ENCODING SPATIAL IMAGES - A FUZZY SET THEORY APPROACH

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ABSTRACT

As the use of fuzzy set theory continues to grow, there. is an increased need for methodologies and formalisms to manipulate obtained fuzzy subsets. Concepts involving relative position of fuzzy patterns are acknowledged as being of high importance in many areas.

In this paper, we present an approach based on the concept of dominance in fuzzy set theory for modelling relative positions among fuzzy subsets of a plane. In particular, we define the following spatial relations: to the left (right), in front of, behind, above, below, near, far from, and touching.

This concept has been implemented to define spatial relationships among fuzzy subsets of the image plane. Spatial relationships based on fuzzy set theory, coupled with a fuzzy segmentation should therefore yield realistic results in scene understanding.

INTRODUCTION

One of the main difficulties in computer vision is the difference between how a human sees a scene and how a computer sees it. A human may see a large red building between two trees, but the computer "sees" only a two-dimensional array of pixel values.

To design a user interface for computer vision that can be used without extensive special training we have to translate from the computer's view to the human's. We must segment the image, properly label the objects in it, and then describe the objects both in terms of their absolute properties and in terms of their properties relative to each other.

This paper proposes to examine ways of defining and deriving the relative spatial properties of the objects in an arbitrary scene.

A Need of Fuzzy Set Theory in Computer Vision

In computer vision, the standard approach to image analysis and

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recognition is to segment the image into regions and to compute various properties of and relationships among these regions. However, the regions are not always "crisply" defined. It is sometimes more appropriate to regard them as fuzzy subsets of the image.

In the last several years, there has been increased attention given to the use of fuzzy set theory in image segmentation [1, 2, 3, 4].

When the objects in a scene are represented by crisp sets, the all-or-nothing definitions of the subsets actually add to the problem of generating such relational descriptions. It is our belief that definitions of spatial relationships based on fuzzy set theory, coupled with a fuzzy segmentation will yield realistic results.

For the purpose of this work we assume that we deal with an image of objects, that is, the scene has already been segmented and the objects have been labelled. The segmentation may be either crisp or fuzzy.

Using the above considerations the problem may be looked at in three different ways:

i. Given a scene, describe (linguistically) the spatial relations between the objects in the scene,

ii. Given a scene and a spatial description of an object, find that object in the scene,

iii. Given the spatial relations between the objects , construct a scene, locating the objects so as to satisfy those spatial relations (this is the "layout" problem).

This work concentrates on the first two problems, although the resulting definitions of spatial relations will be useful for the "layout" problem.

SPATIAL RELATIONS AMONG FUZZY SUBSETS

Spatial relationships between regions in an image play important role in scene understanding. Humans are able to quickly ascertain the relationship between two objects, for example "B is to the right of A", or "B is in front of A", but this has turned out to be a somewhat illusive task for automation [5, 6, 7].

When the objects in a scene are represented by crisp sets, the all-or-nothing definitions of the subsets actually add to the problem of generating such relational descriptions. It is our belief that definitions of spatial relationships based on fuzzy set theory, coupled with fuzzy segmentation will yield realistic results.

The Idea of Projections

This work proposes an initial approach at defining spatial relationships among fuzzy subsets of the image plane.

The idea is to project the fuzzy subsets onto two orthogonal coordinate axes and to utilize fuzzy dominance relations to capture the approximate relationships.

Let A be a fuzzy subset of an image. Then $A \subseteq U \times V$, where U is the first spatial coordinate axis and V is the second one. In our case, both U and V are subsets of the reals (assumed to be the interval [0, 1] for convenience). Then $\mu_A(x, y)$ is a fuzzy relation in $U \times V$. The projection of A onto U, denoted A_U is that fuzzy subset of U given by

$$\mu_{Au}(x) = \sup_y \langle \mu_A(x,y) \rangle$$

for each $x \in U$.
A similar equation defines the projection of A onto V, that is

$$\mu_{Av}(y) = \sup_x \langle \mu_A(x,y) \rangle$$

for each $y \in V$.

For a fuzzy subset C of U, the α-level set C^α is defined by

$$C^\alpha = \langle x \in U \mid \mu_C(x) \geq \alpha \rangle$$

for $\alpha \in [0, 1]$.
When $\alpha = 0$, the inequality is usually considered to be strict and the C^α is called the support of C.

Definitions of Spatial Relations for Fuzzy Objects

Once the two fuzzy subsets A and B are projected onto U and V axes, methods must be defined to access their relative position.
In this paragraph we introduce definitions for spatial relations.

<u>Definition 1</u> : We say that subset A is to the right of subset B if the projection of A onto the U axis dominates the projection of B, while the projections onto the V axis are (ideally) identical. In other words $\mu_v(\alpha)$ should stay near zero for all α (especially for small α).
Similar definitions are suggested for all other spatial relations [13, 14].
The definitions are for antisymmetric and transitive relations, that is TO THE LEFT (RIGHT) OF, IN FRONT OF (BEHIND), ABOVE (BELOW), INSIDE (OUTSIDE). They are strict partial order relations (i.e. reflexive, antisymmetric and transitive) and every one has a semantic inverse.

Separation Measure

Let A_U, B_U, A_V, B_V be the projections of A and B onto U and V, respectively. Since these projections are fuzzy numbers, their α-level sets are intervals, i.e.,

$A_U^{\alpha} = [A_U^{\alpha l}, A_U^{\alpha r}]$, etc.

For the projections of A and B onto the U axis, the α-separation of A and B is defined by

$$S_U^{\alpha} = (\overline{A}_U^{\alpha} - \overline{B}_U^{\alpha})^2 / (W_{Au}^{\alpha} + W_{Bu}^{\alpha})^2$$

where

$$\overline{A}_U^{\alpha} = (A_U^{\alpha l} + A_U^{\alpha r}) / 2,$$

$$\overline{B}_U^{\alpha} = (B_U^{\alpha l} + B_U^{\alpha r}) / 2,$$

$$W_{Au}^{\alpha} = (A_U^{\alpha r} - A_U^{\alpha l}) / 2, \text{ and}$$

$$W_{Bu}^{\alpha} = (B_U^{\alpha r} - B_U^{\alpha l}) / 2.$$

Now, S_U^{α} is the ratio of the square of difference between the midpoints of the α-level sets and the square of the sum of the half-widths of these intervals. Similar equations are used for the projection of A and B onto the V axis.

<u>Definition 2</u> : We say that A_U and B_U are α-separated if $S_U^{\alpha} > 1$.

<u>Definition 3</u> : We say that A_U and B_U are α-just separated if $S_U^{\alpha} = 1$.

<u>Definition 4</u> : We say that A_U and B_U are α-overlapping if $S_U^{\alpha} < 1$.

<u>Theorem 1</u> : i) A_U and B_U are α-separated if and only if $A_U^{\alpha r} < B_U^{\alpha l}$.

ii) A_U and B_U are α-just separated if and only if $A_U^{\alpha r} = B_U^{\alpha l}$.

iii) A_U and B_U are α-overlapping if and only if $A_U^{\alpha r} > B_U^{\alpha l}$.

The proof of the theorem can be found in [9].

The value of these definitions and theorems is two-fold. First, they incorporate the fuzziness in the description of image regions, i.e., they use fuzzy subsets of the plane. Second, they deal with the ambiguity of defining spatial relationships in the plane. By this we mean that it is possible that parts of the two sets can overlap (small α) and yet be well separated for large α.

The values of S_U^α can get arbitrarily large as the widths of the level set intervals get small. In order to create a fuzzy membership function, we will map the interval $[0, \omega)$ into $[0, 1]$ by an "S-shaped function" [15] as follows. For a given α, suppose $A_U^\alpha = [0. 0.2]$ and $B_U^\alpha = [0.8, 1]$. (Recall that we have scaled the domain of the image into the unit square). Then $S_U^\alpha = 16$. This amount of separation (or more) will be considered complete, i.e., $\mu(S_U^\alpha) = 1$ if

$S_U^\alpha \geq 16$. Also we will require that $\mu(0) = 0$, $\mu(1) = 0.5$ and $\mu'(16) = 0$. Such a function is defined in our case by:

$$
\mu(S) = \left\{
\begin{array}{ll}
0.5\ S^2 & 0 \leq S \leq 1 \\
-0.0022\ S^2 + 0.0711\ S + 0.4311 & 1 < S \leq 16 \\
1 & S > 16
\end{array}
\right.
$$

The Model for Spatial Relationships

The model for given spatial relationships can now be defined from the fuzzy subsets μ_U and μ_V of $[0, 1]$. For example, to model the relationship "A IS TO THE RIGHT OF B", we would like the projection of A onto the U axis to dominate that of B; wheares the projections should (ideally) be identical on the V axis. That is, $\mu_V(\alpha)$ should stay near zero for all α (especially for small α). Similar observations can be made for "ABOVE", and "BELOW".

Instead of dealing with two fuzzy subsets, μ_U and μ_V can be combined into a single set from which the relationship can be determined. Fuzzy set theory offers an infinite number of aggregation operators, which, given two pieces of evidence (values in $[0, 1]$) can produce essentially any composite value between 0 and 1, depending on the type of connective and the parameters chosen. Union operators produce values greater than or equal to the maximum of the two numbers; intersection operators give a result less than or equal to the minimum; and generalized means fill the gap between the minimum and

maximum [11].

"TO THE RIGHT OF" should therefore be a combination of μ_U and the complement of μ_V since its large values signify that the level sets of A are "above or below" those of B.

For the experiments described in the next paragraph, we chose a generalized mean

$$m(\mu_U, \mu_V) = [W \mu_U^P + (1 - W) (1 - \mu_V)^P]^{1/P}$$

as the aggregation connective [16]. In this way, higher weight can be associated with the horizontal component with decreased compensation as the level sets diverge vertically. Note, that if P -> ∞, then we have [11]:

$$\lim_{p \to \infty} m(\mu_U, \mu_V) = \max(\mu_U, \mu_V).$$

Either the two fuzzy sets μ_U and $(1-\mu_V)$ or the single aggregated set $m(\mu_U, \mu_V)$ can be used to define the relation "A IS TO RIGHT OF B". If a single value for the degree to which the two sets satisfy the relation is desired, we can construct a fuzzy measure from the sets - such as the integral of the fuzzy number, or the output of an ordered weighted average (OWA) [12]. An alternate approach is to use the curves directly to define a linguistic assessment of the relation. Here, it is necessary to define fuzzy sets representing terms used in the relation, such as "to the right of", "somewhat to the right of", "barely to the right of", "very to the right of", etc. These sets could be defined by the designer of the system, or perhaps, by utilizing a group of humans to give relative comparisons of a set of examples. The actual curve is then matched to the closest term available to give the linguistic assessment. This process is known as linguistic approximation [13].

Results of Sample Systems

All the definitions and theorems listed above were tested using simulated data on a computer workstation. Fuzzy subsets with two-sided drum like shaped membership functions on projections were used. The experiments were as follows. Let us consider an image containing two fuzzy subsets A and B whose

membership functions are identical gaussians, but with different mean locations. The set B will be fixed with mean (0.5, 0.5). Table 1 shows the fuzzy set μ_U generated from eight choices of locations for the mean of A (assume that the V coordinate for the mean is 0.5). As can be seen, as the set A moves to the right, the fuzzy set μ_U increases for all α. Recall that the value $\mu_U(\alpha) = 0.5$ represents the just separated condition. The seven α-values are 0.011, 0.135, 0.258, 0.606, 0.796, 0.882, 0.923. They were chosen in order to get the following ranges from the mean of gaussian functions: $-$ 0.4σ, $-$ 0.5σ, $-$ 0.6745σ, $-$ σ, $-$ 1.645σ, $-$ 2σ, $-$ 3σ, where σ is a standard deviation.

Mean of Projections of A	α_1	α_2	α_3	α_4	α_5	α_6	α_7
0.525	.001	.002	.003	.008	.017	.031	.049
0.550	.014	.031	.046	.125	.275	.500	.516
0.575	.070	.158	.234	.508	.543	.579	.623
0.600	.222	.500	.514	.564	.622	.680	.731
0.625	.503	.536	.558	.631	.712	.788	.851
0.650	.532	.580	.609	.706	.806	.891	.950
0.675	.567	.628	.665	.783	.893	.968	.998
0.700	.604	.680	.724	.857	.962	1.00	1.00

Table 1. Membership functions generated from the projection of A onto U axis.

Since the projections onto V for these sets are the same as the projections onto U, the fuzzy sets from Table 1 can be used to simulate other placings of A relative to B, e.g., to the northeast or southeast. Table 2 shows four cases for the placement of the center of set A along with the aggregated fuzzy set generated from both projections. Generalized mean with W = 0.75 and P = 2 was used. The first case represents a set A which is east of B. Here, the combined values are larger than those for the U projections only. In fact, even the smallest α (0.011) gives rise to a membership larger than 0.5 (the just separated crossover point). In case 2, the set A has moved to the north east of B. The movement north effectively decreases the membership in the fuzzy set "A is to the right of B". Cases 3 and 4 depict the situation where A is directly above B. As the centers move further apart, the membership drops dramatically.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7
(.6, .5)							
μ_U	.222	.500	.514	.564	.622	.680	.731
$1 - \mu_V$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$m(\mu_U, \mu_V)$	0.54	0.66	0.67	0.70	0.73	0.77	0.81
(.6, .6)							
μ_U	.222	.500	.514	.564	.622	.680	.731
$1 - \mu_V$	.778	.500	.486	.436	.378	.320	.269
$m(\mu_U, \mu_V)$	0.43	0.50	0.51	0.53	0.57	0.61	0.65
(.5, .6)							
μ_U	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$1 - \mu_V$	.778	.500	.486	.436	.378	.320	.269
$m(\mu_U, \mu_V)$	0.39	0.25	0.24	0.22	0.19	0.16	0.13
(.5, .7)							
μ_U	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$1 - \mu_V$	.396	.320	.276	.143	.038	0.00	0.00
$m(\mu_U, \mu_V)$	0.20	0.16	0.14	0.07	0.02	0.00	0.00

Table 2. Combined membership function for the relation "A is to the right of
B" (W = 0.75, P = 2).

If we change either the weight W or the exponent P, we can alter the
shape of the resultant fuzzy set. For more details see [9].

Summary and Conclusions

A new approach, based on the concept of dominance in fuzzy set theory, for modelling spatial relationships among fuzzy subsets of an image has been proposed. Simulation results were presented to corroborate the theory and demonstrate the power of the approach for image description.

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