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MAPPING METHODS FOR COMPUTATIONALLY EFFICIENT AND ACCURATE STRUCTURAL RELIABILITY

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The structures community has recognized that uncertainties in the structural parameters as well as in the service environments need to be considered in evaluating structural integrity and reliability. Probabilistic structural analyses are formal methods to include those uncertainties. However, these methods are inherently computational intensive because of the large number of deterministic analyses required to accurately simulate the effect of the uncertainties on the desired structural response (such as stress, displacement, and frequency) required for structural reliability assessment. Moreover, modern structures are often analyzed by finite element methods. Probabilistic structural analyses using finite element models can be economical when relatively coarse meshes are employed. Finite element analyses using coarse meshes not only raise questions regarding the convergence to the deterministic values, but also significantly alter the true probabilistic distributions of the structural responses.

It is important, therefore, to evaluate the influence of mesh coarseness on the accuracy of structural reliability. Several alternatives were recently examined at Lewis. The objectives of this presentation are to briefly describe these alternatives and to demonstrate their effectiveness. The results show that special mapping methods can be developed by using (1) deterministic structural responses from a fine (convergent) finite element mesh, (2) probabilistic distributions of structural responses from a coarse finite element mesh, (3) the relationship between the probabilistic structural responses from the coarse and fine finite element meshes, and (4) probabilistic mapping. The structural responses from different finite element meshes are highly correlated. Using this correlation together with the probabilistic potential energy variation principle (ref. 1) that determines the mean and standard deviation of a structural response for a given finite element mesh, one can obtain the linear relationships shown in equation (1):

$$X_f = A\, X_c + B \qquad (1)$$

where X_f and X_c are the random responses from fine and coarse meshes, respectively, and A and B are constants. Three relationships are derived for three different mapping methods. Shift mapping considers only the first moment correction: A in equation (1) is equal to 1, and B is the difference between the deterministic responses from fine and coarse meshes. Shift mapping states that the shapes of the probabilistic distribution functions of structural responses are the same for different meshes, but their mean values vary. Ratio mapping is from a second moment correction: A in equation (1) is the ratio of fine and coarse mesh deterministic responses, and B is equal to zero. Ratio mapping represents not only the fact that the mean values are different but also that the scatters around the mean can be either wider or narrower. Therefore, only the reduced response variables are considered to be the same. Mixed mapping is the average of shift and ratio mapping. Once this relationship is developed, probabilistic mapping can be applied.

Four examples were studied to verify this methodology. The computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) (ref. 2) was used to perform the probabilistic structural analyses. In the first example, a cantilever plate subjected to lateral pressure was analyzed. Plate thickness and the uniform pressure were considered to be random variables. In the second example, a buckling analysis of a simply supported composite plate was performed. The random variables were the coefficients of the stiffness matrix for the stress resultants/generalized strains relations. The probabilistic distributions of those coefficients were computed by the computer code PICAN (Probabilistic Integrated Composite Analyzer). In the third example, a cantilever plate subjected to thermal and mechanical loads was analyzed. Three random fields (uncertainties) were considered - thickness, modulus, and temperature. Each field consisted of correlated nodal random variables, and the loads at the free edge were also considered to be random. The structural parameters, such as modulus and strength, deteriorated under the aggressive service environments. These effects were characterized by the Multi-Factor Interaction Model (ref. 3). In this example, even with a poor mesh (16 percent error in the deterministic response), the probabilistic distribution of the response and the structural reliability using ratio mapping compare very well with those using fine finite element mesh. In addition, the ratio of the computational time using coarse and fine mesh was about 1:20. In the final example, a tapered cantilever plate with variable thickness was studied. The uncertainties considered were the same as in the previous example. In each example, only the maximum effective stresses from coarse and fine meshes using different mapping methods were compared. From examples 1 and 2, we verified the equality of the ratios r_1 and r_2 as defined in the viewgraph "Comparisons Between Probabilistic Distributions from Different Mapping Methods, Cantilever Plate Subjected to Uniform Load." This is essential to the derivation of the ratio mapping. We also found that (1) the shift mapping works well only with a good coarse mesh; (2) ratio mapping, which provides very accurate probabilistic distribution and the structural reliability even with a very coarse mesh, is highly recommended; and (3) results from mixed mapping always lie between those from shift and ratio mapping.

In conclusion, mapping methods were developed to perform probabilistic structural analyses by using coarse finite element meshes. High accuracy was achieved, and computational time was saved. Therefore, the dilemma experienced using either coarse or fine meshes for the probabilistic structural analyses was resolved.

REFERENCES

1. Liu, W.K.; Mani, A.; and Belytschko, T.: Finite Element Methods in Probabilistic Mechanics. Probabilistic Engineering Mechanics, vol. 2, no. 4, Dec. 1987, pp. 201-213.

2. Probabilistic Structural Analysis Methods for Select Propulsion System Components (PSAM). Vols. 1-3, NASA CR-185125-Vol-1,-2,-3, 1989.

3. Shiao, M.C.; and Chamis, C.C.: A Methodology for Evaluating the Reliability and Risk of Structures Under Complex Service Environments. 31st Structures, Structural Dynamics and Materials Conference, part 2, AIAA, 1990, pp. 1070-1080.

Presentation Outline

- Background
- Objective
- Mathematical foundation and derivation
- Numerical examples
- Concluding remarks

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Background

- Probabilistic structural analysis using convergent finite element mesh is computationally inefficient
  - Large amount of CPU time is needed
  - Long turn-around time is expected
- Probabilistic structural analysis using coarse finite element mesh may not provide accurate results
- Accuracy can be improved by
  - Shifting the mean
  - Adjusting the scatter around the mean

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Objective

Develop methods to increase the efficiency and to improve the accuracy of probabilistic structural analysis using coarse finite element mesh

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Mathematical Foundation for Various Mapping Methods

- Probabilistic structure responses from different finite element meshes are highly correlated
- Mean and standard deviation are determined by probabilistic potential energy variation principle (Liu, Mani, and Belytschko (ref. 1))

$$\boxed{\begin{aligned} E(X) &= X^o - \sum_{k=1}^{N} (X_k'' \sigma_k^2) \\ \sigma_X^2 &= \sum_{k=1}^{N} (X_k')^2 \sigma_k^2 \end{aligned}}$$

where

X probabilistic structural response for given mesh

X^o deterministic value

X_k' $\partial X / \partial U_k$ where U_k is k^{th} independent random variable

X_k'' $\partial^2 X / \partial U_k^2$

σ_k standard deviation of U_k

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Probabilistic Relationship for Shift Mapping

First Moment Correction

From

$$X_f - E(X_f) = X_c - E(X_c)$$

$$\boxed{X_f = X_c - X_c^o + X_f^o + \sum_{k=1}^{N} (X_c)_k'' \sigma_k^2 - \sum_{k=1}^{N} (X_f)_k'' \sigma_k^2}$$

Since

$$\sum_{k=1}^{N} (X_c)_k'' \sigma_k^2 - \sum_{k=1}^{N} (X_f)_k'' \sigma_k^2 \text{ is small,}$$

$$\boxed{X_f \equiv X_c - X_c^o + X_f^o}$$

- X_f and X_c are the probabilistic structural responses from fine and coarse finite element meshes.

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Probabilistic Relationship for Ratio Mapping

Second Moment Correction

From

$$\frac{X_f - E(X_f)}{\sigma_{X_f}} = \frac{X_c - E(X_c)}{\sigma_{X_c}}$$

Letting

$$r_1 = \frac{X_f^o}{X_c^o} \quad \text{and} \quad r_2 = \sqrt{\frac{\Sigma(X_f)_k'^2\,\sigma_k^2}{\Sigma(X_c)_k'^2\,\sigma_k^2}}$$

$$\boxed{X_f = r_2 X_c - r_2\left(X_c^o - \Sigma(X_c)_k''\,\sigma_k^2\right) + \left(X_f^o - \Sigma(X_f)_k''\,\sigma_k^2\right)}$$

Since

$$r_1 \cong r_2$$

$$\boxed{X_f \cong \frac{X_f^o}{X_c^o} X_c}$$

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Probabilistic Relationship for Mixed Mapping

Combined First and Second Moment Mapping

Letting

$$X_f \cong \frac{1}{2}\,(\text{shift mapping}) + \frac{1}{2}\,(\text{ratio mapping})$$

$$\boxed{X_f \cong \frac{1}{2}\left[1 + \frac{X_f^o}{X_c^o}\right] X_c + \frac{X_f^o - X_c^o}{2}}$$

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Probabilistic Response Can Be Accurately Predicted Using Coarse Finite Element Mesh By—

(1) Computing the convergent deterministic response

(2) Computing the probabilistic distribution of the response using coarse finite element mesh

(3) Determining the probabilistic relationship between the coarse and fine finite element responses

(4) Computing the true probabilistic distribution of the response using the results obtained from steps (1) to (3)

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Probability of Damage Initiation

$$P_f = P(\sigma \geq s)$$

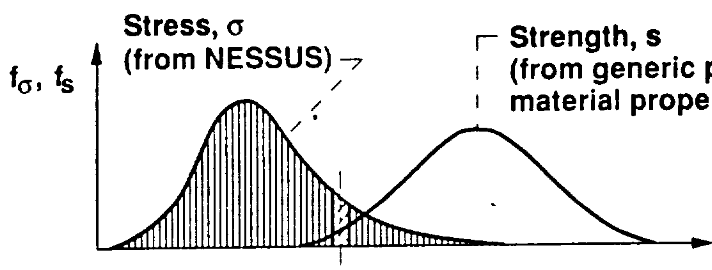
where

P_f probability

σ stress

s strength

$$P_f = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{x} f_s(s)\, ds \right) f_\sigma(x) dx$$



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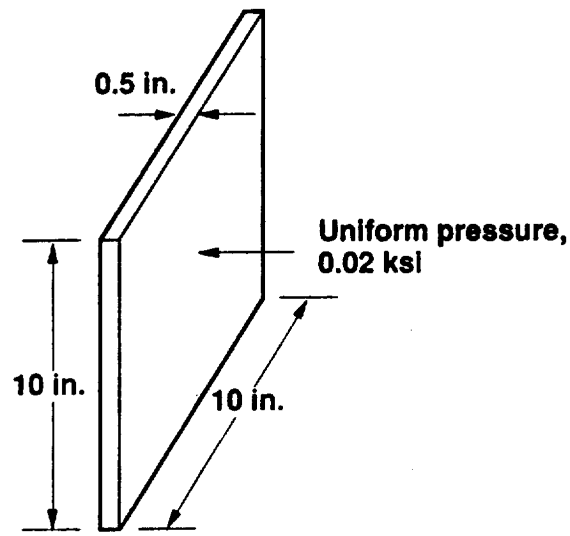
Probabilistic Stress Analysis of Cantilever Plate

- Uncertainties (random variables): plate thickness, lateral pressure



Geometry and Load

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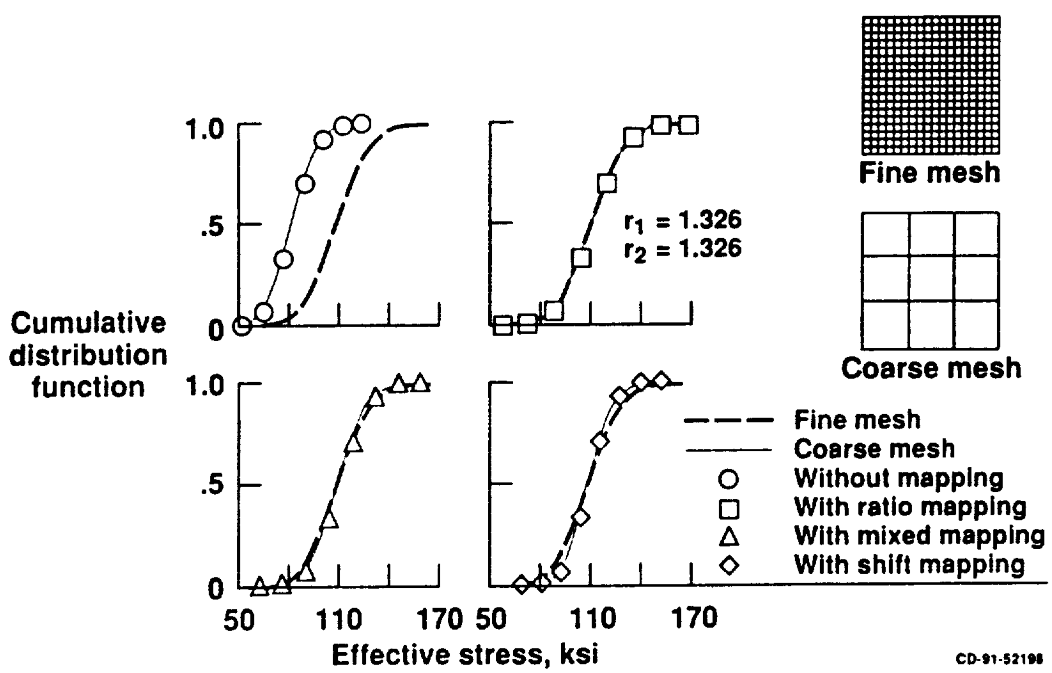
Comparisons Between Probabilistic Distributions From Different Mapping Methods

Cantilever Plate Subjected to Uniform Load



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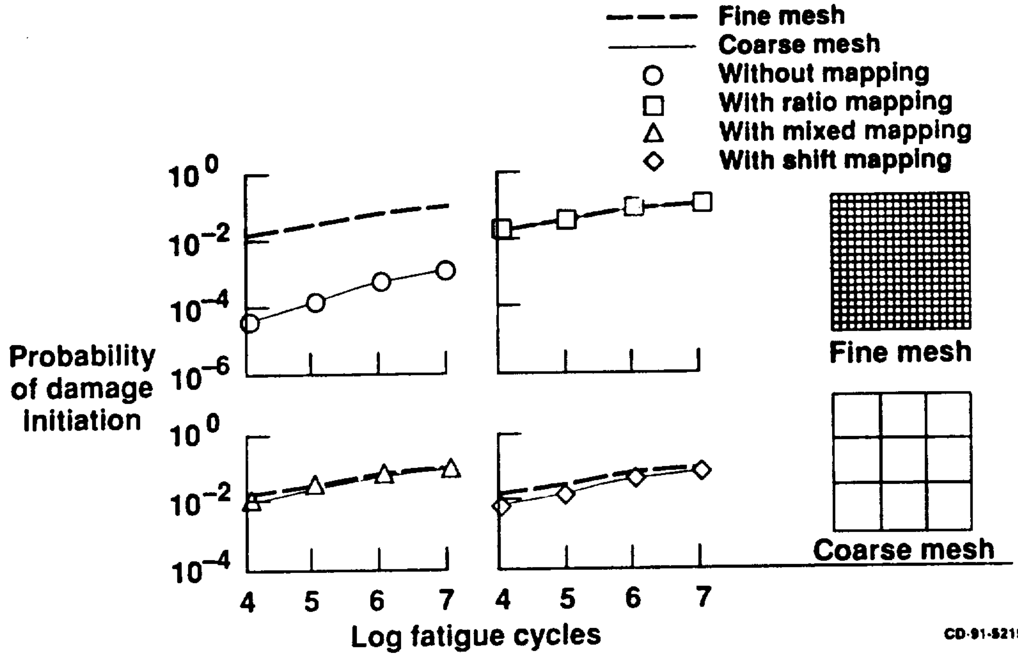
Comparisons Between Structural Reliabilities From Different Mapping Methods

Cantilever Plate Subjected to Uniform Load



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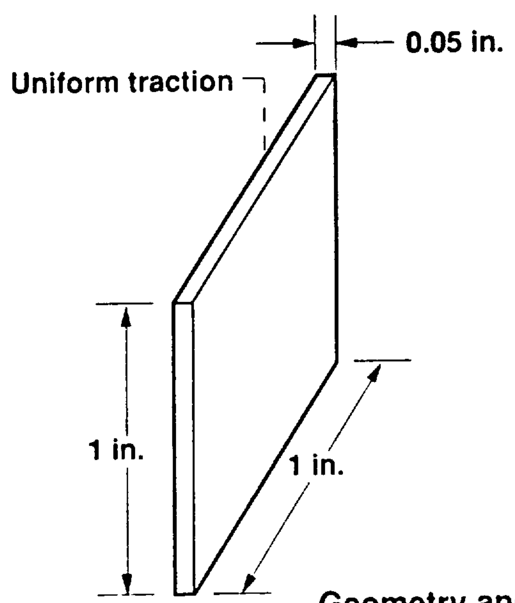
Probabilistic Buckling Analysis of a Simply Supported Composite Plate

- Uncertainties (random variables): coefficients in [D] matrix ($[M_x\, M_y\, M_{xy}]^T = [D]\,[K_x\, K_y\, K_{xy}]^T$)



Geometry and Load

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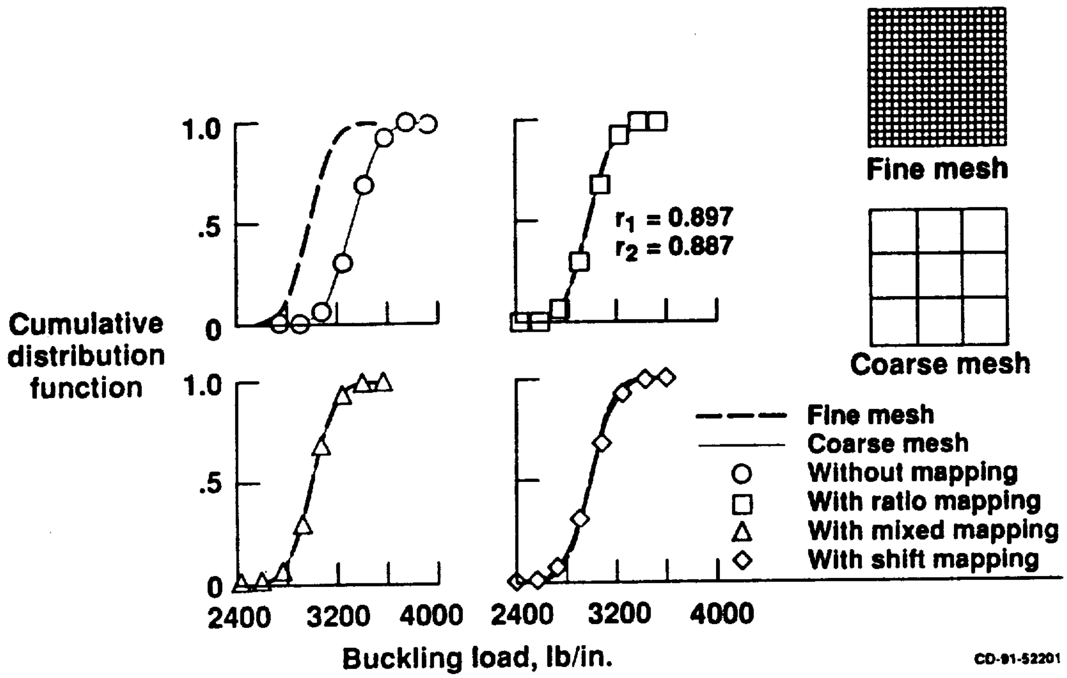
Comparisons Between Probabilistic Distributions From Different Mapping Methods

Buckling Analysis of Composite Plate



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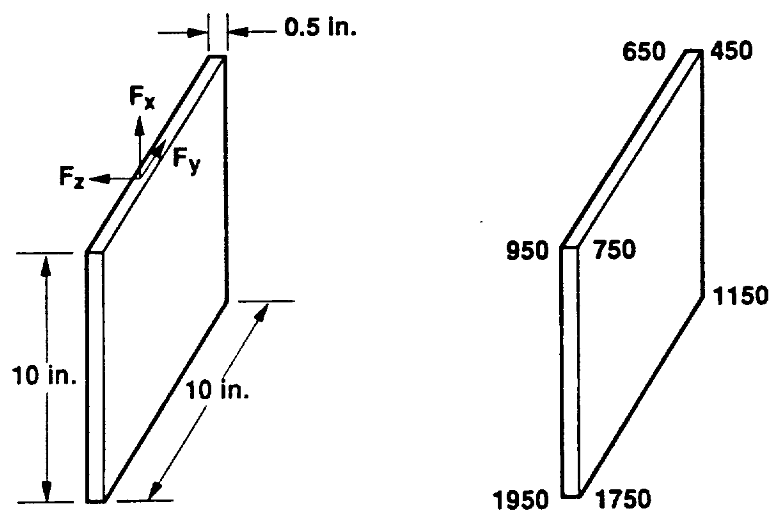
Probabilistic Stress Analysis of Cantilever Plate Subjected to Thermal and Mechanical Loads

- Uncertainties (random fields): thickness, modulus, temperature, and thermal and mechanical loads



Geometry and Mechanical Loads

Temperature Profile (temperatures in degrees Fahrenheit)

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Deterministic Stress Analyses Using Different Finite Element Meshes

Nodes	Deterministic stress	Error, percent	CPU time, sec
16 (4 by 4)	75.4	16	0.9
25 (5 by 5)	81.0	9	1.2
36 (6 by 6)	83.9	6	1.7
49 (7 by 7)	85.6	4	2.2
64 (8 by 8)	86.7	3	2.9
81 (9 by 9)	87.5	2	3.5
100 (10 by 10)	88.1	1.5	4.5
361 (19 by 19)	89.4	0	19.1

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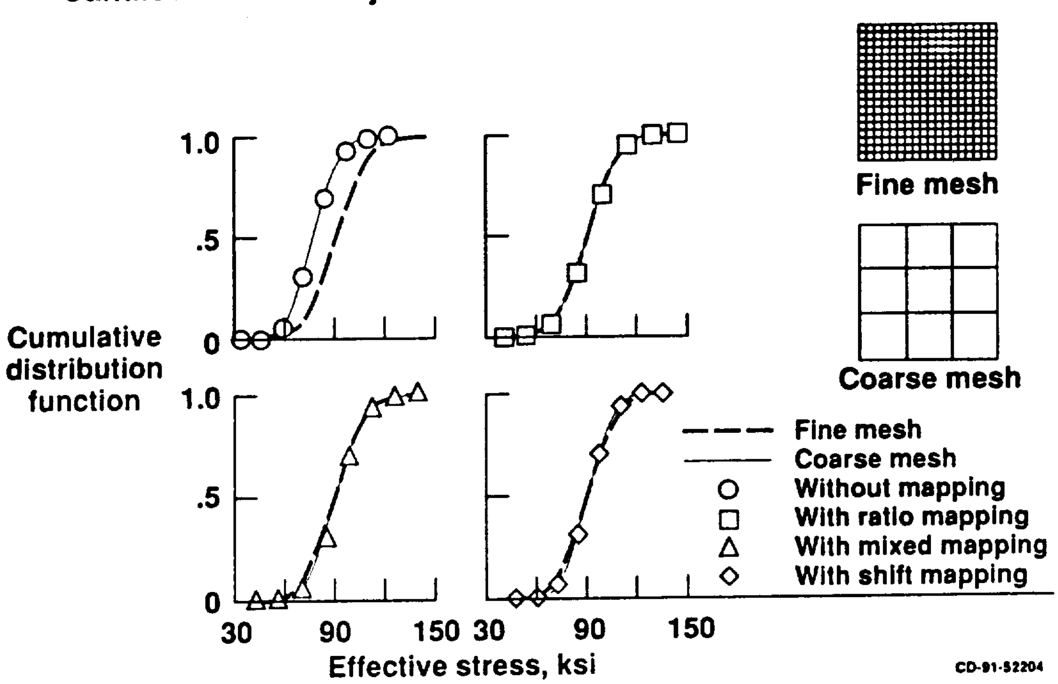
Comparisons Between Probabilistic Distributions From Different Mapping Methods

Cantilever Plate Subjected to Thermal and Mechanical Loads



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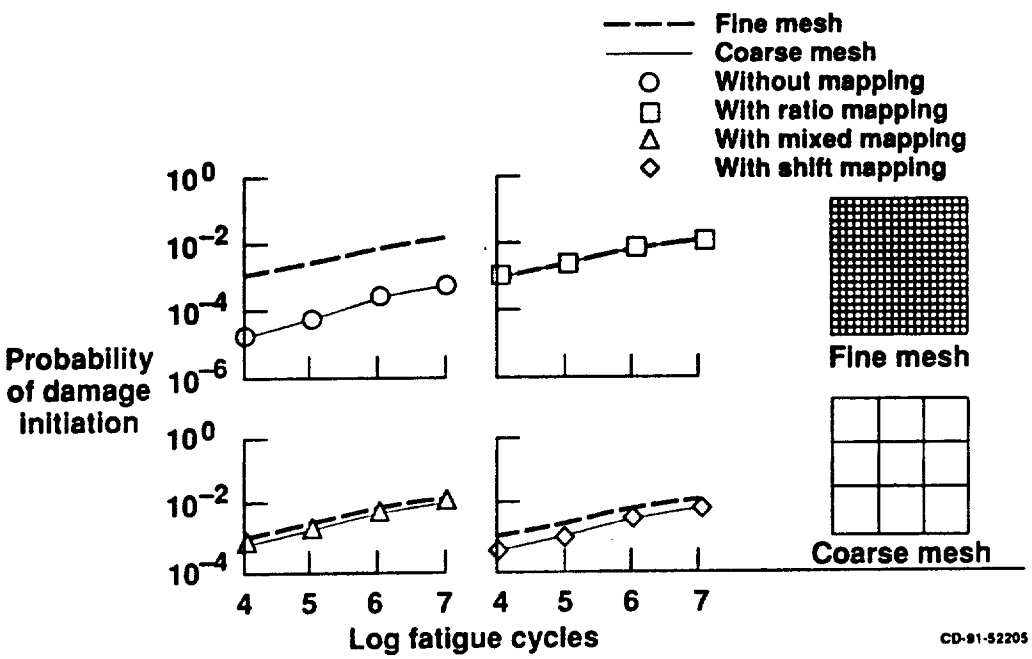
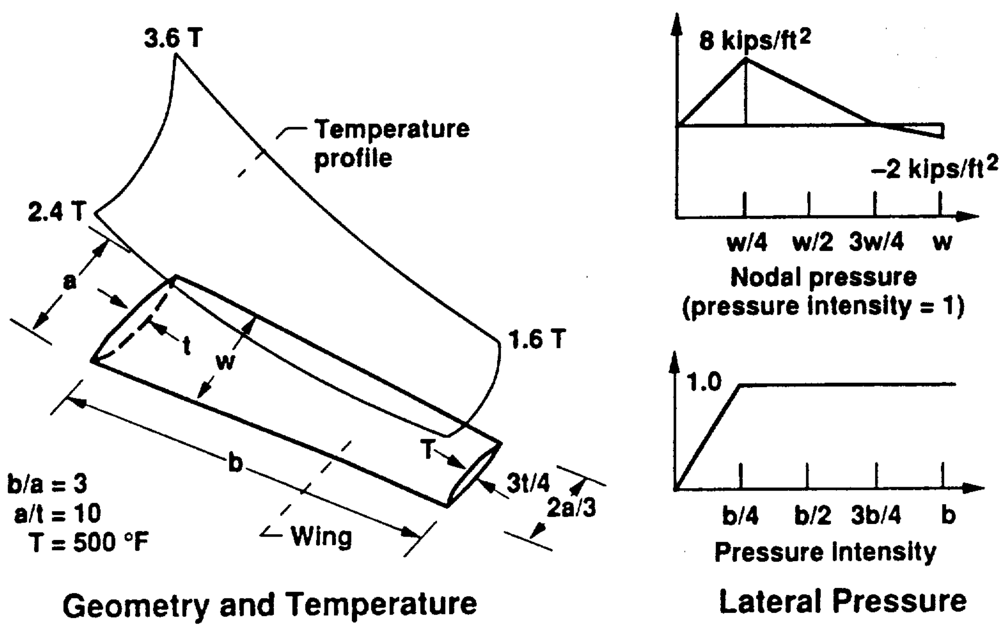
Comparisons Between Structural Reliabilities From Different Mapping Methods

Cantilever Plate Subjected to Thermal and Mechanical Loads



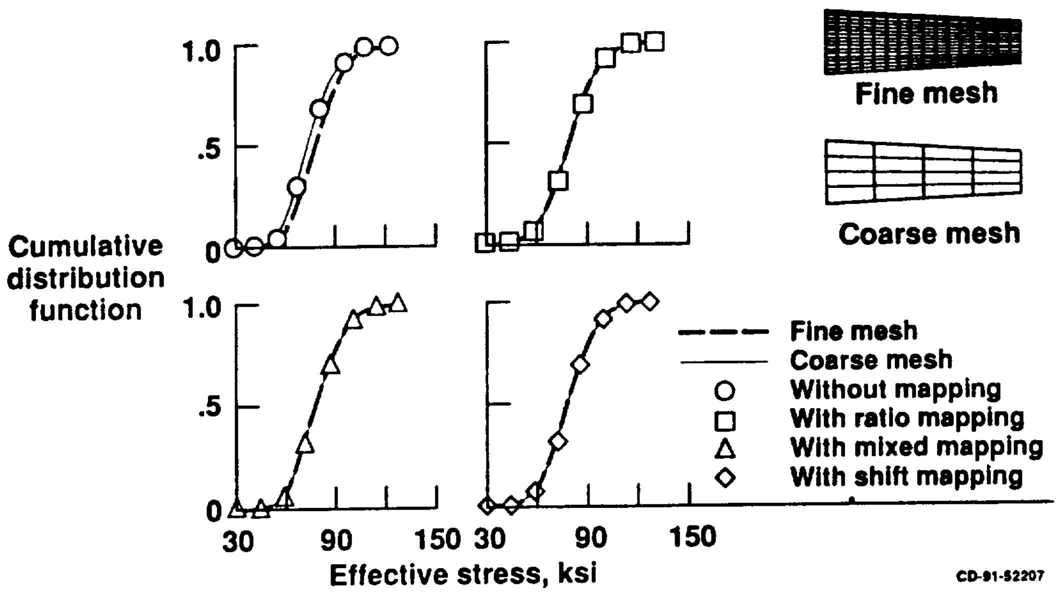
Probabilistic Stress Analysis of Tapered Cantilever Plate

- Uncertainties (random fields): thickness, modulus, temperature, and thermal and mechanical loads



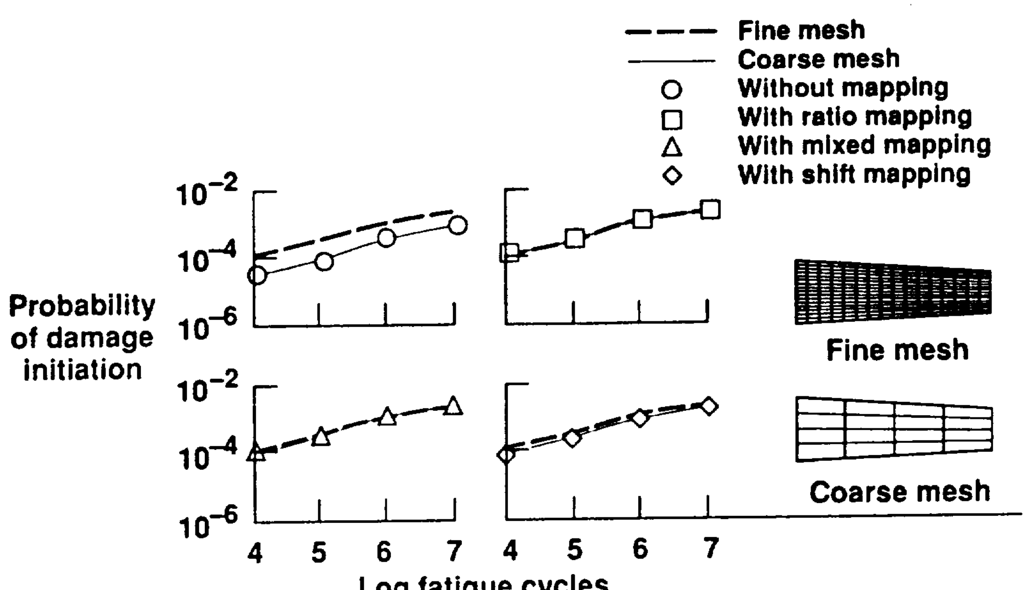
Comparisons Between Probabilistic Distributions From Different Mapping Methods

Tapered Cantilever Plate Subjected to Thermal and Mechanical Loads



Comparisons Between Structural Reliabilities From Different Mapping Methods

Tapered Cantilever Plate Subjected to Thermal and Mechanical Loads



Concluding Remarks

Mapping methods have been developed—

(1) To improve the accuracy of structural reliability using coarse finite element meshes

(2) To save computational and turn-around time

(3) To evaluate reliability of large structural systems

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