

# DISTRIBUTION OF PHOTONS IN "SQUEEZED" POLYMODE LIGHT

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## Abstract

The distribution functions of photons in squeezed and correlated light for one-mode and multimode cases are obtained based on the method of integrals of motion. Correlation coefficient and squeezing parameter are calculated. The possibility to generate squeezed light using nonstationary Casimir effect is discussed. Quantum parametric Josephson junction is proposed as quantum vacuum generator of electrical vibrations.

## 1 Introduction

The aim of this work is to discuss integrals of the motion and uncertainty relations and to obtain the distribution function of photons in squeezed and correlated light for one-mode and multimode cases. The distribution function of photons in squeezed light for one-mode fields was discussed by Schleich and Wheeler [1], by Agarwal and Adam [2], and by Chaturvedi and Srinivasan [3]. The photon distribution function for squeezed and correlated light [4] and [5] was discussed by Dodonov, Klimov and Man'ko [6]. This distribution function depends not only on the squeezing parameter, but also on the correlation parameter connected with Schrödinger uncertainty relation [7] as well,

$$\delta q \delta p \geq \frac{\hbar}{2\sqrt{1-r^2}}, \quad (1)$$

where the parameter  $r$  is the correlation coefficient of the position and momentum

$$r = (\delta q \delta p)^{-1} \left\{ \frac{\langle \hat{q} \hat{p} + \hat{p} \hat{q} \rangle}{2} - \langle \hat{q} \rangle \langle \hat{p} \rangle \right\}. \quad (2)$$

The states with nonzero parameter  $r$  we call the correlated states. In the section below we'll consider the problem how to find the states which minimize the Schrödinger uncertainty relation. For such states instead of the Schrödinger inequality we have the equality

$$\delta q \delta p = \frac{\hbar}{2\sqrt{1-r^2}}. \quad (3)$$

These states describe squeezed and correlated light. We will demonstrate in the next section how these states are naturally created for quantum parametric oscillator. The case of the photon

distribution function for the two-mode squeezed light was considered by Caves, Zhu, Milburn and Schleich [8]. Multidimensional generalization of the expression for the distribution of photons in squeezed light in terms of Hermite polynomials of several variables may be reformulated. We derive this expression on the bases of the result obtained in [17], [9] and [10] for a nonstationary parametric multidimensional oscillator.

The squeezing phenomenon in quantum optics is closely related to the oscillator models described by relativistic wave equations for elementary particles with mass spectrum. These relativistic models have been studied by Yukawa [11], by Markov [12], by Ginzburg and Man'ko [13], and by Kim and Noz [14]. As shown in [14], the Lorentz boost applied to relativistic oscillator gives the squeezing whose mathematics is identical to that of the squeezing in quantum optics. The statistical properties of such squeezed relativistic oscillators have been studied by Kim and Wigner [15].

To obtain the photon distribution function we will consider the nonstationary multidimensional oscillator. We shall discuss first the one-mode case in Sec. 2.

## 2 One-mode Light

The Hamiltonian for one-mode light is given by the formula

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \quad (4)$$

This mode of the electromagnetic field in a resonator may be described by the model of the mechanical oscillator with the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{q}^2}{2}. \quad (5)$$

In this case the annihilation and creation operators with boson commutation relations

$$\hat{a} = \frac{1}{\sqrt{2}} \left( \frac{\hat{q}}{l} + i \frac{\hat{p}}{p_0} \right), \quad (6)$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}} \left( \frac{\hat{q}}{l} - i \frac{\hat{p}}{p_0} \right), \quad (7)$$

where

$$l = \left( \frac{\hbar}{m\omega} \right)^{\frac{1}{2}}, \quad p_0 = (\hbar m\omega)^{\frac{1}{2}}, \quad (8)$$

connect both Hamiltonians and forms, together with the identity operator, the basis of the Heisenberg-Weyl algebra. In coordinate representation the complete set of coherent states  $|\alpha\rangle$  satisfying the equation

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle, \quad (9)$$

where  $\alpha$  is any complex number, is given by the formula

$$\langle q | \alpha \rangle = \pi^{-\frac{1}{4}} l^{-\frac{1}{2}} \exp \left[ -\frac{q^2}{2l^2} - \frac{|\alpha|^2}{2} + \frac{\sqrt{2}\alpha q}{l} - \frac{\alpha^2}{2} \right]. \quad (10)$$

The dispersions of the positions  $\delta q$  and the momentum  $\delta p$  do not depend on the parameter  $\alpha$ , and are given by the relation

$$\delta q = \frac{l}{\sqrt{2}}, \quad (11)$$

$$\delta p = \frac{p_0}{\sqrt{2}}. \quad (12)$$

For the coherent states the product of these dispersions minimizes the Heisenberg inequality

$$\delta q \delta p = \frac{\hbar}{2}. \quad (13)$$

The time evolution of the coherent state  $|\alpha, t\rangle$  may be obtained by simple replacement of the parameter  $\alpha$  in the formula (10) by the term  $\alpha \exp(-i\omega t)$  and the phase of the wave function. We have

$$\langle q | \alpha, t \rangle = \langle q | \alpha \exp(-i\omega t) \rangle \exp\left(\frac{-i\omega t}{2}\right). \quad (14)$$

The correlation coefficient of the position and momentum is equal to zero for arbitrary coherent state. It is also equal to zero for stationary Fock state  $|n, t\rangle$  satisfying the eigenvalue equation

$$\hat{a}^\dagger \hat{a} |n, t\rangle = n |n, t\rangle, \quad n = 0, 1, 2, \dots \quad (15)$$

This state has the following wave function in the coordinate representation

$$\langle q | n, t \rangle = \pi^{-\frac{1}{4}} l^{-\frac{1}{2}} 2^{-\frac{n}{2}} (n!)^{-\frac{1}{2}} H_n\left(\frac{q}{l}\right) \exp\left[-\frac{q^2}{2l^2} - i\omega t\left(n + \frac{1}{2}\right)\right] \quad (16)$$

The photon distribution function  $W_n(\alpha)$  for the coherent state  $|\alpha, t\rangle$  is determined by the overlap integral

$$|\langle n, t | \alpha, t \rangle|^2 = W_n(\alpha) \quad (17)$$

and coincides with the Poisson distribution function

$$W_n(\alpha) = \frac{|\alpha|^{2n}}{n!} \exp(-|\alpha|^2). \quad (18)$$

The mode has the following time-dependent integral of the motion

$$\hat{A}(t) = \exp(i\omega t) \hat{a}. \quad (19)$$

We now discuss how the influence of the dependence of the oscillator frequency  $\Omega(t)$  on time will change the photon distribution function and the dispersions of the conjugate variables  $\hat{q}$  and  $\hat{p}$ . The Hamiltonian of the mechanical parametric oscillator depends on time and has the form

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2(t)\hat{q}^2}{2}. \quad (20)$$

This system has the linear integral of the motion [16]

$$\hat{A}(t) = \frac{i}{\sqrt{2}} \left( \frac{\epsilon(t)\hat{p}}{p_0} - \frac{\dot{\epsilon}(t)\hat{q}}{l\omega} \right). \quad (21)$$

Here  $\omega = \Omega(0)$ , and the complex function  $\epsilon(t)$  satisfies the equation of classical oscillator motion

$$\ddot{\epsilon} + \Omega^2(t)\epsilon = 0. \quad (22)$$

The integrals of motion  $\hat{A}(t)$  and  $\hat{A}^\dagger(t)$  satisfy the boson commutation relation

$$[\hat{A}(t), \hat{A}^\dagger(t)] = 1, \quad (23)$$

if the Wronskian for the equation (22) is given by the equality

$$\epsilon\dot{\epsilon}^* - \epsilon^*\dot{\epsilon} = 2i\omega. \quad (24)$$

The initial condition for the function  $\epsilon(t)$  may be taken as follows

$$\epsilon(0) = 1, \quad \dot{\epsilon}(0) = i\omega. \quad (25)$$

If the frequency of the oscillator is constant, the function  $\epsilon = \exp(i\omega t)$  and the formula (21) gives the integral of the motion (19). The normalized state  $|0, t\rangle$  satisfying the Schrödinger equation and the relation

$$\hat{A}(t) |0, t\rangle = 0 \quad (26)$$

has the following wave function in the coordinate representation

$$\langle q | 0, t \rangle = \pi^{-\frac{1}{4}} (l\epsilon)^{-\frac{1}{2}} \exp\left(\frac{i\dot{\epsilon}q^2}{2\omega\epsilon l^2}\right). \quad (27)$$

The state  $|\alpha, t\rangle$  which is the eigenstate of the integral of the motion  $\hat{A}(t)$  given by formula (21)

$$\hat{A}(t) |\alpha, t\rangle = \alpha |\alpha, t\rangle \quad (28)$$

has the following wave function in the coordinate representation

$$\langle q | \alpha, t \rangle = \langle q | 0, t \rangle \exp\left(-\frac{1}{2} |\alpha|^2 + \frac{\sqrt{2}\alpha q}{l\epsilon} - \frac{\alpha^2 \epsilon^*}{2\epsilon}\right). \quad (29)$$

Here  $\alpha$  is an arbitrary complex number and

$$\langle \beta, t | \alpha, t \rangle = \exp\left(-\frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2} + \beta^* \alpha\right). \quad (30)$$

The wave function (29) satisfies the Schrödinger equation. The Fock states of the parametric mode  $|n, t\rangle$  satisfy the eigenvalue equation

$$\hat{A}^\dagger(t)\hat{A}(t) |n, t\rangle = n |n, t\rangle, \quad n = 0, 1, 2, \dots \quad (31)$$

The solutions to this equation have the following form in the coordinate representation

$$\langle q | n, t \rangle = \frac{1}{\sqrt{n!}} \langle q | 0, t \rangle \left(\frac{\epsilon^*}{2\epsilon}\right)^{n/2} H_n\left(\frac{q}{l|\epsilon|}\right). \quad (32)$$

Since the state  $|\alpha, t\rangle$  is the generating state for the Fock states  $|m, t\rangle$

$$|\alpha, t\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{m=0}^{\infty} \frac{\alpha^m |m, t\rangle}{\sqrt{m!}}, \quad (33)$$

the transition probability from the initial state  $|n\rangle$  may be calculated

$$W_n^m = W_0^0 \frac{n!}{m!} \left( P_{\frac{(m+n)}{2}}^{\frac{(m-n)}{2}}(W_0^0) \right)^2, \quad m \geq n. \quad (34)$$

Here the transition probability  $W_0^0$  is the probability to be in the ground state

$$W_0^0 = 2\left(|\epsilon|^2 + \omega^{-2}|\dot{\epsilon}|^2 + 2\right)^{-1/2}. \quad (35)$$

For  $n > m$  the formula (34) must be changed to

$$W_n^m = W_0^0 \frac{m!}{n!} \left( P_{\frac{(n+m)}{2}}^{\frac{(n-m)}{2}}(W_0^0) \right)^2. \quad (36)$$

The numbers  $n$  and  $m$  in the formulae (34) and (35) are either both even or both odd. If one of these numbers is even and another number is odd the transition probability between such states is equal to zero

$$W_{2k}^{2p+1} = W_{2k+1}^{2p} = 0, \quad k, p = 0, 1, 2, \dots \quad (37)$$

The formulae (34) and (36) describe the photon distribution function for the one-mode electromagnetic field in a resonator either with moving walls or with media with time-dependent refraction index. Thus, we conclude that the squeezing parameters of the parametric oscillator

$$S_q = \left(\frac{2m\omega}{\hbar}\right)^{1/2} \delta q = |\epsilon|, \quad (38)$$

$$S_p = \left(\frac{2}{\hbar m\omega}\right)^{1/2} \delta p = |\dot{\epsilon}| \quad (39)$$

are connected with the photon distribution function by the ratio (35) which may be rewritten in the form

$$W_0^0 = 2(S_q^2 + S_p^2 + 2)^{-1/2}. \quad (40)$$

In the case of vacuum light  $S_p = S_q = 1$  and the vacuum-vacuum transition probability  $W_0^0$  is equal to unity.

Another photon distribution function corresponds to the excitation of light state which may be described by the model of the forced mechanical oscillator with Hamiltonian

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2 - f(t)\hat{q}. \quad (41)$$

This oscillator has the integral of the motion [16]

$$\hat{A}(t) = \exp(i\omega t)\hat{a} + \delta(t), \quad (42)$$

$$\delta(t) = -\frac{il}{\sqrt{2\hbar}} \int_0^t f(\tau) \exp(i\omega\tau) d\tau. \quad (43)$$

If the initial state of a forced oscillator is the coherent state, the squeezing parameters  $S_q$  and  $S_p$  are time-independent. They are equal to unity. The photon distribution function is described by a Poisson distribution. Thus, if the initial state is the vacuum state the Poisson distribution has the form

$$W_m = \frac{|\delta|^{2m}}{m! \exp(|\delta|^2)}. \quad (44)$$

The physical meaning of the parameter  $|\delta|^2$  (43) which determines the integral of the motion (42) is just the mean photon number after the excitation of the vacuum state by the external linear force. The photon distribution function  $W_n^m$  in the case when the initial state was the state  $|n\rangle$  with  $n$  photons is described by the function

$$W_n^m = \frac{n! |\delta|^{2(m-n)}}{m! \exp(|\delta|^2)} [L_n^{m-n}(|\delta|^2)]^2. \quad (45)$$

Here the function  $L_N^M$  is the Laguerre polynomial.

Now consider a general situation when the frequency of an oscillator depends on time and an external force is present. The Hamiltonian of the mechanical oscillator model looks like

$$\hat{H}(t) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2(t)\hat{q}^2 - f(t)\hat{q}. \quad (46)$$

We have taken  $m = \omega = \hbar = 1$ . The linear integral of motion  $\hat{A}(t)$  is equal in this case to

$$\hat{A}(t) = u(t)\hat{a} + v(t)\hat{a}^\dagger + \delta(t), \quad (47)$$

$$u(t) = \frac{1}{2}(\epsilon(t) - i\dot{\epsilon}(t)), \quad (48)$$

$$v(t) = \frac{1}{2}(\epsilon(t) + i\dot{\epsilon}(t)), \quad (49)$$

$$\delta(t) = -\frac{i}{\sqrt{2}} \int f(\tau)\epsilon(\tau) d\tau. \quad (50)$$

The normalized eigenstate  $\psi_\alpha(q, t)$  of the integral of motion (47) has the form

$$\psi_\alpha(q, t) = \psi_0(q, t) \exp\left[-\frac{|\alpha|^2}{2} + \frac{\sqrt{2}\alpha q}{\epsilon} + \frac{\alpha(\delta\epsilon^* + \delta^*\epsilon)}{\epsilon} - \frac{\alpha^2\epsilon^*}{2\epsilon}\right], \quad (51)$$

where

$$\psi_0(q, t) = \frac{1}{\sqrt{\sqrt{\pi}\epsilon}} \exp\left[\frac{iq^2\dot{\epsilon}}{2\epsilon} - \frac{\sqrt{2}q\delta}{\epsilon} - \frac{\delta^2\epsilon^*}{2\epsilon} - \frac{|\delta|^2}{2} + \frac{1}{2} \int_0^t (\dot{\delta}\delta^* - \delta\dot{\delta}^*) d\tau\right]. \quad (52)$$

The squeezing parameters  $S_q$  and  $S_p$  for the states (51) are described by the formulae for the unforced parametric oscillator (38), (39). The correlation coefficient  $r$  is given by the expression

$$r = |\epsilon\dot{\epsilon}|^{-1} [(\epsilon\dot{\epsilon})^2 - 1]^{1/2}. \quad (53)$$

The Fock states which are the eigenstates of the integral of motion  $\hat{A}^\dagger(t)\hat{A}(t)$  are of the form

$$\psi_n(q, t) = \psi_0(q, t) \frac{1}{\sqrt{n!}} \left( \frac{\epsilon^*}{2\epsilon} \right)^{n/2} H_n \left( \frac{q + (\delta\epsilon^* + \delta^*\epsilon)/\sqrt{2}}{|\epsilon|} \right). \quad (54)$$

The photon distribution function for the electromagnetic field created due to the nonstationary Casimir effect is expressed in terms of Hermite polynomials of two variables

$$\frac{W_n^m}{W_0^0} = (n!m!)^{-1} |H_{nm}^{\{R\}}(x_1, x_2)|^2, \quad (55)$$

where

$$x_1 = -\frac{\delta^*}{\zeta^*}, \quad x_2 = \delta - \frac{\eta\delta^*}{\zeta^*}, \quad (56)$$

and the matrix  $R$  has the elements

$$R = \zeta^{-1} \begin{pmatrix} \eta & -1 \\ -1 & -\eta^* \end{pmatrix}.$$

The parameters  $\zeta$  and  $\eta$  are given by the relation

$$\epsilon(t) = \zeta e^{it} - \eta e^{-it}. \quad (57)$$

The photon distribution function (55) has oscillatory behavior due to the oscillatory behavior of the Hermite polynomial of two variables.

The last photon distribution function describes the influence of the nonstationary Casimir effect on the initially thermal equilibrium state

$$\hat{\rho}(0) = Z^{-1} \exp \left[ -\beta \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \right], \quad (58)$$

$$Z^{-1} = 2 \sinh(\beta/2). \quad (59)$$

The distribution of photons in the light mode is expressed by the density matrix diagonal elements

$$\frac{\rho_{nn}}{\rho_{00}} = \frac{1}{n!} H_{nn}^{\{R\}}(\bar{x}_1, \bar{x}_2), \quad (60)$$

where

$$R = D^{-1} \begin{pmatrix} u^*v(1-z^2) & -z \\ -z & uv^*(1-z^2) \end{pmatrix}$$

and

$$\bar{x}_1 = (1-z) \frac{\delta^*uz + \delta v^*}{|u|^2 z^2 - |v|^2}, \quad \bar{x}_2 = \bar{x}_1^*.$$

The photon distribution function has the following deformed Planck distribution form

$$\langle n \rangle = \frac{1}{e^\beta - 1} + |v|^2 \frac{e^\beta + 1}{e^\beta - 1} + |\delta u^* - \delta^* v|^2. \quad (61)$$

If there is no external force the parameter  $\delta = 0$ . The deformed Planck distribution has the form

$$\langle n \rangle = \frac{1}{e^\beta - 1} + |v|^2 \coth(\beta/2). \quad (62)$$

The squeezing parameters  $S_q$  and  $S_p$  depend on temperature

$$S_q = |\epsilon| (\coth(\beta/2))^{1/2}, \quad (63)$$

$$S_p = |\dot{\epsilon}| (\coth(\beta/2))^{1/2}. \quad (64)$$

Thus the parametric excitation of the oscillator may produce the squeezing phenomenon when

$$S_q < 1, \quad (65)$$

or

$$S_p < 1. \quad (66)$$

But the higher the temperature the more difficult to obtain the squeezing.

### 3 Polymode Squeezed Light

We will consider the photon distribution function for polymode squeezed and correlated light using the model of nonstationary parametric multidimensional quantum oscillator with  $N$  degrees of freedom. Its Hamiltonian may be written in the form

$$\hat{H} = \frac{1}{2} \hat{\mathbf{q}} B(t) \hat{\mathbf{q}} + \vec{C}(t) \hat{\mathbf{q}}, \quad (67)$$

where the vector

$$\hat{\mathbf{q}} = (\hat{p}_1, \hat{p}_2, \dots, \hat{p}_N, \hat{x}_1, \hat{x}_2, \dots, \hat{x}_N) = (\hat{\mathbf{p}}, \hat{\mathbf{x}}) \quad (68)$$

contains  $N$  momentum projection operators and  $N$  position projection operators. The  $2N \times 2N$  - matrix  $B(t)$  and  $2N$  - vector  $\vec{C}(t)$  are time-dependent parameters of the system. The model corresponds to  $N$  light modes in the resonator. The interaction of these modes depends on time either due to the motion of the resonator walls or due to the time-dependence of the media refraction index. The system must demonstrate the properties of nonstationary Casimir effect for  $N \mapsto \infty$ . The oscillator has  $2N$  - vector

$$\hat{\mathbf{Q}}(t) = \Lambda(t) \hat{\mathbf{q}} + \vec{\Delta}(t), \quad (69)$$

which is the linear integral of motion if the  $2N \times 2N$ -symplectic matrix  $\Lambda$  satisfies the classical equation of motion

$$\dot{\Lambda} = K \Sigma B(t), \quad (70)$$

where the  $2N \times 2N$ -matrix  $\Sigma$  has the form

$$\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (71)$$



The  $2N$  - vector  $\vec{\Delta}(t)$  obeys the equation

$$\dot{\vec{\Delta}} = \Lambda(t)\Sigma\vec{C}(t). \quad (72)$$

The solution  $(\Lambda, \vec{\Delta})$  of equations (70) and (72) describe the classical trajectories of multidimensional oscillators and may be considered as an element of the inhomogeneous real symplectic group  $\text{ISp}(2N, \mathbb{R})$ . The initial conditions for these equations are

$$\Lambda(0) = 1, \quad \vec{\Delta}(0) = 0. \quad (73)$$

The propagator of the system has the form

$$G(x_2, x_1, t) = \frac{1}{\sqrt{\det(-2i\pi\lambda_3)}} \exp \left\{ -\frac{i}{2} (\mathbf{x}_2 \lambda_3^{-1} \lambda_4 \mathbf{x}_2 - 2\mathbf{x}_2 \lambda_3^{-1} \mathbf{x}_1 + \mathbf{x}_1 \lambda_1 \lambda_3^{-1} \mathbf{x}_1 + 2\mathbf{x}_2 \lambda_3^{-1} \vec{\delta}_2 + 2\mathbf{x}_1 (\vec{\delta}_1 - \lambda_1 \lambda_3^{-1} \vec{\delta}_2) + \vec{\delta}_2 \lambda_1 \lambda_3^{-1} \vec{\delta}_2 - 2 \int_0^t \dot{\vec{\delta}}_1 \vec{\delta}_2 d\tau) \right\}, \quad (74)$$

with  $(\hbar = 1)$ . Here the matrices  $\lambda_i$ ,  $i = 1, 2, 3, 4$  are  $N \times N$  blocks of the matrix  $\Lambda$

$$\Lambda = \begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{pmatrix} \quad (75)$$

and  $N$ -vectors  $\vec{\delta}_1, \vec{\delta}_2$ , are the components of the vector  $\vec{\Delta}$

$$\vec{\Delta} = (\vec{\delta}_1, \vec{\delta}_2). \quad (76)$$

The Hamiltonian of the system may be rewritten in terms of the boson annihilation and creation operators  $\hat{\mathbf{a}} = (\hat{a}_1, \dots, \hat{a}_N)$ ,  $\hat{\mathbf{a}}^\dagger = (\hat{a}_1^\dagger, \dots, \hat{a}_N^\dagger)$ , in the form

$$\hat{H} = \frac{1}{2} (\hat{\mathbf{a}}, \hat{\mathbf{a}}^\dagger) D(t) \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{a}}^\dagger \end{pmatrix} + \vec{f} \hat{\mathbf{a}} + \vec{f}^* \hat{\mathbf{a}}^\dagger. \quad (77)$$

If we introduce the  $2N$  - vector

$$\hat{\mathbf{A}}(t) = \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{a}}^\dagger \end{pmatrix} \quad (78)$$

this vector is connected with the  $2N$  - vector  $\hat{\mathbf{q}} = (\hat{\mathbf{p}}, \hat{\mathbf{x}})$  by the relation

$$\hat{\mathbf{A}} = V \hat{\mathbf{q}}, \quad (79)$$

where the  $2N \times 2N$  - matrix  $V$  has the form

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}, \quad V^\dagger = V^{-1}. \quad (80)$$

Then the matrix  $B(t)$  in (67) is connected with the matrix  $D(t)$  in (77) by the relation

$$D(t) = V^\dagger B V. \quad (81)$$

The integrals of motion of the multimode nonstationary system are given by the formula

$$\hat{\mathbf{A}}(t) = \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{a}}^\dagger \end{pmatrix} = \Lambda_1(t) \begin{pmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{a}}^\dagger \end{pmatrix} + \vec{\Gamma}(t). \quad (82)$$

The  $2N$  - vector  $\vec{\Gamma}(t) = (\vec{\gamma}, \vec{\gamma}^*)$  is connected with the vector  $\vec{\Delta}$

$$\vec{\Gamma}(t) = V\vec{\Delta}(t). \quad (83)$$

The  $2N \times 2N$  - matrix  $\Lambda_1(t)$  with  $N \times N$  - blocks  $\xi$  and  $\eta$

$$\Lambda_1(t) = \begin{pmatrix} \xi(t) & \eta(t) \\ \xi^*(t) & \xi^*(t) \end{pmatrix} \quad (84)$$

is connected with the matrix  $\Lambda(t)$  by the relation

$$\Lambda_1(t) = V\Lambda(t)V^\dagger. \quad (85)$$

The propagator of the system in coherent state representation has the Gaussian form

$$G(\vec{\alpha}^*, \vec{\beta}, t) = \frac{1}{\sqrt{\det \xi(t)}} \exp \left( -\frac{1}{2} \vec{\alpha}^* \xi^{-1} \eta \vec{\alpha}^* + \vec{\alpha}^* \xi^{-1} \vec{\beta} - \vec{\alpha}^* \xi^{-1} \vec{\gamma} + \frac{1}{2} \vec{\beta} \eta^* \xi^{-1} \vec{\beta} \right. \\ \left. + \vec{\beta} (\vec{\gamma}^* - \eta^* \xi^{-1} \vec{\gamma}) + \frac{1}{2} \vec{\gamma} \eta^* \xi^{-1} \vec{\gamma} - \int_0^t \vec{\gamma}(\tau) \dot{\vec{\gamma}}^*(\tau) d\tau \right). \quad (86)$$

The photon distribution function for multimode case may be obtained by expansion of the propagator (86) into a series with respect to the parameters  $\alpha^*$ . We have the distribution of photons in squeezed and correlated light

$$|\langle \vec{m} | \vec{\beta}, t \rangle|^2 = W_{\vec{m}}(\vec{\beta}) = |G(0, \vec{\beta})|^2 \frac{1}{m_1! \dots m_N!} |H_{\vec{m}}^{\{\xi^{-1}\eta\}} [\eta^{-1} [\vec{\beta} - \vec{\gamma}]]|^2. \quad (87)$$

Thus, the distribution function for  $N$  - mode system is described by the Hermite polynomial of  $N$  variables. For squeezed and correlated light the behavior of the function  $W_{\vec{m}}(\vec{\beta})$  is very oscillatory as well as for one- and two- mode cases. The partial cases for two-mode light may be obtained if one uses the formulae for Hermite polynomials of two variables found in [5].

## 4 Nonstationary Casimir Effect

Now let us discuss some possible applications. One of the possible methods to generate squeezed light is to use the nonstationary Casimir effect when moving resonator walls produce continuous time-dependent reconstruction of the electromagnetic vacuum state. The work against the Casimir forces produces two effects. The first effect is the generation of photons from the vacuum. Thus, the resonator with mechanically trembling walls is a quantum vacuum generator of electromagnetic radiation. The second effect is the squeezing of quantized modes in the resonator due to parametric change of vacuum energy. Both effects exist simultaneously. Thus, the plates in the Casimir effect may be moved by external mechanical forces. The refraction index of the media may vary with

time, the geometrical dimensions of the resonator may be influenced by external mechanical forces. In all these cases the vacuum state energy must be changed. This means that the vacuum state is continuously reconstructed. For each of the time moments the state is no longer the vacuum state due to the change of the parameters. If we have a system with photons or other quanta in a box with changing volume, it corresponds to the process with the creation of quanta, e.g. the photons. So, due to the work against the Casimir forces one form of energy may be converted into the other form. Thus for waving neutral plates (due to mechanical external forces) between the plates the photons must be created and this means that the mechanical energy from external sources is converted into electromagnetic energy of photons. It is interesting that this effect must create the quanta of all other fields existing in nature. Thus, due to Casimir forces we can have the generation of photons in a parametric resonator which may be called a quantum vacuum generator.

It is possible to discuss another reduction of nonstationary Casimir effect using the Josephson junction. If there is no external voltage in the Josephson junction but its parameters are time-dependent, the vibrations of current and voltage will be excited in it. This suggestion [17] is based on the analogy of the Josephson junction and a conventional resonant circuit (quantized resonant circuit). In classical resonant circuits it is impossible to excite electrical vibrations without external sources of voltage. But for a quantum resonator circuit due to Casimir nonstationary forces, it is possible to transform mechanical energy which may be the reason for the change of the circuit parameters into electrical energy of current vibrations. If this idea is realized it will be a quantum vacuum generator of electrical vibrations. The current and voltage in this case play the role of conjugate quantum observables and in parametric Josephson junctions they may be squeezed. Thus, the quantum noise in Josephson junctions may be reduced for current. In this case the voltage will have larger noise. The squeezed and correlated states of Josephson junctions may be also excited by changing its parameters with time.

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## References

- [1] W. Schleich and J. A. Wheeler, *J. Opt. Soc. Am.* **134**, 1715 (1987).
- [2] G. S. Agarwal and A. Adam, *Phys. Rev.* **A39**, 6259 (1989).
- [3] S. Chaturvedi and V. Srinivasan, *Phys. Rev.* **A40**, 6095 (1990).
- [4] V. V. Dodonov, E. V. Kurmyshev and V. I. Man'ko, *Phys. Lett.* **A79**, 150 (1980).
- [5] V. V. Dodonov and V. I. Man'ko, *Invariants and Evolution of Nonstationary Quantum Systems*, (Proceedings of Lebedev Physics Institute, vol.183, Nova Science Publishers, Commack, N.Y. 1987).
- [6] V. V. Dodonov, A. B. Klimov and V. I. Man'ko, *Phys. Lett.* **A149**, 225 (1990).

- [7] E. Schrödinger, Ber. Kgl. Akad. Wiss. Berlin , 296 (1930).
- [8] C. M. Caves, C. Zhu, G. J. Milburn and W. Schleich, Phys. Rev. **A43**, 3854 (1991).
- [9] V. I. Man'ko *Proceedings of Hyderabad Conference on Quantum Optics, January 1991*, (Plenum Press, 1992).
- [10] I. A. Malkin, V. I. Man'ko and D. A. Trifonov, J. Math. Phys. **14**, 576 (1973).
- [11] H. Yukawa, Phys. Rev. **91**, 415 (1953).
- [12] M. A. Markov, Nuovo Cim. Suppl. **3**, ser.10, 760 (1956).
- [13] V. L. Ginzburg and V. I. Man'ko, Nucl. Phys., **74**, 577 (1965).
- [14] Y. S. Kim and M. E. Noz, Phys. Rev., **D8**, 3521 (1973).
- [15] Y. S. Kim and E. Wigner, Phys. Lett., **A147**, 343 (1990).
- [16] I. A. Malkin and V. I. Man'ko, Phys. Lett., **A32**, 243 (1970).
- [17] V. V. Dodonov, O. V. Man'ko and V. I. Man'ko, J. of Soviet Laser Research, **10**, 413 (Plenum Press, 1989).