

# A Note on the Computation of Antenna-Blocking Shadows 

R. Levy<br>Ground Antennas and Facilities Engineering Section


#### Abstract

A simple and readily applied method is provided to compute the shadow on the main reflector of a Cassegrain antenna, when cast by the subreflector and the subreflector supports. The method entails some convenient minor approximations that will produce results similar to results obtained with a lengthier, mainframe computer program.


## I. Introduction

The performance of conventional Cassegrain antennas is affected by the interference of the tripod or quadripod structures that support the subreflector. The interference is in the form of a shadow cast on the main reflector surface. The shadow blocks a portion of the microwave energy in either transmission or reception and is caused by the supporting tripod or quadripod legs, and also, to a less serious extent, by the subreflector. Such shadows can be avoided by offset antenna configurations, but the extra difficulties associated with offset construction are usually evaluated to be more objectional than the effects of blocking. Consequently, the symmetrical Cassegrain antenna predominates. Thus, it is important to be able to evaluate the extent of the blocking.

## II. Analysis

The shadowed area consists of two types of blocking: plane wave and spherical wave. The plane-wave blocking is caused by the projections of the subreflector and an upper portion of the support legs on the aperture plane. The spherical-wave blocking is the shadow on the aperture plane of rays emanating from the focal point that
intersect the lower portion of the support leg. Figure 1, which is taken from [1], shows typical shadows from the two effects projected on the aperture plane. J. Herndon [1] developed a comprehensive numerical-integration computer program to calculate the blocked areas. However, results close to those from his computer program, which requires a mainframe computer, can be readily obtained with some simple approximations. The method presented here is suitable for execution on either a programmable calculator or a personal computer.

Figure 2(a) is a profile sketch of the reflector, taken in the plane of one of the support legs. The leg is assumed to have a trapezoidal cross section that is opaque with respect to transmission of microwave energy. Symbols of the figure are
$F=$ focal length
$R=$ main-reflector radius
$R_{S}=$ subreflector radius
$S=$ radial distance to centerline of leg at the intersection with the main-reflector surface
$Z_{o}=Z$ coordinate at $S$

## $Z_{S}=Z$ coordinate at back of subreflector

$h=$ half of the leg depth

$$
\begin{aligned}
w_{I}= & \text { width of leg structure at inside face } \\
w_{O}= & \text { width of leg structure at outside face } \\
\beta= & \text { angle from the focal point to the rim of the main } \\
& \text { reflector } \\
\phi= & \text { slope of surface at intersection with the center- } \\
& \text { line of the leg } \\
\psi= & \text { slope angle of the leg }
\end{aligned}
$$

Figure 2(b) is an expanded detail at the intersection of the leg with the surface. $S_{I}$ and $S_{O}$ are the radial distances to the points where the extensions of the inner and outer faces of the leg would intersect the surface, and $Q$ is the distance along the tangent from the leg centerline to either of the intersection points at $S_{I}$ or $S_{O}$. The relatively small curvature makes it reasonable to replace the curved surface by the tangent in the vicinity of $S . Q$ is given by

$$
\begin{equation*}
Q=\frac{h}{\sin (\psi+\phi)} \tag{1}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
S_{I}=S-Q \cos \phi \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{0}=S+Q \sin \phi \tag{3}
\end{equation*}
$$

Figure 2(c) shows the spherical-wave shadow of the leg as a trapezoid of length $\mathrm{R}-S_{O}$. To find the maximum width of the trapezoid at the rim of the antenna, $w_{M}$, it is necessary to find the distances $X_{I}$ and $X_{O}$ where a ray from the focal point to the rim crosses the inner and outer faces of the leg. To find $X_{I}$, for example, one has

$$
\begin{equation*}
F-Z_{I}=X_{I} / \tan \beta+\left(S_{I}-X_{I}\right) \tan \psi \tag{4}
\end{equation*}
$$

in which $Z_{I}$ is the $Z$ coordinate at $S_{I}$. By introducing $Z_{O}$, the $Z$ coordinate at $S_{O}$, a similar expression can be formed for $X_{O}$, and these expressions can be used to determine $X_{I}$ and $X_{O}$.

If the width at the outer face of the leg governs the spherical-wave shadow, then the width of the trapezoid $w_{B}$ at $S_{O}$ is $w_{O}$, and the width at the rim is

$$
\begin{equation*}
w_{M}=w_{O} R / X_{O} \tag{5}
\end{equation*}
$$

If the width at the inner face of the leg governs, it is necessary to find the width of the trapezoid at $S_{O}$. To do this, one uses the distance $X_{I O}$, which is where a ray from the focal point to the surface at $S_{o}$ intersects the inner leg face. $X_{I O}$ can be found from the following expression:

$$
\begin{equation*}
F-Z_{I}=X_{I O} / \tan \beta_{P}+\left(S_{I}-X_{I O}\right) \tan \psi \tag{6}
\end{equation*}
$$

in which

$$
\begin{equation*}
\tan \beta_{P}=S_{O} /\left(F-Z_{O}\right) \tag{7}
\end{equation*}
$$

and in this case the width of the spherical-wave-blocking trapezoid at its base is

$$
\begin{equation*}
w_{B}=w_{I} S_{O} / X_{I O} \tag{8}
\end{equation*}
$$

and the width at the rim is

$$
\begin{equation*}
w_{M}=w_{I} R / X_{I} \tag{9}
\end{equation*}
$$

The ideal profile for the leg cross-section is when the outer face provides the same width at the rim as the inner face does. In this case, the outer width would be

$$
\begin{equation*}
w_{O}(o p t)=w_{I} S_{I} / X_{I O} \tag{10}
\end{equation*}
$$

The foregoing computations imply several approximations that are expected to have only a minor effect on the results. These are
(1) The leg is assumed to be entirely opaque. This is usually an accurate assumption for the sphericalwave shadow of the leg because the inner, narrowest face is often an opaque solid plate. The plane-wave portion of the leg shadow could be reduced because of a less-than-unity solidity ratio of the projection of the leg trusses on the aperture plane. Consequently, this openness could allow microwave energy to pass through the open spaces. It is conservative to ignore the energy that could pass through the leg regions, and, if it is thought to be significant, the plane area shadow can be reduced by the complement of the solidity ratio. This could possibly reduce the total leg shadow by 10 to 15 percent.
(2) The spherical-wave leg shadow is modelled by the projection of a trapezoid on the aperture plane. The long sides of the trapezoid actually are curved, and the approach here slightly overestimates the shadow.
(3) The curve of the outer reflector rim is replaced by the straight edge of the trapezoid.
(4) The leg profile is taken to have a constant crosssection for the full length, and any customary tapering towards a narrow point at the leg base is ignored.

## III. Method

Figure 3 provides a MATLAB program ${ }^{1}$ to calculate the blocked shadow essentially as described above and to complete some of the omitted details. The total planeand spherical-wave shadow areas and the relative proportions of each are provided. In addition, the user-furnished dimension $Z_{S}$ is used to determine the clearance between the back edge of the subreflector and the inner support leg. A moderate acquaintance with any high-level coding language, such as FORTRAN, should make the code understandable, even for one with no prior exposure to MATLAB. However, the following may help one unfamiliar with the program:
(1) The \% symbol is interpreted as the beginning of a nonexecutable comment.
(2) MATLAB is case sensitive, and almost all instructions and built-in functions require lowercase.
(3) The choice (in this program) was to represent all the variables (including those of Fig. 2) in uppercase. WI represents $w_{I}$, TANBETA represents $\tan \beta$, PSI is $\psi$, and so forth.

[^0](4) At the prompt "supply-," the user can type NLEGS $=\ldots$, PSI $=\ldots$, etc., and when all the requested data are supplied, the user should type "return"; for versions of MATLAB prior to Version 4.0, "CTRL-Z" should be supplied to satisfy the requirements of the command "keyboard." If no data are supplied before providing "return" or "CTRL-Z," the default data, which are for DSS-15 class antenna quadripods, will be executed.

## IV. Summary and Conclusions

The algorithms contained in the program of Fig. 3 were previously executed many times by a programmable desk calculator to compute the blocking during the design stages of the new quadripods for the $70-\mathrm{m}$ antennas, the $34-\mathrm{m}$ high-efficiency antennas, and the $34-\mathrm{m}$ beamwaveguide tripod. The UNIVAC mainframe computer [1] program was used a number of times, and it was established that results agreed within practical requirements. In particular, in [1] an example computation for a $64-\mathrm{m}$ antenna showed a blocking shadow of 7.35 percent and the procedure here showed 7.38 percent. The total leg-shadow area agreement was within 0.5 percent, although the allocations for the plane and spherical shadow contributions had larger differences. These differences could have been caused by the treatment of the pointed bases of the legs in the reference, which may have influenced whether a particular area was assigned to plane- or spherical-wave blocking. Both sets of computations, of course, agreed exactly on the subreflector shadow, which was only about 15 percent of the total.

The sample data built into the Fig. 3 program (data which the user is given the opportunity to replace) will result in a total shadow of 5.478 percent. The effect on the microwave antenna is more severe than the geometric aperture area reduction, perhaps by a factor of about two.

## Reference

[1] J. Herndon, "Efficient Antenna Systems: A Program to Calculate the Optical Blockage by the Quadripod on Large Microwave Antennas," Space Programs Summary 97-48, vol. II, Jet Propulsion Laboratory, Pasadena, California, pp. 5863, November 30, 1967.


Fig. 1. Plane- and spherical-wave blocking.

(c)



Fig. 2. Blocking geometry: (a) overall geometry, (b) detail at base of leg, (c) intersection with leg, and (d) leg-shadow projection.

```
%This is MATLAB\MISCPROBS\BLOCKING.M, Feb.10,1993
% compute blocking of subreflector and tripod or quadripod
% The following two functions are expected to be available to MATLAB:
% function }y=sine(x) function y=cosine(x
% y=sin(x*pi/180); y=cos(x*pi/180);
format compact
% Set some default values for 34M HEF antenna
    NLEGS=4; PSI=61.3967; F=434; ZS=406.7;
    R=669.3; RS=75.; S=328; H=19.45; WI=9.5; WO=14.
disp('supply- NLEGS,PSI,F,ZS,R,RS,S,H,WI,WO, AND "return" ')
keyboard
Tanphi=S/2/F;
PHI=atan(Tanphi)*180./pi;
Q=H/sine(PSI+PHI)
SI=S-Q*cosine(PHI)
SO=S+Q*cosine(PHI)
ZI=SI*SI/4/F
2O=SO*SO/4/F
ZMAX=R*R/4/F;
TANBETA=R/(F-ZMAX)
TANP=sine(PSI)/cosine(PSI)
DEN=1/TANBETA-TANP
XI=(F-ZI-SI*TANP)/DEN
XO}=(F-ZO-SO*TANP)/DEN
TANBETAP=SO/(F-ZO)
DENP=1/TANBETAP-TANP
XIO=(F-ZI-SI*TANP)/DENP
MAGI=R/XI
MAGO=R/XO
MAGIO=SO/XIO
WOPT=MAGIO*WI
AFACT=pi/144.
ASUB=RS*RS*AFACT %SQUARE FEET
AMAIN=R*R*AFACT
if WO>=WOPT
    ASPH=(R-SO)*WO*.5*(1+MAGO);
else
    ASPH=(R-SO)*WI*.5* (MAGIO+MAGI);
end
ASPH=ASPH*NLEGS/144
APLANE=WO* (SO-RS) *NLEGS/144
LEGSHAD=ASPH+APLANE
TOTSHAD=LEGSHAD+ASUB
TOTPCT=TOTSHAD/AMAIN*100
LEGPCT=LEGSHAD/AMAIN*100
% Blocking calculations completed above
% Now get leg-to-subreflector clearances
CLH=SI-(ZS-ZI)/TANP-RS % horizontal clearance
CLP=CLH*sine(PSI) % perpendicular to leg face clearance
```

Fig. 3. Program to calculate blocking.


[^0]:    ${ }^{1}$ MATLAB is a registered trademark of The Math Works, Inc., Co chituate Place, 24 Prime Park Way, Natick, MA 01760.

