

CALCULATION OF THE NUCLEON STRUCTURE FUNCTION FROM THE NUCLEON WAVE FUNCTION

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Abstract

Harmonic oscillator wave functions have played an historically important role in our understanding of the structure of the nucleon, most notably by providing insight into the mass spectra of the low-lying states. High energy scattering experiments are known to give us a picture of the nucleon wave function at high-momentum transfer and in a frame in which the nucleon is travelling fast. This paper presents a simple model that crosses the twin bridges of momentum scale and Lorentz frame that separate the pictures of the nucleon wave function provided by the deep inelastic scattering data and by the oscillator model.

1 Introduction

While a prediction for the nucleon structure functions from first principles in quantum chromodynamics (QCD) seems, even now, a remote prospect, if we are ever to claim a deep understanding of the structure of the nucleon, a clear interpretation of such gross properties as the neutron-proton structure function ratio (R^{np}) and the polarization asymmetry (A^{7p}) of the proton structure function is essential. A notable attempt to relate these features to the nucleon rest-frame wave function was made by Le Yaouanc *et al.* [1, 2, 3], who employed non-relativistic harmonic oscillator spatial wave functions and SU(6) mixing in an attempt to formulate predictions both about the structure functions and the nucleon form factors. While both the large- x behavior of R^{np} and the initial slope of the neutron electric form factor were well accounted for by the inclusion of an admixture 70 excited state in the nucleon wave function, the signs of the mixing angles obtained in the two cases were observed to disagree.

Against the structure-functions calculation of Le Yaouanc *et al.* may be raised the objection that there is no clear prescription for Lorentz-transforming a non-relativistic wave function. It is this concern that will be addressed in this paper. Less widely recognized is an objection that can be raised against the treatment of the form factors by Le Yaouanc *et al.* The latter calculation

involves the assumption that the nucleon's charge (magnetization) density and electric (magnetic) form factor are related by Fourier transformation. The Fourier relationship holds only when the Lorentz transformation of the spatial wave functions is ignored. There are several models which are known to predict a non-zero neutron electric form factor in the absence of SU(6) mixing[4, 5, 6]. In such models, which employ plausible relativistic spin wave functions, the matrix elements involved in the determination of the form factors cannot be factorized into a product of spin, isospin, and spatial matrix elements. Since the spin wave functions play no role in the structure function calculation, the possibility must be considered that the structure functions provide the correct value for the mixing angle.

The spatial wave functions that shall be considered are the "definite metric"[7] solutions to the relativistic harmonic oscillator equation of Feynman *et al.*[8] In their original work, Feynman *et al.* used the non-normalizable "indefinite metric" solutions of their wave equation. These solutions yield divergent form factors as $-q^2$ increases. To remedy this, they multiplied all matrix elements by an ad hoc factor. The "definite metric" solutions are normalizable and, when used to calculate nucleon form factors, yield the proper q^2 behavior, a dipole fall-off for large $-q^2$, without any adjustments. These solutions also help to illuminate features of the structure functions and the parton model, as will be seen later on.

In Section 2 the relativistic harmonic oscillator equation and its normalizable solutions are reviewed. The behavior of these solutions under Lorentz's transformation is discussed, and their form in the infinite momentum frame is exhibited. In Section 3 the infinite-momentum-frame relativistic-oscillator nucleon wave function is combined with QCD momentum scaling incorporated via the valon model of Hwa.[9] The proton and neutron structure functions are considered within the context of the resulting model, and a value for the mixing angle for an admixture of $7Q$ excited state is calculated. In Section 4, the significance of this calculation is reviewed.

2 The Relativistic Oscillator Model

For simplicity of discussion, the relativistic oscillator model is introduced for the two particle case first. This model describes the binding of a pair of quarks to form a meson via the differential equation

$$\{2 [\partial_1^2 + \partial_2^2] - (\omega^2/16) (x_1 - x_2)^2 - m_0^2\} \Psi(x_1, x_2) = 0 \quad (1)$$

where x_1 and x_2 represent the space-time coordinates of the two constituent quarks, and the metric convention is defined by $-g_{00} = g_{ii} = 1$. The quark spin will be ignored here, though versions of the relativistic oscillator model have been formulated to include spin 1/2 quarks.[10, 11] Eq. (1) is readily solved via separation of variables in terms of the coordinates

$$\begin{aligned} X_\mu &= 1/2(x_{1\mu} + x_{2\mu}) \\ x_\mu &= 1/2(x_{1\mu} - x_{2\mu}) \end{aligned} \quad (2)$$

where the X_μ are the space-time coordinates of the meson center of momentum and the x_μ determine the space-time separation of the quarks. The separated equations are

$$(\partial_X^2 - m^2) \psi(X) = 0 \quad (3)$$

and

$$(-\partial_x^2 + \omega^2/4x^2 + m_0^2) \varphi(x) = m^2 \varphi(x) \quad (4)$$

where $\Psi(x_1, x_2) = \psi(X)\varphi(x)$. Eq. (3) is the Klein-Gordon equation for a meson of mass m , while Eq. (4) describes a four-dimensional harmonic oscillator.

Eq. (4) is itself separable in terms of the space-time components x_μ , while the eigenvalue m^2 is given by a linear combination of the eigenvalues corresponding to each of the component equations. In the timelike direction, an increase in the excitation quantum number corresponds to a more negative contribution to the mass squared. To eliminate a degree of freedom which is not observed in nature, and to eliminate, as well, the unphysical possibility of imaginary mass, the oscillator-model solutions are required to obey the subsidiary condition

$$\frac{\partial}{\partial X_\mu} \left(\frac{1}{\sqrt{\omega}} \frac{\partial}{\partial x^\mu} + \frac{\sqrt{\omega}}{2} x_\mu \right) \Psi(X, x) = 0. \quad (5)$$

This condition suppresses timelike excitations in the meson rest frame.

The solutions to Eq. (3) have the familiar form $\exp(iP^\mu X_\mu)$ where P_μ is the four-momentum corresponding to the meson center-of-momentum coordinates X_μ . The solutions to Eq. (4) are products of oscillator solutions in each of the space-time components, with the solution in the timelike coordinate in the restframe being restricted to the fundamental mode via Eq. (5). Such solutions can be written as

$$\begin{aligned} \varphi_n^{b,k,w}(x') &= \omega/2\pi \left(2^{b+k+w} b!k!w! \right)^{-1/2} H_b[x_r \sqrt{\omega/2}] H_k[y_r \sqrt{\omega/2}] H_w[z_r \sqrt{\omega/2}] \\ &\times \exp \left[-\omega/4 (x_r^2 + y_r^2 + z_r^2 + t_r^2) \right] \end{aligned} \quad (6)$$

where H denotes a Hermite polynomial and x' denotes the four vector x , represented in terms of its components, x_r , y_r , z_r and t_r in the meson rest frame. The invariant meson square mass corresponding to P^μ is required by Eq. (3) to be equal to m^2 , while Eq. (4) determines that $m^2 = \omega(b + k + w + 1) + m_0^2$.

The above solutions form a complete set of normalized rest-frame solutions. The wave function in a frame in which the meson is not at rest is specified by the Lorentz transformation between the meson rest frame and the frame in which it is moving. For example, the ground state in an arbitrary frame can be written as

$$\begin{aligned} \Psi(X, x) = & \omega/(2\pi) \left(2^{b+k+w} b!k!w! \right)^{-1/2} e^{iP^\mu X_\mu} \\ & \times \exp \left\{ -\omega/4 [x_\mu^2 - 2(P \cdot x)^2 / P^2] \right\}. \end{aligned} \quad (7)$$

The construction of relativistic-oscillator momentum-space wave functions in arbitrary frames is equally straightforward. Figure 1 provides a pictorial view of the effect of the Lorentz transformation on the rest-frame wave function, both in coordinate space and in momentum space. The bound state quarks are seen to acquire lightlike momenta in the frame where the meson is moving rapidly. The success of the parton model tells us that this should be the case.

Modelling of the nucleon requires that a three particle version of the relativistic oscillator be considered. A harmonic interaction between each pair of quarks is assumed, and the governing differential equation takes the form

$$\left\{ 3 \left[\partial_1^2 + \partial_2^2 + \partial_3^2 \right] - \omega^2/36 \left[(x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2 \right] - U^0 \right\} \Psi(x_1, x_2, x_3) = 0 \quad (8)$$

where x_1 , x_2 and x_3 are the space-time coordinates of three constituent quarks.

Separation of variables can be implemented in terms of the coordinates X , r and s , defined as

$$\begin{aligned} X_\mu &= 1/3 (x_{1\mu} + x_{2\mu} + x_{3\mu}) \\ r_\mu &= 1/6 (x_{1\mu} + x_{2\mu} - 2x_{3\mu}) \\ s_\mu &= -1/(2\sqrt{3})(x_{1\mu} - x_{2\mu}). \end{aligned} \quad (9)$$

QUARKS \longrightarrow PARTONS

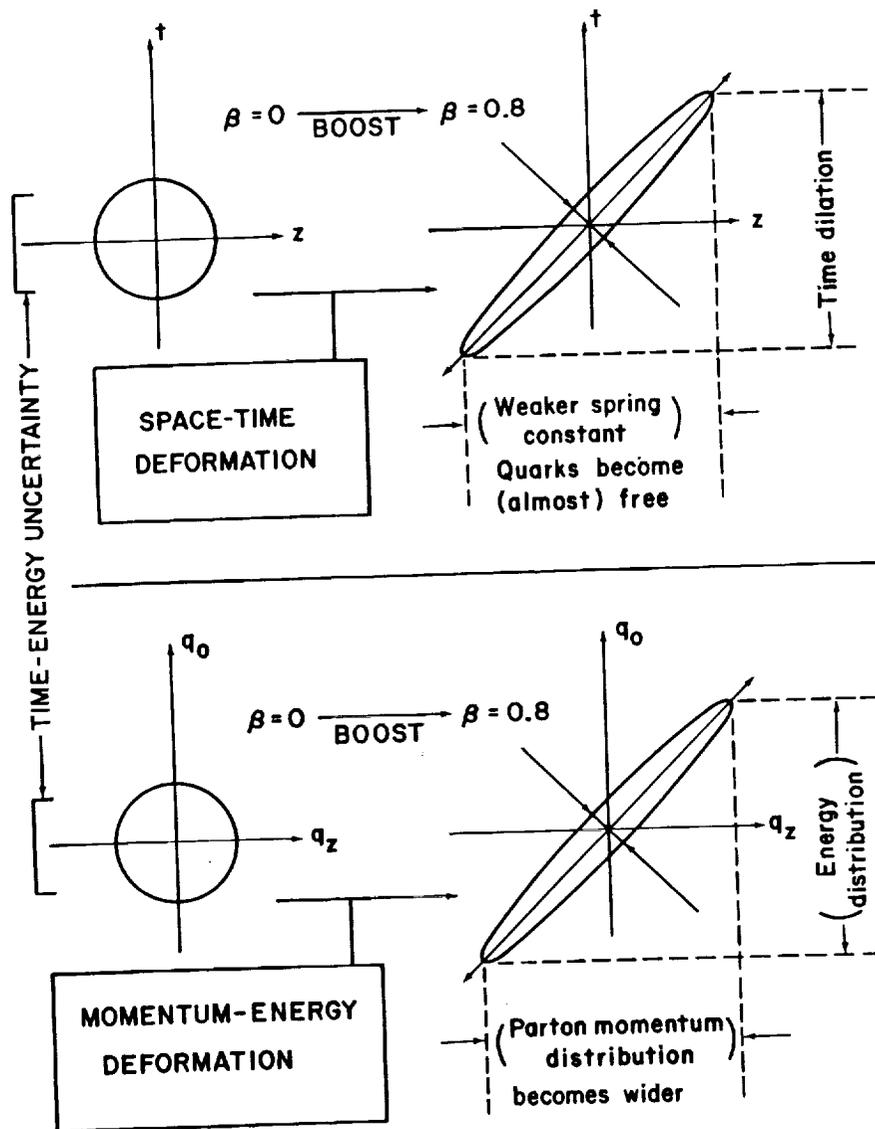


FIG. 1 The Lorentz deformation properties of the relativistic oscillator in coordinate space and in momentum space.

The wave function $\Psi(x_1, x_2, x_3)$ can be written in terms of these variables as $\psi(X)\phi(r)\theta(s)$ where $\psi(X)$ satisfies Eq. (3) while $\phi(r)$ and $\theta(s)$ satisfy respectively

$$\begin{aligned} -1/2(\partial_r^2 - \omega^2 r^2)\phi(r) &= \lambda_r \phi(r) \\ -1/2(\partial_s^2 - \omega^2 s^2)\theta(s) &= \lambda_s \theta(s). \end{aligned} \quad (10)$$

The square mass is Eq. (3) in the nucleon case is then given by $U^0 + \lambda_r + \lambda_s$. To remove the unphysical timelike degree of freedom from the nucleon spectrum of states determined by Eqs. (10), the three-particle relativistic oscillator equation is supplemented by a pair of subsidiary conditions that suppress such excitations in the nucleon rest frame.

Application of the relativistic oscillator model to the structure functions requires construction of the momentum-space wave function in the infinite momentum frame. In a frame in which a meson whose rest-frame wave function is given by (6) is travelling with velocity parameter β along the z -direction, the internal momentum-space wave function is

$$\begin{aligned} \varphi^w(p, \beta) &= \left(\frac{2/\omega}{\pi 2^n w!} \right)^{1/2} H_w \left[\left(\frac{2}{\omega} \right)^{1/2} \left(\frac{p_z - \beta p_0}{(1 - \beta^2)} \right)^{1/2} \right] \\ &\times \exp \left[-1/\omega((p_z - \beta p_0)^2 + (p_0 - \beta p_z)^2)/(1 - \beta^2) \right] \end{aligned} \quad (11)$$

where p represents the momentum conjugate to the internal separation coordinate x , and where transverse degrees of freedom have been neglected. As $\beta \rightarrow 1$, the square magnitude of $\varphi(p, \beta)$ becomes singular along the forward light cone, while vanishing everywhere else. Integrating along the direction perpendicular to the forward light cone results in a distribution for the internal light-cone momentum $p_+ (= p_0 + p_z)$ given by

$$\rho(p_+) = \lim_{\beta \rightarrow 1} \int dp_- |\varphi^w(p, \beta)|^2 \quad (12)$$

where $p_- = p_0 - p_z$. The distribution $\rho(p_+)$ is converted into a distribution in Feynman x by setting $p_{1+} = xP$ and requiring $\rho(x)dx = \rho(p_+)dp_+$.

A similar procedure may be followed in the three-particle case. For three particles the result is[12]

$$\begin{aligned} \rho(x) &= 3m/(2\pi\omega)^{1/2} \sum_{i=1}^w \binom{w}{i} (1/i!)(1/2)^i H_i^2[(m/\sqrt{2\omega})(1 - 3x)] \\ &\times \exp[-(m^2/2\omega)(1 - 3x)^2] \end{aligned} \quad (13)$$

in general, and

$$\rho_0(x) = 3m/(2\pi\omega)^{1/2} \cdot \exp[-(m^2/2\omega)(1 - 3x)^2] \quad (14)$$

if the nucleon is assumed to be described by the oscillator ground state wave function. The variable x in Eqs. (13) and (14) is the momentum fraction variable. A calculation of the proton charge structure function $F_2^{ep}(x)$, can, for example, be based directly on Eq. (14). The result is

$$F_2^{ep}(x) = \langle e_i^2 \rangle mx/(2\pi\omega)^{1/2} \cdot \exp[(-9m^2/2\omega)(x - 1/3)^2] \quad (15)$$

where the average of the charge e_i is taken over the three valence quarks. This calculation ignores scaling effects predicted by QCD and yields only qualitative agreement with experiment.

3 Structure Functions

A valon is a bound state or constituent quark whose internal structure is probed in high energy interactions. To be completely general, valons of different spin as well as flavor should be differentiated. Let $G_{v/N}(x)$ represent a momentum-fraction probability distribution for a valon of type v (v representing spin and flavor) in the nucleon N . A nucleonic structure function $F^N(x, Q^2)$ is expressed in terms of convolutions of $G_{v/N}(x)$ with corresponding structure functions for the valons:

$$F^N(x, Q^2) = \sum_v \int_x^1 dx' G_{v/N}(x') F^v(x/x', Q^2). \quad (16)$$

The Q^2 dependence of the structure functions appears only in $F^v(x, Q^2)$. QCD evolution Eq. (13) for the moments of the structure functions are used to express this dependence. According to Eq. (16), the moments of a nucleon structure function are given by a sum of products of moments:

$$M^N(n, Q^2) = \sum_v M_{v/N}(n) M^v(n, Q^2) \quad (17)$$

where

$$M^{N,v}(n, Q^2) = \int_0^1 dx x^{n-2} F^{N,v}(x, Q^2) \quad (18)$$

and

$$M_{v/N}(n) = \int_0^1 dx x^{n-1} G_{v/N}(x). \quad (19)$$

The evolution equations are the basis for assuming a form for the moments $M^v(n, Q^2)$ of the structure functions $F^v(x, Q^2)$. The $F^v(x, Q^2)$ are understood to be determined by the quark distributions within the valon, v , which distributions can be broken up into components that behave as singlets and as nonsinglets under flavor transformation. The moments $M^v(n, Q^2)$ are correspondingly expressed in terms of singlet and nonsinglet moments, which are defined to be the scaling factors governing the evolution of the moments of such quark distributions in lowest-order, twist-2 QCD. The nonsinglet moments are given by

$$M_{NS}(n, Q^2) = \exp(-d_{NS}^n s) \quad (20)$$

while the singlet moments are

$$M_s(n, Q^2) = 1/2(1 + \rho_n) \exp(-d_+^n s) + 1/2(1 - \rho_n) \exp(-d_-^n s) \quad (21)$$

where

$$s = \ln \left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right). \quad (22)$$

The coefficients d_{NS}^n, d_+^n, d_-^n and ρ_n come from the renormalization group analysis.[13] The constant, Λ , is the usual scaling parameter while Q_0 represents the "starting point" of the evolution.

Since valons of different helicity as well as flavor are to be distinguished, four separate valon distributions will be required to characterize the nucleon. The corresponding moments are denoted as

$$\begin{aligned} U_{\uparrow}(n) &= M_{U_{\uparrow/p}}(n) = M_{D_{\uparrow/n}}(n) \\ D_{\uparrow}(n) &= M_{D_{\uparrow/p}}(n) = M_{U_{\uparrow/n}}(n) \\ U_{\downarrow}(n) &= M_{U_{\downarrow/p}}(n) = M_{D_{\downarrow/n}}(n) \\ D_{\downarrow}(n) &= M_{D_{\downarrow/p}}(n) = M_{U_{\downarrow/n}}(n) \end{aligned} \quad (23)$$

where the symbol \uparrow (\downarrow) denotes that the valon's helicity is parallel (antiparallel) to that of the nucleon, and where the identification of valon distributions within the neutron with the corresponding isospin-reversed distributions in the proton follows from charge symmetry. In terms of the singlet and nonsinglet moments Eqs. (20) and (21) and the valon moments Eq. (23), the moments of the nucleon structure functions $F^{2p}(x, Q^2)$ and $F^{2n}(x, Q^2)$ are given by

$$M^{2p}(n, Q^2) = 2/9[2U(n) + D(n)]M_s(n, Q^2) + 1/9[4U(n) - D(n)]M_{NS}(n, Q^2) \quad (24)$$

$$M^{2n}(n, Q^2) = 2/9[2U(n) + D(n)]M_s(n, Q^2) - 2/9[U(n) - D(n)]M_{NS}(n, Q^2) \quad (25)$$

with

$$\begin{aligned} U(n) &= U_{\uparrow}(n) + U_{\downarrow}(n) \\ D(n) &= D_{\uparrow}(n) + D_{\downarrow}(n). \end{aligned} \quad (26)$$

It is easily verified that these equations describe the lowest-order twist-2 QCD evolution of the moments of F^{2p} and F^{2n} from a starting point at which the nucleon is viewed as consisting of its three bound-state quarks.

Eqs. (24) and (25) were used by Hwa[9] in conjunction with experimental moments of F^{2p} and F^{2n} to obtain fitted values for the parameters Q_0 and Λ . These equations are first order, and will therefore not be accurate for low Q^2 . Ideally we would like to evolve the bound-state momentum-space wave function from the energy scale Q_0^2 at which the nucleon is describable as a bound state of its three constituent quarks (with, perhaps, an oscillator-like momentum distribution), out to high Q^2 where the structure functions are observed. The fitted parameter Q_0^2 is an approximation for Q_0^2 in the sense that the lowest-order evolution equations are used. This approximation is a key feature of the valon model and is discussed in detail in [14]. The goal of Hwa's fitting procedure was to obtain estimates for the functions $G_{v/N}(x)$. In Figure 2, an "average" valon distribution obtained in [9] by neglecting spin and flavor dependence is compared with $\rho_0(x)$ given by Eq. (14)

Let us now introduce a $\underline{70}$ component of SU(6) into the nucleon wave function in the form

$$\Psi = [\cos \theta \phi_0 | 56 \rangle, + (\sin \theta / \sqrt{2})(\phi_\alpha | 70 \rangle_\alpha + \phi_\beta | 70 \rangle_\beta)] \cdot \exp(-iP \cdot X). \quad (27)$$

The ϕ 's represent the spatial wave functions; ϕ_0 is the harmonic oscillator ground state, while ϕ_α and ϕ_β are taken to be excited states with total harmonic oscillator quantum number $n = 2$ and zero orbital angular momentum. The subscripts α and β refer to the two possible types of mixed symmetry which are characterized by the behavior of the (three quark) wave function under exchange of the first and second quarks. The form of the excited-state component is uniquely determined in the oscillator model. The $\underline{70}$ state that involves $n = 1$ oscillator wave functions is disallowed because it is of the wrong parity. No other $n = 2$ state with the same quantum

numbers as the nucleon interferes with the ground state to produce the SU(6) breaking effects that are observed in the structure functions. The wavefunction Eq. (27) leads to spin-and-flavor dependent valon distributions of the form

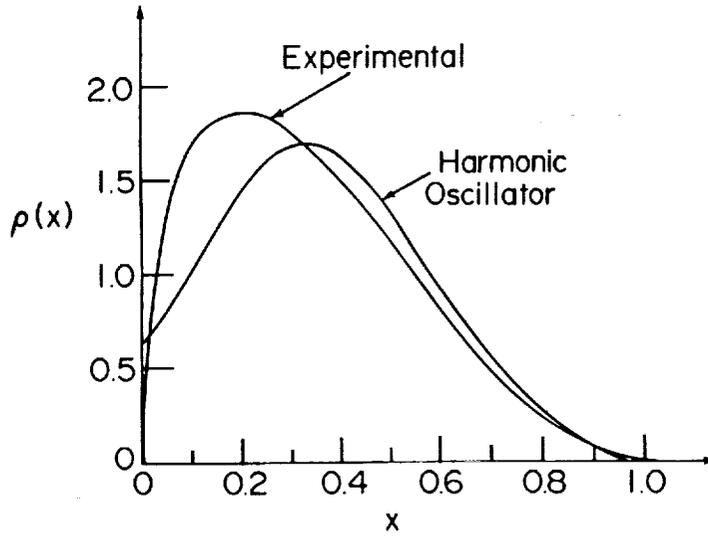


FIG. 2 A comparison of Hwa's "average" valon distribution with $\rho_0(x)$ defined by the infinite-momentum-frame relativistic-oscillator momentum-space wavefunction.

$$\begin{aligned}
 G_{U1/p}(x) &= (3m/\sqrt{2\pi\omega}) \left[5/6 \cos^2 \theta + \sin^2 \theta \{ 5/36h(x) + 1/3i(x) \} - 2\sqrt{6}/9 \cos \theta \sin \theta j(x) \right] \\
 &\quad \times \exp \left[-(m^2/2\omega)(1-3x)^2 \right] \\
 G_{U1/p}(x) &= (3m/\sqrt{2\pi\omega}) \left[1/6 \cos^2 \theta + \sin^2 \theta \{ 1/36h(x) + 1/3i(x) \} + \sqrt{6}/18 \cos \theta \sin \theta j(x) \right] \\
 &\quad \times \exp \left[-(m^2/2\omega)(1-3x)^2 \right] \\
 G_{D1/p}(x) &= (3m/\sqrt{2\pi\omega}) \left[1/3 \cos^2 \theta + \sin^2 \theta \{ 1/18h(x) + 2/3i(x) \} + \sqrt{6}/9 \cos \theta \sin \theta j(x) \right] \\
 &\quad \times \exp \left[-(m^2/2\omega)(1-3x)^2 \right] \\
 G_{D1/p}(x) &= (3m/\sqrt{2\pi\omega}) \left[2/3 \cos^2 \theta + 1/9 \sin^2 \theta + 2\sqrt{6}/9 \cos \theta \sin \theta j(x) \right] \\
 &\quad \times \exp \left[-(m^2/2\omega)(1-3x)^2 \right]
 \end{aligned} \tag{28}$$

where

$$\begin{aligned}
 h(x) &= 43/16 + m^2/8\omega(1-3x)^2 + m^4/16\omega^2(1-3x)^4 \\
 i(x) &= 5/8 + m^2/8\omega(1-3x)^2 \\
 j(x) &= 1/4 - m^2/4\omega(1-3x)^2.
 \end{aligned} \tag{29}$$

Moments $U(n)$ and $D(n)$ determined from the above distributions were used in Eqs. (24) and (25) to obtain fits for experimental moments[15] of $F^{2p}(x)$ and $F^{2n}(x)$ derived from the CHIO

muon data[16] and SLAC electron data[17] at $Q^2 = 22.5 \text{ GeV}^2$. A somewhat large value of Q^2 was chosen to minimize target mass and higher twist effects that may be present in the data. The ratios $R^{np}(x)$ and $A^{np}(x)$ do not appear to show any appreciable Q^2 dependence. The extension of the tails of the distributions into the unphysical regions $x < 0$ and $x > 1$ was ignored for purposes of computing the moments. The resulting small deviation from the Adler sum rule does not appear to lead to noticeable discrepancies.

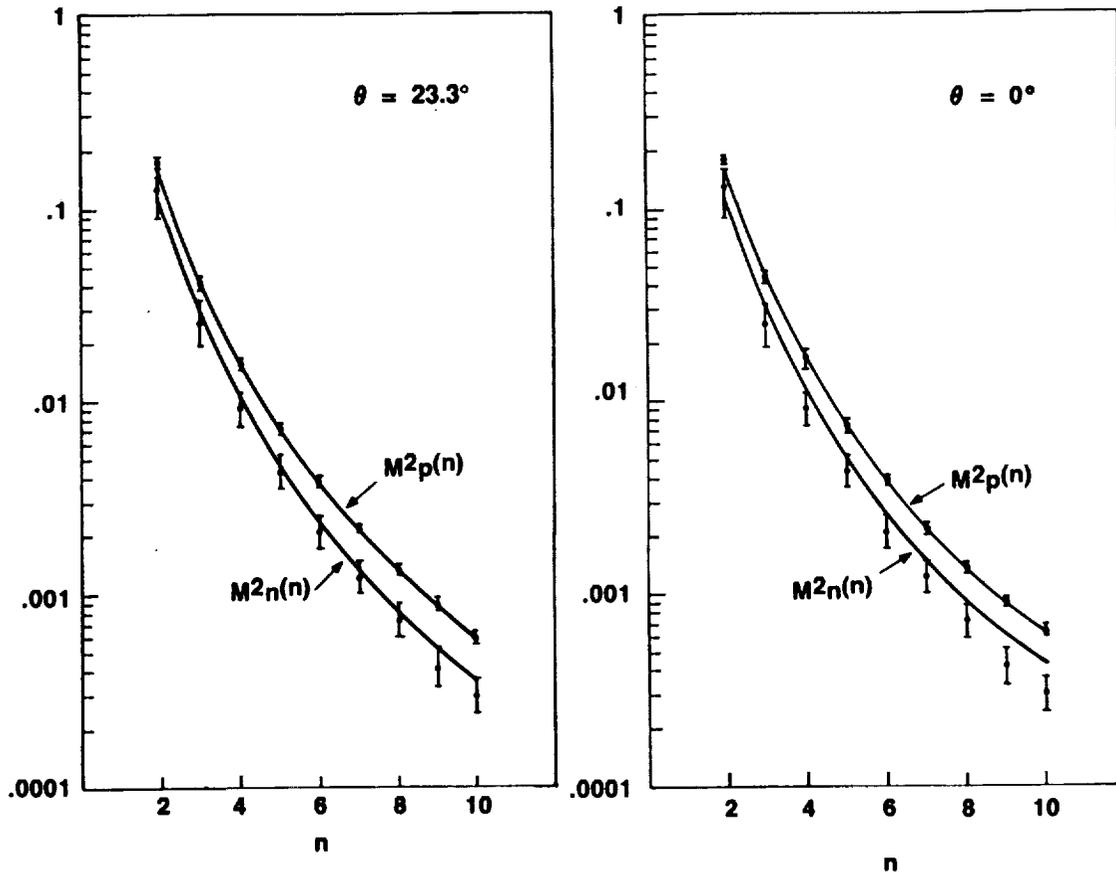


FIG. 3 The moments of the nucleon structure functions vs. n as fitted by Eqs. (24) - (26) in conjunction with Eq. (29). Fitted moments at $\theta = 0^\circ$ and at $\theta = 23.3^\circ$ are represented by the solid curves and are compared with data from[15].

The fitted moments were functions of two parameters - the mixing angle θ and the scaling variable s defined in Eq. (22). χ^2 minimization was used to determine the best fit. The χ^2 function in this case cannot be taken as an absolute indication of the quality of the fit due to the statistical interdependence among the moments of F^{2p} and F^{2n} . \ln^2 was used, rather, as a relative determinant of merit, so that the quality of the fit as a function of θ could be evaluated. The minimum of χ^2 occurs at $\theta = 23.3^\circ$, and a positive mixing angle is clearly preferred. Figure 3 compares the best-obtainable predicted moments from Eqs. (24) and (25) for $\theta = 0^\circ$ and for $\theta = 23.3^\circ$ with

the experimental moments. At $\theta = 0^\circ$, the fitted moments of F^{2n} fall outside the error limits for large n . With the inclusion of the $\overline{70}$ state in the wave function at a mixing angle of 23.3° , a simultaneous fit to the moments of F^{2p} and F^{2n} appears more reasonable, although the fitted moments of F^{2n} remain somewhat large for large n .

4 Conclusion

The simple model presented in this paper falls short of providing us with the ability to draw precise numerical correspondences between nucleonic bound state properties and the structure function data. The model does, however, address the crucial questions of Lorentz transformation and momentum scaling that must be considered if such correspondences are ever to be drawn. The approximate agreement between $\rho_0(x)$ and Hwa's phenomenologically-determined valon distribution (see Figure 2) allows us to believe that some of the essential physics is being captured. The value of the SU(6) $\overline{70}$ state mixing coefficient t obtained in this model via a simultaneous fit to proton and neutron structure function moments is very close to the original value determined by Le Yaouanc *et al.* This fact, together with the dependence of the form factors on the nucleon spin wavefunction, lends credence to the idea that the observed behavior of R^{np} and A^{np} can be reliably interpreted as evidence of SU(6) mixing in the nucleon wavefunction.

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13. ABSTRACT (Maximum 200 words) This conference publication contains the proceedings of a workshop on harmonic oscillators held at the College Park Campus of the University of Maryland on March 25-28, 1992. The harmonic oscillator formalism has been and still is playing an important role in many branches of physics. This is the simplest mathematical device which can connect the basic principle of physics with what we observe in the real world. The oscillator formalism is, therefore, a very useful language in establishing communications among the physicists interested in fundamental principles and those interested in describing what we observe in laboratories. Researchers in different branches of physics, such as atomic, nuclear and particle physics, quantum optics, statistical and thermal physics, foundations of quantum mechanics and quantum field theory, and group representations, are developing possible future theories. The harmonic oscillator is the bridge between pure and applied physics.				
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