

## NEAR-WALL RECONSTRUCTION OF HIGHER ORDER MOMENTS AND LENGTH SCALES USING THE POD

*M. N. Glauser*

Department of Mechanical and Aeronautical Engineering  
Clarkson University  
Potsdam, NY 13699, USA

### ABSTRACT

An analysis of the near-wall behavior of the proper orthogonal decomposition (POD) eigenfunctions<sup>1</sup> derived from direct numerical simulation (DNS) of channel flow<sup>2</sup> is performed. Consistent with previous studies, a low order multi-mode reconstruction of the kinetic energy and Reynolds shear stress suffices. A similar reconstruction of the isotropic dissipation rate is shown to be insufficient, however. An analysis is performed of the multi-mode composition of the dissipation rate in the near-wall region, and it is shown that a significant number of higher-order modes are required to achieve the correct asymptotic consistency in the near-wall region. In an attempt to avoid this problem, a length scale definition is proposed in terms of an integration of the correlation tensor which factors in the presence of the wall. The wall is accounted for by only integrating out to  $2y^+$  and not over the entire domain. Viscous and inviscid estimates for the dissipation were used in the near-wall and core regions respectively, in conjunction with this length scale representation to obtain an estimate of the dissipation throughout the domain. The resulting dissipation exhibits the proper behavior near the wall and in the inertial layer. A 1 POD mode estimate of the length scale is computed and found to agree quite well with the length scale obtained when the entire correlation tensor is used.

Figure 1 shows the near-wall asymptotic consistency of the POD reconstruction of the kinetic energy and shear stress. As can be seen, even a first mode reconstruction has the correct functional behavior in both cases. Higher order modes contribute to the correct amplitude level so that with the seven mode reconstruction the correct near-wall asymptotic behavior results.

Figure 2 shows the reconstruction of the turbulent dissipation rate ( $\epsilon^+ = \overline{\left[\frac{\partial u_i}{\partial x_j}\right]^2}$ ). The figure shows only the contribution from the terms which are derivatives of distance from the wall. The 1D POD reconstruction can only handle these terms directly while the remaining terms can must be obtained using symmetry assumptions. It suffices here to present only the contribution that can be obtained directly. As the figure shows several modes are needed to accurately reproduce the near-wall behavior from the DNS results.

Figure 3 shows the length scale  $L_{ii}$  computed 2 different ways; the dashed line represents the scale obtained by accounting for the wall and the solid line that obtained by integrating out over the entire domain. Note how the latter definition blows up at the origin which demonstrates the need to factor in the wall. Figure 4 shows the a calculation of the dissipation from the length scale which factors in the wall and the kinetic energy, compared to the dissipation computed directly from the DNS results. Note the proper behavior for  $y^+ < 5$  and  $y^+ > 30$  and the discrepancy for  $5 < y^+ < 30$ . It is possible to argue that this discrepancy is due to an inappropriate definition for the dissipation (in terms of the kinetic energy and length scale) in this buffer layer and *not* the length scale. A formal matching procedure may be a useful approach for this region. Figure 5 shows a 1 POD mode estimate of the length scale (the dashed line) compared to the length

scale computed earlier using the full tensor in the near-wall region. The comparison is remarkable when compared to the number of modes required to obtain the dissipation as shown in Figure 5.

The results presented confirm the ability of the POD to properly reconstruct the second moments in the near-wall region of a turbulent flow. It appears that the present one-dimensional reconstruction is insufficient to properly account for the near-wall treatment of the turbulent dissipation rate. In an attempt to avoid this problem, a length scale definition has been proposed in terms of an integration of the correlation tensor which factors in the presence of the wall. The dissipation computed from this length scale and the kinetic energy exhibits the proper behavior near the wall and in the inertial layer. A 1 POD mode estimate of the length scale is computed and found to agree quite well with the length scale obtained when the entire correlation tensor is used.

The question that naturally arises at this point is whether the extra processing of simulation data is necessary since the various turbulent stresses and budgets can be directly obtained from simulation data. The answer lies in the relatively low Reynolds number range of the simulations to date. It has recently been shown by Sirovich and Rodriguez<sup>3</sup> for the Ginzburg-Landau equation that the lower-order eigenfunctions are somewhat Reynolds number independent. If this were to hold for the Navier-Stokes equations, then the model development, based on the eigenfunction reconstruction would have a wider range of applicability.

## References

1. Lumley, J.L., 1967 "The structure of inhomogeneous turbulent flows," *Atmospheric Turbulence and Radio Wave Propagation*, (ed. A.M. Yaglom and V.I. Tatarski), Moscow: Nauka, pp. 166-178.
2. Dinavahi, S. and Zang, T.A., 1992 "Reynolds stress budgets in a transitional channel flow," *To be submitted for publication*.
3. Sirovich, L. and Rodriguez, J.D., 1987 "Coherent structures and chaos: a model problem," *Physics Letters A*, vol. 120, pp. 211-214.

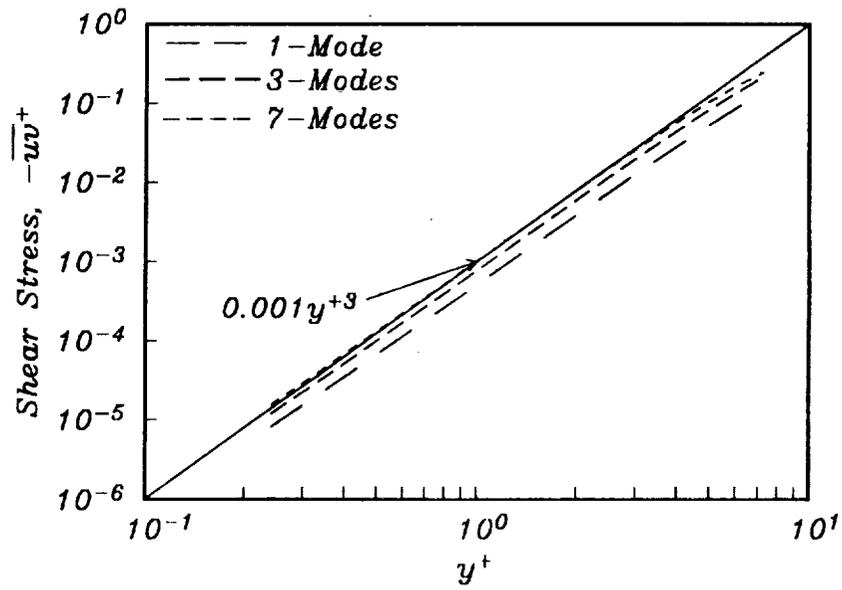
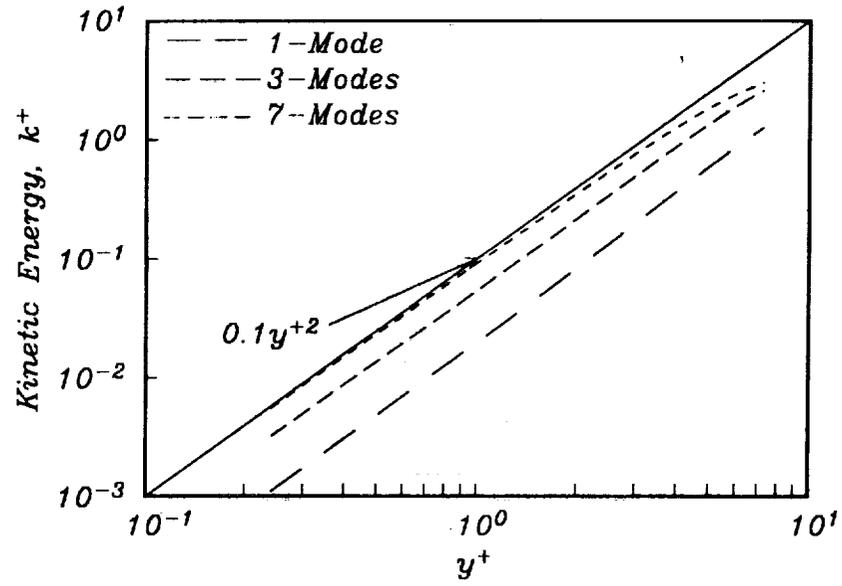


Figure 1: Asymptotic consistency of kinetic energy and shear stress with  $y^+$  in near-wall region.

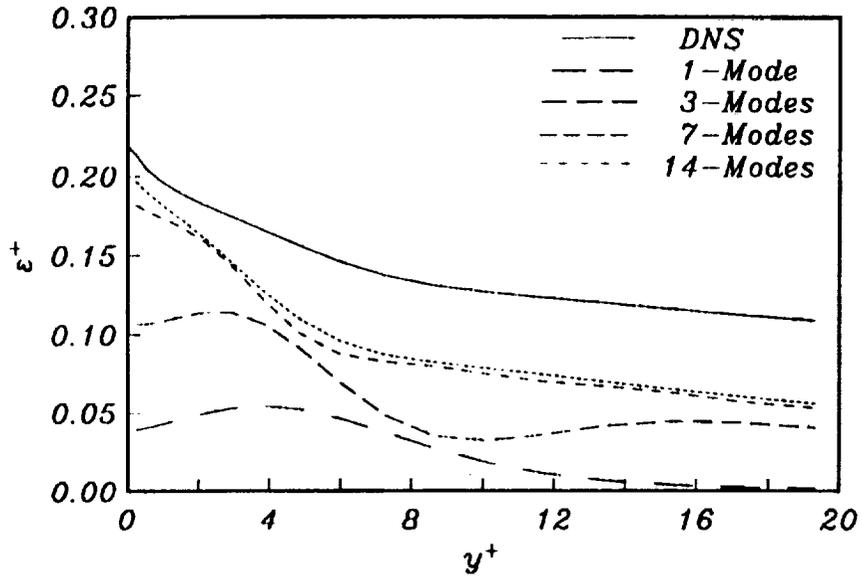


Figure 2: Effect of mode addition on  $\epsilon^+$  in close proximity to wall.

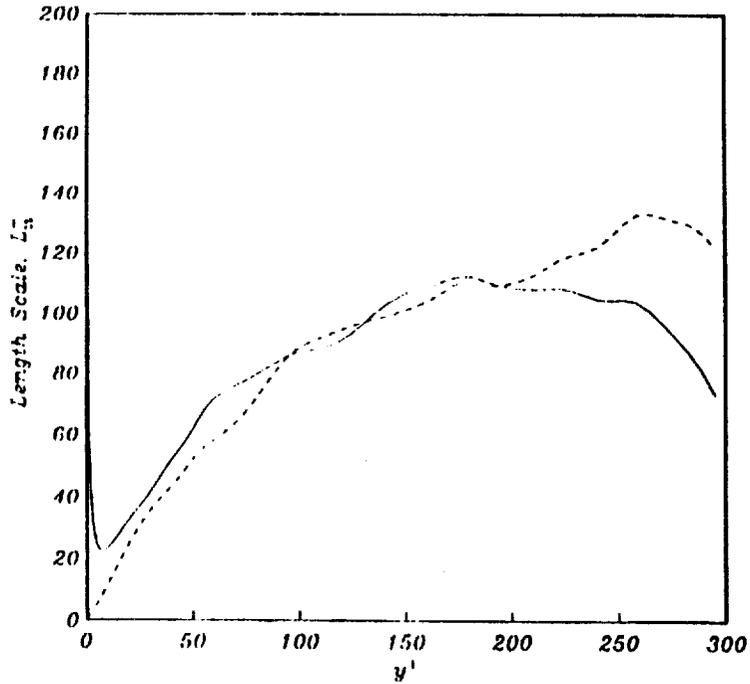


Figure 3: Length Scales computed with and without accounting for the wall

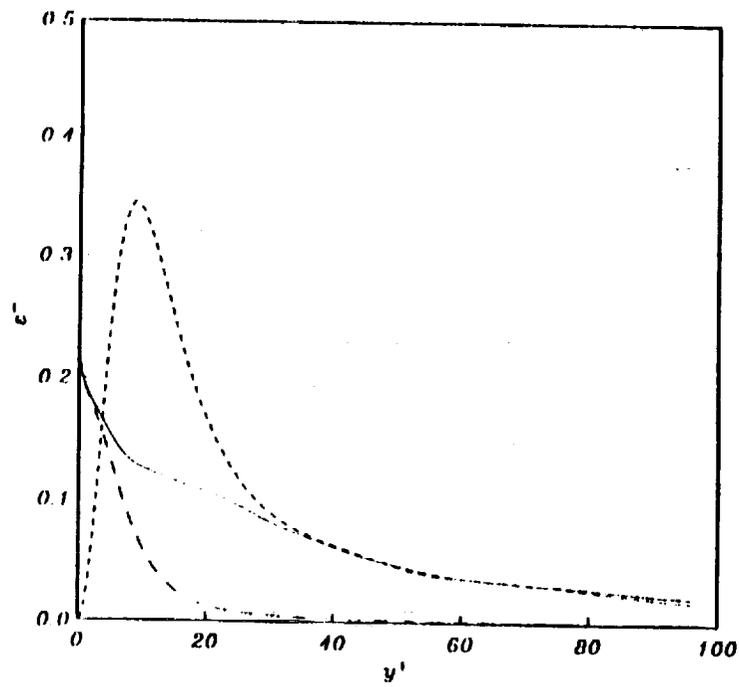


Figure 4: Dissipation computed from length scale compared to DNS.

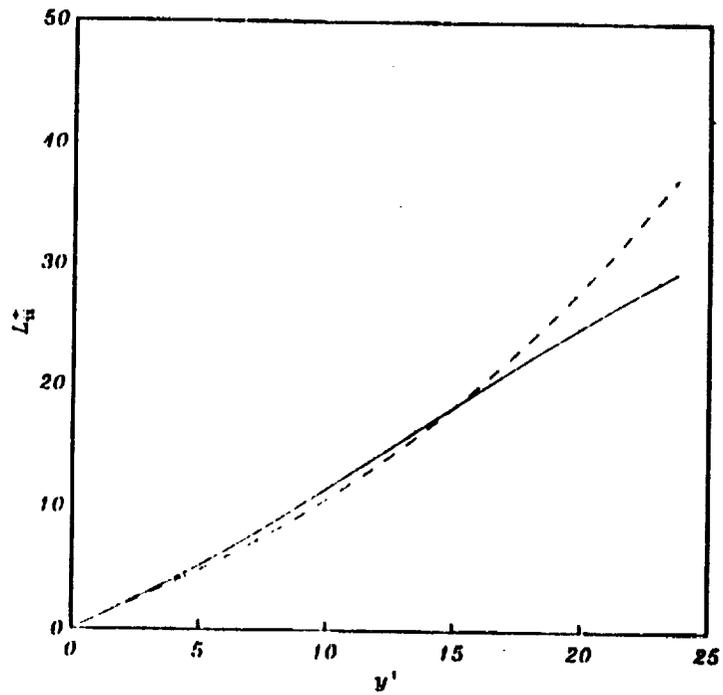


Figure 5: 1st POD estimate of length scale compared to that from the full tensor.