

Omega from the Anisotropy of the Redshift Correlation Function

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ABSTRACT. Peculiar velocities distort the correlation function of galaxies observed in redshift space. In the large scale, linear regime, the distortion takes a characteristic quadrupole plus hexadecapole form, with the amplitude of the distortion depending on the cosmological density parameter Ω . Preliminary measurements are reported here of the harmonics of the correlation function in the CfA, SSRS, and IRAS 2 Jansky redshift surveys. The observed behavior of the harmonics agrees qualitatively with the predictions of linear theory on large scales in every survey. However, real anisotropy in the galaxy distribution induces large fluctuations in samples which do not yet probe a sufficiently fair volume of the Universe. In the CfA 14.5 sample in particular, the Great Wall induces a large negative quadrupole, which taken at face value implies an unrealistically large $\Omega \sim 20$. The IRAS 2 Jy survey, which covers a substantially larger volume than the optical surveys, and is less affected by fingers-of-god, yields a more reliable and believable value, $\Omega = 0.5^{+0.5}_{-0.25}$.

1. Theory

Peculiar velocities distort the pattern of clustering of galaxies in redshift space. The most familiar distortion is the finger-of-god effect, caused by large orbital velocities of galaxies in the virialized cores of clusters. A more subtle effect, which is the subject of this paper, is the squashing effect induced on large scales by peculiar infall towards overdense regions. Both finger-of-god and squashing effects are illustrated in Figure 1.

The same distortions affect the redshift correlation function. Hamilton (1992), building on work by Kaiser (1987), showed that the anisotropy of the redshift correlation function ξ in the linear regime has a characteristic quadrupole plus hexadecapole form (the redshift correlation function ξ is printed in bold face to distinguish it from the true underlying correlation function ξ)

$$\xi(r, \mu) = \xi_0(r)P_0(\mu) + \xi_2(r)P_2(\mu) + \xi_4(r)P_4(\mu) \quad (1)$$

Here r is the separation of a pair, μ is the cosine of the angle of the pair relative to the line of sight, and P_l are Legendre polynomials. The quadrupole harmonic ξ_2 of the redshift correlation function should be negative in the linear regime, reflecting the line of sight compression of overdense regions caused by peculiar motions toward them. The hexadecapole harmonic ξ_4 , which is induced by the dispersion of peculiar infall velocities, should be small and positive. All the higher order harmonics, ξ_l for

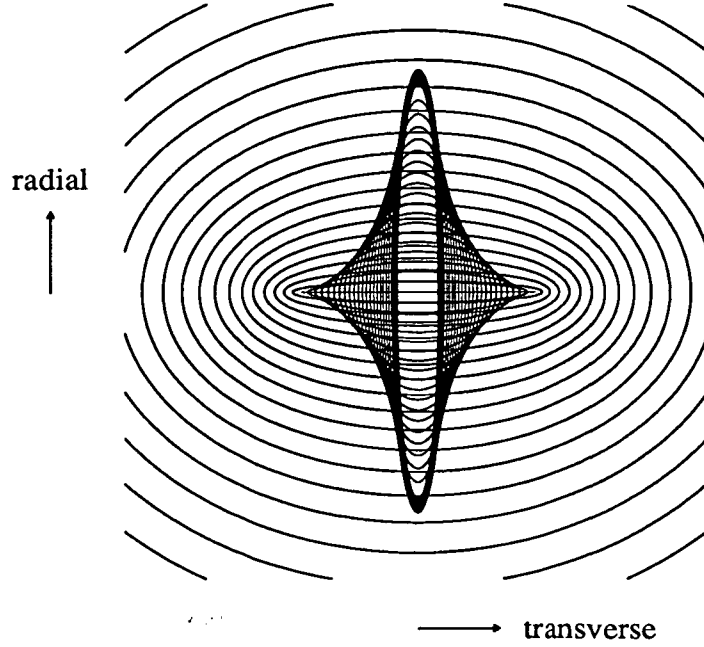


Figure 1. A spherical overdense region appears distorted by peculiar velocities when observed in redshift space. On large scales (linear regime) the region appears squashed, producing a negative quadrupole harmonic. On small scales (nonlinear regime), fingers-of-god appear, producing a positive quadrupole harmonic.

$l \geq 6$, should be zero in the linear regime. Note that non-zero azimuthal harmonics, $m \neq 0$, vanish identically by symmetry about the line of sight, while odd l harmonics vanish by pair exchange symmetry.

The relative amplitudes of the monopole, quadrupole, and hexadecapole harmonics ξ_l depend on the dimensionless growth rate f of growing modes in linear theory, which in the standard gravitational instability picture in the standard pressureless Friedmann cosmology is (Peebles 1980, eq. [14.8])

$$f \approx \Omega^{0.6} \quad (2)$$

where Ω is the density of the Universe relative to the critical density. The relative amplitudes of ξ_l also depend on the shape of the underlying correlation function $\xi(r)$, but by weighting the harmonics in an appropriate fashion, it is possible to eliminate this shape dependence. An appropriately weighted set of harmonics $\tilde{\xi}_l$ is (Hamilton 1992):

$$(1 + 2/3f + 1/5f^2) [\bar{\xi}(r) - \xi(r)] = \tilde{\xi}_0(r) \equiv -\xi_0(r) + 3 \int_0^r \xi_0(s) (s/r)^3 ds / s \quad (3)$$

$$(4/3f + 4/7f^2) [\bar{\xi}(r) - \xi(r)] = \tilde{\xi}_2(r) \equiv -\xi_2(r) \quad (4)$$

$$8/35f^2 [\bar{\xi}(r) - \xi(r)] = \tilde{\xi}_4(r) \equiv -\xi_4(r) + 7 \int_r^\infty \xi_4(s) (r/s)^2 ds / s \quad (5)$$

where $\bar{\xi} \equiv 3r^{-3} \int_0^r \xi s^2 ds$ denotes the mean correlation function interior to r . The ratio

of any pair of the weighted harmonics ξ_i yields a value of f , hence Ω through equation (2). The dependence of the ratios on Ω is illustrated in Figure 4. In practice, the ratio ξ_2/ξ_0 is the important one, since ξ_4 is small and its relative uncertainty therefore large.

2. Results — Optical Surveys

Figure 2 shows the harmonics of the redshift correlation function measured in a number of optical redshift surveys: the CfA 14.5 survey (Huchra et al. 1983; Huchra 1991); the first CfA slice (Huchra 1991); and the Southern Sky Redshift Survey (da Costa et al. 1991). All the surveys show qualitatively the expected behavior. On small scales the quadrupole harmonic, along with all the other harmonics, becomes large and positive, which is the nonlinear finger-of-god effect. The quadrupole harmonic goes negative somewhere around $5-10h^{-1}\text{Mpc}$, which is the squashing effect. In linear theory, the quadrupole harmonic is expected to become more negative to smaller scales, so the separations in Figure 2 up to the point where the quadrupole goes through its minimum are presumably contaminated by nonlinearity. In the northern samples, CfA 14.5 North, and CfA Slice, the nonlinear contamination by fingers-of-god apparently extends to separations of around $20h^{-1}\text{Mpc}$, whereas in the southern samples, CfA 14.5 South, and SSRS, the nonlinear contamination extends only to about $8h^{-1}\text{Mpc}$.

Judged from the scatter in the higher order harmonics, the least noisy of the optical samples is the CfA 14.5 North sample. A calculation of Ω in this sample using equations (2)-(5) yields $\Omega \sim 20$, an implausibly large value. Analysis of various angular and radial cuts from CfA 14.5 North reveals that the large negative quadrupole and large positive hexadecapole visible in Figure 2 arises almost entirely from a radial shell $60-80h^{-1}\text{Mpc}$ away. The region coincides physically with the location of the Great Wall (Geller & Huchra 1989). This result highlights what may be the main problem with this method for measuring Ω , which is that real anisotropic structure in the galaxy distribution can mimic the effect of peculiar velocities.

The results shown in Figure 2 should be regarded as preliminary, since error analyses are still in progress, and there is at least one important source of bias which has not been removed. This is a bias caused the local overdensity, which is not a random piece of the Universe (see Hamilton 1993 for a discussion).

3. Results — IRAS 2 Jansky Survey

Figure 3a shows the harmonics of the redshift correlation function measured in the IRAS 2 Jansky survey (Strauss et al. 1990, 1992; Strauss 1992). To exclude the local bias (Hamilton 1993), only galaxies with recession velocities $> 2,500\text{km s}^{-1}$ have been retained. As in the optical surveys, the quadrupole harmonic in the IRAS survey is large and positive at small separations, which is the finger-of-god effect, and it goes negative at about $5h^{-1}\text{Mpc}$, which is the squashing effect.

Although the number of galaxies, 2658, in the IRAS 2 Jy survey is not much greater than in any of the optical surveys discussed in §2, it covers a substantially larger volume, albeit in a substantially sparser fashion. The larger volume means that the correlation function is more accurately measured in the IRAS survey on the larger scales which are the focus of this paper. The IRAS survey has the further advantage

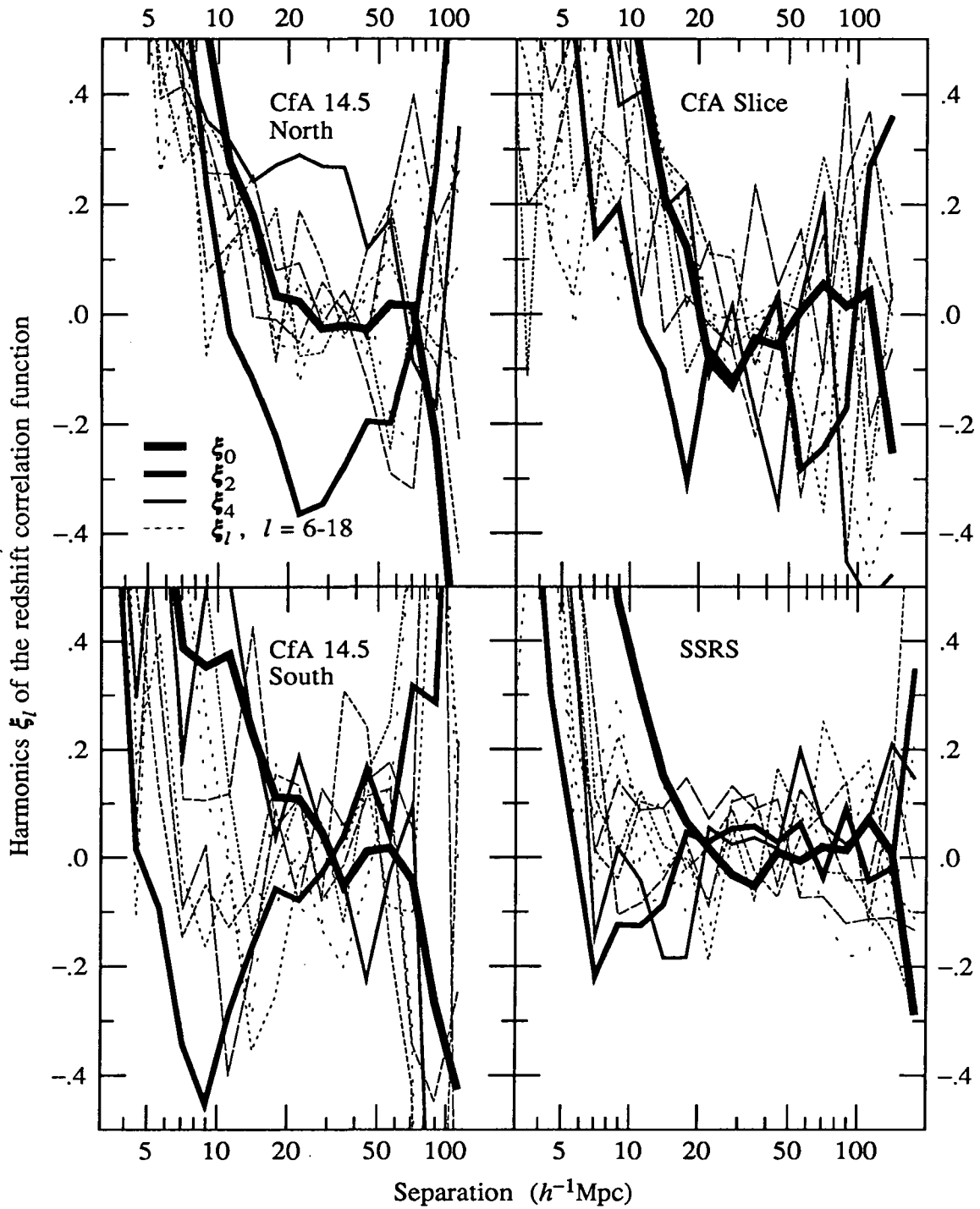


Figure 2. Harmonics ξ_l of the redshift correlation function from several optical surveys: the northern and southern parts of the CfA 14.5 survey, the first CfA slice, and the Southern Sky Redshift Survey. The thickest line is the usual monopole redshift correlation function; successively thinner and less complete lines are higher order harmonics, up to $l = 18$.

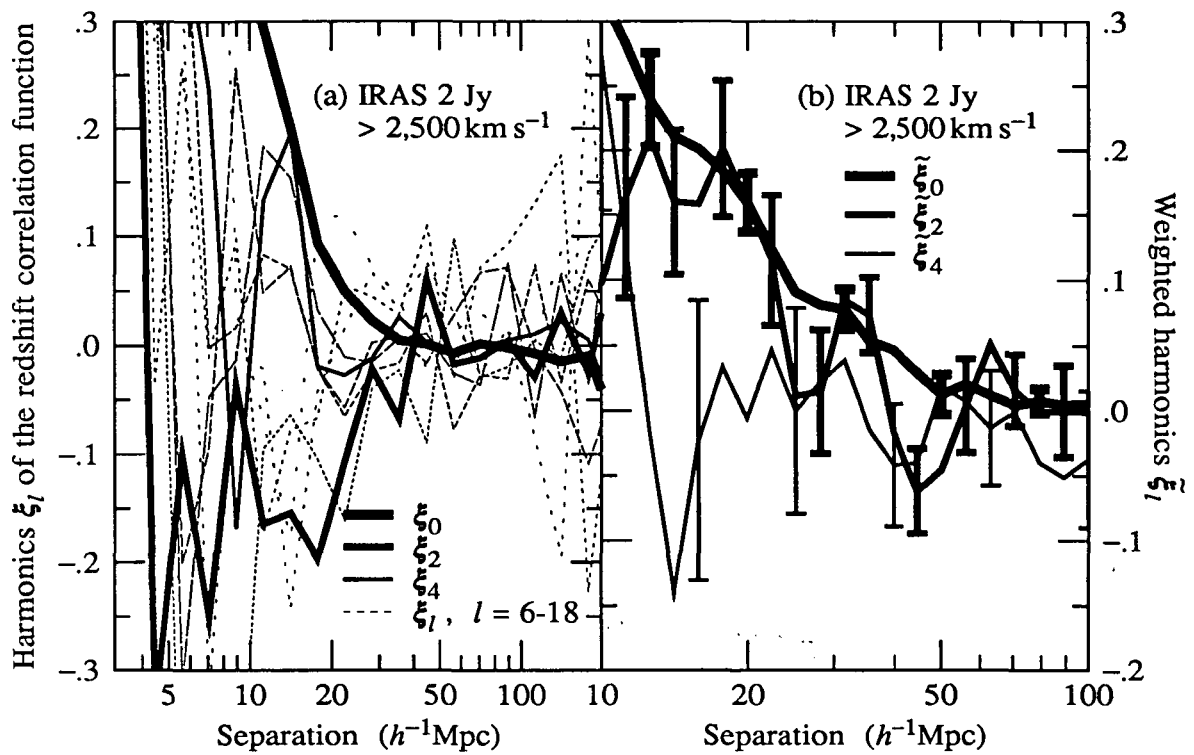


Figure 3. Harmonics of the redshift correlation function in the IRAS 2 Jy survey. (a) The harmonics ξ_l . (b) Weighted harmonics ξ_l , weighted in accordance with equations (3)-(5).

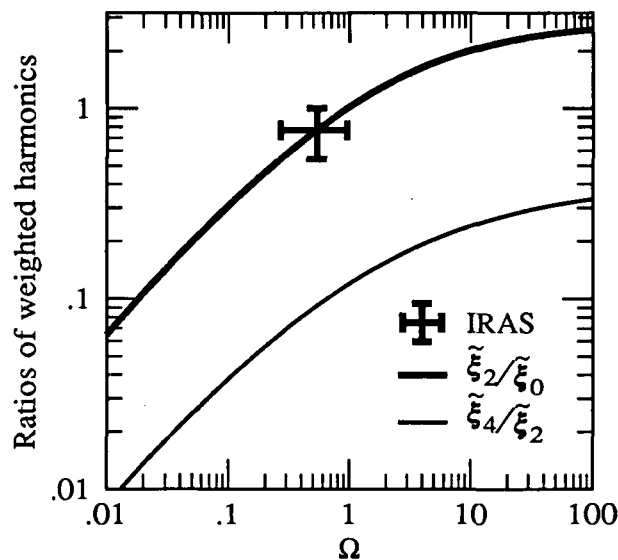


Figure 4. Relation between Ω and the ratios of the weighted harmonics ξ_l , from equations (2)-(5). The ratio ξ_2/ξ_0 measured from the IRAS 2 Jy sample is plotted as a cross with error bars.

over optical samples that it is likely to be subject to fewer systematic uncertainties arising for example from galactic absorption, or from calibration of fluxes/magnitudes. In addition, since IRAS galaxies tend to avoid the cores of rich clusters, fingers-of-god are less prominent than in optical samples. All of these considerations lead to the conclusion that IRAS samples are likely to yield more reliable values of Ω using the present method than are optical samples. Whether this view is borne out by future work remains to be seen.

Figure 3b shows the harmonics of the correlation function in the IRAS 2 Jy survey weighted in accordance with equations (3)-(5). The prediction of linear theory is that these weighted harmonics ξ_l should be proportional to each other, with their ratios depending on the value of Ω . The predicted proportionality appears to hold approximately on scales $\geq 12h^{-1}\text{Mpc}$.

Stacking the data shown in Figure 3b over $12.5-45h^{-1}\text{Mpc}$, below which nonlinearity is important, and above which there is no signal to speak of, yields $\xi_2/\xi_0 = 0.77 \pm 0.23$, which is plotted on Figure 4. The corresponding value of the parameter f , equation (1), is $f = 0.69^{+0.27}_{-0.23}$, and the inferred value of Ω is, in round numbers, $\Omega = 0.5^{+0.5}_{-0.25}$.

More details of the analysis of the IRAS 2 Jy survey are given by Hamilton (1993).

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