

Mode Interaction in Stiffened Composite Shells under Combined Mechanical and Thermal Loadings

by

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Abstract

Stiffened Shells of various configurations, fabricated out of composite materials find extensive applications in aircraft structures. Two distinctive modes of buckling dominate structural response of stiffened panels, viz. the short-wave local mode in which the shell skin buckles essentially between the stiffeners and the and the long-wave overall mode in which the shell skin buckles carrying the stiffeners with it. In optimized designs, the critical stresses corresponding to these modes of buckling would be close to each other. This leads to a nonlinear mode interaction which is recognized to be the principal cause of the failure of stiffened structures. If the structure is subjected to through-the-thickness thermal gradients, then large-wave bending effects would begin to occur well below the overall critical load and these would play the role of overall imperfections. The load carrying capacity would be significantly diminished as a result of interaction of local buckling with overall thermal distortions. The analysis of this problem using standard finite element techniques can be shown to be prohibitively expensive for design iterations.

In this research a novel concept which would greatly facilitate the analysis of mode interaction is advanced. We note that the local buckling occurs in a more or less periodic pattern in a structure having regular spacings of stiffeners. Thus it is a relatively simple matter to analyze the local buckling and the second order effects (which are essential for modelling postbuckling phenomena) using a unit cell of the structure. Once analyzed, these deformations are embedded in a shell element. Thus a shell element could span several half-waves of local buckling and still be able to depict local buckling effects with requisite accuracy. A major consequence of the interaction of overall buckling/bending is the slow variation of the local buckling amplitude across the structure - the phenomenon of "amplitude modulation" - and this is accounted for in the present model by letting the scaling parameter of the local mode vary according to a "slowly varying" function.

The construction of the analytical model involves essentially two stages and these are described briefly in the sequel.

Stage I: In this stage, the local buckling problem is first solved, thus obtaining the buckling mode and the associated second order field. This problem is solved "locally" for the given state of stress without reference to the boundary conditions of the structure and taking advantage of symmetry/periodicity of the arrangement of the stiffeners. For example, in the case of stringer stiffened shells, a finite strip technique with the displacements characterized by appropriate trigonometric functions is employed. The second order field is produced using an asymptotic procedure.

Stage II : The displacement at any point can be viewed as the sum of local and overall contributions. Thus :

$$\{u\} = \{u_{ov}\} + \xi_i f_i \{u_1\} + \xi_j \xi_j f_j \{u_2\}$$

where u_{ov} , u_1 and u_2 stand respectively for the overall displacement field, the local buckling mode, and the associated second order field; ξ_i and f_i are respectively the degrees of freedom (d.o.f.) and functions representing the amplitude modulation. A finite element is constructed to describe the overall displacement variations and the amplitude modulation in terms of appropriate number of d.o.f. For a stringer-stiffened shell, the element would span from stiffener to stiffener and cover several half-waves of buckle in the longitudinal direction. Drastic simplifications arise in setting up the tangent stiffness matrix and the unbalanced load vector at any stage in the loading history, by the recognition of the "slowly varying" nature of the overall displacement and amplitude modulating functions in contrast to the local buckling functions.

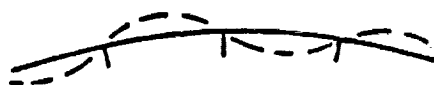
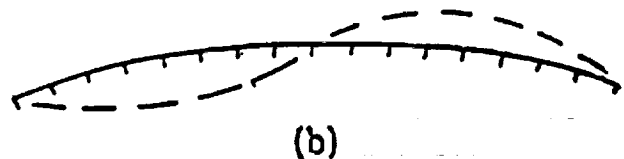
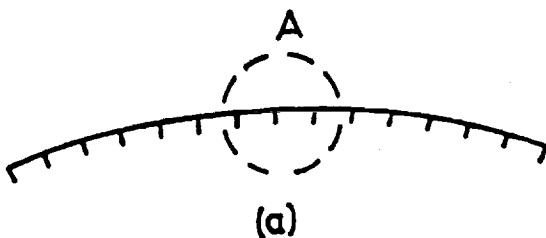
Current status :

Stage I, which comprises of local and postlocal buckling analysis is complete. Stringer-stiffened shells were considered, with appropriate trigonometric functions characterizing the displacement variations in the longitudinal direction and p-version type polynomials in the transverse direction. The advantages of p-version finite strips are the absence of shear-locking and the rapid rate of convergence of the solutions.

Work on Stage II is currently under way. p-version finite elements have been employed to perform full range nonlinear analysis. An arc-length scheme has been implemented to negotiate the limit points. Several bench mark problems were studied with the object of assessing the performance of the elements in highly nonlinear situations. Local buckling displacement fields were embedded in the elements in terms of the scaling factor of the buckling mode, which is given freedom to vary spatially. Preliminary results on panels having blade-stiffeners are found to closely agree with the established results in the field.

Future Work :

The next phase of the study will focus on modal interaction in the context of combined mechanical and thermal loading of composite stiffened plate and shell structures.



Cross-section of a Stiffened Shell and the Modes of Buckling :
 (a) The local mode (b) The overall mode

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