# Practical Aspects of Transfer from GTO to Lunar Orbit <br> by <br> Chauncey Uphoff <br> Ball Space Systems Division <br> Boulder, Colorado 


#### Abstract

: This paper is a presentation of some practical aspects of orbital transfer from Geosynchronous Transfer Orbit (GTO) to close, near-circular orbits of the Moon. The intent is to identify the important parameters affecting the problem and to bound (approximately) the range of required $\Delta \mathrm{V}$ for a spacecraft that has been placed in GTO. The basic geometric relationships are described and the dynamics are simulated by use of the Zero-Sphere-of Influence Patched Conic method. It is found that the inclination of the transfer orbit to the Earth-Moon plane is relatively unimportant while the position of the line of apsides with respect to the Moon's orbit is the main geometric parameter of interest. It is shown that this parameter can be controlled by selecting the time of day for launch and that two launch windows of approximately 45 minutes duration are available each day of the year if use is made of the recommended phasing orbit transfer. The phasing orbit transfer not only provides twice-daily launch windows, but also provides a mechanism for efficacious correction of GTO injection errors. $\Delta \mathrm{V}$ penalties for out-of-plane transfer and for late launch are evaluated and the method is recommended for use as an affordable means of achieving lunar orbit.


## Introduction:

It is not generally recognized that daily launch windows are available for launch to GTO that are compatible with reasonably efficient transfer from GTO to lunar orbit. This study described in this paper (Reference 1) was undertaken for a private company that has compelling reasons for minimizing the funding requirements for the launch vehicle. The study revealed that transfer from GTO to lunar orbit is not only viable but that it may be the most affordable means of such transfer because of the relatively high traffic to GTO. The recent renewal of interest in lunar exploration suggests the need for a wider distribution of the study results.

It is pointed out that the correct relationship between the transfer orbit line-of-apsides and the EarthMoon plane can be established by waiting in GTO until the Earth's oblateness rotates the orbit into position. For some initial orientations, this wait is not practical as the rotation proceeds at only about 0.8 degrees per day. If , however, the daily launch windows are chosen as suggested in this paper, it is possible to define realistic GTO waiting periods ( 10 to 20 days) that permit near minimal energy transfer from GTO to lunar orbit that extend the twice-daily launch windows.


One of the concerns for the viability of this mode of transfer is the possibility of radiation damage to the spacecraft during repeated passages through the Earth's radiation belts as the orbit rotates into position for final insertion into the lunar transfer orbit. This effect may be one of the major tradeoffs in mission design for some missions and it is suggested that it is important to weigh the requirements for radiation shielding against those for propulsive adjustment of the perigee position to account for launch time variations.

It is recommended that the propulsion system be a restartable bipropellant hydrazine/NTO system (or the equivalent) so that the transfer impulses can be applied at various points on the GTO and phasing orbits. Lunar orbit insertion is accomplished using about $850 \mathrm{~m} / \mathrm{s}$ of $\Delta V$ to yield a near circular orbit 100 km above the lunar surface with any inclination in the range $30^{\circ}<\mathrm{i}<150^{\circ}$ to the lunar equator.

It is suggested that orbit sustenance requirements will be at least $100 \mathrm{~m} / \mathrm{s}$ per year due to the uncertainty in our knowledge of the lunar gravity field for close, high inclination orbiters. The judicious use of this impulse to adjust the eccentricity and argument of perilune may yield a "frozen" polar lunar orbit that will not only provide stability but will aid in discrimination between the various models of the lunar gravity.

## Geometry of the Earth-Moon System:

Figure 1 is a diagram of the geometry of the Earth-Moon system showing the pertinent relationships between the various orbits required for transfer from GTO to lunar orbit. The figure is an edge-on view of the Earth-Moon system and shows the (dark) GTO, the lighter shaded geosynchronous orbit (GSO), the lightest Earth-Moon plane, and the unshaded phasing and lunar transfer orbits. The GTO is assumed to be inclined to the Equator (shown by the GSO) by about $7^{\circ}$ which is compatible with a typical launch on the Ariane launch vehicle. It is assumed that the perigee of the GTO is within about $5^{\circ}$ of the Earth-Moon plane so that, with a reasonable waiting period ( 10 to 20 days), the orbit can be allowed to precess into a favorable alignment for initiation of the phasing orbit and then the lunar transfer orbit. The figure shows the line of apsides of the phasing and translunar orbits along the line of intersection of the equator and Earth-Moon plane. It is not necessary to have this alignment but it is most probable because the perigee of the GTO will be near the equator. In case it is required to wait while the GTO precesses until its line of apsides is near the Earth-Moon plane, the orientation could be quite different from that shown in the figure. What is important is that the transfer from GTO to phasing orbit be done when their common perigee is near the Earth-Moon plane.


Fig. 1 Geometry of GTO to Lunar Orbit Transfer
The Moon's orbit plane is inclined about $5.14^{\circ}$ to the ecliptic and, because of the solar gravitational perturbations, its node (on the ecliptic) regresses about $20^{\circ}$ per year. This regression causes a change in the inclination, $\eta^{\prime}$, of the Earth-Moon plane to the Earth's equator. This inclination varies between about $18.5^{\circ}$ and $28.6^{\circ}$ with a period of about 18 years. In the early $1990^{\prime}$ s, the inclination $\eta^{\prime}$ is about midway between these extremes at about $24^{\circ}$. Thus, a GTO orbit with an inclination of $7^{\circ}$ to Earth's equator will have an inclination, $\eta$, with respect to the Earth-Moon plane of no less than about $17^{\circ}$ and no more than about $31^{\circ}$. It is recommended that this inclination be a free variable in preliminary analyses because its effect on the total $\Delta V$ required to achieve lunar orbit is less than $75 \mathrm{~m} / \mathrm{s}$. If the transfer orbits and vehicle sizing exercises use the larger value of $31^{\circ}$, then launch may be accomplished on any day of the year with varying degrees of payload margin.

## The Phasing Orbit:

Use of an intermediate phasing orbit is recommended so as to provide the capability for launching any day of the month and to eliminate the need for a plane change maneuver. It also provides the capability to make midcourse corrections at near-optimal positions. For the "nominal" case, it is assumed that transfer from GTO to the phasing orbit is made at perigee (of both orbits) and at a time when perigee is in the Earth-Moon plane. The period of the phasing orbit is chosen so that, after an integral number of revolutions in that orbit, a second impulse is applied at perigee to place the spacecraft on the lunar transfer orbit (LTO) at a time which will insure intercept with the Moon when the spacecraft reaches the Moon's orbit. It should be noted that the impulse to go from GTO to the
phasing orbit is not "wasted", it is an impulse that would have to be added in any case to effect the complete transfer from GTO to the LTO. Then, after the proper timing has been established by selection of the period of the phasing orbit, the remainder or the GTO to LTO impulse is applied at perigee of the phasing orbit and the lunar encounter is assured.


Fig. 2 Two-Impulse Lunar Transfer
Fig. 2 is a diagram of the GTO, the phasing orbit and the LTO. The impulse $\Delta V_{1}$ takes the spacecraft from GTO to the phasing orbit and $\Delta V_{2}$ provides the transfer from the phasing orbit to LTO. Later studies will probably reveal an optimal split between these two impulses but, for these studies, it is sufficient to require only that their sum equal the $\Delta V$ required to go from GTO to LTO. Assuming that GTO is a typical Ariane-launched orbit, ( $200 \mathrm{~km} \times 35975 \mathrm{~km}$ with an inclination of $7^{\circ}$ and an argument of perigee of $178^{\circ}$ wrt the equator) and that the LTO has a semi-major axis of $198,000 \mathrm{~km}\left(\mathrm{C}_{3}=-2.013\right.$ $\mathrm{km}^{2} / \mathrm{s}^{2}$ ), we obtain for the total of the two impulses the difference between the perigee speeds of the initial and final orbits, that is,

$$
\begin{aligned}
\Delta \mathrm{V}_{1}+\Delta \mathrm{V}_{2} & =\mathrm{V}_{\mathrm{P}_{\mathrm{LIO}}}-\mathrm{V}_{\mathrm{p}_{\mathrm{GTO}}} \\
& =\sqrt{\mu}\left\{\sqrt{\frac{2}{\mathrm{r}_{\mathrm{p}}}-\frac{1}{\mathrm{a}_{\mathrm{LTO}}}}-\sqrt{\frac{2}{\mathrm{r}_{\mathrm{p}}}-\frac{1}{\mathrm{a}_{\mathrm{GTO}}}}\right\}
\end{aligned}
$$

where $r_{p}$ represents the common perigee radius of the two orbits, agTO and aLTO represent their semimajor axes, and $\mu$ is the gravity constant (GM) of the Earth ( $=398600.5 \mathrm{~km}^{3} / \mathrm{s}^{2}$ ). Using the typical values quoted above and taking the equatorial radius of the Earth as 6378.14 km , we obtain,

$$
\Delta V_{1}+\Delta V_{2}=0.675 \mathrm{~km} / \mathrm{s},
$$

to be applied so as to ensure lunar encounter near apogee of the LTO.

## The Zero-Patched Conic Method:

One of the most powerful and simple methods for estimating the energy during Moon passage is the zero sphere of influence or point to point patched conic method. It can be shown that this method conserves the Jacobian integral in the restricted three-body problem and is capable of predicting the Moonrelative energy at perilune to within terms of the order of the Moon/Earth mass ratio and the ratio of the passage distance to the Moon's distance from the Earth. For preliminary analysis of an orbiter mission, it yields the excess speed of the lunar encounter hyperbola to within a few percent and permits estimation of the orbit insertion requirements to the same level of accuracy.

The method is basically the same as that used by Professor Rutherford in his famous analysis of the nuclear scattering problem except, in the orbital case, there is no appreciable recoil and, therefore, no need to transform to "laboratory" coordinates. The essence of the method is to determine a Keplerian ellipse that goes from the launch point to the center of the Moon in some desired transfer time (Lambert's problem). The Keplerian velocity at encounter is transformed to a Moon-centered frame and that transformed velocity is taken to be the hyperbolic excess velocity of the Moon-passage trajectory. This method cannot yield any information about the lunar passage distance and the transfer times will be in error by several hours but the Moon-relative energy is surprisingly accurate. This technique is used for most preliminary lunar and interplanetary transfer studies.

Let $V_{M}$ represent the velocity vector of the Moon at the time of encounter (when the spacecraft is assumed to have the same position as the center of the Moon) and let $\mathbf{V}_{\mathrm{A}}$ represent the Earth-relative velocity vector on the Keplerian transfer orbit at encounter. The Moon-relative excess velocity, $\mathbf{V}_{\infty}$, is simply

$$
\mathbf{V}_{\infty}=\mathbf{V}_{\mathrm{A}}-\mathbf{V}_{\mathrm{M}},
$$

where the bold quantities represent vectors. In the nomenclature of spherical astronomy, if $\eta$ represents the inclination of the Keplerian transfer orbit to the Earth-Moon plane, and $\gamma$ is the elevation path angle of the Keplerian velocity vector at encounter (see Fig. 1), then

$$
V_{\infty}{ }^{2}=\left|V_{\infty}\right|^{2}=V_{M}^{2}+V_{A}^{2}-2 V_{M} V_{A} \cos \eta \cos \gamma .
$$

The excess speed is related to the selenocentric energy (the vis-viva energy of the spacecraft with respect to the Moon) as

$$
E_{M}=V_{\infty}^{2} / 2=v^{2} / 2-\mu_{M} / r,
$$

where $\mu_{M}$ is the gravitational constant (GM) of the Moon ( $=4903.2 \mathrm{~km}^{3} / \mathrm{s}^{2}$ ) and v and r represent the speed and radial distance of the spacecraft with respect to the Moon during the encounter.

Now, with an accurate estimate of the Moon-relative energy, it is a simple matter to determine the orbit insertion requirements. Assume that the lunar transfer orbit has been targeted for a closest approach distance, $\mathrm{r}_{\mathrm{p}}$, of 1838 km ( 100 km above the lunar surface). This targeting is achieved by selection of launch time and does not appreciably affect the value of $\mathrm{V}_{\infty}$ given above. The Moonrelative speed at closest approach, then, is

$$
v_{p}=\left[V_{\infty}^{2}+2 \mu_{M} / r_{p}\right]^{1 / 2},
$$

and the (circular) orbit insertion impulse required is

$$
\Delta V_{C}=v_{p}-v_{C}=\left(V_{\infty}^{2}+2 \mu_{M} / r_{p}\right)^{1 / 2}-\left(\mu_{M} / r_{p}\right)^{1 / 2}
$$

For a typical value of $V_{\infty}$ of $0.85 \mathrm{~km} / \mathrm{s}$, we obtain

$$
\Delta V_{C}=\left\{(0.85)^{2}+2 \cdot(4903) / 1838\right\}^{1 / 2}-\{4903 / 1838\}^{1 / 2}=0.828 \mathrm{~km} / \mathrm{s} .
$$

Fig. 3 shows the orbit insertion $\Delta \mathrm{V}$ as a function of $\mathrm{V}_{\infty}$ throughout the range of values to be expected for a GTO to lunar orbit mission.


Fig. 3 Circular Orbit Insertion $\Delta V$ vs Hyperbolic Excess Speed

## Sensitivity of Requirements to Launch Window:

It is important in studies of this type to identify the sensitivity of the performance to various parameters that may have to change from the nominal situation during an actual mission. Perhaps the most important of these is the time of launch. It is recommended that the launch vehicle contractor be asked to provide a launch to GTO at a time of day that will place the perigee of the orbit within $5^{\circ}$ of the Earth-Moon plane anytime within 15 days after launch. If there were no other constraints, this could be accomplished twice per day by launching just before the GTO injection point crosses the EarthMoon plane. But for a shared launch, there will be constraints on launch windows as a result of requirements placed by the primary payload such as solar aspect angles at injection or apogee passage. Primary payload constraints may eliminate one or both of the lunar transfer options during certain times of the year and should be the subject of more careful consideration than given here. Because of this kind of interaction between primary and secondary payload constraints, it is of interest to evaluate the penalties associated with launch at non-optimal times and to consider practical (near-optimal) strategies for correcting the effects of variations in the initial argument of perigee with respect to the Earth-Moon plane.

Fig. 4 shows the positions of the relevant planes as they will be in Jan of 1993. The best time to launch is just before the GTO injection point ( which can be assumed to rotate with the Earth) reaches the Earth-Moon plane. This can be accomplished twice per day (see Reference 2), once near the ascending node of the Earth-Moon plane on the equator or just prior to crossing the descending node about 12 hours later than the situation shown in Fig. 4. During the 1990's, the ascending node of the Earth-Moon plane never gets far from the vernal equinox as it oscillates from about $-13^{\circ}$ to $+13^{\circ}$ in Right Ascension. In the early 1990's the ascending node of the Earth-Moon plane on the equator regresses from about $-7^{\circ}$ relative to the vernal equinox (in Jan 1990) to its minimum at about $-13^{\circ}$ (in Jan 1993). The node then advances from $-13^{\circ}$ to $-7^{\circ}$ during the next three years and will not cross into positive longitudes until near the turn of the century. Thus the optimum injection point in Jan 1993 would have a celestial longitude of $-15^{\circ}$ (that is 15 degrees West of the vernal equinox) which would allow for about $2^{\circ}$ of perigee advance or about 2.5 days of waiting time until the oblateness rotates the perigee into the Earth-Moon plane at which time the spacecraft could be injected into the phasing orbit.


Fig. 4 Geometry of the Earth-Moon System in Jan 1993

But launch windows are not instantaneous and should be about 45 minutes wide to account for weather and other types of delays and still provide a reasonable probability of launch. Notice that the text of Fig. 4 calls for a $15^{\circ}$ window. This corresponds to a one hour launch window ending with an injection exactly at the Earth-Moon plane. However, in the example above, the launch window should open about 45 minutes before the example time or about 11.25 degrees of Earth rotation earlier than the optimum which is to say that the launch window should open at a time which yields a GTO injection point whose celestial longitude is $-26.25^{\circ}$. Then, in case the vehicle is launched at the beginning of the window, there would be a waiting time of about 16 days (approximately $13.25^{\circ} / 0.816^{\circ}$ per day) while the oblateness rotates the perigee into the Earth-Moon plane (see discussion of correction method 1 below). The other launch window (near the descending node of the Earth-Moon plane on the equator) would open at a time which would yield an injection celestial longitude of about $180^{\circ}-13.25^{\circ}=166.75^{\circ}$ East of the vernal equinox. Either option would yield a 45 minute launch window ending with a trajectory whose perigee point is about $2^{\circ}$ out of the Earth-Moon plane and approaching it. Thus, even for a launch at the end of either window, there would be about 2.5 days of waiting while the perigee advanced to the Earth-Moon plane. This would give ample time for orbit trim, orbit and attitude
determination, and spacecraft checkout and maneuvers prior to insertion into the phasing orbit. The example above is for illustrative purposes. It is recommended that the trajectory requirements be stated as in a previous section, i.e. that the GTO perigee shall be within $5^{\circ}$ of the Earth-Moon plane at some time within 15 days of GTO injection. The exact values of these requirements may change but the method of specification leaves nothing to the imagination.

## Correction Strategies:

The first method using the simplest strategy, as suggested earlier, is to wait until the Earth's oblateness rotates the perigee into the Earth-Moon plane. Such a strategy would involve no direct performance penalty except perhaps a few meters per second for orbit maintenance. But this strategy may require six or seven month waiting periods in the worst cases. As such a wait would almost certainly be unacceptable because of the long time spent in the Earth's radiation belts, it seems wise to investigate alternate techniques. The rates of the orbital ascending node and argument of perigee with respect to the Earth's equator for an orbiter under the influence of the Earth's oblateness parameter, $\mathrm{J}_{2}$, are given by (see e.g. Ref. 3)

$$
\frac{d \Omega}{d t}=-\frac{3}{2} \mathrm{~J}_{2} n\left(\frac{\mathrm{R}_{\mathrm{e}}^{2}}{a^{2}(1-e)^{3^{2}}}\right) \cos i, \quad \text { and } \frac{d \omega}{d t}=-\frac{3}{2} \mathrm{~J}_{2} n\left(\frac{\mathrm{R}_{\mathrm{e}}^{2}}{a^{2}(1-e)^{2}}\right)\left[\frac{1}{2}-\frac{5}{2} \cos ^{2} i\right]
$$

where $\Omega$ and $\omega$ are the longitude of the ascending node and argument of perigee, $n$ is the orbital mean motion ( $n=\left(\mu / a^{3}\right\}^{1 / 2}$ ), $a$ and $e$ are the orbital semi-major axis and eccentricity, $i$ is the equatorial inclination, $\mathrm{R}_{\mathrm{e}}$ is the Earth's equatorial radius, and $\mathrm{J}_{2}$ is the Earth's dynamical oblateness parameter ( $\mathrm{J}_{2}=1.082 \times 10^{-3}$ ). For a GTO with a $7^{\circ}$ inclination (see Ref. 4), these rates are

$$
\frac{d \Omega}{d t}=-0.413 \mathrm{deg} / \text { day }, \text { and } \frac{d \omega}{d t}=+0.816 \mathrm{deg} / \text { day } .
$$

Because the perigee advances (moves in the direction of spacecraft motion) it is better to launch early than late with respect to the Earth-Moon plane crossing.

A second method for adjusting the argument of perigee with respect to the Earth-Moon plane is to seek a large phasing orbit that is strongly perturbed by the gravitational perturbations of the Sun and Moon in such a way as to move the line of apsides into the Moon's orbit plane. This may be quite a complex solution because if the perturbations are strong enough to rotate the orbit quickly, they will also change the shape and probably the energy of the orbit. Such a solution to extend the launch window could alleviate any radiation problems associated with remaining in GTO for long periods of time. A large phasing orbit could be selected so that the spacecraft spends most of its time outside the severe
radiation, and an optimal multi-impulse transfer be selected to complete the transfer to the TLO. It is suggested that this possibility be investigated in more detailed studies as a possible means of improving overall performance in case margins are considered inadequate or if the radiation dose is considered excessive as the mission plan and spacecraft system become more mature.

The third technique considered is the brute force method in which the spacecraft propulsion is used to correct for the effects of an early or late launch on the argument of perigee.


Fig. 5 Total $\Delta V$ versus Angular Perigee Offset

Fig. 5 shows the total $\Delta \mathrm{V}$ required to transfer from GTO $(200 \times 35975 \mathrm{~km})$ to a 100 km circular lunar orbit as a function of the perigee offset from the Earth-Moon plane. The perigee offset is the angle from the Earth-Moon plane (measured along the orbit) to perigee of the phasing orbit at the time of injection into the phasing orbit. It is assumed that perturbations acting on the phasing orbit itself are negligible and, therefore, the perigee of the phasing orbit and the LTO are at the same point in space. In the calculations, it is assumed that the inclination of the phasing and lunar transfer orbits is $31^{\circ}$ (the greatest possible) and that $200 \mathrm{~m} / \mathrm{s} \Delta \mathrm{V}$ budget has been allocated for transfer orbit corrections and lunar orbit sustenance maneuvers. The calculations for Fig. 5 also include the assumption that the Moon's orbit is circular at $384,000 \mathrm{~km}$ from the Earth. This corresponds to the mean distance of the Moon from the Earth which varies by $\pm 5 \%$ during any month. The strategy used to compensate for perigee offset is to increase the size of the LTO by a small additional impulse applied at perigee. The additional impulse is just that required to increase the radial distance from the Earth at the largest node on the

Earth-Moon plane. This increase exactly compensates for the decrease in radial distance caused by the angular offset of the perigee at injection. Although this is not necessarily the optimal strategy, it is a practical one and the figure shows that offsets of 4 to 6 degrees are probably tolerable as they can be accounted for by the use of less than $100 \mathrm{~m} / \mathrm{s}$. Of course, such a strategy would increase the moonrelative excess speed and some (small) additional $\Delta \mathrm{V}$ would be required for lunar orbit insertion. The point here is that there are many transfer strategies available that will permit an adequate launch window for achieving the objectives of both the primary and secondary payloads on the launch vehicle.

## Preliminary Mass Calculations:

It is instructive to estimate the amount of payload mass that can be delivered to lunar orbit for various levels of required $\Delta \mathrm{V}$ as estimated above. Fig. 6 shows the net payload delivered to end of mission assuming a single on-board propulsion system with a specific impulse of 310 seconds and for stage propellant mass fractions from 0.65 to 0.85 . (This is the ratio of the mass of propellant to the total wet mass of the stage not including payload). The performance is given as payload mass as a percent of the spacecraft liftoff mass. This is the mass of the spacecraft after separation from the launch vehicle and jettison of any adaptors or extra mass that will not be accelerated by the spacecraft propulsion system. The analyst should be forewarned that stage propellant mass fractions of 0.85 are not generally achievable with very small spacecraft ( $<100 \mathrm{~kg}$ ). The minimum mass of existing valves, tanks, and other necessary propulsion system hardware dictate a stage propellant mass fraction of the order of 0.65 to 0.70 for spacecraft in the 50 to 100 kg range. As improvements in small spacecraft propulsion systems become available to the general user, these values will improve but, for current studies, it is suggested that the performance be calculated using the masses of the propulsion system component parts that are actually available for use.

Preliminary studies of optimal staging indicate that very little is to be gained by going to a two-stage propulsion system in the cases of the larger propellant mass fractions. The flexibility afforded by a restartable, single-stage system will probably turn out to be the deciding factor in selection of the propulsion system. For smaller spacecraft systems, it may prove wise to use a small solid for one of the larger maneuvers. Based on these preliminary deliberations, it appears that an on-orbit payload mass of from $50 \%$ to $20 \%$ of the mass in GTO can be expected, depending upon the exact time of launch, vehicle size, and final strategies selected.


Fig. 6 Payload Fraction vs. Total Impulse

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