Generalized Probability Model for Calculation of Interference to the Deep Space Network Due to Circularly Earth-Orbiting Satellites

C. J. Ruggier
Telecommunications Systems Section

The probability of exceeding interference power levels and the duration of interference at the Deep Space Network (DSN) antenna is calculated parametrically when the state vector of an Earth-orbiting satellite over the DSN station view area is not known. A conditional probability distribution function is derived, transformed, and then convolved with the interference signal uncertainties to yield the probability distribution of interference at any given instant during the orbiter's mission period. The analysis is applicable to orbiting satellites having circular orbits with known altitude and inclination angle.

I. Introduction

Knowing the orbital parameters and transmitting power level of a potentially interfering satellite is prerequisite to estimating the received interference power level incident at any DSN tracking antenna. However, there are occasions when the satellite launch and orbit injection data are uncertain or unavailable at the time of the analysis. In this case, the satellite orbital position relative to the ground station antenna beam cannot be predicted using the usual deterministic analytical methods: A probabilistic approach to the analysis is more practicable. Reasonable probabilities of interference can be calculated if the geometrical and signal parameter uncertainties are defined for the orbiting satellite. The calculated probabilities can then be useful in determining whether any significant in-terference threats to the DSN will exist during the orbiting satellite mission period.

The probability of interference is sought from the probability of the orbiting satellite being incident at some point in the antenna beam, for some DSN tracking antenna pointing angle, during the period when the satellite is in view. This conditional probability function is derived from the geometrical relationships of the satellite's orbit. From the conditional probability, the off-beam antenna gain probability can be determined and then convolved with the interference signal transmission uncertainties to yield the probability of received interference power. Finally, the probability of received interference power is scaled by the ratio of view area contained within the satel-

lite's ground track coverage area and the probability of tracking time of the DSN antenna during a 24-hour period. The probability of the DSN station exceeding a given interference power level (density) at any given time can then be determined. In addition to the probability of interference, the period of the interference signal and its probability of occurrence is also considered, given that the interference event has occurred.

II. Rationale for the Interference Model

Figure A-1 (Appendix A) shows the geometrical relationships involved in the derivation of the probability model. It is assumed that the interfering satellite trajectory over the DSN station view area is constrained on the surface of a spherical sector with radius $(R_e + h)$ and swept out by a given antenna off-beam angle, α. R_e is the mean Earth radius, and h is the altitude of the orbiting satellite above ground. Furthermore, the uncertainty of the satellite's orbiting position is equivalent to the satellite entering the view area randomly with uniformly distributed probability.

The probability model is derived from both the random and periodic variables which interplay during the encounter between the DSN station antenna beam and the orbiting satellite. There exist two fundamental random variables: the receiving antenna pointing angle and the satellite's position relative to the antenna off-beam angle. The antenna pointing angle variable can be further resolved into the elevation and azimuth components. The azimuth angle is primarily a function of the Earth's diurnal rate motion, and has been assumed to be nonrandom. However, the elevation angle is considered to be random and to have some probability density, relative to the spacecraft trajectory being tracked. Deterministic and periodic variations are associated with the azimuth angle, the angle of inclination of the Earth's axis with respect to the ecliptic (23.44 deg), and the rapid changes in the elevation angle at the rise and set times. Thus, if averaged over long periods of time, the periodic variations can be neglected without affecting the overall derived probabilities.

The circular orbiting system is also assumed to be ergodic, by virtue of the satellite's constant angular velocity in a circular orbit. This property enables one to transform the satellite's time-averaged periods within the view area into probabilities; these probabilities can then be used to scale the derived probability distribution function. Similarly, DSN spacecraft tracking period statistics can be utilized as additional probability scaling factors.

III. Statistical Interference Model

The probability density of the satellite interference signal power P_r at the DSN antenna is defined as

$$P(p_r) = [P(p_t) * P(g_a) * P(-L)] \, P_o \, P_V \qquad (1)$$

where * denotes convolution and

$P(p_t) \triangleq$ probability density function of the satellite transmit power (dBm), where the satellite antenna is assumed to have isotropic gain

$P(g_a) \triangleq$ probability density function of the DSN tracking off-beam antenna gain (dBi)

$P(-L) \triangleq$ probability density function of free space loss (−dB)

$P_o \triangleq$ probability of DSN antenna tracking time over a 24-hour period

$P_V \triangleq$ ratio of the view area contained within the total satellite ground track area

$\quad = h/2R(\sin\xi)$

$h \triangleq$ satellite altitude, km

$R \triangleq R_e$ (Earth radius) $+h$, km

$\xi \triangleq$ satellite inclination angle, $0 \deg < \xi \le 90 \deg$

The probability of exceeding the satellite interference signal power at the DSN antenna can be defined as the integral of $P(p_r)$:

$$\mathcal{P}(p_r) = \int_{P_r}^{\infty} P(p_r) \, dp_r$$

The probabilities arise from the uncertainty of the satellite's orbital position and transmission parameters. $P(p_t)$ can be represented by a triangular probability density function defined from the transmitter power minimum, maximum, and most probable values; $P(g_a)$ is the off-beam antenna gain probability density which is derived in this analysis; and $P(-L)$ is the uncertainty associated with the free space loss and can be represented by a uniform probability density function, with space loss values corresponding to line-of-sight range between the antenna and the interfering satellite. The aggregate effect of the system uncertainties can then be determined from the convolution of these probability densities. Note that when the

satellite transmit power variability is negligible, the probability density function can be replaced with a Dirac Delta function, which sums the mean value of p_t with the random variable values of $P(g_a)$.

The probability density $P(g_a)$, is the derivative of the probability distribution $\mathcal{P}(g_a)$. $P(g_a)$ is transformed using the International Radio Consultative Committee (CCIR) antenna gain pattern Eq. (2); $P(\alpha)$ represents the probability of the satellite being inside the antenna beam cone, bounded by the off-axis angle α and is derived in Appendix A.

The CCIR generalized antenna gain pattern is given as[1]

$$
\left.
\begin{aligned}
g &= G_{\max}, \text{ dBi (maximum antenna gain)} \\
&\quad 0 \leq \alpha < 1 \text{ deg} \\[6pt]
g &= 32 - 25 \log(\alpha), \text{ dBi, } 1 \text{ deg} \leq \alpha < 48 \text{ deg} \\[6pt]
g &= -10 \text{ dBi, } 48 \text{ deg} \leq \alpha \leq 180 \text{ deg}
\end{aligned}
\right\} \quad (2)
$$

The probability distribution of α is

$$
\mathcal{P}(\alpha) = \int_0^{90} \mathcal{P}(\alpha \mid \epsilon) \, P(\epsilon) \, d\epsilon \tag{3}
$$

where $\mathcal{P}(\alpha \mid \epsilon)$ is the conditional probability distribution of α, given ϵ, which is the DSN station antenna elevation angle.

$P(\epsilon)$ is the probability density function of the DSN antenna elevation angle. If this probability density function is not known, it can be approximated with a triangular probability density function, similar to that shown in Appendix B.

Figure A-1 in Appendix A shows the geometrical relationships used to determine $\mathcal{P}(\alpha \mid \epsilon)$. It is required to know the probability of finding the satellite anywhere inside the antenna beam bounded by the off-beam angle and the portion of the antenna beam which is in view of the satellite. The antenna beam cuts the satellite's orbital sphere at r_1 and r_2, for any given elevation angle ϵ; this sphere contains orbital surfaces bounded by an ellipse projection whose eccentricity is a function of the antenna pointing angle. Surface asymmetry is also affected by the

---

[1] The equations apply to systems with antenna gains >48 dBi. Due to the very small probabilities associated with the angle α values between 0 deg and 1 deg, only values equal to or greater than 1 deg are considered in the model.

variable off-beam angle α (representative of the satellite probable position inside the antenna beam cone). For the ease of calculation, first-order ellipses are assumed.

Furthermore, the portion of the antenna beam that blocked by the ground and out of sight of the satellit is a major consideration. Instead of directly calculatin the antenna beam cone surface area blockage, the ratio c the projected beam blockage to the total projected area i calculated and used as a scaling factor.

IV. Interference Period Probability

Given that the interference occurs, it becomes useful t determine the interference period and its associated prob ability. The probability of t seconds of interference for given interference threshold level, corresponding to som fixed off-beam angle α_o, can be written as

$$
\mathcal{P}(t) = \int_0^{\epsilon_{mp}} \left(\frac{p(\epsilon)}{t_{\alpha_o}} \right) d\epsilon = \frac{\epsilon_{mp}}{\epsilon_{max}} \, t_{\alpha_o} \tag{4}
$$

where

$t_{\alpha_o} \triangleq$ longest interference period at α_o

$\epsilon_{mp} \triangleq$ most probable antenna elevation angle

$\epsilon_{max} \triangleq$ maximum antenna elevation angle

the period t is computed piecewise to also account for the Earth-blocked segment of the antenna beam. For the unblocked view area,

$$
t_{\alpha_o} = \frac{s}{V_s} = \cos^{-1} \left(1 + \frac{2r_1 r_2 \cos(2\alpha_o) - r_1^2 - r_2^2}{2R^2} \right)
$$

$$
\times \frac{R}{V_s}, \qquad \alpha_o < \epsilon \leq 90 \text{ deg} \tag{5}
$$

where s is the longest possible path length (km) traversed by the satellite over the antenna beam cone bounded by α_o, and V_s is the mean orbital velocity of the satellite (km/sec). For the blocked view area,

$$
t_{\alpha_o} = \frac{s}{V_s} = \cos^{-1} \left(1 + \frac{2r_1 \rho \cos(\epsilon + \alpha_o) - r_1^2 - \rho^2}{2R^2} \right)
$$

$$
\times \frac{R}{V_s}, \quad 0 \leq \epsilon \leq \alpha_o, \, \rho \triangleq \sqrt{R^2 - R_e^2} \tag{6}
$$

where the inverse cosine functions are in radians, and the space loss is averaged over the region bounded by a fixed antenna off-axis angle.

V. Application of Model

An example is now given of the model's application to determine the probability of interference when the satellite orbiting mission period and frequency band coincide with those of a DSN mission. Consider a circularly Earth-orbiting satellite and a DSN station with the following parameters:

Earth radius, R_e	6378 km
Satellite inclination angle, ξ	+75 deg
Satellite altitude above Earth, h	637.8 km
Satellite transmit power spectral density[2]	−26.9 dBm/Hz
Satellite orbital velocity	7.6 km/sec
Satellite/DSN station frequency band	8.0 GHz
DSN antenna diameter	70 m

P_o, the probability of tracking time over a 24-hour period, is assumed to be 1.0; P_V, the ratio of the view area, is calculated as 0.06. It is also assumed that the satellite transmitter is switched on throughout the satellite's Earth-orbiting mission period.

---

[2] Assume that satellite equivalent isotropic radiated power is at 1 W (0 dBW) and the transmit data bandwidth is 500 kHz.

By applying these specified parameters to the model, the derived $P(g_a)$ can be convolved with $P(p_t)$ and $P(-L)$ to yield the probability of exceeding a given power spectral density level. In this example $P(p_t)$ is treated as a constant and can be convolved as a Dirac Delta function. $P(-L)$ is derived in Appendix B.

Figures 1 and 2 show the probability of exceeding the total received power and power spectral density, respectively, for the above orbital and transmission parameters.

From Fig. 2, it can be noted that the probability of exceeding the CCIR recommended interference threshold level of −190 dBm/Hz is 4.5×10^{-6} or 0.00045 percent.

The interference periods and the probabilities of occurrence of those periods for the most probable antenna elevation angles are given in Table 1.

VI. Conclusion

It has been shown that when the instantaneous position of an Earth-orbiting satellite is unknown, one can determine the probability of interference to a DSN tracking station from the conditional probability of the satellite's signal being within the antenna beam cone, given any elevation angle. The analytical derivation of the interference probability function is made possible entirely from the known orbital geometrical properties and transmission parameters of the orbiting satellite.

The probability model provides an early first-order assessment of the potential interference to the DSN, and enables one to determine whether more comprehensive analysis will be necessary.

Table 1. Interference periods and probabilities.

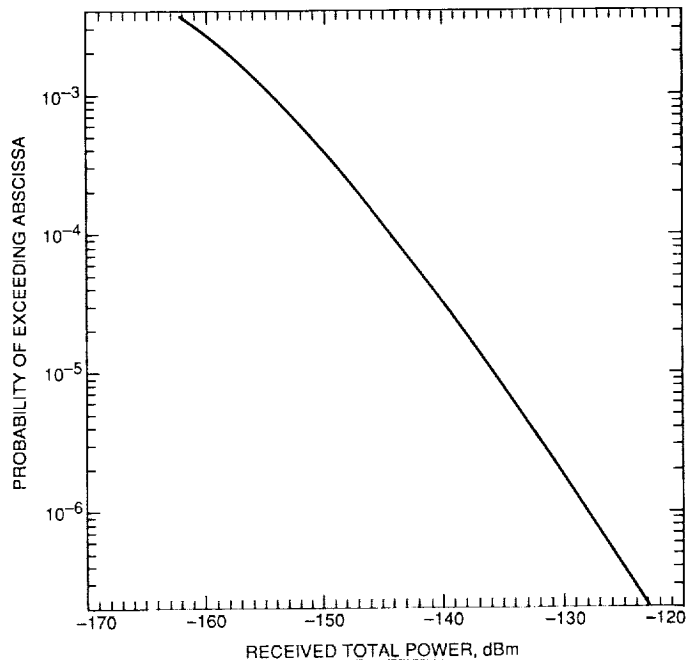| Most probable elevation angle, deg	Interference period, sec	Probability, percent
15	102.8	0.16
25	65.2	0.42
35	44.1	0.88
45	32.3	1.50
55	25.5	2.40
65	21.5	3.30
75	19.3	4.30

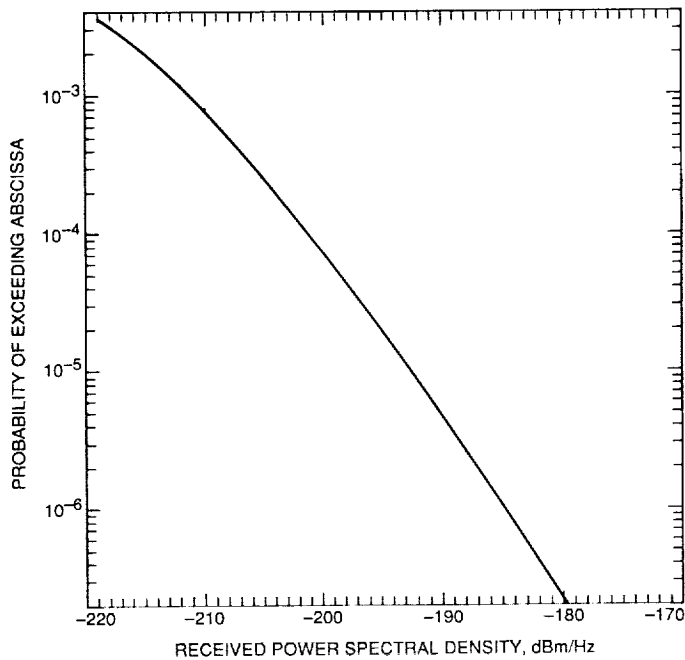Fig. 1. Probability of exceeding the total received power.

Fig. 2. Probability of exceeding the power spectral density.

Appendix A

Calculation of $P(g_a)$

The projected surface area is given by [1]

$$A = \frac{1}{2}\left[s'r' - c(r' - h')\right] \qquad \text{(A-1)}$$

where

$$s' = 2r'\cos^{-1}\left(\frac{r' - h'}{r'}\right) \qquad \text{(A-2)}$$

arc length of projected segment (not shown)

$$r' = r_1 \sin(\alpha) \qquad \text{(A-3)}$$

radius of projected segment

$$c = \left[4(2h'r' - h'^2)\right]^{1/2} \qquad \text{(A-4)}$$

chord of projected segment (not shown)

$$h' = \frac{r_1 \sin(\epsilon + \alpha)}{\cos(\epsilon)}$$

$$\text{for } 0 \le \epsilon \le \alpha,$$

$$1 \le \alpha \le 48 \text{ deg} \qquad \text{(A-5)}$$

Case 1. In this case, $\epsilon \le \alpha$ (occurrence of view area blockage).

The conditional probability distribution of the satellite being in the antenna beam cone bounded by the off-beam angle α for a given elevation angle ϵ can be approximated by expressions for the blocked and unblocked view area. For the blocked view area, the conditional probability distribution $\mathcal{P}(\alpha \mid \epsilon)$ is given as

$$\mathcal{P}(\alpha \mid \epsilon) \approx \frac{S(\alpha, \epsilon) A_p(\alpha, \epsilon) \mid \sec \delta \mid}{2\pi R^2 \left[1 - R_e/R\right] A_T}$$

$$\text{for } \epsilon \le \alpha,\ 0 \le \epsilon \le \alpha,$$

$$1 \le \alpha \le 48 \text{ deg} \qquad \text{(A-6)}$$

where $A_T = \pi a b$ is the total area of the ellipse projected from the orbital surface area bounded by the angle α; a and b are computed from Eqs. (A-12) and (A-13), respectively

$$R \triangleq R_e + h$$

and the visible projected area A_P is determined as

$$A_P = r_1^2 \sin^2(\alpha) \cos^{-1}\left[-\frac{\tan(\epsilon)}{\tan(\alpha)}\right]$$

$$- \left[2r_1^2 \frac{\sin(\alpha)\sin(\epsilon + \alpha)}{\cos(\epsilon)} - \frac{r_1^2 \sin^2(\epsilon + \alpha)}{\cos^2(\epsilon)}\right]^{1/2}$$

$$\times \frac{r_1}{2}\left[\sin(\alpha) - \frac{\sin(\epsilon + \alpha)}{\cos^2(\epsilon)}\right],$$

$$\text{for } 0 \le \epsilon \le \alpha,\ 1 \le \alpha \le 48 \text{ deg} \qquad \text{(A-7)}$$

Note: The inverse sine function is in radians.

$$\mid\sec(\delta)\mid = \left|\frac{2xy}{y^2 + x^2 - z^2}\right| \qquad \text{(A-8)}$$

where

$$x = \left(r_1^2 + r_2^2 - 2r_1 r_2 \cos 2\alpha\right)^{1/2}$$

$$y = \left[2r_1^2(1 - \cos 2\alpha)\right]^{1/2}$$

$$z = r_2 - r_1$$

and

$$r_1 = -R_e \sin(\epsilon + \alpha)$$
$$\quad + \left[R^2 - R_e^2 \cos^2(\epsilon + \alpha)\right]^{1/2} \qquad \text{(A-9)}$$

$$r_2 = -R_e \sin(\epsilon - \alpha)$$
$$\quad + \left(R^2 - R_e^2 \cos^2(\epsilon - \alpha)\right)^{1/2} \qquad \text{(A-10)}$$

The surface area covered by the orbiting satellite for a given α and ϵ is the surface area of the sphere (satellite orbital space) cut by the ellipse (antenna beam).

The surface integral $S(\alpha,\epsilon)$ is given as

$$S(\alpha,\epsilon) = 2\pi R^2$$

$$- 4R \int_0^{\pi/2} \left[R^2 - \frac{b^2(\alpha,\epsilon)}{M(\alpha,\epsilon)\sin^2\theta + 1} \right]^{1/2} d\theta$$

$$(A-11)$$

where

$$M = \frac{b^2 - a^2}{a^2}$$

and

$$a^2(\alpha,\epsilon) = \frac{r_1^2}{4}(\alpha,\epsilon) + \frac{r_2^2}{4}(\alpha,\epsilon)$$

$$- \frac{r_1}{2}(\alpha,\epsilon)\, r_2(\alpha,\epsilon)\cos 2\alpha \quad (A-12)$$

$$b^2(\alpha,\epsilon) = \frac{r_1^2}{2}(\alpha,\epsilon)(1 - \cos 2\alpha) \qquad (A-13)$$

By expanding $S(\alpha,\epsilon)$ as a power series [2],

$$S(\alpha,\epsilon) = 2R^2 \int_0^{\pi/2} \frac{b^2/R^2}{(M\sin^2\theta + 1)}\, d\theta$$

$$+ \frac{R^2}{2} \int_0^{\pi/2} \frac{(b^2/R^2)^2}{(M\sin^2\theta + 1)^2}\, d\theta$$

$$+ \frac{R^2}{4} \int_0^{\pi/2} \frac{(b^2/R^2)^3}{(M\sin^2\theta + 1)^3}\, d\theta + \cdots \quad (A-14)$$

Due to the fast convergence of the series, two terms will adequately approximate $S(\alpha,\epsilon)$ and can be further simplified as

$$S(\alpha,\epsilon) \approx \frac{\pi b^2}{\sqrt{1+M}} \left\{ 1 + \left(\frac{b^2}{R^2}\right) \frac{2+M}{8(1+M)} \right\}$$

$$(A-15)$$

Case 2. In this case, $\epsilon > \alpha$.

For the unblocked view area, the conditional probability distribution is given as

$$\mathcal{P}(\alpha \mid \epsilon) = \frac{S(\alpha,\epsilon)}{2\pi R^2 \left(1 - \dfrac{R_e}{R}\right)}, \text{ for } \epsilon > \alpha,$$

$$\alpha < \epsilon \leq 90 \deg, 1 \leq \alpha \leq 48 \deg$$
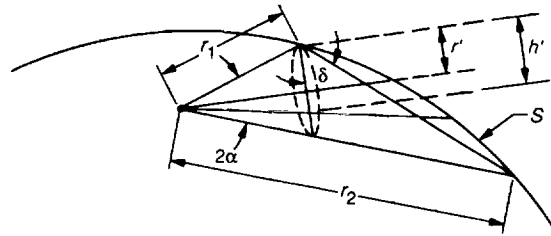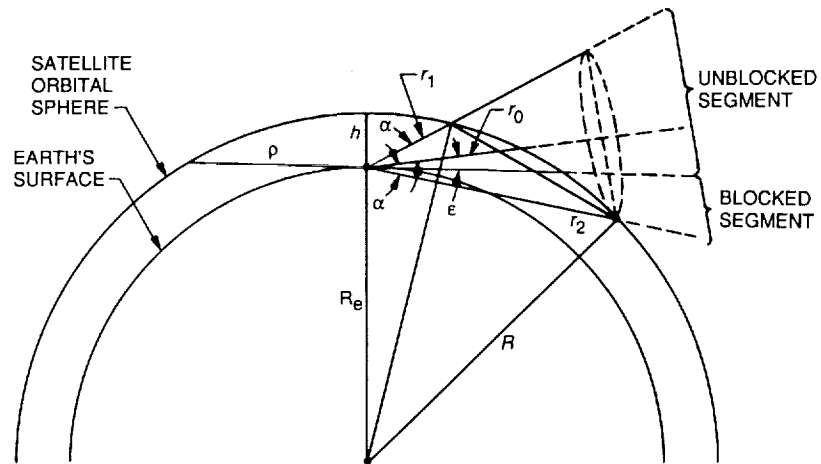
$$(A-16)$$

where $S(\alpha,\epsilon)$ is given in Eq. (A-15). Then, from Eq. (3),

$$\mathcal{P}(\alpha) = \int_0^{90} \mathcal{P}(\alpha \mid \epsilon)\, p(\epsilon)\, d\epsilon$$

the probability density $P(\alpha)$ is obtained by differentiating $\mathcal{P}(\alpha)$; hence,

$$P(\alpha) = \frac{d}{d\alpha}[\mathcal{P}(\alpha)] \qquad (A-17)$$

and $P(\alpha)$ is tranformed into $P(g_a)$ by using Eq. (2).

r_1, r_2 ≜ INTERFERENCE CONE BOUNDS	R_e ≜ RADIUS OF EARTH
r_0 ≜ ANTENNA BORESIGHT	α ≜ ANTENNA OFF-BEAM ANGLE
	ϵ ≜ ANTENNA ELEVATION ANGLE
ρ ≜ SATELLITE ALTITUDE ABOVE GROUND (MAXIMUM DISTANCE)	S ≜ SURFACE AREA BOUNDED BY α
h ≜ SATELLITE ALTITUDE ABOVE GROUND (MINIMUM DISTANCE)	δ ≜ PROJECTION ANGLE

Fig. A-1. Orbital geometry of an Earth-orbiting satellite over a DSN station.

Appendix B

Consideration of *P(p_t)* and *P(L)*

In the cases where the uncertainty of the satellite transmit power becomes a significant part of the analysis, a triangular probability density function can be used. The satellite transmit power probability density can be represented as [3]

$$P(p_t) = \begin{cases} \dfrac{2(p_t - a)}{(b - a)(c - a)} & \text{if } a \le p_t \le c \\[3mm] \dfrac{2(b - p_t)}{(b - a)(b - c)} & \text{if } c < p_t \le b \end{cases}$$

where

a $\stackrel{\Delta}{=}$ minimum power

b $\stackrel{\Delta}{=}$ maximum power

c $\stackrel{\Delta}{=}$ most probable power

The variability of free space loss due to the continuous changes in the satellite's range to the DSN tracking antenna can introduce an additional uncertainty in the received interference power. This uncertainty becomes very significant for lower Earth-orbiting satellites where fluctuations in space loss can be several orders of magnitude.

The probability density function of the free space loss uncertainty *P(L)* can be modeled as a uniform probability density:

$$P(L) = \begin{cases} \dfrac{1}{b - a} & \text{if } a \le L \le b \\[3mm] 0 & \text{otherwise} \end{cases}$$

where

$a \stackrel{\Delta}{=}$ minimum loss $= 20 \log_{10}(4\pi/\lambda) + 20 \log_{10} h, \text{dB}$

$b \stackrel{\Delta}{=}$ maximum loss $= 20 \log_{10}(4\pi/\lambda) + 10 \log_{10}(R^2 - R_e^2), \text{dB}$

$\lambda \stackrel{\Delta}{=}$ wavelength of interference signal, meters

$h \stackrel{\Delta}{=}$ satellite altitude above ground, meters

$R_e \stackrel{\Delta}{=}$ Earth's radius, meters

$R \stackrel{\Delta}{=} R_e + h$, meters

References

[1] C. E. Jordan, *Reference Data for Engineers: Radio Electronics, Computer, and Communications*, 7th ed., 3rd printing, Indianapolis, Indiana: Howard W. Sams and Co., 1986.

[2] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, New York: Academic Press, 1980.

[3] A. M. Law and D. Kelton, *Simulation Modeling and Analysis*, New York: McGraw-Hill, 1982.