graphite-reinforced material with a fiber volume fraction of roughly 65% is available: How can the material be used most effectively to construct a plate with the geometry and loading of fig. 1? The term "most effectively" implies that the load transmitted is the maximum, or that the weight of the plate is a minimum. That the words 'maximum' and 'minimum' are being used is unfortunate because it implies something akin to optimal design is being studied. Optimum design is not the issue being discussed. The issue being discussed is to use fundamental knowledge regarding the response of fiber-reinforced composite materials, coupled with analysis, to design the plate to carry more load than conventional straightline fiber formats allow. Herein the designs focus specifically on layered plates in which the fiber direction can vary from point to point within a layer or group of layers.

Design Philosophy

The basic philosophy used to design the plate for tensile loading is to assume that some or all of the fibers should, in some sense, be aligned with the principal stress directions in the plate. Strictly speaking, principal stress directions are meaningless when discussing fiber-reinforced materials. More meaningful are the principal material directions. However, here principal material directions, in conjunction with principal stress directions, will be utilized. Specifically, using an iteration technique, in certain layers the principal material directions and the principal stress directions of those layers will be aligned. With this accomplished, two failure criteria are then used to evaluate the effectiveness of the tensile designs. The noninteracting maximum strain criterion, and the interacting stress-based Tsai-Wu criterion are used to study allowable load levels. Two diverse criteria are used to determine if predicted behavior or performance gains are a function of the criterion used. As will be seen, the predicted performance gains are not a function of the criterion.

Method of Analysis

All results discussed here are based on finite-element analyses. For the tensile load analysis, because of the lack of any overall shear-extension coupling, i.e., A_{16}, A_{26}, a quarter-plate model is employed. For the buckling analysis, because of the presence of D_{16} and D_{26} terms, a full plate model is used (3). In applying the maximum strain criterion to the tensile designs, lamina strains in the principal material system in the fiber direction, perpendicular to the fibers, and in shear are used as indicators of failure. Operationally, the maximum strain criterion may be expressed as follows: A laminate has not failed if at every point in every layer

$$-\frac{\varepsilon_1^C}{\varepsilon_1} < 1 < \frac{\varepsilon_1^T}{\varepsilon_1}$$

$$-\frac{\varepsilon_2^C}{\varepsilon_2} < 1 < \frac{\varepsilon_2^T}{\varepsilon_2} \tag{1}$$

$$-\frac{\gamma_{12}^S}{\gamma_{12}} < 1 < \frac{\gamma_{12}^S}{\gamma_{12}} \quad .$$

where ε_1^C and ε_1^T are the failure strains in the fiber direction in compression and tension, ε_2^C and ε_2^T are the failure strains perpendicular to the fiber in compression and tension, and γ_{12}^S is the failure strain in shear. The Tsai-Wu criterion (4) may be expressed as follows: A laminate has not failed if at every point in every layer

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2$$

$$+ 2F_{12} \sigma_1 \sigma_2 + F_{66} \tau_{12}^2 < 1 \quad . \tag{2}$$

where

$$F_1 = \frac{1}{\sigma_1^T} - \frac{1}{\sigma_1^C} \quad ; \quad F_2 = \frac{1}{\sigma_2^T} - \frac{1}{\sigma_2^C}$$

$$F_{11} = \frac{1}{\sigma_1^T \sigma_1^C} \quad ; \quad F_{22} = \frac{1}{\sigma_2^T \sigma_2^C} \quad ; \quad F_{66} = \frac{1}{\tau_{12}^{S}} \tag{3}$$

$$F_{12} = -0.5\sqrt{F_{11} F_{22}} \quad .$$

The σ_1^C and σ_1^T are the failure stresses in the fiber-direction, σ_2^C and σ_2^T are the failure stresses perpendicular to the fibers, and τ_{12}^S is the failure stress in shear.

For buckling, the lowest buckling load as computed from a linear buckling analysis is considered to be the compressive load capacity. Since the plates studied are quite thin, this load is much lower than any material compressive failure loads.

With the finite-element discretization used here, several approximations are inherent in the analysis. First, it is assumed that within an element the fiber direction is constant. This is not correct because fiber direction should be continuously changing in an actual plate. Second, the calculations for stresses and strains, and hence principal stress directions and fiber directions, key on computations at the Gauss points in the element. For the element used there are nine points to choose from. Using the results from just one could be viewed as somewhat arbitrary. Alternatively, the results at all nine Gauss points could be averaged in some sense. For simplicity all results are based on computations at just one Gauss point. There is no compromise in the conclusions with this limitation.

Finally, there is a problem with the principal stress calculations at the so-called isotropic point at the hole edge. The isotropic point is as follows: The circumferential stress around the hole changes from tension at the net section, point A in fig. 1, to compression on the horizontal centerline of the plate, point B. At some point around the circumference the circumferential stress is zero. Since the hole edge is traction free, all stresses are zero at this point. At this point principal stress directions are meaningless. All directions are principal stress di-

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rections. In the analyses here the fiber directions at this point are chosen to be consistent with the directions of neighboring elements.

Determination of Fiber Angles

The first idea that comes to mind with the notion of using a curvilinear fiber format is to orient the fibers in the principal stress directions. It must be kept in mind that principal stress directions depend on material properties. What may be thought of as principal stress directions for an isotropic material, such as aluminum, will change when fibers are introduced into the material. In addition, the idea of principal stress directions for a laminate must be applied carefully. Each layer has principal stress directions and these directions vary from layer to layer. Here an iterative approach is used to find the fiber directions within a group of layers such that the fibers in those layers are everywhere aligned with the principal stress directions of those layers. The first iteration assumes that the fibers are everywhere aligned with the principal stress directions for an isotropic plate. The stresses in each element are recomputed and another set of element principal stress directions determined. The fibers are aligned with these new directions and the stress analysis repeated. A third set of element principal stress directions is computed and the fibers realigned. This process of stress calculation and fiber realignment is repeated until the principal stress directions and the fiber directions are aligned to within a given tolerance. Specifically, the solution is considered converged when the quantity

$$\frac{\theta - \phi}{\theta} \tag{4}$$

is less than 0.01 for each element. In the above θ denotes the principal stress direction and ϕ denotes the fiber orientation. In practice, convergence was realized in 4 or 5 interactions. For the laminates studied here the result of the iteration process was a principal stress direction for each element that was not significantly different than the direction from the original isotropic analysis.

Material Properties Used

The material properties used throughout the study are chosen to represent AS4/3501, an organic matrix graphite composite (4). The elastic properties used are:

$$E_1 = 138\,\text{GPa} \ (20.0\,\text{Msi}) \qquad E_2 = 8.96\,\text{GPa} \ (1.30\,\text{Msi})$$
$$G_{12} = 7.10\,\text{GPa} \ (1.03\,\text{Msi}) \qquad v_{12} = 0.30 \tag{5}$$
$$\text{layer thickness} = 0.1\,mm \ (0.005\,\text{in.})$$

The failure strains used are:

$$\varepsilon_1^C = 10,500 \times 10^{-6} \ ; \quad \varepsilon_1^T = 10,500 \times 10^{-6} \ .$$
$$\varepsilon_2^C = 23,000 \times 10^{-6} \ ; \quad \varepsilon_2^T = 5,800 \times 10^{-6} \ : \tag{6}$$
$$\gamma_{12}^S = 13,100 \times 10^{-6} \ .$$

The failure stresses used are:

$$\sigma_1^C = 210\,\text{Ksi} \ ; \quad \sigma_1^T = 210\,\text{Ksi} \ :$$
$$\sigma_2^C = 30\,\text{Ksi} \ ; \quad \sigma_2^T = 7.5\,\text{Ksi} \ : \tag{7}$$
$$\tau_{12}^S = 13.5\,\text{Ksi} \ .$$

Results

All laminates studied consist of 16 layers. The discussion to follow is for square plates, $L/W = 1$, with a plate width to hole diameter ratio, W/D, of 3.33. To provide a basis for comparison, the results obtained are normalized by the results obtained for a quasi-isotropic ($\pm 45/0/90)_{2s}$ plate with the identical geometry. Since the quasi-isotropic configuration re-

presents an acceptable conventional design, it is used as a datum. With this normalization, if the load capacity of a particular design, be it tensile load capacity or buckling load capacity, is greater than unity, then the load capacity is predicted to be greater than a quasi-isotropic laminate with identical geometry. If the normalized capacity of a particular design is less than unity, then the design is predicted to have less capacity than a quasi-isotropic laminate with identical geometry.

Curvilinear Design for Tensile Loads

Both failure criteria are used to predict the tensile load capacity of the various designs. The maximum strain criterion is also used to determine the failure mode, i.e., fiber tension failure, matrix tension failure, etc. The Tsai-Wu criterion can also be used to predict the failure node, particularly interactive failure nodes. The Tsai-Wu criterion is not, however, used here in this fashion. The tensile load capacity predicted by the Tsai-Wu criterion is simply compared with the tensile load capacity predicted by the maximum strain criterion. In this regard the Tsai-Wu criterion is used as a check, or backup. If large discrepancies in the predicted capacities between the two criterion were to occur, a further investigation would be warranted. Of all the cases studied, the tensile load capacity as predicted by the Tsai-Wu criterion was practically identical to the tensile load level predicted by the maximum strain criterion. No major discrepancies occurred.

The first curvilinear design discussed aligns all 16 layers with the principal stress direction at each point: an all-curvilinear design. This laminate is denoted $(C_8)_S$, the C standing for curvilinear. The laminate may be considered somewhat academic but much can be learned from it, and it is a good starting point. The $(C_8)_S$ laminate is the curvilinear counterpart to a $(0_8)_S$ laminate. A $(0_8)_S$ laminate fails at a load of 0.59, relative to the quasi-isotropic case, due to a shear failure of the matrix at the location shown in fig. 2. The shear stress is generated because the uniform stress state at the ends of the plate must redistribute to a nonuniform state in the region of the hole. This realignment generates shear stresses. Aligning the fibers with the principal stress directions should eliminate that problem. The iteration process, to determine the fiber angles in each element, and subsequent analysis of the $(C_8)_S$ laminate confirms that this is the case. The failure analysis predicts that the material will fail in tension perpendicular to the fibers at the net section, as shown in fig 2. but away from the hole edge, at a tensile load of 1.01. Since there are no fibers in another layer to suppress this matrix failure, this matrix failure constitutes failure of the laminate. Indeed, with the matrix cracking, the laminate will disintegrate much like the $(0_8)_S$, namely, with many cracks in the matrix following the fiber direction. Table 1 summarizes these findings, as well as results discussed in the ensuing sections. The normalized failure loads as predicted by the Tsai-Wu criterion are shown in parenthesis in the table. Pertinent comments regarding failure of each laminate are included.

The next logical step in the curvilinear design is to eliminate the matrix tension failure perpendicular to the fibers in the $(C_8)_S$ laminate. If this can be done, then the load capacity of the plate would be increased by a significant factor relative to the quasi-isotropic plate. In the $(C_8)_S$ the matrix tension failure perpendicular to the fibers is a result of a Poisson effect in the plate. The high strain in the x direction at the net-section hole edge causes a large contraction strain in the y direction at the hole edge. Away from the hole edge but still at the net-section, the strain in the x direction is not as large. As a result, the Poisson contraction in the y direction is not as large. With a large y-direction Poisson contraction at the hole edge, and a smaller y-direction Poisson contraction away from the hole edge, the material is actually subjected to a tensile stress in the y direction. This tension is enough to crack the matrix. If a small amount of reinforcement is added perpendicular to the fibers in this region, the matrix tension cracking would be suppressed there. This idea is pursued next.

To keep all laminates 16 layers, so the weights and thicknesses of all laminates studied are identical, 14 layers in the primary load direction are chosen to be curvilinear and 2 are chosen to be orthogonal to these layers. The notation adopted is $(O/C_7)_S$, the O being an 'oh' not a zero. The O denotes layers orthogonal to the curvilinear layers. Though it is felt this change

in the laminate construction would not significantly alter the principal stress directions relative to the $(C_8)_s$ laminate, the iteration procedure was repeated, starting with the fiber directions for the $(C_8)_s$ design. As expected, after the iteration produced a converged solution, and the fiber directions of the $(O/C_7)_s$ and the $(C_8)_s$ were practically identical. However, for the $(O/C_7)_s$, there were two layers that were everywhere orthogonal to the C layers.

The addition of the orthogonal layers more than suppresses the matrix tension cracking. The $(O/C_7)_s$ laminate is predicted to fail at a normalized load of 1.89, the fibers at the net-section hole edge limiting the load. Having fibers limiting the load, as opposed to having matrix limiting the load, is making good use of the concept of fiber reinforcing. Having a fiber-reinforced material fail in a way other than fiber failure would seem to be an inefficient use of the material. In Table 1 the superior load capacity of the orthogonal-curvilinear design contrasts the load capacities of $(0_8)_s$ and $(C_8)_s$ designs.

Though the $(O/C_7)_s$ laminate accomplishes the goal of preventing matrix failure, it suffers from two distinct problems. First, it is very susceptible to a shear failure. Any slight misalignment of the tensile loading, or any amount of shear introduced at the plate boundaries, would most likely crack the matrix. The likelihood of having a pure tensile loading, as has been assumed, is small and so shear is an issue. Second, the manufacturing of a laminate with fibers in an orthogonal grid could be a problem. It is probably safe to say that any manufacturing technique would result in less than perfect placement of the fibers. The fibers could only be oriented in the desired direction to within a specified tolerance. This would result in a slight misalignment of the fibers. Because of this slight misalignment, there could be unwanted tensile or shear stresses in the matrix. This would lead to matrix failure and negate any gains made by using curvilinear fibers. To counter any unwanted shear loading, or any misalignment of the fibers, a pair of $\pm 45°$ layers are used in the laminate instead of using an orthogonal layer arrangement. The result is a $(\pm 45/C_6)_s$ laminate. Adding the $\pm 45°$ layers to a laminate would be straightforward from a manufacturing viewpoint. Furthermore, the off-axis layers tend to serve as a sandwich to hold the curvilinear layers together, a desirable feature. With the off-axis layers in the laminate, the directions of the curvilinear layers would not necessarily be the same as the directions of the curvilinear layers with the orthogonal layers. Therefore the iteration procedure is again used to find the specific fiber directions in each element.

The iteration procedure applied to the $(\pm 45/C_6)_s$ laminate results in a curvilinear fiber format that was not appreciably different from the curvilinear format for the $(C_8)_s$ and $(O/C_7)_s$ designs. The analysis indicates that the $(\pm 45/C_6)_s$ laminate sustains a load level of 1.60. Failure is due to tension in the fiber direction in the curvilinear layers at the net-section hole edge.

Figure 3 illustrates the directions of the curvilinear fibers in the $(\pm 45/C_6)_s$ design. By way of this figure, the finite-element discretization is also shown. The figure is presented to illustrate several important points. First, the figure illustrates the assumption in the analysis that the fiber direction within each element is constant. However, even with this restriction, the fiber trajectories are smooth. Second, the figure illustrates that the fiber trajectories involve fairly gentle curves. For a structure with a window-sized hole, radical changes in fiber direction with location do not occur. It is therefore conceivable that laminates with the curvilinear format can be manufactured.

The straightline counterpart to the $(\pm 45/C_6)_s$ design is a $(\pm 45/0_6)_s$ design. Both designs have four off-axis layers at $\pm 45°$ and 12 load-bearing layers. Application of the maximum strain criterion to this straightline counterpart indicates that it does not have as much tensile capacity as the curvilinear design. Specifically the normalized load capacity of the $(\pm 45/0_6)_s$ is 1.27. The failure mode of the straightline design is fiber failure in the $0°$ layers at the net section hole edge. The Tsai-Wu criterion predicts some shear interaction in the failure process and hence a load less than the maximum strain load. What is puzzling is the following: Near the net-section hole edge, the $(\pm 45/C_6)_s$ and the $(\pm 45/0_6)_s$ designs look identical. In the curvilinear design the curvilinear fibers pass by the net section perpendicular to a line from the hole edge to the plate edge, line AA' in fig. 3. In the straightline design the $0°$ fibers also

pass by the net section perpendicular to that same line. Both designs have $\pm$ 45° layers at the net section hole edge. Locally, then, near the net-section hole edge, where failure is predicted to occur in both cases, the two laminates look identical. Yet their load capacities are different!

Two other designs further illustrate this point, and also demonstrate the advantage of the curvilinear design. These two designs are a $(\pm 45/C_2)_{2S}$ laminate and a $(\pm 45/0_2)_{2S}$ laminate. Both designs have the same number of $\pm$ 45° layers and both have the same number of what might be considered load-bearing layers. Yet the curvilinear design has a normalized load capacity of 1.33, and the straightline design has a capacity of 1.20. As can be seen in Table 1, both designs fail due to fiber failure, though according to the Tsai-Wu criterion, there is some shear failure in the $(\pm 45/0_2)_{2S}$ laminate. Again these two laminates have similar fiber orientations near the net section hole edge, point A, where they both fail, yet the curvilinear design has a higher load capacity.

Figure 4 provides insight into this dilemma. This figure illustrates contours of the stress resultant N_x for two different laminates. For exaggeration of the effect and to make a point, the two laminates chosen are the $(0_8)_S$ and the $(C_8)_S$ laminates. The value of N_x applied at the ends of the plate is the same in each case, and the contours in the plate have been normalized by this load. Hence these contours illustrate a stress concentration effect. Two characteristics are clear from the figure. First, the stress concentration is lower for the $(C_8)_S$ laminate than the $(0_8)_S$ laminate. The contour number at the net-section hole edge of the $(C_8)_S$ laminate is lower than the contour number at the net-section hole edge of the $(0_8)_S$ laminate. Second, the distribution of the contours is different. The contours for the $(C_8)_S$ laminate are not as concentrated at the net-section hole edge as they are for the $(0_8)_S$ laminate. Additionally, the shapes of the $(C_8)_S$ contours are different than the shapes of the $(0_8)_S$ contours. Specifically the contour "loops" of the $(C_8)_S$ design are broader and less narrow than the contours of the $(0_8)_S$ design. This indicates that the curvilinear fibers of the $(C_8)_S$ design tend to move the load outwards around the hole more than the straightline fibers of the $(0_8)_S$ design do. With this influence, the material away from the hole is being used more effectively in the $(C_8)_S$ than with the $(0_8)_S$ laminate. This effect translates to the other curvilinear designs.

It should be mentioned that recently Katz, et al. [5] used sequential linear programming in an optimization scheme to investigate performance increases using varying fiber direction in a plate with a hole. The geometry and material properties used by Katz, et al. were identical to those considered here. A maximum strain criterion was also used as a measure of performance. The fiber orientation was allowed to vary in seven regions of the plate. Fiber orientations in the seven regions were varied in a systematic way by the optimization scheme. Predicted performance increases and predicted fiber directions compared quite well with the results discussed here.

Buckling Considerations

With a design for improved tensile performance using the curvilinear established, the influence of these designs on buckling resistance is investigated. The approach here is to simply compute the buckling loads of the plates studied, i.e., the plates of Table 1. This is accomplished with a finite-element analysis, specifically, an analysis using the code EAL (6). Other than the fact the fiber orientation varies from element to element for the curvilinear designs, the use of this code for this problem follows normal procedures. Here it is assumed that the plates are simply supported on all four edges.

Table 2 summarizes the buckling load results for the plate summarized in Table 1, i.e., a plate with $L/W = 1$ and W/D of 3.33. Again the quasi-isotropic laminate with identical geometry is used as a normalization datum.

Several interesting results are found when considering the buckling strength. First, the $(0/C_7)_S$ laminate, which is so much better than the quasi-isotropic laminate in tension, has a buckling load about 45% less than the buckling load of a quasi-isotropic laminate. Second, none of the laminates studied have as high a buckling load as the quasi-isotropic laminate. The $(\pm 45/C_2)_{2S}$ and $(\pm 45/0_2)_{2S}$ laminates have buckling loads quite close to the buckling load

for a quasi-isotropic laminate. This is presumable due to these two laminates having the same number of ($\pm$ 45) layers as the quasi-isotropic laminate.

A third point to be observed is that though the buckling loads of the ($\pm 45/C_6)_s$ and ($\pm 45/0_6)_s$ laminates are lower than the buckling load of the quasi-isotropic laminate, the laminate with the curvilinear fiber format has a slightly higher buckling load. If this information is combined with the fact that in tension, the ($\pm 45/C_6)_s$ is better than the ($\pm 45/0_6)_s$, and clearly better than the quasi-isotropic laminate, then the ($\pm 45/C_6)_s$ is a laminate that deserves consideration. The gains in using a ($\pm 45/C_2)_{2s}$ rather than a ($\pm 45/0_2)_{2s}$ are not as significant.

Concluding Comments

Presented has been an idea for possibly improving structural efficiency using fiber-reinforced materials. Specifically the notion of using a curvilinear fiber format in flat plates with central circular holes has been studied. The paper is not all-encompassing, and it was not intended to be that way. The paper simply suggests that the curvilinear format has the potential for using reinforcing fibers more effectively. Designs using the curvilinear format have been contrasted with conventional straightline design counterparts. Results have been compared with a standard quasi-isotropic design. Tensile and compressive buckling loads have been studied. It can be concluded that in tension the curvilinear designs studied lead to improved performance. In compression, the buckling loads are not as high as they are for quasi-isotropic laminates, but the buckling loads for the curvilinear design are no lower than the buckling loads for their straightline non-quasi-isotropic counterparts. Though the results presented are for a specific geometry, as stated earlier, the results are similar for other geometrics. Refs. 7 and 8 document these findings.

Before closing a comment should be made regarding manufacturing. As with many design exercises, the curvilinear configurations discussed here may present some manufacturing problems. For example, the ($\pm 45/C_6)_s$ design calls for the fiber angles along line BB' to not be horizontal. When considering a complete plate, rather than just one quarter, this is a problem. For a complete plate, the fibers along line BB' must be horizontal. Along line BB' there cannot be a slightly negative angle specified by the top quarter of the plate, and a slightly positive angle specified by the bottom quarter. In practice, the fiber angles in the elements along line BB' must be adjusted to make them horizontal. This may not impact the increased efficiency. Sensitivity analyses, to determine which areas of the plate are least sensitive to deviations of the fiber angle from the ideal, can be conducted. It is suspected that the load capacity is not particularly sensitive to fiber angle along line BB'. Thus the fiber angles can be adjusted to achieve manufacturing compatibility and not significantly effect performance.

Also regarding manufacturing, Knoblach [9] recently investigated the use of electric fields to align short chopped fibers in specific directions while the material was being manufactured. There were problems with the electrodes and fiber-fiber interaction but the concept worked. Plates were produced which had better stiffness and strength properties than plates with random fiber orientation, i.e., quasi-isotropic.

More work needs to be done. Studies are needed to pursue laminates with other combinations of straight and curvilinear layers. Currently studies are underway to investigate the use of the curvilinear format to increase the buckling resistance. Manufacturing issues are also being addressed. The character of post-buckling performance and failure with the curvilinear format also deserves attention. These developments will be reported on at a later date as work progresses.

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Table 1
Tensile Failure Loads

design	failure load[1]	failure node
$(\pm 45/0/90)_{2s}$	1.00 (0.99)	fiber failure in $0°$ layer at net section hole edge
$(0_8)_s$	0.59 (0.51)	shear failure in matrix near hole but away from net section (see fig. 2)
$(C_8)_s$	1.01 (0.93)	tension failure perpendicular to the fibers at net section but away from hole edge (see fig. 2)
$(0/C_7)_s$	1.89 (1.78)	fiber failure in C layers at net section hole edge
$(\pm 45/C_6)_s$	1.60 (1.66)	fiber failure in C layers at net section hole edge
$(\pm 45/0_6)_s$	1.27 (0.94)	fiber failure in $0°$ layers at net section hole edge[2]
$(\pm 45/C_2)_{2s}$	1.33 (1.38)	fiber failure in C layers at net section hole edge
$(\pm 45/0_2)_{2s}$	1.20 (1.22)	fiber failure in $0°$ layers at net section hole edge[2]

[1] normalized by maximum strain criterion failure load for a quasi-isotropic laminate, number in parenthesis is Tsai-Wu prediction.

[2] Tsai-Wu criterion predicts some shear interaction.

Table 2

design	buckling load[1]
$(\pm 45/0/90)_{2s}$	1.00
$(0_8)_s$	0.46
$(C_8)_s$	0.52
$(O/C_7)_s$	0.55
$(\pm 45/C_6)_s$	0.87
$(\pm 45/0_6)_s$	0.84
$(\pm 45/C_2)_{2s}$	0.93
$(\pm 45/0_2)_{2s}$	0.90

[1] normalized by buckling load for quasi-isotropic laminate
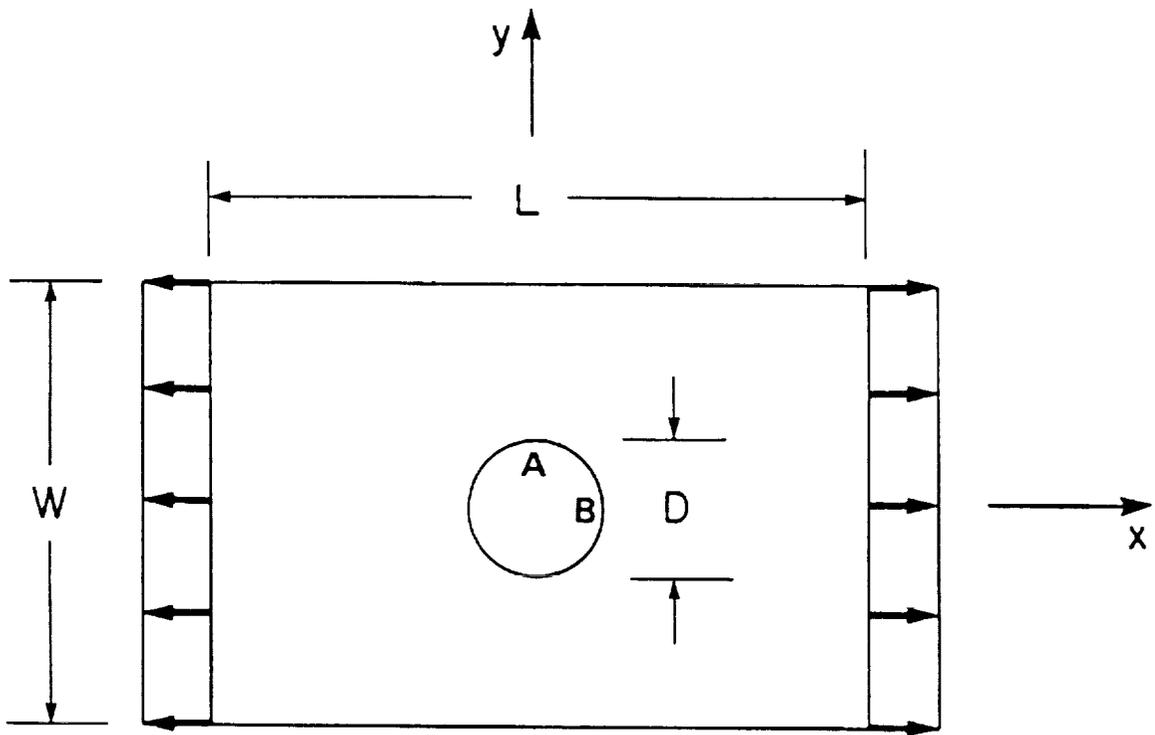
Fig. 1 Plate Geometry and Nomenclature.
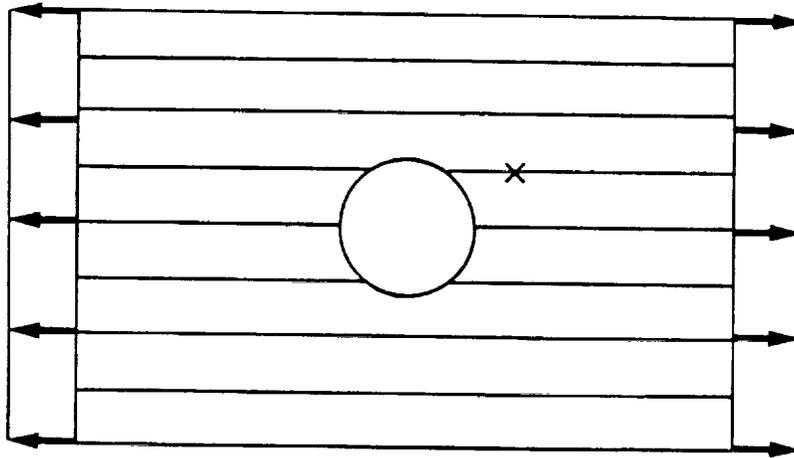
$(O_8)_S$
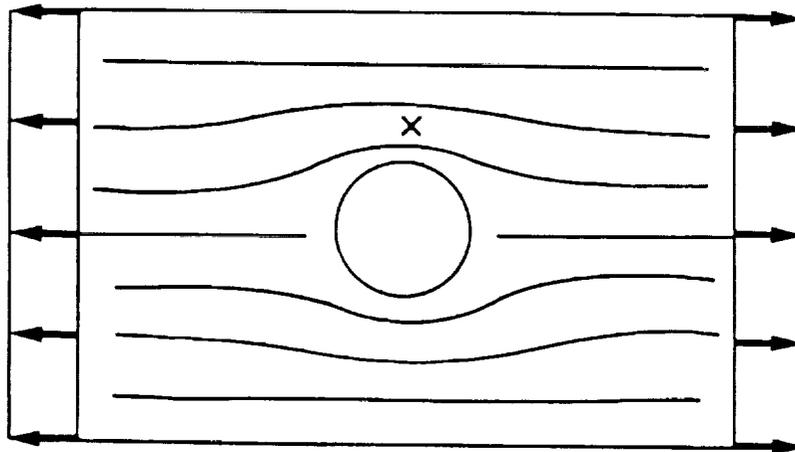
$(C_8)_S$

Fig. 2 Failure Locations (x) for $(O_8)_S$ and $(C_8)_S$ Plates.
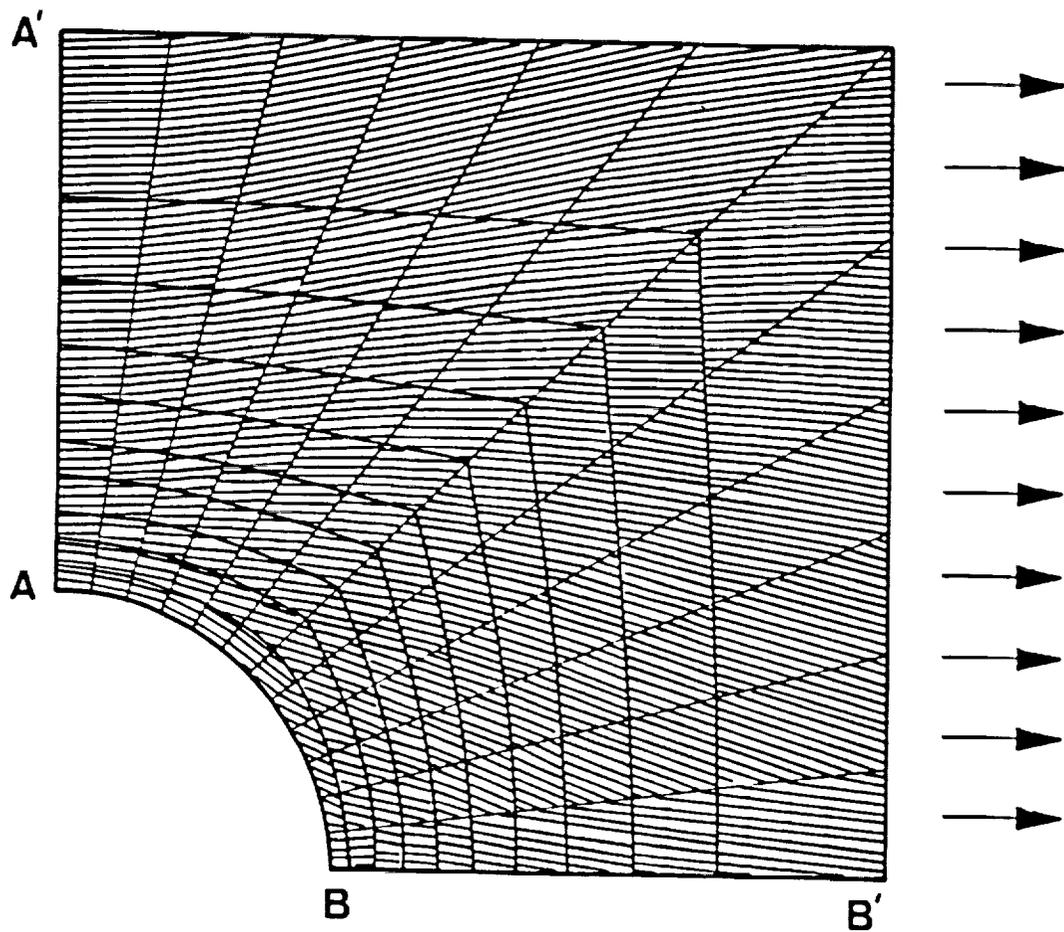
Fig. 3 Directions of Curvilinear Fibers in (± 45/C_6)$_S$ Plate.
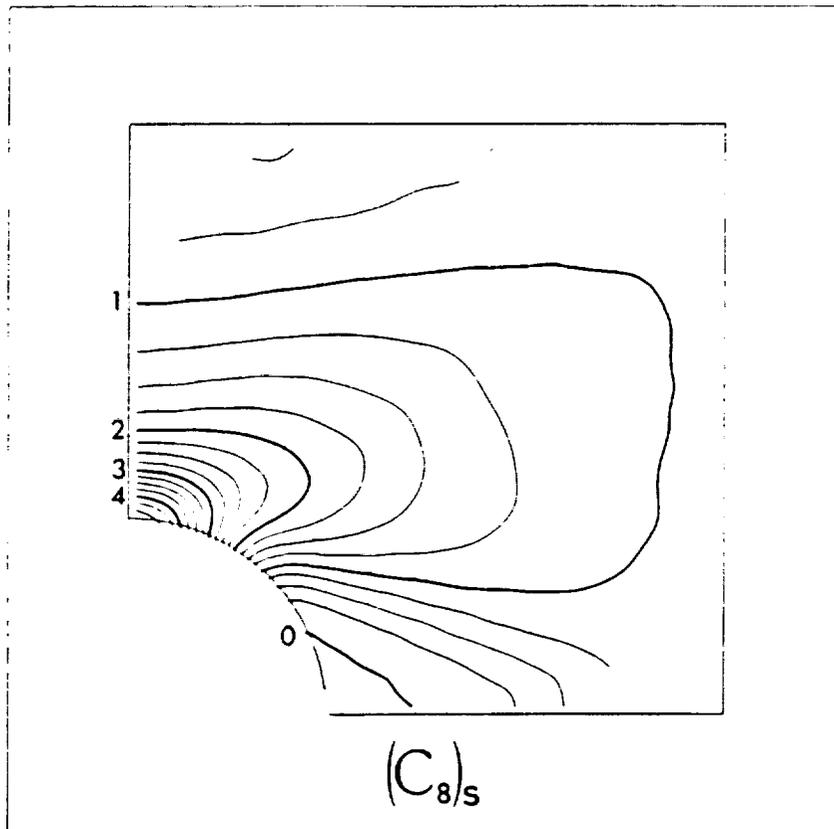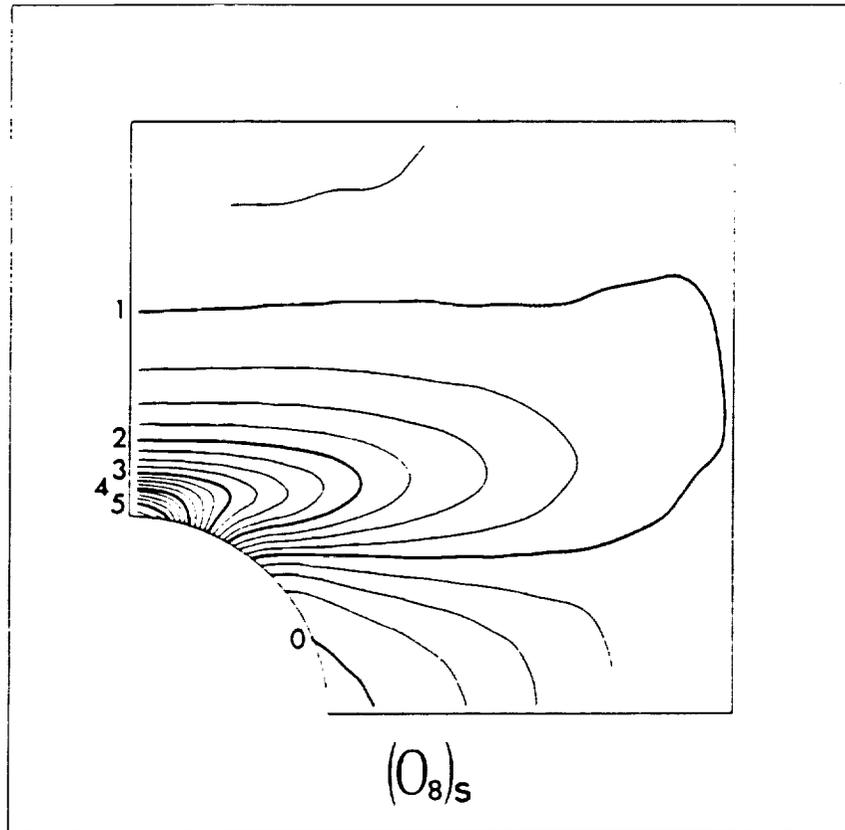
Fig. 4 Contours of Normalized Stress Resultant N_x