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A Method for Partitioning Centralized Controllers

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Summary

The notion of controller partitioning is described. Conditions are developed under which the input/output behavior of a multi-input multi-output centralized controller can be exactly matched by two separate subsystem controllers interconnected through output crossfeed. A systematic method is developed for determining a controller partitioning which best approximates the input/output behavior of the centralized controller for the general case when the exact matching conditions are not satisfied. The controller partitioning procedure is demonstrated for a centralized integrated flight/propulsion controller designed in a previous study.

Introduction

Large interconnected systems often exhibit a significant amount of coupling between the various subsystems thus requiring an integrated approach to controller design. Short takeoff and vertical landing (STOVL) aircraft are an example of such a system. In STOVL aircraft, the forces and moments generated by the propulsion system provide control and maneuvering capabilities for the aircraft at low speeds thus creating the need for integrated flight propulsion control (IFPC) system design. One approach to integrated control design is to partition the overall system into loosely coupled subsystems and then design a decentralized control system considering one subsystem at a time. A survey of decentralized control design techniques can be found in reference 1, and an example application of decentralized control design techniques to IFPC design is available in reference 2.

Although the decentralized approach to integrated control design is intuitively appealing in that it results in low-order, easy to implement subsystem controllers (hereinafter referred to as subcontrollers), its major drawback is that accounting for all the interactions between the various subsystems, especially when the intercoupling is strong, is quite cumbersome. The strengths and weaknesses of a decentralized, hierarchial approach to IFPC design are further discussed in reference 3.

Another approach to integrated control design is to design a centralized controller considering the plant to be the overall integrated system with all its interconnections. An IFPC design based on a centralized approach is discussed in reference 4. Although such an approach will lead to an "optimal" design from a systems point of view, since it accounts for all the subsystem interactions, it results in one high-order controller which is difficult to implement. Often the design, manufacture, and testing of different subsystems are performed by different companies which are accountable only for individual subsystem performance. For instance, in an aircraft design, the engine manufacturer ensures that the propulsion system will provide the desired performance when installed in the airframe. The subsystem manufacturer performs extensive tests with an independent subcontroller to assure an adequate design. The testing and accountability of each individual subsystem can be formidable with a centralized controller since closed-loop performance evaluation would require all the subsystems to be assembled without previous independent testing.

An approach to integrated control design which combines the "best" aspects of the centralized and decentralized approaches was suggested in reference 5. This approach consists of first designing a centralized controller, so that all subsystem interconnections are accounted for in the initial design stage, and then partitioning the centralized controller into separately implementable, decentralized subcontrollers for individual subsystems. By *partitioning* here is meant representing the high-order centralized controller with two or more lower-order subcontrollers which have input/output intercoupling such that the overall controller representation obtained on assembling the subcontrollers closely approximates the input/output behavior of the centralized controller. A partitioning with subcontroller output crossfeed, suggested in reference 5, was shown to lead to much simplified subcontrollers for an IFPC design without significant loss in closed-loop performance and stability robustness as compared with that obtained with the high-order centralized controller.

The objectives of this paper are to provide a mathematically rigorous approach to controller partitioning with output crossfeed and to develop stepwise procedures to implement such a partitioning. In the following, the notion of controller partitioning is further discussed, exact matching conditions for controller partitioning are derived, and a methodology is developed for determining the controller partitioning that "best" approximates the centralized controller when the exact matching conditions are not satisfied. A step-wise algorithm for controller partitioning is presented and demonstrated via a numerical example for the centralized IFPC design of reference 5. The research described in this paper was motivated by the authors' IFPC studies, and thus the results are presented with respect to partitioning a centralized integrated flight propulsion controller into separate airframe and engine subcontrollers. The procedures developed in this paper are, however, relevant to any general centralized controller and can easily be extended to the case of more than two subcontrollers.

Controller Partitioning

The problem.—A plant consisting of engine and airframe subsystems is to be controlled in an efficient and effective manner. It is assumed that a centralized controller for this plant has already been designed. The transfer matrix for the plant is denoted as $G(s)$, and that of the centralized controller is $K(s)$. The control loop for the system is shown in figure 1. Given the predetermined optimal centralized controller with state space realization

$$\left.\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{e} \\ \mathbf{u} &= C\mathbf{x} + D\mathbf{e} \end{aligned}\right\} \qquad (1)$$

the problem is to determine partitioned subcontrollers that are interconnected in such a way as to match the $\mathbf{e} \rightarrow \mathbf{u}$ performance of $K(s)$. We assume without loss of generality that the direct feedthrough matrix is zero ($D = 0$).

The controller in partitioned form.—The inputs, $\mathbf{e}$, to the partitioned subcontrollers are assumed to be the same as for the centralized controller, and the outputs associated with the airframe and engine are denoted as $\mathbf{u}_a$ and $\mathbf{u}_e$ of dimensions m_a and m_e, respectively. The variables represented by $\mathbf{u}_a$ and $\mathbf{u}_e$ are exclusive and together exhaust those of $\mathbf{u}$. The only interconnections that we consider here are the crossfeed of outputs between the two subcontrollers. Such a partitioned controller is shown in figure 2. This form has the following advantages:

(1) It approximates the centralized compensator as a whole.

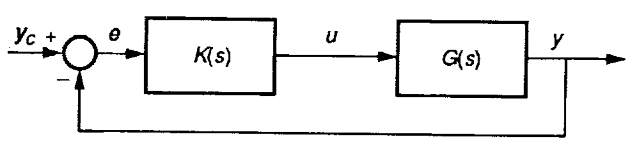

Figure 1.—Control loop.

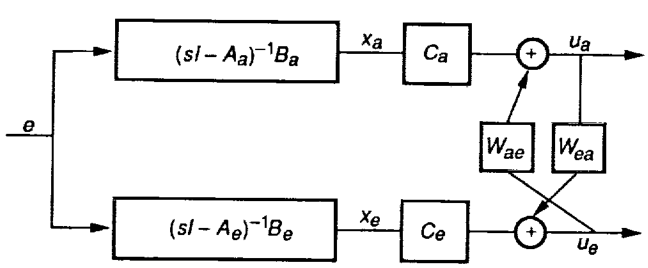

Figure 2.—Partitioning by output crossfeed.

(2) The order of the centralized controller may be preserved by the assembled partitioned controller but internal state coupling is approximated by output coupling.

(3) The controller is amenable to subsequent simplification because of the structure of the subcontrollers.

Assume that state variable vectors, $\mathbf{x}_a$ and $\mathbf{x}_e$, have been assigned to the airframe and engine subcontrollers, respectively; a specific manner for doing this will be discussed later. The partitioned controller with output crossfeeds has the state space representation

$$\left.\begin{aligned} \dot{\mathbf{x}}_a &= A_a\mathbf{x}_a + B_a\mathbf{e} \qquad & \dot{\mathbf{x}}_e &= A_e\mathbf{x}_e + B_e\mathbf{e} \\ \mathbf{u}_a &= C_a\mathbf{x}_a + W_{ae}\mathbf{u}_e \qquad & \mathbf{u}_e &= C_e\mathbf{x}_e + W_{ea}\mathbf{u}_a \end{aligned}\right\} \qquad (2)$$

The transfer matrix for the partitioned controller is denoted as $\tilde{K}$, satisfying the input/output relation

$$\begin{pmatrix} \mathbf{u}_a \\ \mathbf{u}_e \end{pmatrix} = \tilde{K}(s)\mathbf{e}(s)$$

Algebraic manipulation of equation (2) under the assumption that $I - W_{ea}W_{ae}$ is invertible (in which case $I - W_{ae}W_{ea}$ is also invertible) results in a state space representation which better demonstrates the input/output characteristics of the partitioned controller

$$\begin{pmatrix} \dot{\mathbf{x}}_a \\ \dot{\mathbf{x}}_e \end{pmatrix} = \begin{pmatrix} A_a & 0 \\ 0 & A_e \end{pmatrix}\begin{pmatrix} \mathbf{x}_a \\ \mathbf{x}_e \end{pmatrix} + \begin{pmatrix} B_a \\ B_e \end{pmatrix}\mathbf{e}$$

$$\begin{pmatrix} \mathbf{u}_a \\ \mathbf{u}_e \end{pmatrix} = \begin{pmatrix} (I - W_{ae}W_{ea})^{-1}C_a & W_{ae}(I - W_{ea}W_{ae})^{-1}C_e \\ W_{ea}(I - W_{ae}W_{ea})^{-1}C_a & (I - W_{ea}W_{ae})^{-1}C_e \end{pmatrix}\begin{pmatrix} \mathbf{x}_a \\ \mathbf{x}_e \end{pmatrix} \qquad (3)$$

Exact partitioning condition.—We conclude from equation (3) that if a controller is in partitioned form, then it has a state space representation of the form

$$\left.\begin{aligned} \begin{pmatrix} \dot{\mathbf{x}}_a \\ \dot{\mathbf{x}}_e \end{pmatrix} &= \begin{pmatrix} A_a & 0 \\ 0 & A_e \end{pmatrix}\begin{pmatrix} \mathbf{x}_a \\ \mathbf{x}_e \end{pmatrix} + \begin{pmatrix} B_a \\ B_e \end{pmatrix}\mathbf{e} \\ \begin{pmatrix} \mathbf{u}_a \\ \mathbf{u}_e \end{pmatrix} &= \begin{pmatrix} C_{aa} & W_{ae}C_{ee} \\ W_{ea}C_{aa} & C_{ee} \end{pmatrix}\begin{pmatrix} \mathbf{x}_a \\ \mathbf{x}_e \end{pmatrix} \end{aligned}\right\} \qquad (4)$$

for some W_{ae} and W_{ea} with $I - W_{ae}W_{ea}$ invertible.

It is easy to check that if an output matrix, as in equation (4), exists for a controller with state dynamics and input matrices in the block form, then $C_a = (I - W_{ae}W_{ea})C_{aa}$ and $C_e = (I - W_{ea}W_{ae})C_{ee}$ are the output matrices for the partitioned subcontrollers as in equation (2) with output crossfeeds via W_{ea} and W_{ae}. Therefore, the condition for exactly partitioning a controller into subcontrollers with output crossfeeds is that there exist a state space representation of the transfer matrix $K(s) = C(sI - A)^{-1}B$ with some assignment of the state variables $\mathbf{x} = \begin{pmatrix}\mathbf{x}_a \\ \mathbf{x}_e\end{pmatrix}$ so that

$$A = \begin{pmatrix} A_a & 0 \\ 0 & A_e \end{pmatrix} \qquad B = \begin{pmatrix} B_a \\ B_e \end{pmatrix}$$

$$C = \begin{pmatrix} C_{aa} & C_{ae} \\ C_{ea} & C_{ee} \end{pmatrix}$$

where C_{ea} and C_{ae} are related to C_{aa} and C_{ee}, respectively, by

$$C_{ea} = W_{ea}C_{aa} \quad \text{and} \quad C_{ae} = W_{ae}C_{ee} \tag{5}$$

for some W_{ea} and W_{ae} with $I - W_{ae}W_{ea}$ invertible.

The condition in equations (5) is thus a necessary and sufficient condition for exact partitioning with output crossfeed.

Approximation by a Partitioned Controller

The condition developed in equations (5) for exact controller partitioning will not in general be satisfied by a given centralized controller. The approach that we use is to seek a matrix $\tilde{C}$ which satisfies conditions of equations (5) and which closely approximates the C matrix of an appropriate realization for the centralized controller.

We first put the state dynamics matrix into a "modal canonical form" so that the appropriate block structure for A and B can be easily determined. This also guarantees that the order of the assembled partitioned controller will be the same as the order of the centralized controller. Next, the modal state variables are assigned to two sets, the airframe and engine variables, according to the physical insight of the designer coupled with controllability and observability analyses such as those in reference 5. The resulting state vectors are designated as $\mathbf{x}_a$ and $\mathbf{x}_e$. Notice that many such assignments may be reasonable. Any particular state variable assignment can be represented by multiplication of the modal state vector by a permutation matrix, P.

Now the problem reduces to finding a matrix $\tilde{C}$ which satisfies (5) and which is an approximation to the transformed output matrix, $\hat{C} = CP$ with

$$\hat{C} = \begin{pmatrix} \hat{C}_{aa} & \hat{C}_{ae} \\ \hat{C}_{ea} & \hat{C}_{ee} \end{pmatrix}$$

where, for example, the block $\hat{C}_{ae}$ corresponds to the airframe output response to the engine modal state.

The approach to approximation is to separately approximate the two block matrices $\begin{pmatrix}\hat{C}_{aa} \\ \hat{C}_{ea}\end{pmatrix}$ and $\begin{pmatrix}\hat{C}_{ae} \\ \hat{C}_{ee}\end{pmatrix}$ by matrices of ranks m_a and m_e, respectively (recall that m_a and m_e are the numbers of output variables of the two types). Determine the best rank m_a (or rank m_e) approximations to these submatrices by using singular value decompositions (ref. 7). Denote the resulting block matrices as $\begin{pmatrix}\tilde{C}_{aa} \\ \tilde{C}_{ea}\end{pmatrix}$ and $\begin{pmatrix}\tilde{C}_{ae} \\ \tilde{C}_{ee}\end{pmatrix}$.

These approximations satisfy the condition as equation (5) for the following reason: The rank of a matrix describes the maximum number of rows which are linearly independent (normally there are many such sets). It is easy to check whether the first m_a rows of $\begin{pmatrix}\tilde{C}_{aa} \\ \tilde{C}_{ae}\end{pmatrix}$ are linearly independent. If they are, then the remaining rows are a linear combination of these, so the relation $\tilde{C}_{ea} = W_{ea}\tilde{C}_{aa}$ holds for some matrix W_{ea}. Similarly, if the matrix $\tilde{C}_{ee}$ is also of full rank, then there is a matrix W_{ae} such that $\tilde{C}_{ae} = W_{ae}\tilde{C}_{ee}$. These matrices are computed using pseudoinverses $W_{ea} = \tilde{C}_{ea}\tilde{C}^{\#}_{ae}$ and $W_{ae} = C_{ae}C^{\#}_{ee}$. The condition that $(I - W_{ea}W_{ae})$ is invertible must now be checked.

One measure of goodness of approximation which bounds the maximum difference in output responses between the centralized controller and an assembled partitioned approximation over all possible inputs and at all possible frequencies is the H_∞ norm of the difference of transfer matrices,

$$\max_{\omega} \bar{\sigma}(\tilde{C} - \hat{C})P^{-1}(j\omega I - A)^{-1}B$$

where by $\bar{\sigma}(M)$ is meant the largest singular value of the matrix M. Note that the centralized controller should be well-scaled in order for this measure to give meaningful results. For further reference to this norm, its computation, and usage, see the text by Francis (ref. 8) and references therein. This difference should now be calculated, and, if this norm is sufficiently small, then this approximation may be used.

The block submatrices C_a and C_e are constructed from W_{ea}, W_{ae}, $\tilde{C}_{aa}$, and $\tilde{C}_{ee}$. The partitioned system in the form of equation (2) is assembled. Here, A_a and A_e refer to the blocks of the transformed matrix A corresponding to the chosen state vectors $\mathbf{x}_a$ and $\mathbf{x}_e$, and B_a and B_e refer to the corresponding rows of the transformed B matrix. It is this partitioned representation that will eventually be implemented.

There is no guarantee that without an exhaustive search of all possible modal state variable assignments one will have found the closest assembled partitioned controller to the centralized controller. Nonetheless, by examining reasonable candidate variable assignments as determined by the designer's physical insight aided by controllability and observability analysis, one should be able to find the best reasonable partitioned controller. The only mathematical guidelines that one should use in determining these candidates is that the columns of $\hat{C}$ should be assigned to airframe or engine (corresponding to the assignments of the modal state variables) so that the subblocks $\hat{C}_{aa}$ and $\hat{C}_{ee}$ are of maximal rank and so that the other subblocks don't contribute significantly to the singular values of the block matrices.

The procedure discussed above is outlined in a stepwise manner as follows:

(1) The preliminary step is to compute the modal decomposition of A, $T^{-1}AT = A_M$ where A_M may contain either real entries or real 2×2 blocks along the diagonal. The matrix T transforms the modal states to physical states.

(2) The designer uses physical insight and controllability and observability analyses to propose an assignment of the modal blocks into two sets corresponding to n_a and n_e modal variables. The permutation matrix P is determined by this assignment.

(3) The $\hat{C} = C_M P$ is computed, and the two blocks $\hat{C}_a$ and $\hat{C}_e$ corresponding to the first n_a columns and the last n_e columns of $\hat{C}$ are formed. These two matrices should have ranks at least m_a and m_e, respectively; otherwise, return to step 2.

(4) Determine the best rank m_a approximation to $\hat{C}_a$ as follows:

(a) Perform the singular value decomposition

$$\hat{C}_a = U\Sigma V^{\mathsf{T}} \sum_{i=v}^{r} \sigma_i u_i v_i^{\mathsf{T}}$$

where σ_i are the singular values and u_i and v_i^{T} are the left and right singular vectors of $\hat{C}_a$.

(b) The best rank m_a approximation is

$$\tilde{C}_a = \begin{pmatrix} \tilde{C}_{aa} \\ \tilde{C}_{ea} \end{pmatrix} = \sum_{i=1}^{m_a} \sigma_i u_i v_i^{\mathsf{T}}$$

which is formed by retaining only the largest m_a singular values.

(5) Determine the best rank m_e approximation

$$\tilde{C}_e = \begin{pmatrix} \tilde{C}_{ae} \\ \tilde{C}_{ee} \end{pmatrix}$$

to $\hat{C}_e$ by applying the procedure of step 4 to $\hat{C}_e$.

(6) Check that the $m_a \times n_e$ block $\tilde{C}_{aa}$ has rank m_a and that the $m_e \times n_e$ block $\tilde{C}_{ee}$ has rank m_e; if not, then return to step 2.

(7) Calculate $W_{ea} = \tilde{C}_{ea}\tilde{C}^{\#}_{aa}$ and $W_{ae} = \tilde{C}_{ae}\tilde{C}^{\#}_{ee}$, where # refers to the Moore-Penrose psuedo-inverse of the corresponding matrix. Check that $I - W_{ea}W_{ae}$ is invertible; if not, return to step 2.

(8) Form the system difference matrix

$$K(j\omega) - \tilde{K}(j\omega) = (\hat{C} - \tilde{C})P^{-1}(j\omega I - A)^{-1}B$$

where A and B are the original state and input matrices and where P represents the partitioning of the modal states. Compute the largest singular value (or matrix norm) for this matrix for each ω in the desired frequency range, and find the maximum of these singular values. If this measure of goodness of approximation is not sufficiently small, then return to step 2.

(9) Calculate $C_a = \tilde{C}_{aa} - W_{ea}\tilde{C}_{ea}$ and $C_e = \tilde{C}_{ee} - W_{ae}\tilde{C}_{ae}$ and form the partitioned state space representation

$$\dot{\mathbf{x}}_a = A_a \mathbf{x}_a + B_a \mathbf{e}$$

$$\mathbf{u}_a = C_a \mathbf{x}_a + W_{ae} \mathbf{u}_e$$

$$\dot{\mathbf{x}}_e = A_e \mathbf{x}_e + B_e \mathbf{e}$$

$$\mathbf{u}_e = C_e \mathbf{x}_e + W_{ea} \mathbf{u}_a$$

This completes the procedure for constructing a partitioned system using crossfeeds of outputs with the property that it approximates the original system to within a prescribed accuracy.

Example of Partitioning by Output Crossfeed

Here, we apply the steps of controller partitioning to the centralized flight propulsion controller obtained in reference 5. This controller has the form of equation (1) with the error vector $\mathbf{e}$ consisting of errors in following pitch rate, velocity, engine fan speed and engine pressure ratio commands, $\mathbf{e} = [e_q, e_v, e_{N2}, e_{EPR}]^{\mathsf{T}}$. The control input vector $\mathbf{u}$ consists of rates of thrust vectoring, fuel flow, thrust reverser port area, and nozzle throat area, $\mathbf{u} = [\dot{\delta}_{TV}, \dot{WF}, \dot{A}78, \dot{A}8]$. Note that $\mathbf{u}$ consists of rates because integrators were appended to the control inputs during the process of control design (see ref. 5 for details). Based on open-loop control effectiveness studies for the plant, the partitioned airframe and engine controllers are desired to have inputs $\mathbf{e}$ and outputs $\mathbf{u}_a = [\dot{\delta}_{TV}]$ and $\mathbf{u}_e = [\dot{WF}, \dot{A}78, \dot{A}8]$, respectively. The centralized controller matrices A, B, and C are available in reference 5.

After the transformation to modal form, the centralized controller matrices A_M, B_M, and C_M as in equation (1) are formed. The last is especially important since an analysis of its rows determines a potential assignment of the state variables.

$C_M =$

$$
\begin{pmatrix}
-2.23E^{-7} & -2.18E^{-4} & 1.77E^{-2} & -1.94E^{-2} & 1.39E^{-1} \\
2.54E^{-8} & 2.99E^{-6} & -1.66E^{-5} & -3.57E^{-2} & -1.65E^{-2} \\
-3.28E^{-6} & -6.56E^{-4} & -1.66E^{-3} & -5.75E^{+0} & 1.08E^{+1} \\
3.26E^{-6} & 6.03E^{-4} & 1.14E^{-3} & 4.78E^{+0} & -9.05E^{+0}
\end{pmatrix.
$$

$$
\begin{matrix}
3.20E^{-5} & -1.35E^{-3} & -1.69E^{-3} & -9.58E^{-4} \\
7.31E^{+0} & -8.69E^{+0} & -2.57E^{+0} & -1.28E^{+1} \\
1.68E^{-2} & -7.26E^{-2} & -1.02E^{-1} & -8.43E^{-2} \\
2.24E^{-2} & -1.03E^{-1} & -1.52E^{-1} & -1.69E^{-1}
\end{matrix}
$$

$$
\left.
\begin{matrix}
4.50E^{+1} & -7.22E^{+1} & -5.51E^{-4} & -5.33E^{-3} \\
-1.37E^{-2} & 1.55E^{-2} & -2.97E^{+0} & 9.05E^{+0} \\
-4.88E^{-1} & 8.88E^{-1} & 6.06E^{+1} & -2.21E^{+2} \\
4.08E^{-1} & -7.38E^{-1} & 7.28E^{+1} & -2.65E^{+2}
\end{matrix}
\right)
$$

Based on the choices of $\mathbf{u}_a$ and $\mathbf{u}_e$ as above and on an inspection of C_M from the point of view of controller observability, the two possible state variable assignments of interest are, for P_1

$$x_a = \{x_1, x_2, x_3, x_{10}, x_{11}\} \text{ and } x_e \text{ is the rest}$$

and, for P_2

$$x_a = \{x_3, x_{10}, x_{11}\} \text{ and } x_e \text{ is the rest}$$

where x_i denotes the i^{th} modal state variable of the 13th order centralized controller.

The results in terms of the approximation errors for the two assignments P_1 and P_2 are shown in figure 3. This figure shows the minimum singular value for the global controller, $\underline{\sigma}(K(j\omega))$, and the maximum singular value of the difference, $\bar{\sigma}(E(j\omega)) = \bar{\sigma}(K - \tilde{K}_i)(j\omega)$, for the assignments P_1 and P_2 where $\tilde{K}_i(j\omega)$ is the transfer matrix for the assembled partitioned controller corresponding to partitioning P_i.

As can be seen in figure 3, both the assignments P_1 and P_2 lead to a good approximation of the centralized flight/propulsion controller. Apart from matching the centralized controller itself, it is also of interest to see whether the assembled partitioned controllers match the stability and performance robustness characteristics that are achieved with the centralized controller. Based on the work of Doyle et al. (ref. 9), with the error in assignment P_i represented as $E_i(s) = K(s) - \tilde{K}_i(s)$, it can be shown that the closed-loop system with partitioned controller K_i will remain stable if

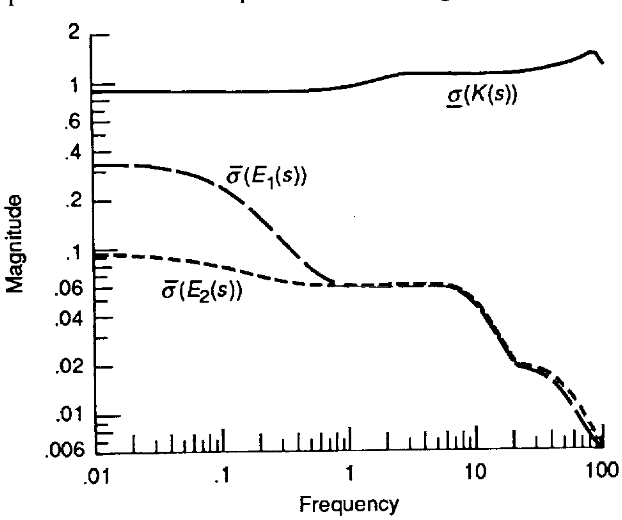

Figure 3.—$\underline{\sigma}(k)$ and $\bar{\sigma}(E_i)$ for the two partitionings.

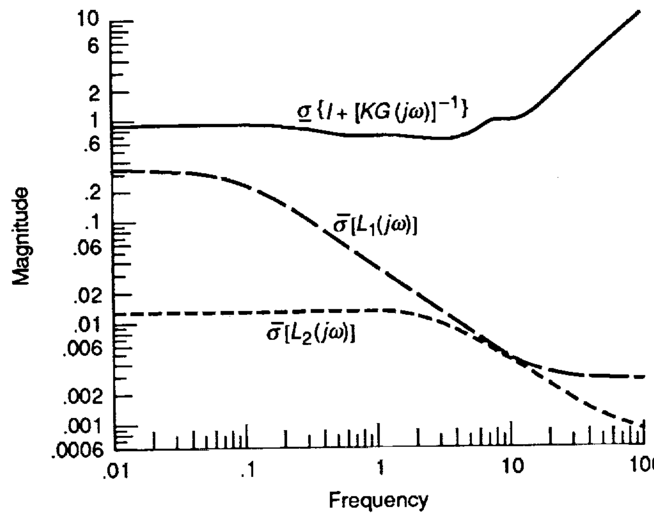

Figure 4.—Stability check for partitioning controllers.

$$\bar{\sigma}\left[L_i\ (j\omega)\right] < \underline{\sigma}\left[I + (KG(j\omega))^{-1}\right]$$

where $L_i(s)$ is the multiplicative error for the partitioned controller given by $L_i(s) = E_i(s) \cdot K^{-1}(s)$. The plots in figure 4 show that this stability condition is satisfied for both assignments discussed above. Here, $G(s)$ is the transfer matrix for the integrated flight and propulsion plant considered in reference 5. Although detailed results are not presented here, closed-loop performance with either of the two assembled, partitioned controllers closely matched the performance with the centralized controller.

Conclusion

The idea of partitioning a centralized controller into interconnected subcontrollers by output crossfeed was introduced. Conditions were developed for a centralized controller to be representable as a partitioned controller. A procedure for approximating a centralized controller by interconnected, decentralized subcontrollers was presented. An example was presented to demonstrate a procedure wherein an integrated flight and propulsion controller was partitioned into separate airframe and engine subcontrollers. Results were presented showing that the assembled, partitioned subcontrollers closely match the response of the centralized controller in the frequency domain.

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15. Supplementary Notes

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16. Abstract

The notion of controller partitioning is described. Conditions are developed under which the input/output behavior of a multi-input multi-output centralized controller can be exactly matched by two separate subsystem controllers interconnected through output crossfeed. A systematic method is developed for determining a controller partitioning which best approximates the input/output behavior of the centralized controller for the general case when the exact matching conditions are not satisfied. The controller partitioning procedure is demonstrated for a centralized integrated flight/propulsion controller designed in a previous study.

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