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LOCALIZATION OF AEROELASTIC MODES IN MISTUNED HIGH-ENERGY TURBINES

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Perfect periodicity, or cyclic symmetry, is a convenient, frequent assumption when analyzing the dynamics of bladed-disk assemblies. A primary reason for taking advantage of cyclic symmetry is that the blade response and excitation can always be expressed in terms of constant *interblade phase angle* modes that uncouple the equations of motion, thereby reducing the size of the problem to that of one blade. Unfortunately, such ideal periodicity is always disrupted by differences in the blade structural properties and modes of vibration, which are a result of manufacturing and material tolerances. This phenomenon, known as mistuning, not only increases tremendously the size and cost of the analysis of blade assemblies, but may also alter *qualitatively* their dynamics.

The present work investigates the effects of blade mistuning on the aeroelastic characteristics of a class of bladed-disk assemblies, namely high-energy turbines. The specific rotor we analyze is the first stage of turbine blades of the oxidizer turbopump (HPOTP) in the space shuttle main rocket engine (SSME). The common occurrence of fatigue cracks for these turbine blades indicates the

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possibility of high dyamic loading. Since mistuning, under conditions of weak interblade coupling, has been shown to increase blade response amplitudes drastically for simple structural models of blade assemblies, it provides a plausible explanation for the occurrence of cracks. In this work, we are focusing on the effects of frequency mistuning on the aeroelastic stability of the assembly and on the aeroelastic mode shapes.

The rotor consists of 78 blades equally spaced on the disk. A three-dimensional finite element model is used to calculate the first four free vibration natural frequencies and mode shapes of individual blades in a vacuum. To account for blade root flexibility linear springs are included at the surfaces of the firtree lobes. However, we do not include in our model the structural coupling between blades through the disk. Thus we assume that coupling between blades arises solely from aerodynamic effects. The unsteady, motion-dependent aerodynamic forces are calculated from a two-dimensional, linearized, unsteady aerodynamic theory applied in axisymmetric strips along the airfoil span. This results in a matrix of aerodynamic influence coefficients. No structural energy dissipation is included in the present models, although linear structural damping could be added easily. The aeroelastic analysis for a rotor of blades having identical frequencies, known as a tuned rotor, has been developed in ref 1.

Applying component mode analysis to the finite element equations, we obtain a set of 78 x M differential equations in the modal amplitudes of the blades, where M is the number of component modes for each blade (in our analysis, M=4). Assuming harmonic motion, we obtain the aeroelastic eigenvalue problem:

$$\left([K] + \lambda^2([M]+[A])\right)\{u\} = 0$$

where {u} is the 78 x M-dimensional complex eigenvector of blade modal amplitudes, [M] is the diagonal mass matrix (which reduces to the identity matrix for orthonormalized modes), [K] is the diagonal stiffness matrix of the individual blades' natural frequencies squared. The matrix [A] is the fully-populated aerodynamic influence matrix that provides interblade coupling ([A] depends on the flutter frequency), and λ is the complex eigenvalue. The real part of λ represents the aerodynamic damping in the corresponding aeroelastic mode and thus determines stability and occurrence of flutter. The imaginary part of λ represents the damped natural frequency of oscillation, and should be equal to the assumed flutter frequency where [A] was evaluated.

For a tuned system, all blades are identical and the stiffness matrix has only M distinct diagonal elements (for example, for a one mode per blade model, [K] is proportional to the identity matrix). In the tuned case all matrices in the above equation are block-circulant. For a mistuned system, we assume that the individual blade frequencies are random and uniformly distributed about the frequency of the nominal blade with a small standard deviation. For a multi-mode per blade model, we take the patterns of frequency mistuning in the various component modes to be identical. For a mistuned assembly, the stiffness matrix [K] is no longer circulant, but with the assumption of frequency mistuning, the mass and aerodynamic matrices remain circulant.

We have solved the aeroelastic eigenvalue problem for the tuned rotor and for various strengths of mistuning. We have used four modes of vibration in the component mode analysis to represent the motion of each blade. For the tuned system, we have found that most of the interblade phase angle modes in the second group (corresponding to blades vibrating primarily in their second normal mode) feature an unstable motion. This means that flutter occurs for an edgewise motion of the blades. No flutter was found in the other groups of modes. Therefore, we have focused our investigation of the effects of mistuning on the edgewise aeroelastic modes. It is important to note at this point that the current analysis assumes that their is no damping of the blade due to either structural or hysteretic means. The operational HPOTP contains blade-to-blade friction dampers which provide sufficient damping to stabilize the rotor in terms of flutter (ref 1). Mechanical damping was neglected in order to investigate the effect of mistuning on rotors which were already unstable, although all results apply equally well to an initially stable rotor.

The primary finding of our study is that the dynamics of the first stage turbine stage of the SSME turbopump is severely sensitive to small blade mistuning. Both aeroelastic eigenvalues and eigenvectors undergo *qualitative* changes when perfect cyclicity is altered, due to frequency mistuning. First, the root locus of the aeroelastic eigenvalues (in the damping-frequency plane) loses the regular pattern that characterizes the tuned system to become apparently randomly scattered for small mistuning. Second, the constant interblade phase angle mode shapes of the tuned system become *strongly localized* to a few blades of the assembly when mistuning is present. The transition from extended to localized modes is very rapid in the turbopump's first turbine stage. In all cases, the standard deviations of mistuning we consider are smaller than 2%. The results from the present analysis are similar to those presented for a generic rotor in Reference 2, although the HPOTP results in considerably higher sensitivity and localization than the rotor of the prior work.

Figure 1 displays the variation of the real part of the most unstable aeroelastic eigenvalue versus the standard deviation of the frequency mistuning of the randomized blade frequencies. Note that the least stable eigenvalue becomes more stable when mistuning is present, indicating the well-known stabilizing effect of mistuning. Furthermore, observe the drastic alteration of the corresponding aeroelastic mode shape by mistuning. As expected, the mode shape in the tuned configuration features identical amplitudes for all blades. When mistuning increases, the whole assembly ceases to participate in the motion and the vibration is confined to a few of the blades, i.e., localization occurs. The extreme sensitivity of the assembly dynamics to mistuning is clearly illustrated in Figure 1: even for a very small mistuning of 0.03%, the blade amplitudes vary widely throughout the rotor; *for a mistuning of 1%, only a couple of blades participate in the motion!* This indicates that the transition from extended to localized modes is very rapid. Also this transition is doubtlessly very complex, as shown by the rather irregular variation of the most unstable eigenvalue with mistuning in Figure 1a. An important focus of continued work is examining this transition region between a tuned rotor to a localized rotor in more detail.

Figure 2 displays the root locus of the 78 aeroelastic eigenvalues in the edgewise component mode for various mistuning values. Note the regular pattern of the root locus for very small mistuning (0.01%), which is characteristic of a tuned assembly. As mistuning increases, the regularity of the root locus is lost, and for small mistuning of 0.1% the locus consists of a constellation of eigenvalues with little discernible pattern. This is another illustration of the extreme sensitivity of the structure of the eigensolution to mistuning. Finally, note that mistuning results in a narrowing of the range of the real parts of the eigenvalues (damping) but in a stretching of the root locus along the imaginary direction (frequency).

The above results demonstrate that the aeroelastic characteristics of the first turbine stage of the SSME turbopump are highly sensitive to frequency mistuning. Strong mode localization and scattering of the root locus occur for values of mistuning that cannot be avoided in practice. This means that, for the turbopump model studied, most tuned aeroelastic calculations are probably invalid. We expect the forced response of the assembly to be affected by mistuning in a similar way, because the response can be expressed as a combination of motions in the free aeroelastic modes.

Continued research into this topic will focus on the following points; first, we will perform an in-depth examination of the transition region between constant interblade phase angle modes (tuned rotor) to localized modes for the mistuned system. Second, we will apply perturbation schemes to predict and understand the onset of mode localization. These methods will also allow us to quantify the degree of localization of the modes. We are also hoping they will lead to the definition of a *sensitivity measure* that will enable us to predict the occurrence of localization and the sensitivity of the assembly in a simple way.

REFERENCES

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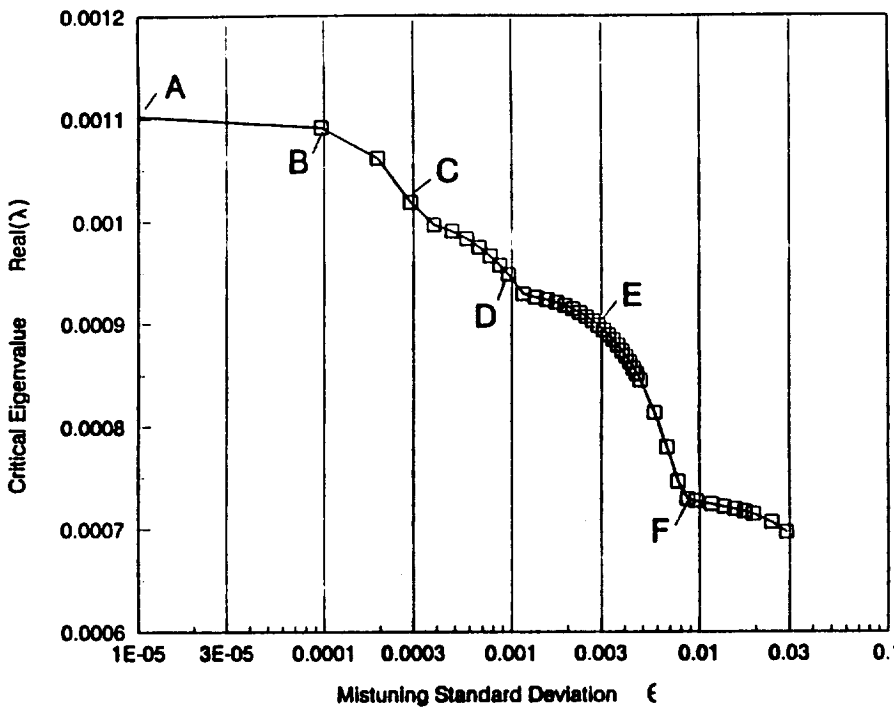

Figure 1.a Real part of the most unstable eigenvalue (damping) versus mistuning standard deviation, ϵ

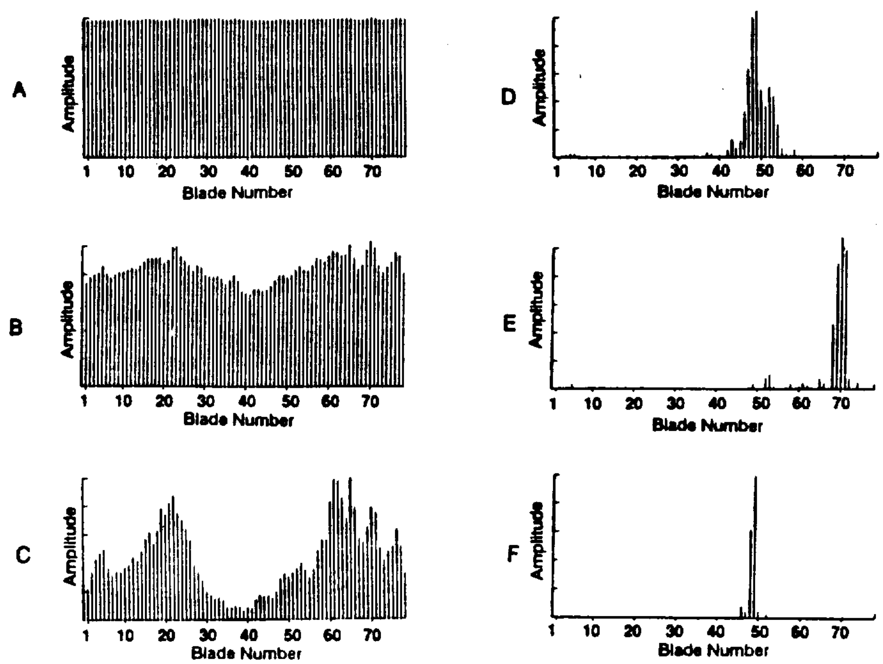

Figure 1.b Amplitude of the rotor aeroelastic mode shapes corresponding to most unstable eigenvalue for various values of mistuning

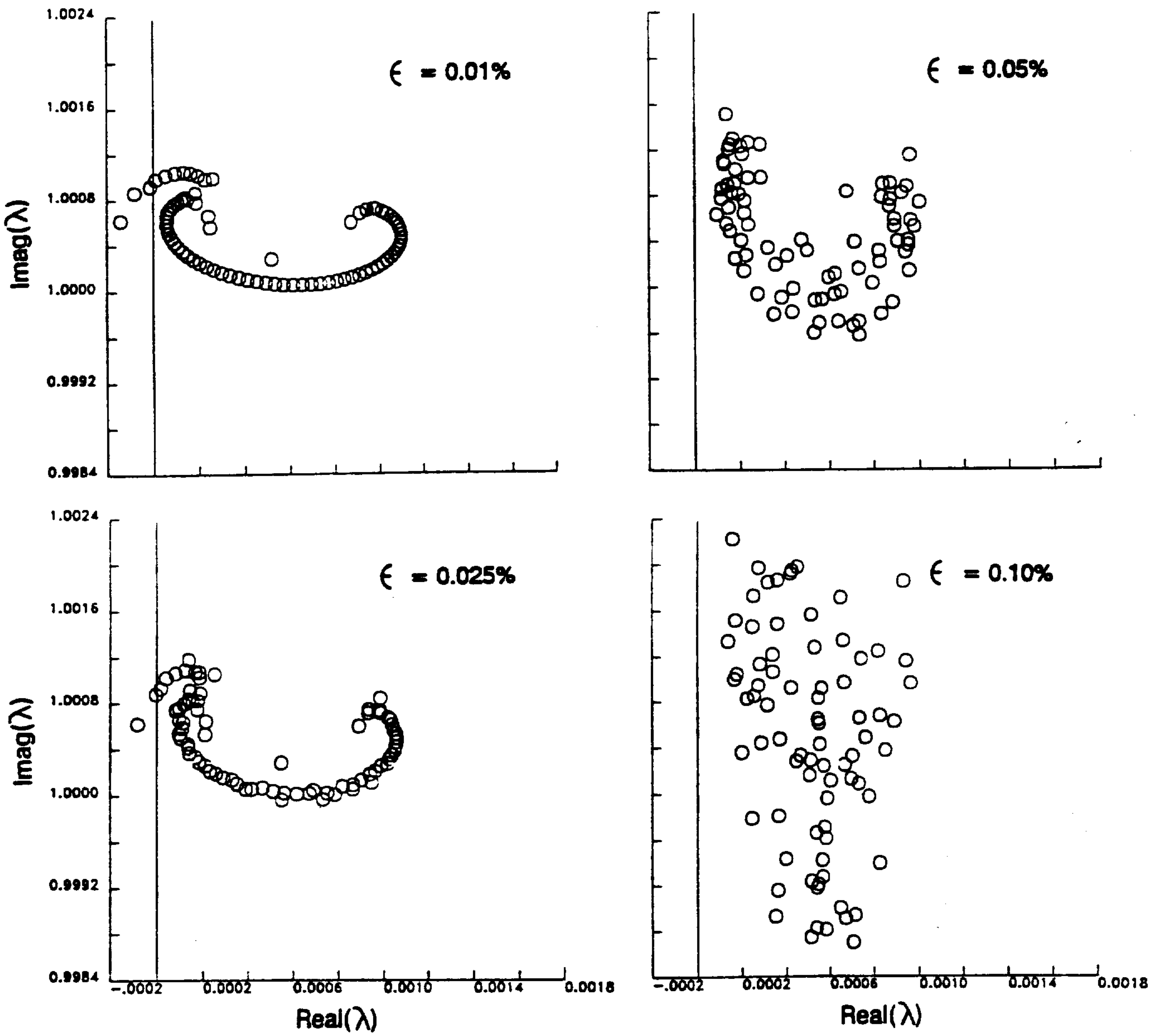

Figure 2 Root locus of the 78 aeroelastic eigenvalues (frequency vs damping) for various values of mistuning (Edgewise motion only)