TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 780

MEASURED MOMENTS OF INERTIA OF 32 AIRPLANES

By William Gracey
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SUMMARY

A compilation of the experimentally determined moments of inertia of 32 airplanes is presented. The measurements were obtained at the laboratories of the NACA by means of a pendulum method. The airplanes tested are representative of several types of aircraft of gross weight less than 10,000 pounds.

The results are presented in coefficient as well as in dimensional form. An elementary analysis of the data disclosed the possibility of grouping the results according to wing type of the airplane, as low-wing monoplanes, parasol and high-wing monoplanes, and biplanes. The data are shown to provide a convenient means of rapidly estimating the moments of inertia of other airplanes. A three-view drawing of each of the 32 airplanes is included.

This note supersedes NACA Technical Note No. 375.

INTRODUCTION

The determination of the moments of inertia of airplanes by means of a pendulum method has been described in detail in reference 1. The precision of the results obtained by this method is within ±2.5 percent, ±1.3 percent, and ±0.8 percent, for the X, the Y, and the Z axes, respectively; whereas, the precision of estimates computed on the basis of a weight schedule has been shown to be about 10 percent (reference 2). The pendulum method has been in use at the laboratories of the NACA for the past 12 years, during which time measurements have been obtained on several types of airplane of gross weight less than 10,000 pounds. Because these measurements represent the most accurate data available, it appeared desirable to compile and publish all the data accumulated up to the present time.

The results of the tests of a few of the airplanes have already been published in an appendix of reference 1 but are included herein for completeness. These same data had also been published earlier in reference 3 but were slightly in error because the methods of correcting for the additional-mass effect had not been developed at that time. As the corrected values of reference 1 are presented herein, the information contained in the present paper should be considered as superseding that of reference 3. Since the publication of reference 3 the practice of determining the angle between the principal and the reference axes has been abandoned, this angle having been found to be less than 3° for conventional airplanes.

METHOD AND APPARATUS

The measurements reported herein were obtained by the method described in reference 1. According to this procedure, the moments of inertia are determined about three reference axes, the origin of which is the center of gravity of the airplane. These axes are: the X axis, parallel to the thrust line in the plane of symmetry; the Y axis, perpendicular to the plane of symmetry; and the Z axis, perpendicular to the thrust line in the plane of symmetry.

The moments of inertia about the X and Y axes are found by swinging the airplane as a compound pendulum, whereas the moment of inertia about the Z axis is determined by oscillating the airplane as a bifilar-torsional pendulum. In each case the true moments of inertia are determined by correcting the measured moments of inertia for (1) the buoyancy of the structure, (2) the air entrapped within the structure, and (3) the additional-mass effect. The apparent additional moment of inertia about each axis is evaluated on a basis of (1) the size and the shape of the airplane normal to the direction of motion and (2) the results of tests of the additional-mass effect of flat plates (reference 1).

The airplanes tested are listed in table I. Most of the airplanes are representative of several types of military aircraft, both Army and Navy. A few commercial and experimental types are also included. With the exception of the twin-engine OA-4A, all of the airplanes tested were of the single-engine type and, except for the Hammond Y-1, the airplanes were all of the tractor type. All of the airplanes except the amphibian OA-4A were landplanes.

In general, the airplanes were tested for the normal full-load condition. In all cases the gas and the oil tanks were filled. As a rule the pilot and each passenger of the airplane was represented by 175 to 200 pounds of ballast. In some cases, however, only the pilot was so represented and, in other cases, no ballast at all was added. For this reason both the weight of the airplane as tested and the weight of the airplane minus the ballast for the pilot and the passengers will be noted.

The airplanes with fixed landing gear were usually tested with the landing gear in flying position, that is, with the oleo extended. For an airplane with a retractable landing gear, tests were conducted with the landing gear either retracted or extended (with the oleo extended). In some few instances, the wheels were fixed in the taxying condition, that is, with the oleo compressed.

RESULTS

The results of the tests on the various airplanes are summarized in table I. The data presented include the true moments of inertia of the airplane and the additional moments of inertia about the reference axes. The true moments of inertia are based on the weight of the airplane as tested.

The data are also presented as radii of gyration and in coefficient form. The radii of gyration are computed from the true moments of inertia from the expressions:

$$k_X = \sqrt{\frac{A}{W/g}}$$

$$k_Y = \sqrt{\frac{B}{W/g}}$$

$$k_Z = \sqrt{\frac{C}{W/g}}$$

where

k_X, k_Y, k_Z radii of gyration about X, Y, and Z axes, respectively

A, B, C the true moments of inertia about the X, Y, and Z axes, respectively

W weight of the airplane as tested

Nondimensional coefficients, useful for comparing the moments of inertia of airplanes whose size and weight vary considerably, are expressed in terms of the wing span, b, and are calculated from the expressions:

$$C_X = \frac{k_X}{b}$$

$$C_Y = \frac{k_Y}{b}$$

$$C_Z = \frac{k_Z}{b}$$

where C_X, C_Y, and C_Z are the coefficients for the moments of inertia about the X, the Y, and the Z axes, respectively. For convenience all the coefficients are expressed in terms of the wing span even though the over-all length of the airplane may be the more rational parameter for C_Y.

In order to facilitate the comparison of data from similar airplanes, the results in table I are arranged in three groups according to wing type, namely, low-wing monoplanes, parasol and high-wing monoplanes, and biplanes. This grouping permits a graphical presentation of certain portions of the data. Thus, for any one of the groups, the radii of gyration about the X and the Z axes may be plotted as functions of the wing span, and the radii of gyration about the Y axis may be plotted against the over-all length of the airplane. Charts of this type are given for the low-wing monoplane and the biplane groups (figs. 1 to 4, inclusive). No charts are shown for the high-wing monoplane group because the available data were insufficient to define the curves. It should be noted that the curves shown in figures 1 to 4 were derived from data of similar airplanes, namely, military airplanes of comparatively recent design. The data for commercial and experimental airplanes obviously do not apply to these curves. The biplane data given in reference 1 are omitted from figures 3 and 4 because the airplanes were generally of older design and because the more recent airplanes were tested with improved apparatus.

In order to give some indication of the mass distribution about the various axes, a three-view diagram of each airplane tested is included. (See figs. 5 to 36.)

DISCUSSION

The information presented provides a convenient method of rapidly approximating the moments of inertia of airplanes similar to those for which measurements are given. The method involves simply the selection from table I of an airplane which is sufficiently like the airplane considered that the radii of gyration or coefficients of the airplane in table I can be used to compute the moments of inertia of the airplane under consideration. The convenience of the method is obvious, because the only numerical data required for its application are the weight and the over-all dimensions of the airplane considered. The method can be applied when the airplanes are similar as regards general type, shape, and structural characteristics but are different in size and weight. That the results from one airplane can be applied to a similar airplane of different size may be seen from the fact that the data of similar airplanes vary uniformly with the over-all dimensions (figs. 1 to 4). In reference to these figures it is interesting to note that, for both low-wing monoplanes and biplane groups, the curves of k_X and k_Z are parallel and that the curve of k_Y for biplanes is parallel to and above the k_Y curve for the low-wing monoplanes.

It should be appreciated that the indiscriminate application of the method given may lead to very erroneous results. In order to emphasize this point, the data of the P-35 and the NF-1, two very similar airplanes, will be considered. In spite of the close similarity as regards size, shape, and structural design, the moments of inertia of the NF-1 airplane are considerably higher than those of the P-35, particularly about the X and the Z axes. These differences are readily accounted for by the fact that the NF-1 was tested with a 100-pound bomb under each wing. Deducting the moments of inertia of the bombs reduces the values of A, B, and C to 2653, 4620, and 5795 slug-feet^2 for the case with the landing gear extended.

The values for the two airplanes are thus shown to be in better agreement than the data in table I indicate. The radii of gyration of the NF-1 plotted in figures 1 and 2 were calculated from these corrected values. None of the other airplanes carried bombs or other concentrated loads not included in the normal load condition of the airplane.

The precision of the moments of inertia approximated by the method just described is difficult to estimate because it depends on the degree of similarity between the two airplanes considered and on the exactness with which any dissimilarities can be accounted for. If the method is used with due regard to its limitations, it is believed that the precision obtained will in many cases approach that obtained by computation methods.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 13, 1940.

REFERENCES

1. Soulé, Hartley A., and Miller, Marvel P.: The Experimental Determination of the Moments of Inertia of Airplanes. Rep. No. 467, NACA, 1933.

2. Kirschbaum, H. W.: Estimation of Moments of Inertia of Airplanes from Design Data. T.N. No. 575, NACA, 1936.

3. Miller, Marvel P., and Soulé, Hartley A.: Moments of Inertia of Several Airplanes. T.N. No. 375, NACA, 1931.

TABLE I.-MOMENTS OF INERTIA OF 32 AIRPLANES

Figure	Airplane	Wing type	Type	Seating capacity	Structural details (a)	Landing gear	Wing span (ft)	Over-all length (ft)	Weight of airplane without passengers (lb)	Weight of ship as tested (lb)	True moments of inertia (slug-ft²) A	B	C	Additional moments of inertia (slug-ft²) I_{A_X}	I_{A_Y}	I_{A_Z}	Radii of gyration (ft) k_X	k_Y	k_Z	Coefficient of moments of inertia C_X	C_Y	C_Z
5	P-35	Low-wing monoplane	Army pursuit	1	Am	Retracted	36.00	26.83	5676	5877	2321	4,414	5872	223	71	59	3.56	4.91	5.66	0.099	0.136	0.157
						Extended	--	--	5676	5877	2497	4,610	5862	223	71	59	3.70	5.03	5.67	.103	.140	.158
6	NF-1	-- do --	Navy fighter	1	Am	Retracted	35.00	25.17	[b]5792	5992	2935	4,460	6252	206	69	57	3.97	4.89	5.80	.113	.140	.166
						Extended	--	--	5792	5992	3117	4,640	6259	206	69	57	4.09	4.99	5.79	.117	.143	.166
7	BT-9A	-- do --	Army trainer	2	Wm, Ff	Fixed	42.00	27.33	4072	4472	2534	4,163	6098	357	76	71	4.27	5.47	6.62	.102	.130	.158
8	PB-2	-- do --	Army pursuit bomber	2	Am	Extended	44.00	29.29	4869	5247	3351	5,202	7739	510	113	87	4.53	5.65	6.88	.103	.128	.156
9	YP-29A	-- do --	Army pursuit	1	Am	Retracted	29.30	25.00	3258	3433	926	2,124	2856	104	42	41	2.95	4.46	5.17	.101	.152	.176
10	P-26A	-- do --	-- do --	1	Am	Fixed	34.00	23.60	2916	3091	980	1,826	2512	76	41	31	3.19	4.36	5.11	.094	.128	.150
11	XSB2U-2	-- do --	Navy scout bomber	2	W & F mf	Extended	42.00	23.98	6104	6305	3877	8,296	8767	575	171	180	4.45	6.51	6.69	.106	.155	.159
12	Hammond Y-1	-- do --	Commercial	2	Wf, Fm	Oleo compressed	40.00	26.98	1893	1893	1115	1,402	2310	357	50	29	4.35	4.88	6.27	.109	.122	.157
13	J-2	-- do --	Experimental	1	Wpw, Ff[c]	Fixed	34.00	22.00	1535	1535	898	--	--	47	--	--	4.34	--	--	.128	--	--
14	McDonnell	-- do --	-- do --	2	Af[d]	-- do --	35.00	21.33	1516	1708	1345	1,101	2279	157	42	26	5.04	4.55	6.55	.144	.130	.187
15	XR2K-1	Parasol monoplane	Navy transport	2	Af	-- do --	32.83	22.00	1544	1729	703	1,186	1663	139	28	22	3.62	4.71	5.56	.110	.143	.169
16	F-22	-- do --	Commercial	2	Af	-- do --	32.83	22.00	1213	1388	697	--	--	139	--	--	4.02	--	--	.122	--	--
17	Doyle O-2	-- do --	-- do --	2	Af	-- do --	30.00	19.00	1038	1388	596	659	971	97	14	9	3.72	3.91	4.75	.124	.130	.158
18	OA-4A	High-wing monoplane	Army observation	8	Wpw, Fm[e]	Oleo compressed	60.00	45.42	8553	8553	--	14,908	--	--	556	--	--	7.50	--	--	.125	--
19	Aeronca C-2N	-- do --	Commercial	1	Af	Fixed	36.00	20.00	584	584	--	373	--	--	22	--	--	4.53	--	--	.126	--
20	XF13C-3	-- do --	Navy fighter	1	Wf, Fm	Retracted	35.00	27.50	4487	4662	2288	4,267	5989	190	58	74	3.97	5.43	6.43	.113	.155	.184
21	XSBF-1	Biplane	Navy scout bomber	2	Wf, Fm	-- do --	34.50	25.82	4473	4773	2622	4,204	--	238	72	--	4.20	5.32	--	.122	.154	--
22	XSB3U-1	-- do --	-- do --	2	Wf, Fm	-- do --	33.25	28.20	5125	5330	2364	5,270	--	217	99	--	3.78	5.64	--	.114	.170	--
23	F3F-2	-- do --	Navy fighter	1	Wf, Fm	Retracted	32.00	23.17	4470	4673	1546	2,998	3887	139	45	39	3.26	4.54	5.17	.102	.142	.162
						Extended	--	--	4470	4673	1621	3,033	3907	139	45	39	3.34	4.57	5.19	.105	.143	.160
24	XF2F-1	-- do --	-- do --	1	Wf, Fm	-- do --	28.50	21.42	3375	3550	1003	2,198	2631	111	34	39	3.01	4.46	4.89	.106	.157	.172
25	XBM-1	-- do --	Navy bomber	2	Wf, Fm	Fixed	41.00	28.46	4769	4769	5029	5,574	8460	487	135	82	5.82	6.13	7.56	.142	.150	.185
26	O-11	-- do --	Army observation	2	Af	-- do --	38.00	28.33	3908	4258	2507	4,133	6231	370	105	59	4.35	5.59	6.86	.115	.147	.181
27	F4B-1	-- do --	Navy fighter	1	Af	-- do --	30.00	20.61	2365	2540	1023	1,560	2077	124	30	15	3.60	4.45	5.13	.120	.148	.171
28	F4B-2	-- do --	-- do --	1	Af	-- do --	32.00	20.00	2637	2816	1063	1,795	2378	124	30	15	3.49	4.53	5.21	.109	.142	.163
29	PW-9	-- do --	Army pursuit	1	Af	-- do --	32.08	22.85	2510	2885	1299	1,888	2648	101	42	25	3.80	4.59	5.44	.119	.143	.170
30	PT-1	-- do --	Army trainer	2	Af	-- do --	34.79	27.67	2162	2512	1967	2,088	3289	238	80	54	5.02	5.17	6.49	.144	.149	.187
31	NY-1	-- do --	Navy trainer	1	Af	-- do --	34.48	27.75	2447	2622	2098	2,450	3807	238	80	80	5.07	5.48	6.84	.147	.159	.198
32	XN2Y-1	-- do --	-- do --	1	Af	-- do --	28.00	20.75	1392	1567	732	922	1299	86	22	17	3.88	4.35	5.16	.139	.155	.184
33	O2U-3	-- do --	Navy observation	2	Af	-- do --	34.50	24.63	3200	3550	2482	2,796	4481	335	84	49	4.75	5.03	6.38	.138	.146	.185
34	O3U-1	-- do --	-- do --	2	Af	-- do --	36.00	26.06	3882	4057	2740	3,283	5112	335	84	49	4.66	5.11	6.36	.130	.142	.177
35	VE-7	-- do --	Navy trainer	2	Af	-- do --	34.11	24.45	1858	2208	1227	1,434	2478	242	64	32	4.23	4.57	6.01	.124	.134	.176
36	NB-1	-- do --	-- do ---	2	Af	-- do --	36.83	25.17	2194	2544	2409	2,239	4099	347	67	44	5.52	5.32	7.20	.150	.144	.195

[a] The symbols in this column have the following significance: **Am**, all-metal construction; **Af**, all-fabric covered; **Wm**, metal-covered wing; **Wf**, fabric-covered wing; **Wpw**, plywood-covered wing; **Fm**, metal-covered fuselage; **Ff**, fabric-covered fuselage; **W & F mf**, wing and fuselage covered partly with metal and partly with fabric.

[b] Weight includes two 100-pound bombs under wings.

[c] Wing equipped with several flaps; F-22 fuselage.

[d] Wing equipped with slots and flaps.

[e] Converted amphibian, tricycle landing gear replacing standard gear.

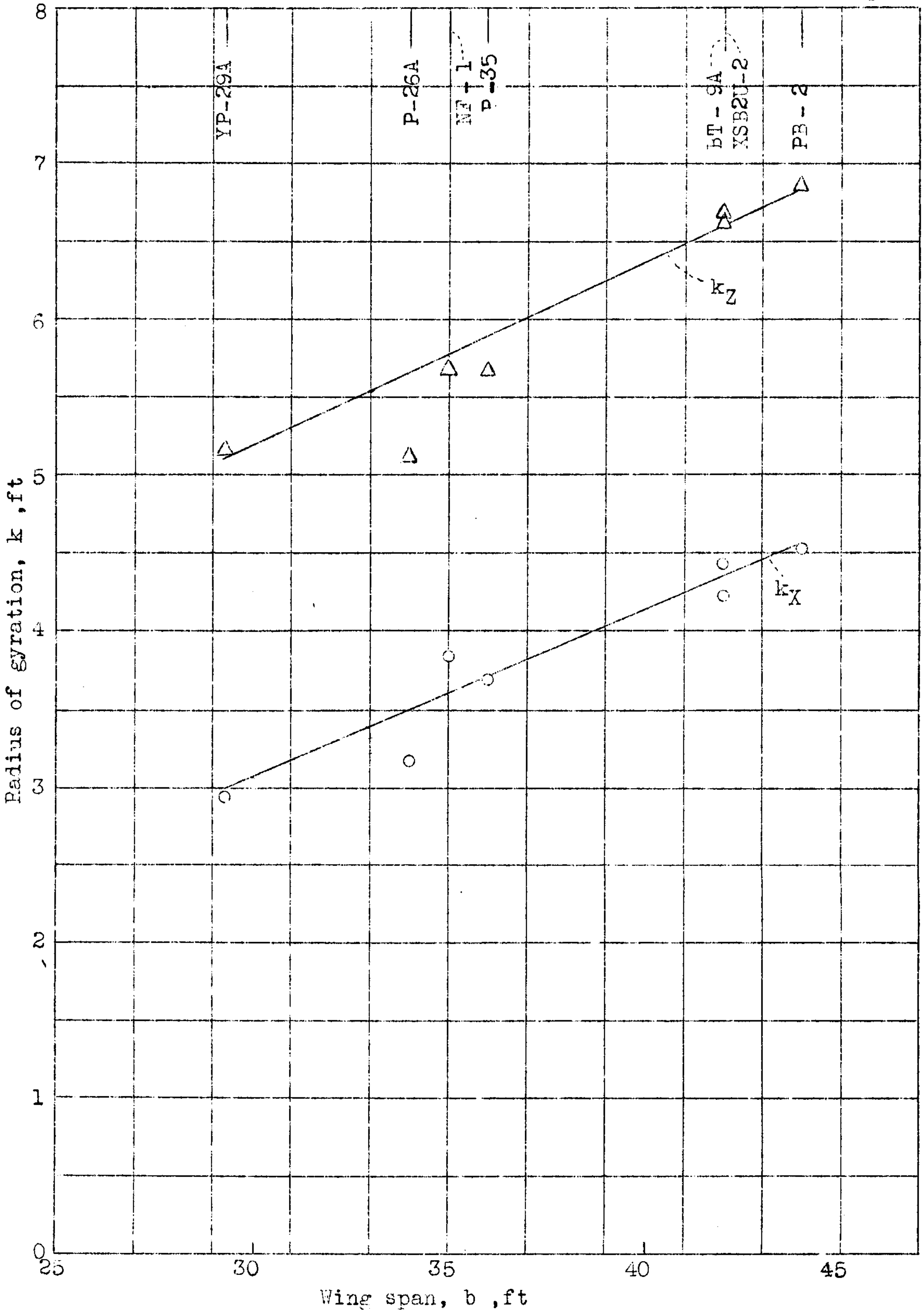

Figure 1.- Variation of k_X and k_Z with wing span for seven low-wing monoplanes.

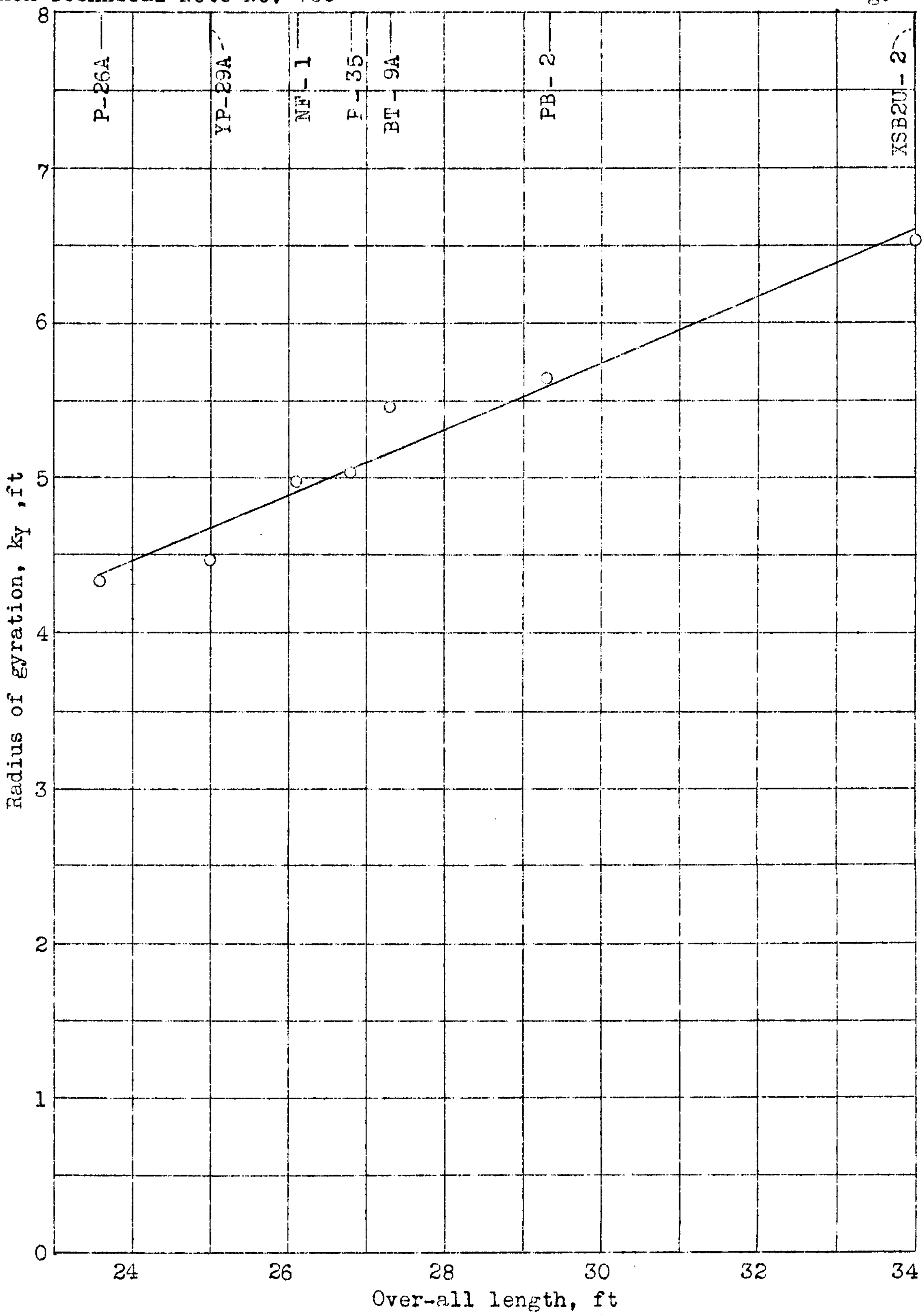

Figure 2.- Variation of k_Y with over-all length for seven low-wing monoplanes.

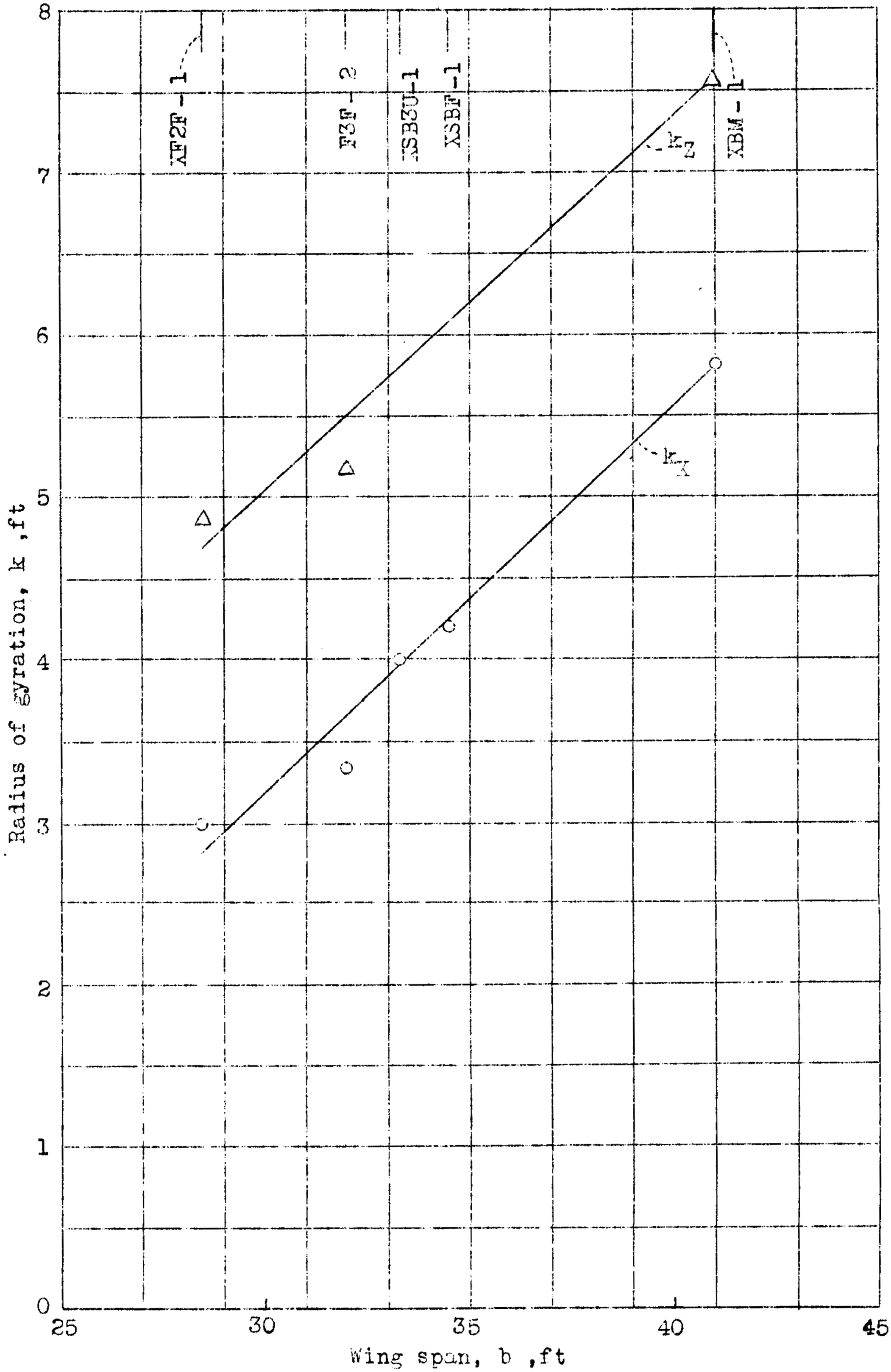

Figure 3.- Variation of k_X and k_Z with wing span for five biplanes.

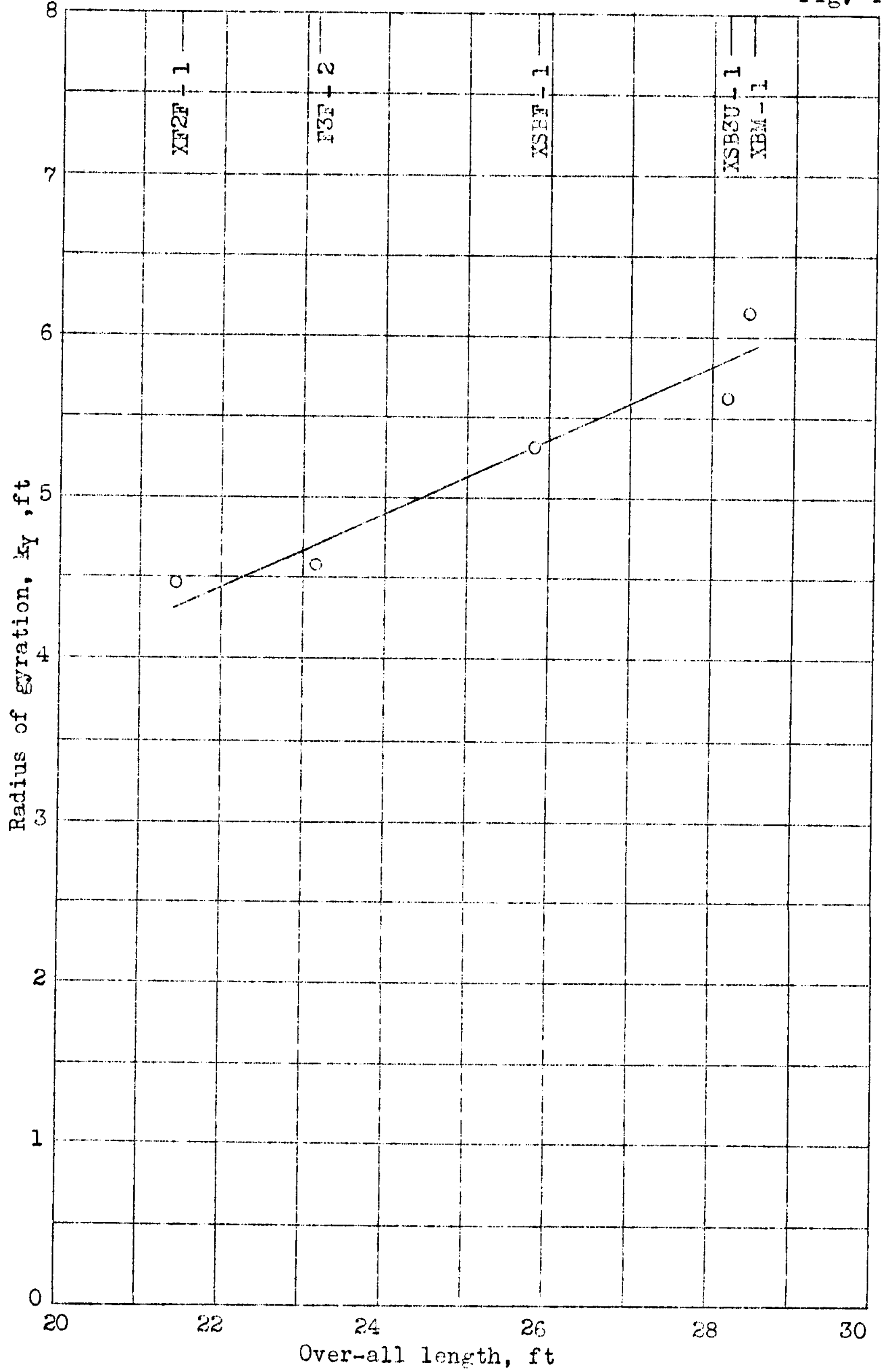

Figure 4.- Variation of k_Y with over-all length for five biplanes.

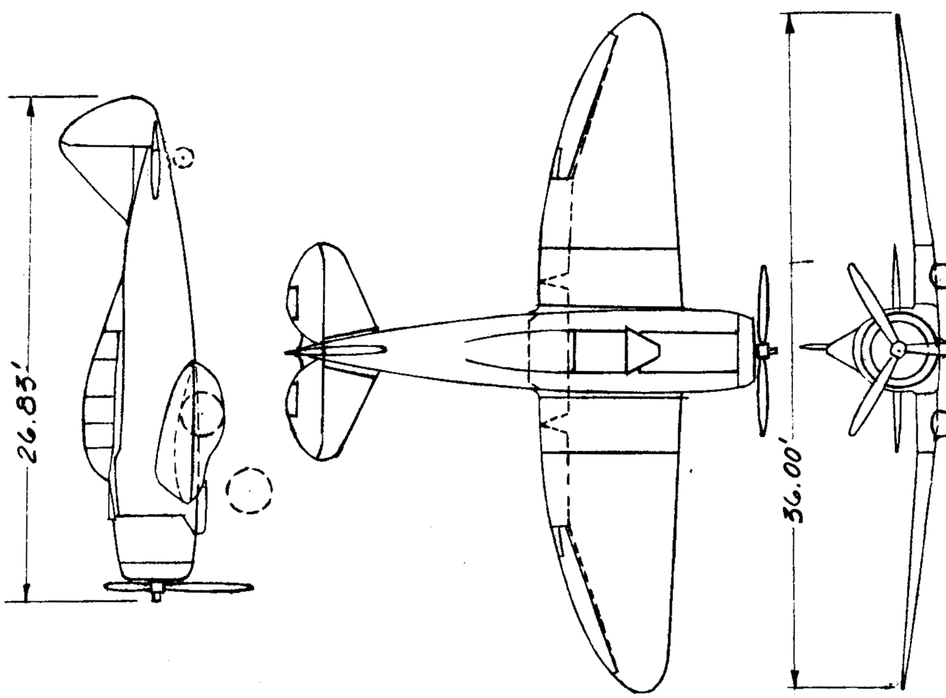

Figure 5.- The P-35 airplane
Test weight, 5877 pounds.

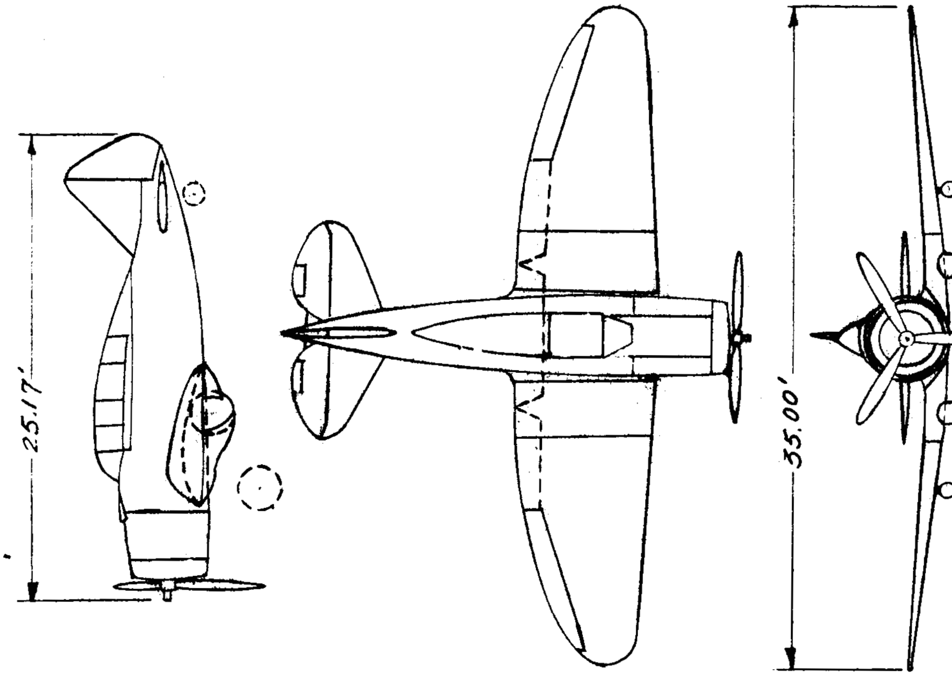

Figure 6.- The NF-1 airplane
Test weight, 5992 pounds

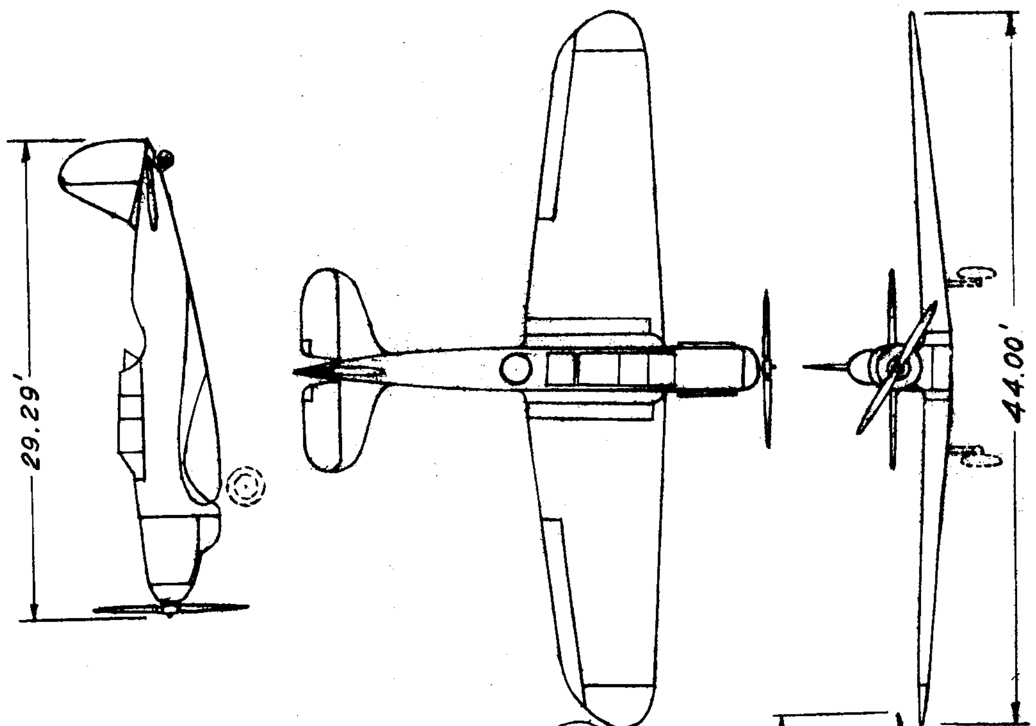

Figure 8.- The PB-2 airplane.
Test weight 5247 pounds.

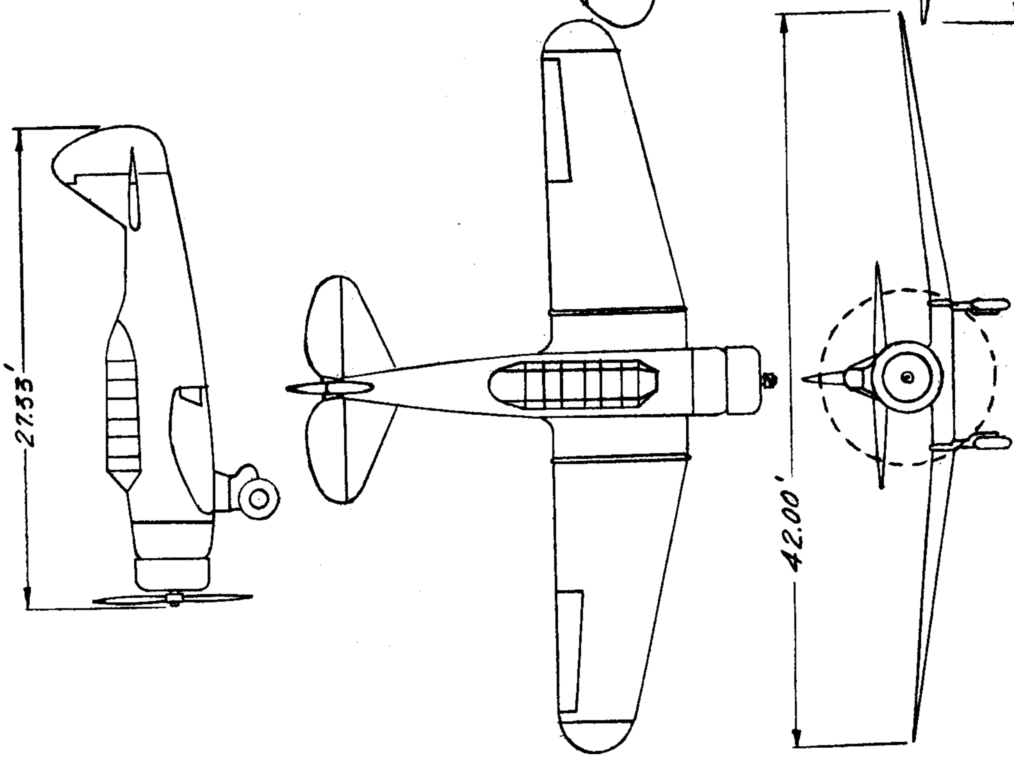

Figure 7.- The BT-9A airplane.
Test weight 4472 pounds

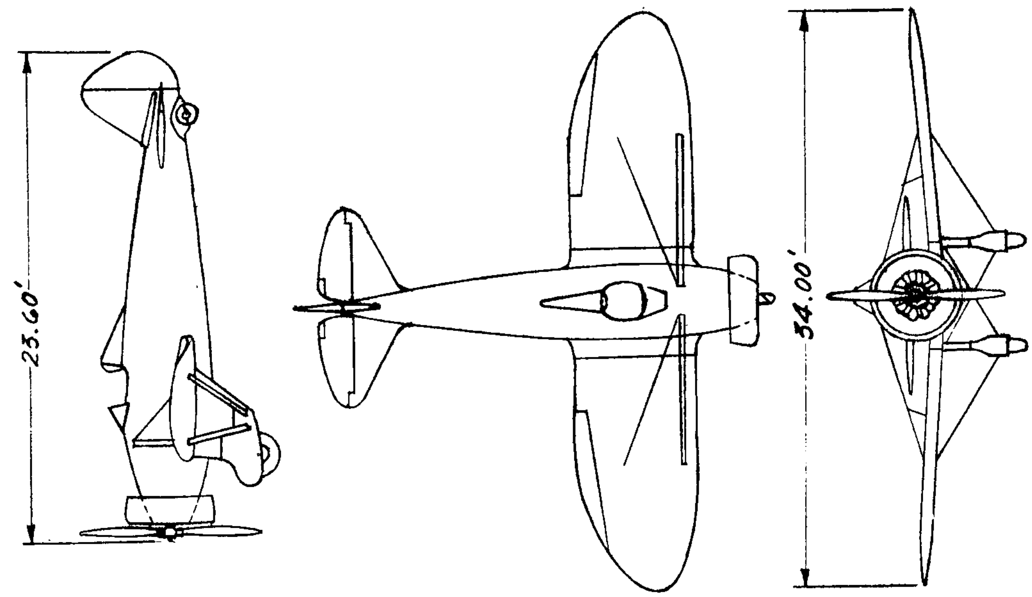

Figure 10.- The P-26A airplane.
Test weight, 3091 pounds

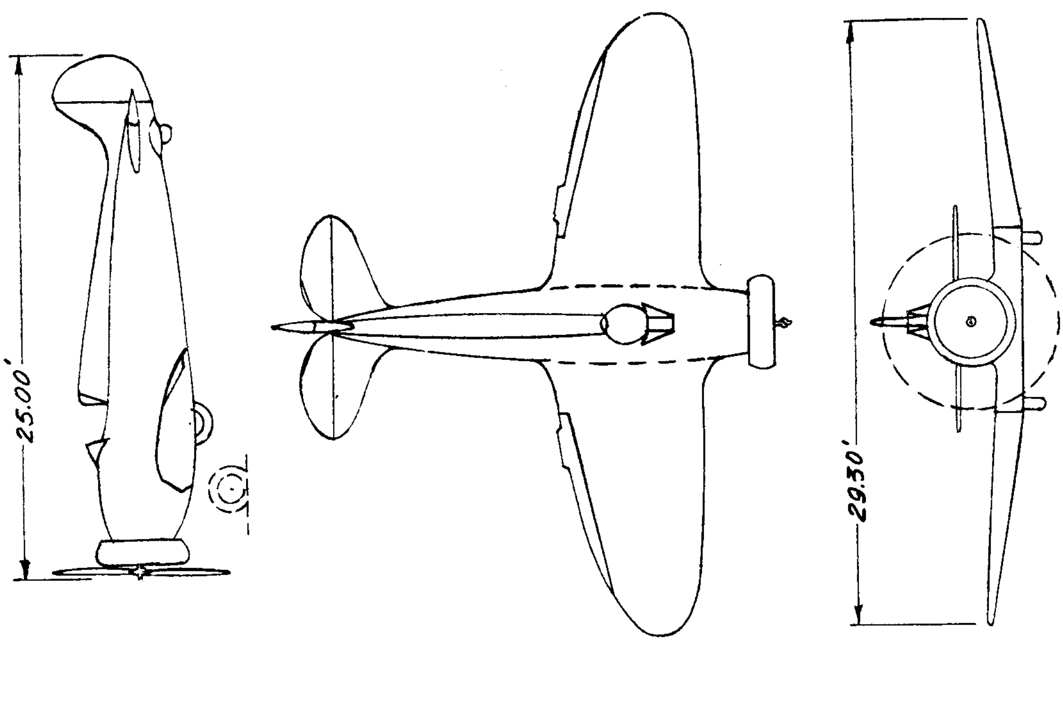

Figure 9.- The YP-29A airplane.
Test weight, 3433 pounds

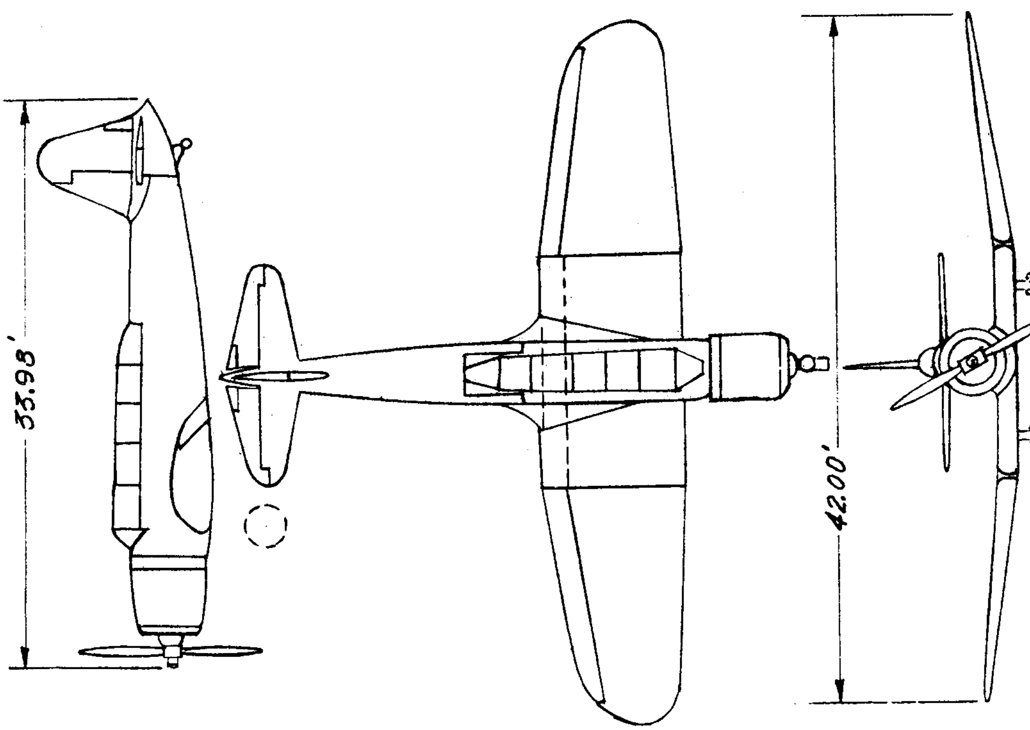

Figure 11.- The XSB2U-2 airplane.
Test weight, 6305 pounds

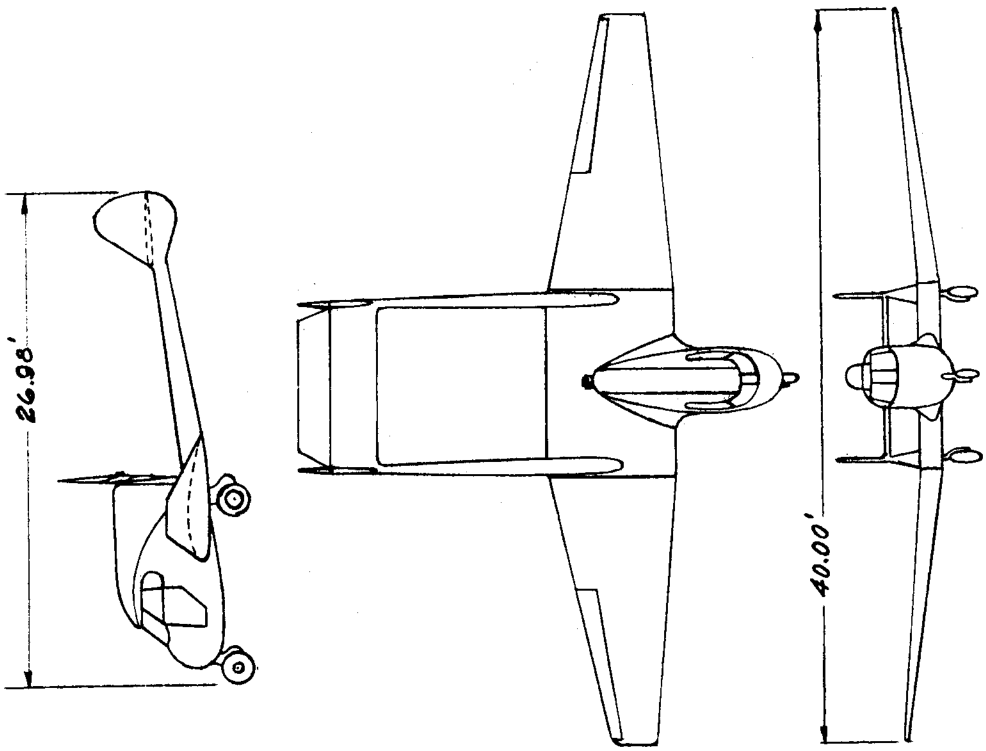

Figure 12.- The Hammond Y-1 airplane.
Test weight, 1893 pounds

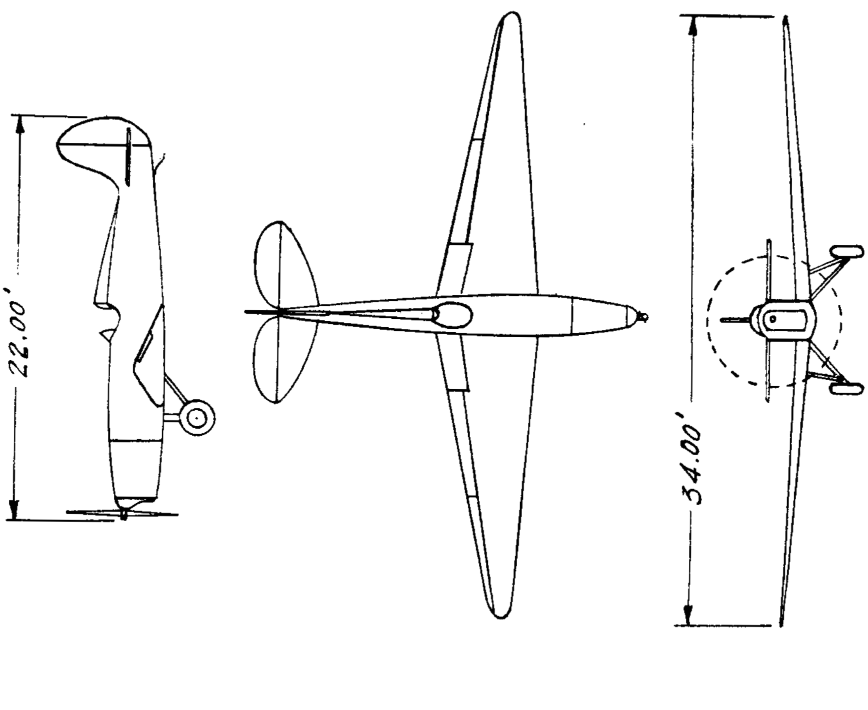

Figure 13.- The J-2 airplane.
Test weight 1535 pounds

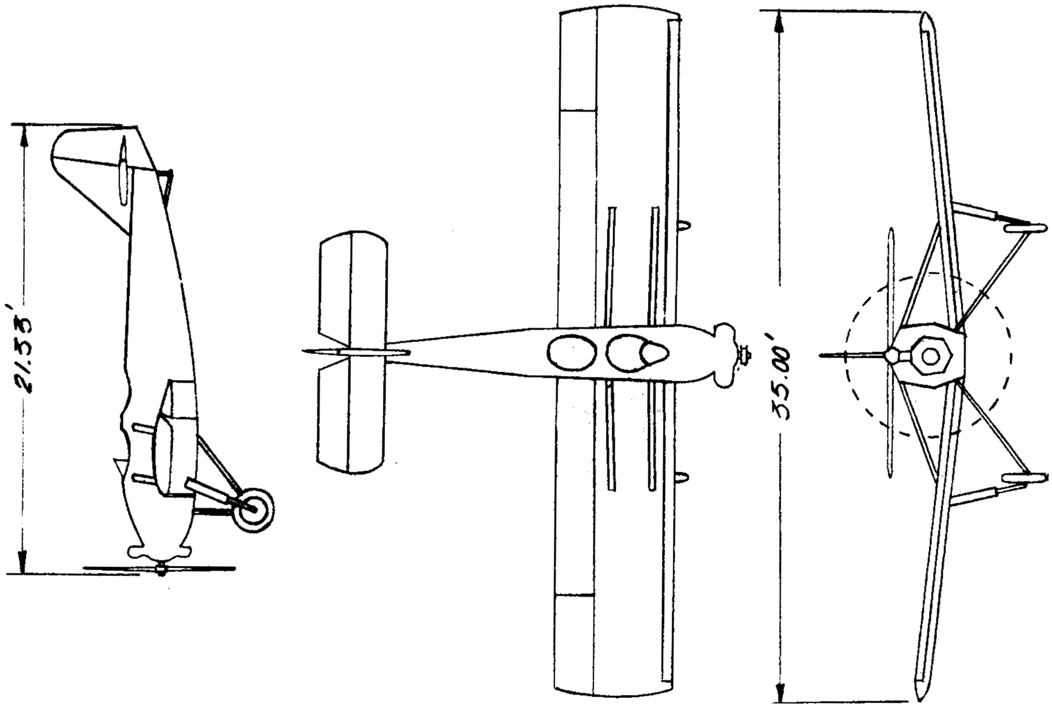

Figure 14.- The McDonnell airplane.
Test weight, 1708 pounds

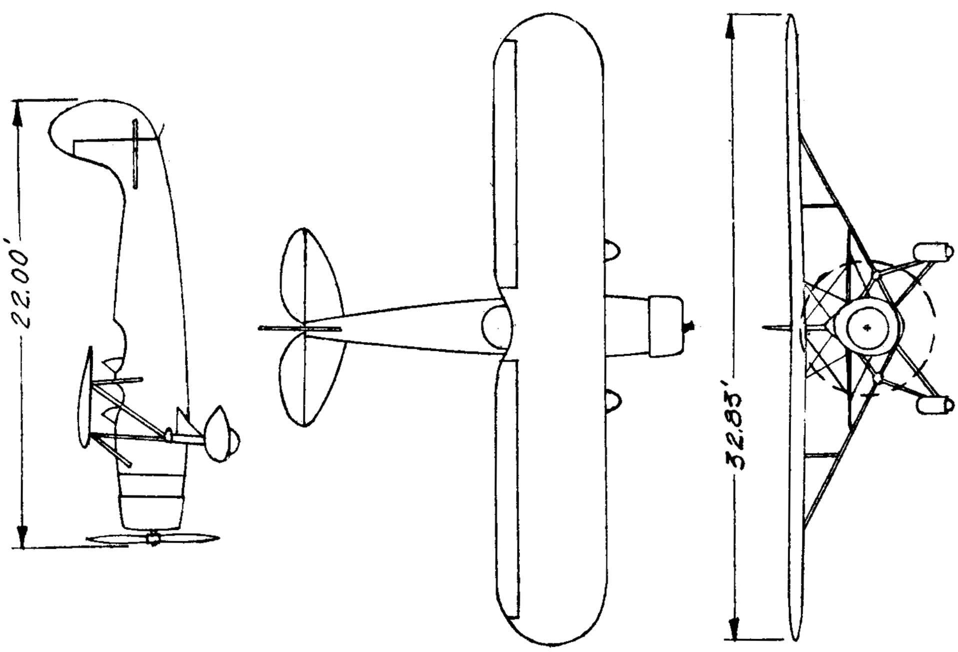

Figure 15.- The XR2K-1 airplane. Test weight, 1729 pounds

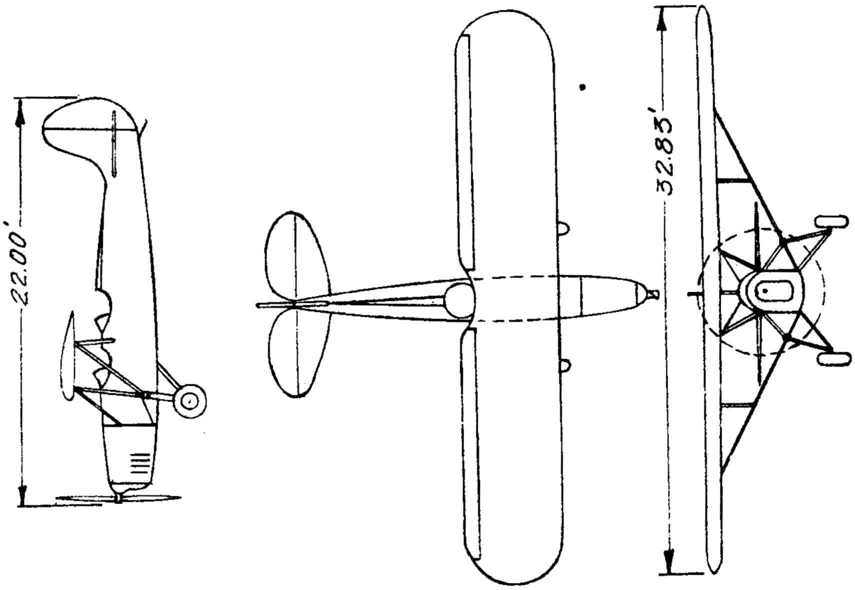

Figure 16.- The F-22 airplane. Test weight, 1388 pounds

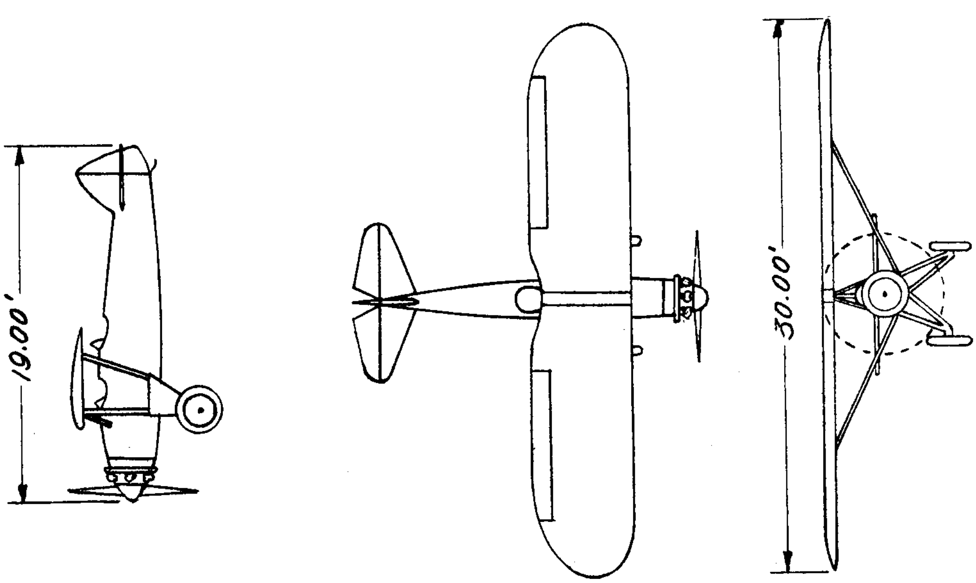

Figure 17.- The Doyle O-2 airplane.
Test weight, 1388 pounds

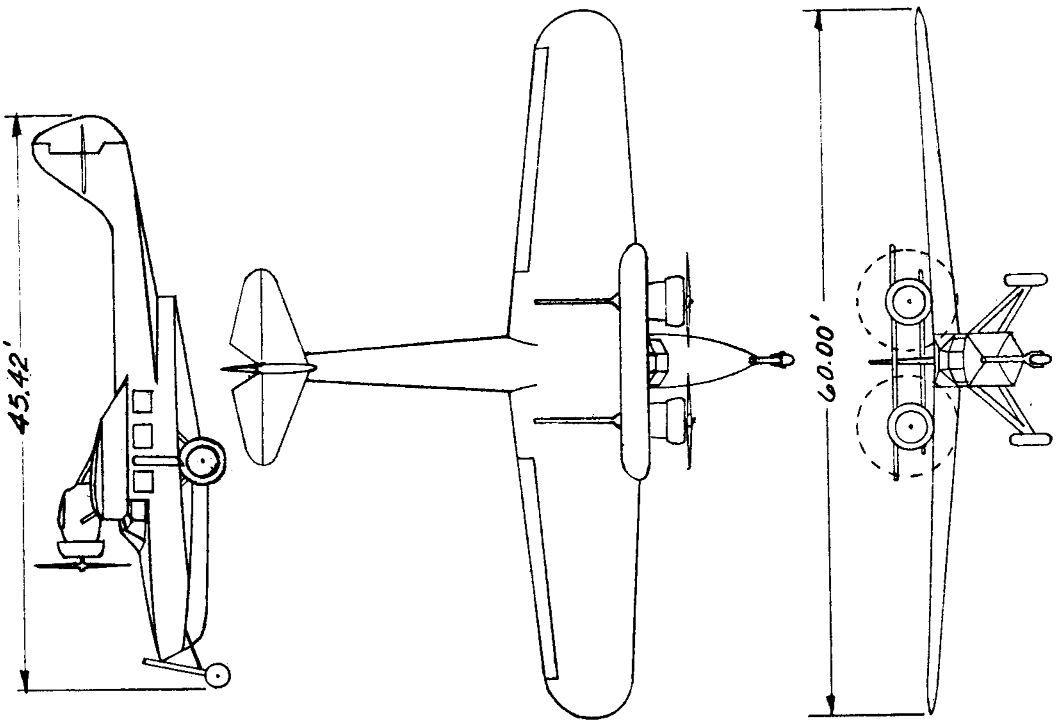

Figure 18.- The OA-4A airplane.
Test weight, 2553 pounds

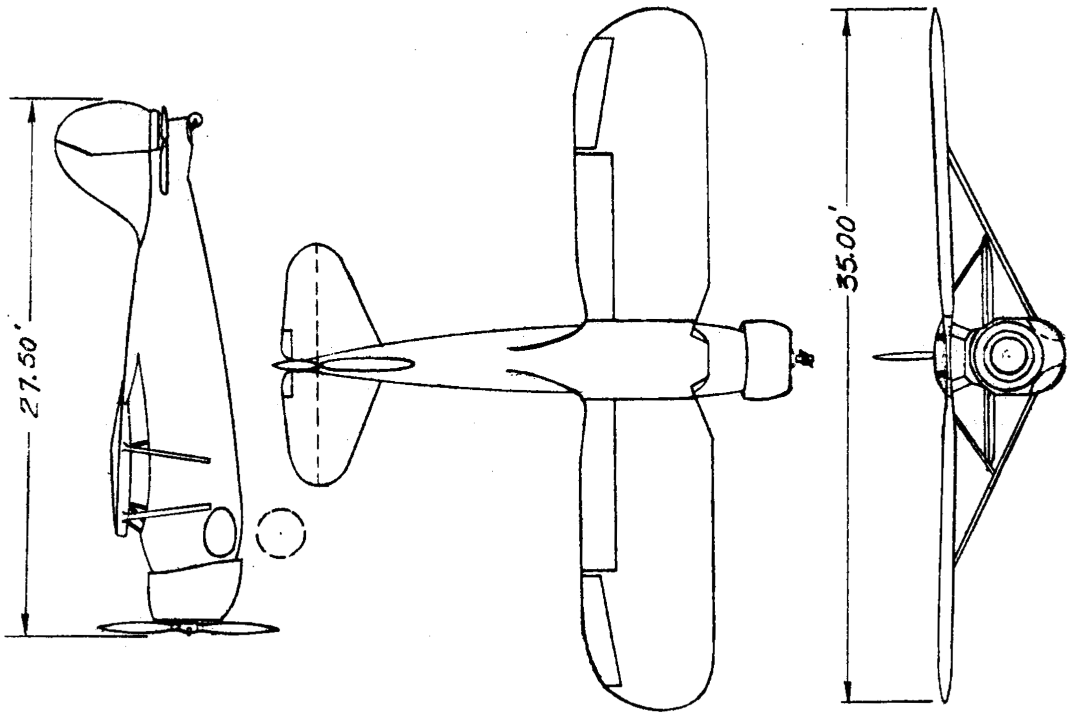

Figure 20.- The XF13C-3 airplane. Test weight, 4662 pounds

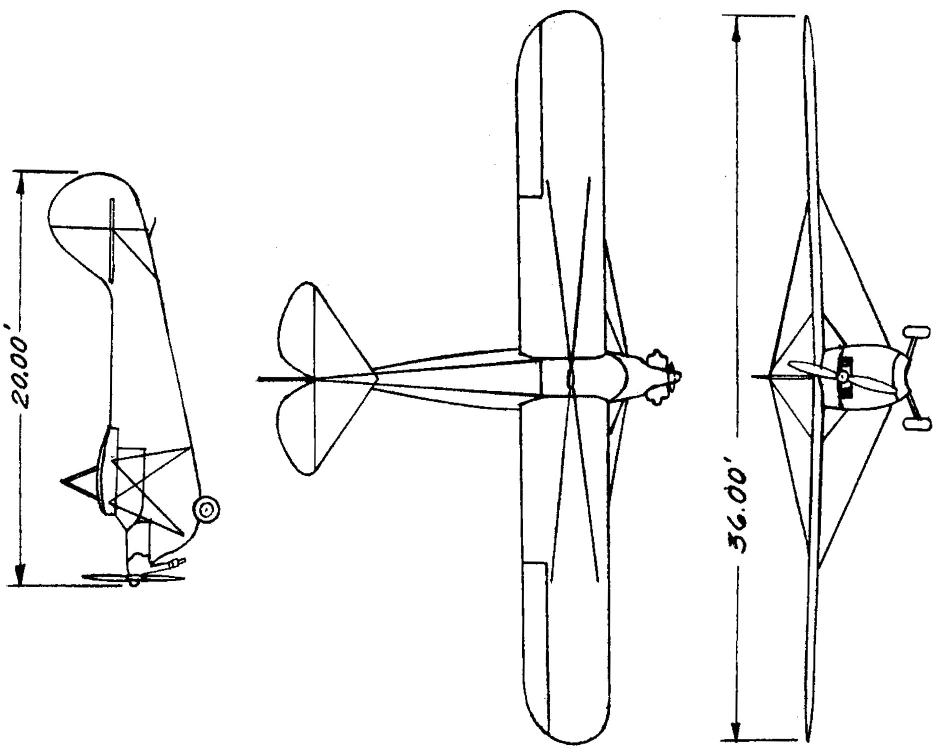

Figure 19.- The Aeronca C-2N airplane. Test weight, 584 pounds

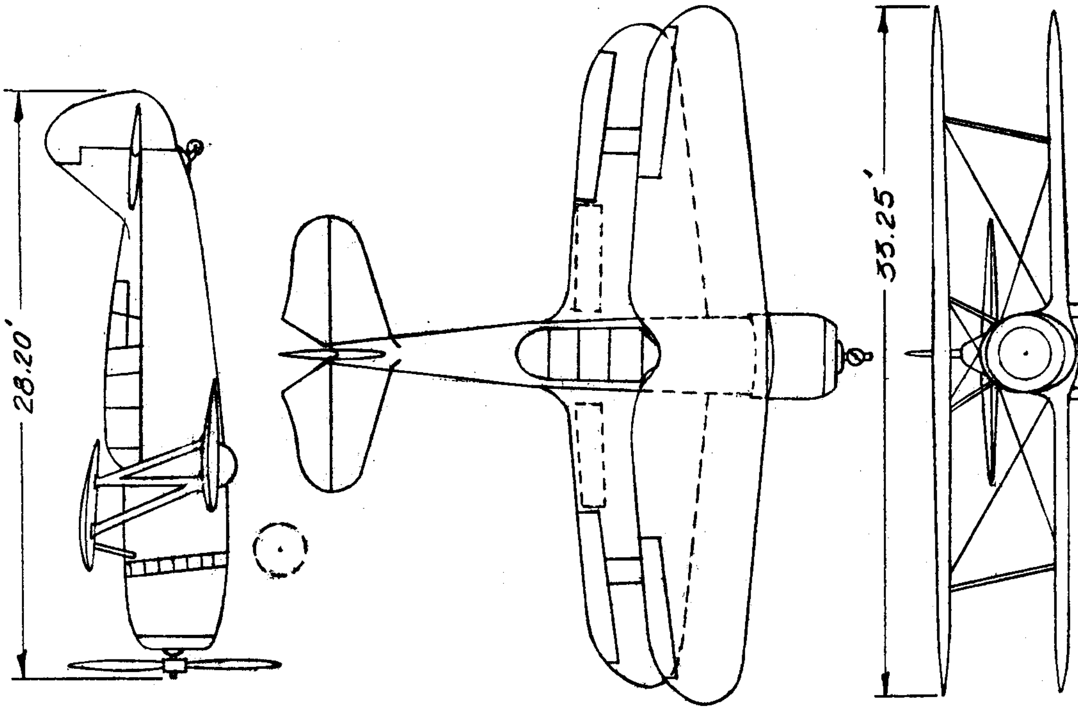

Figure 22.- The XSB3U-1 airplane.
Test weight, 5330 pounds

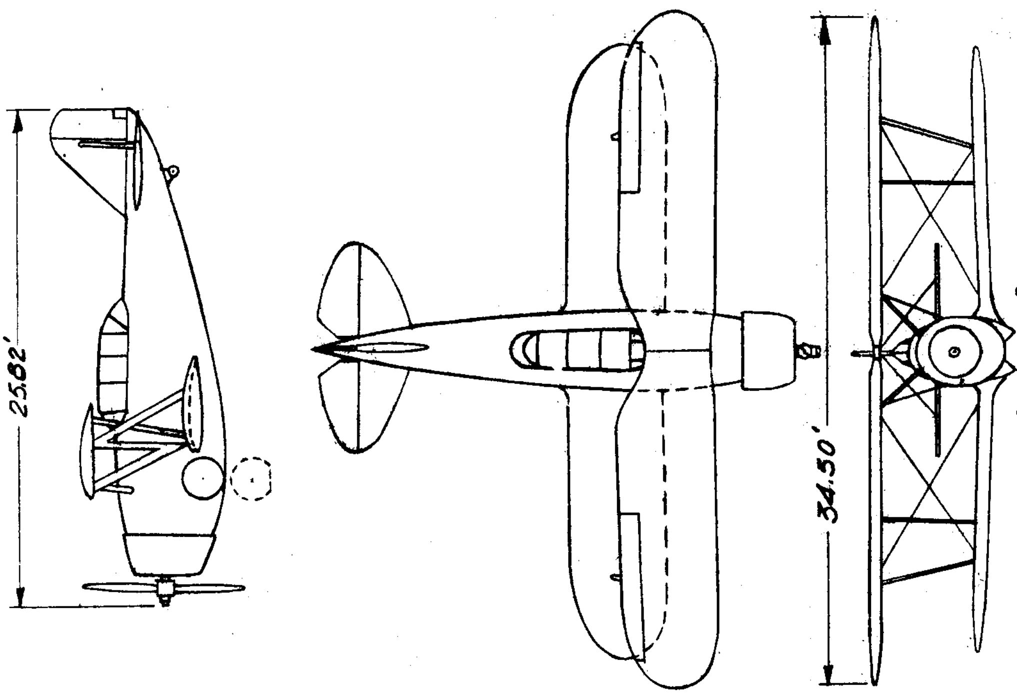

Figure 21.- The XSBF-1 airplane.
Test weight, 4473 pounds

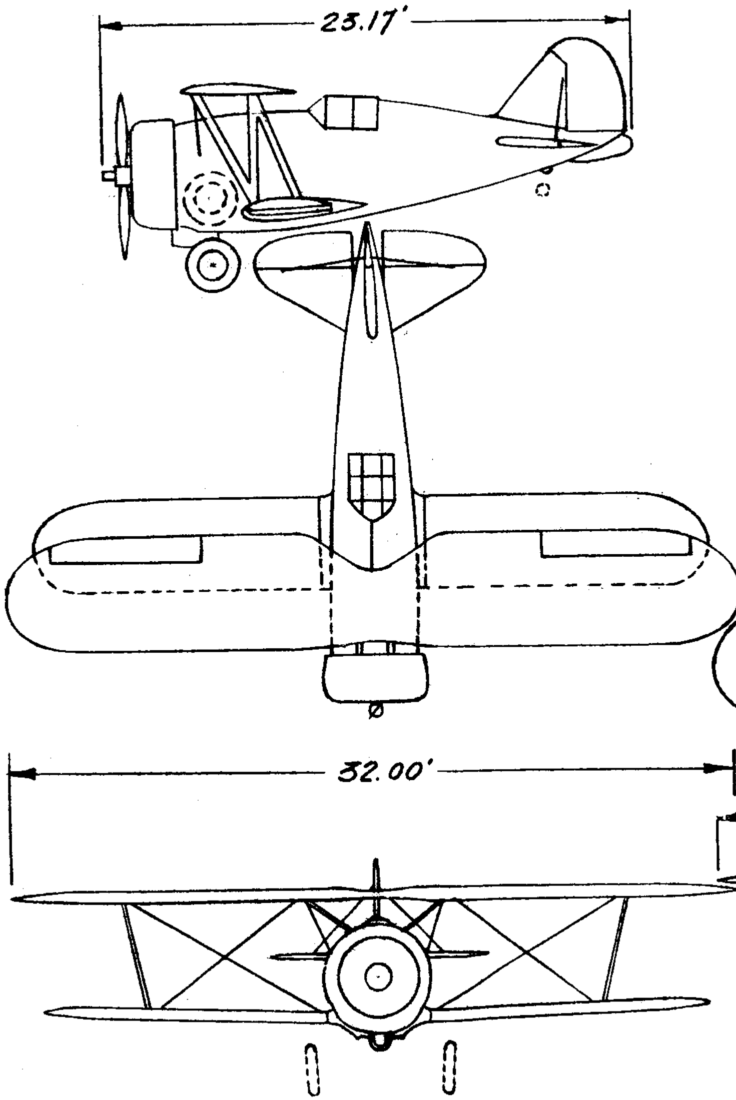

Figure 23. - The F3F-2 airplane.
Test weight, 4673 pounds.

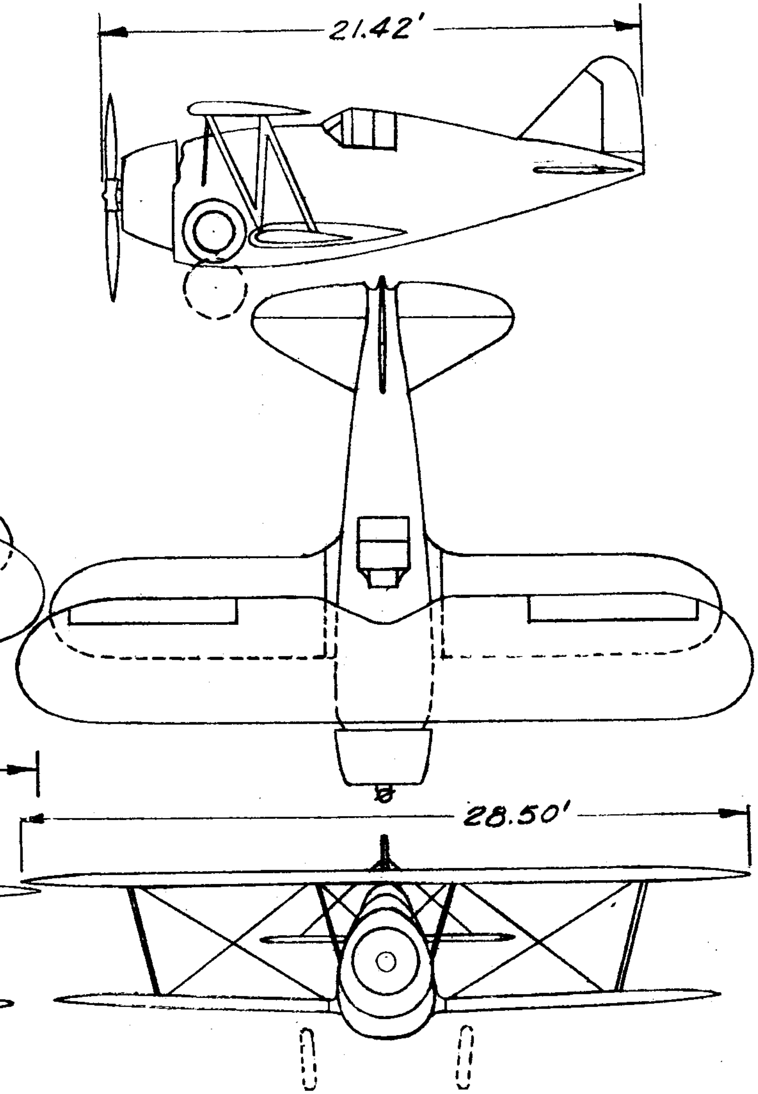

Figure 24. - The XF2F-1 airplane.
Test weight, 3550 pounds.

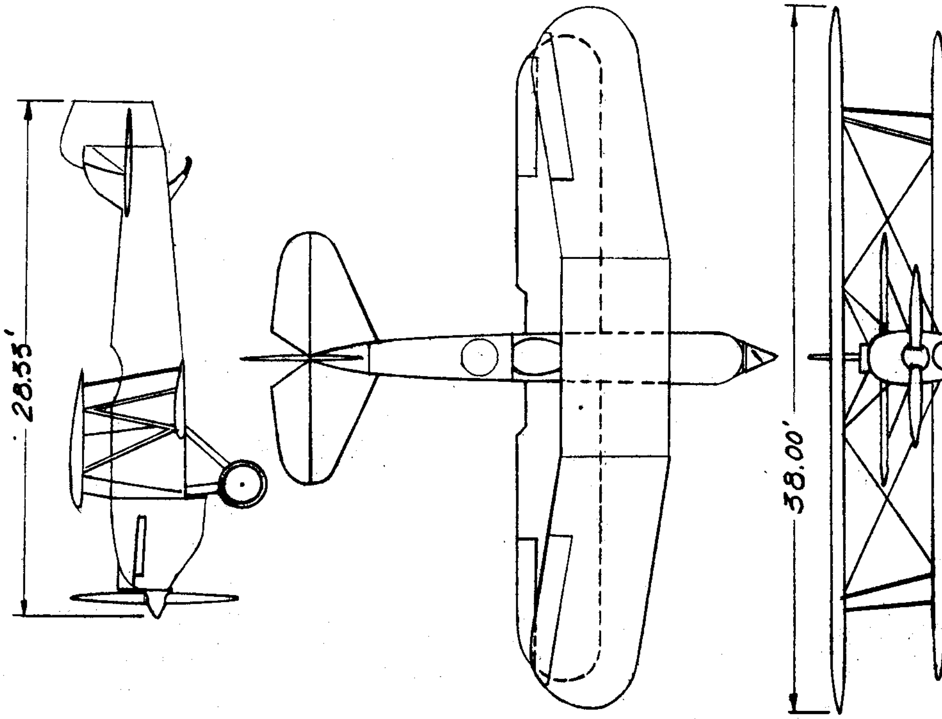

Figure 26.- The O-11 airplane.
Test weight, 4258 pounds.

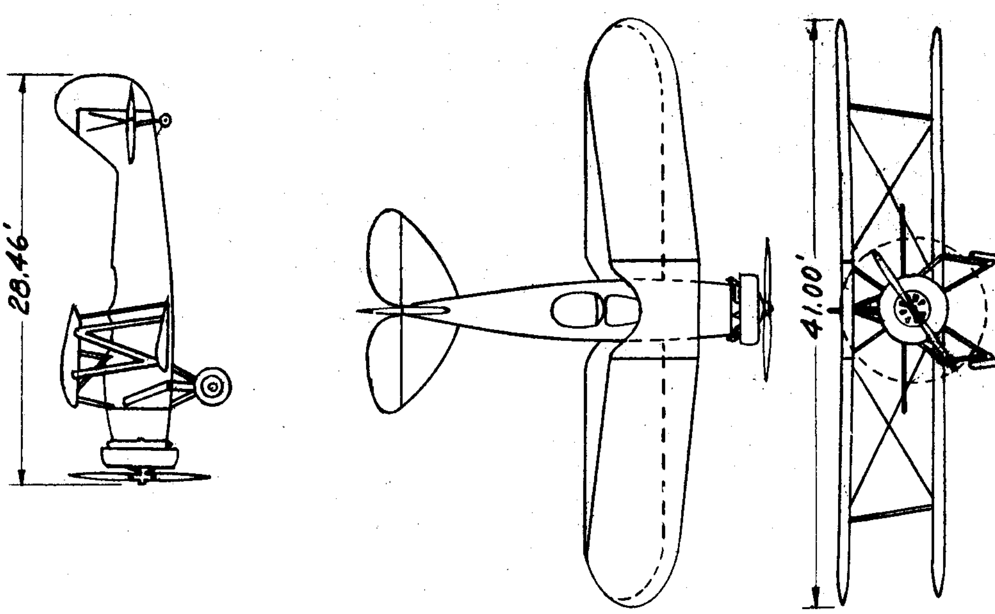

Figure 25.- The XBM-1 airplane.
Test weight, 4769 pounds.

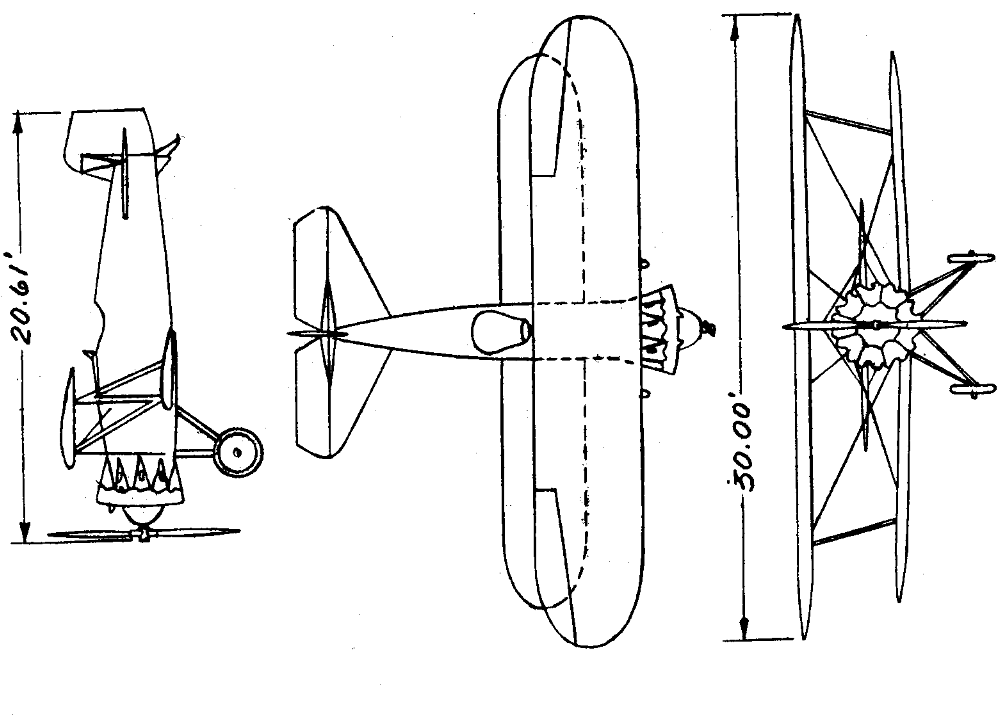

Figure 27.- The F4B-1 airplane.
Test weight, 2540 pounds.

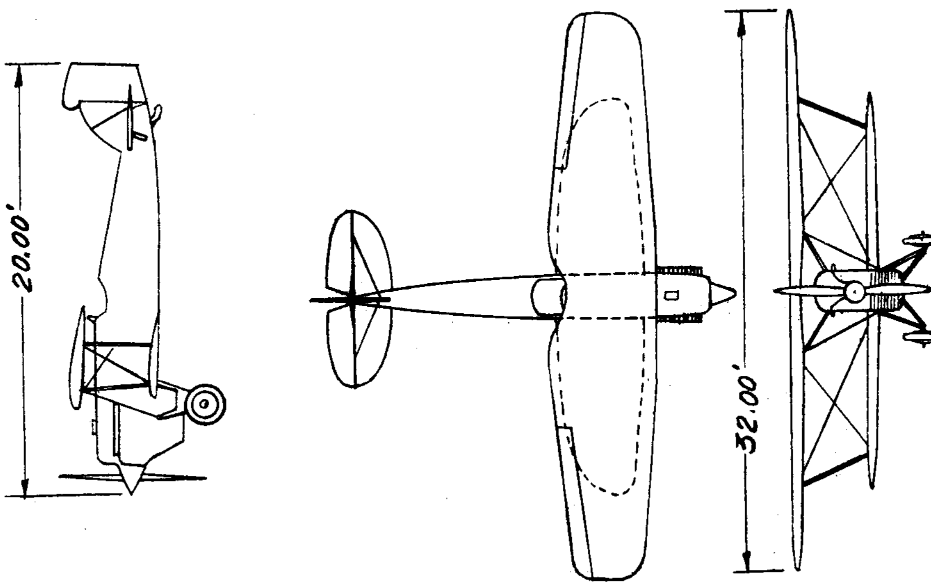

Figure 28.- The F4B-2 airplane.
Test weight, 2816 pounds.

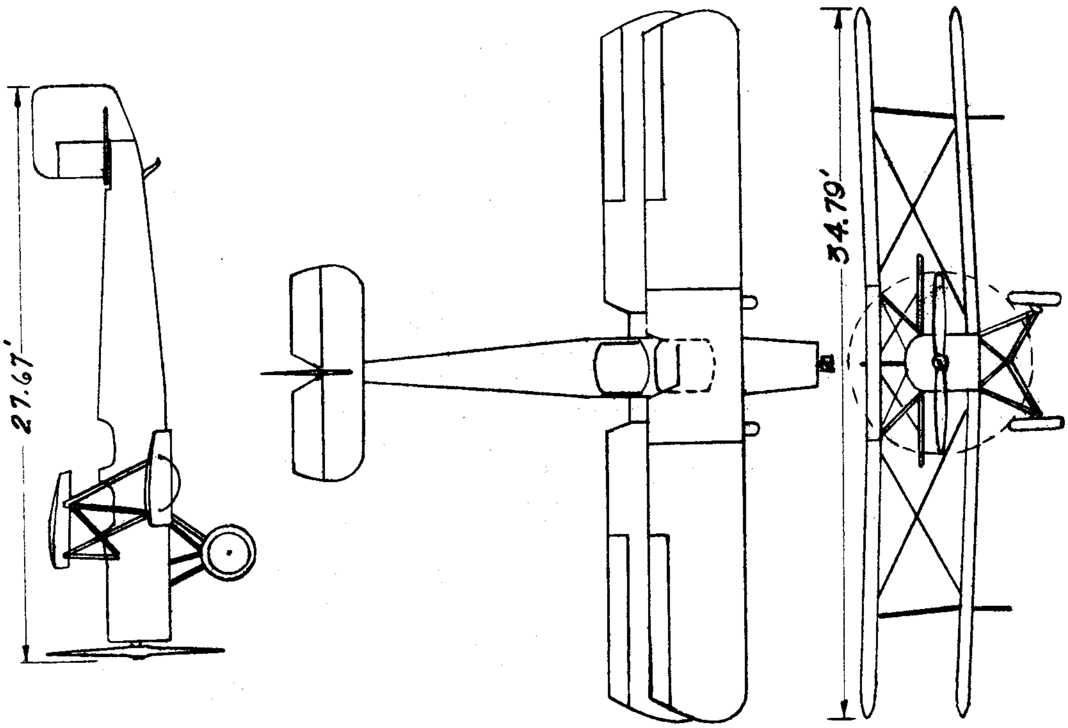

Figure 30.- The PT-1 airplane.
Test weight, 2512 pounds.

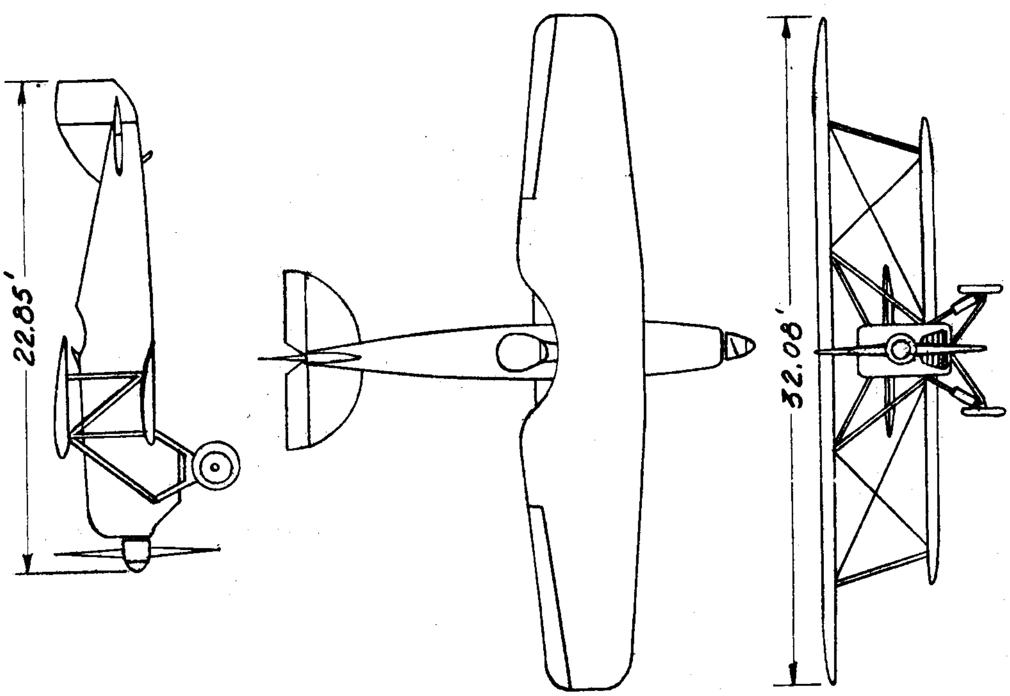

Figure 29.- The PW-9 airplane.
Test weight, 2885 pounds.

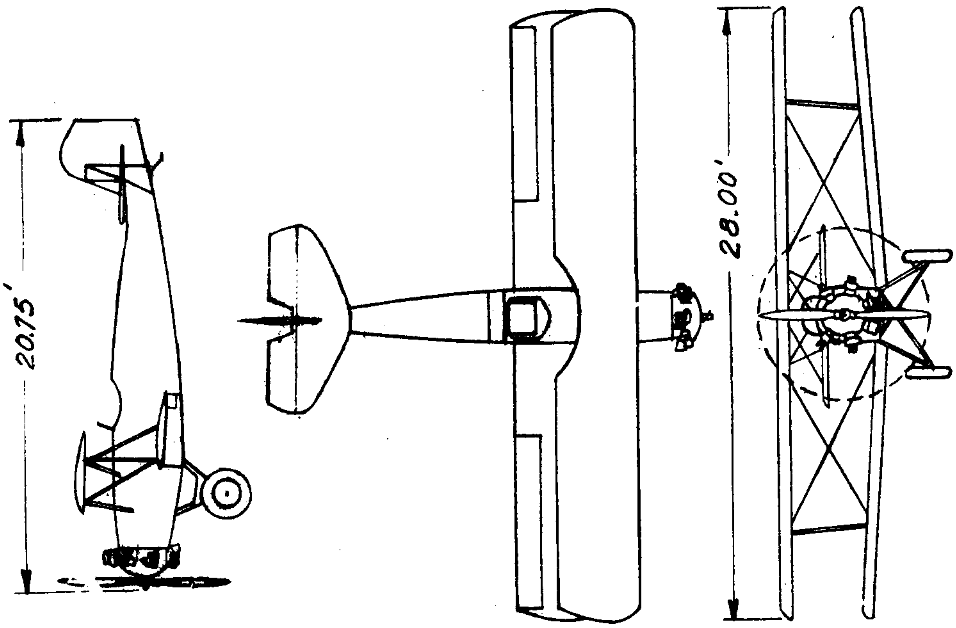

Figure 32.- The XN2Y-1 airplane.
Test weight 1567 pounds.

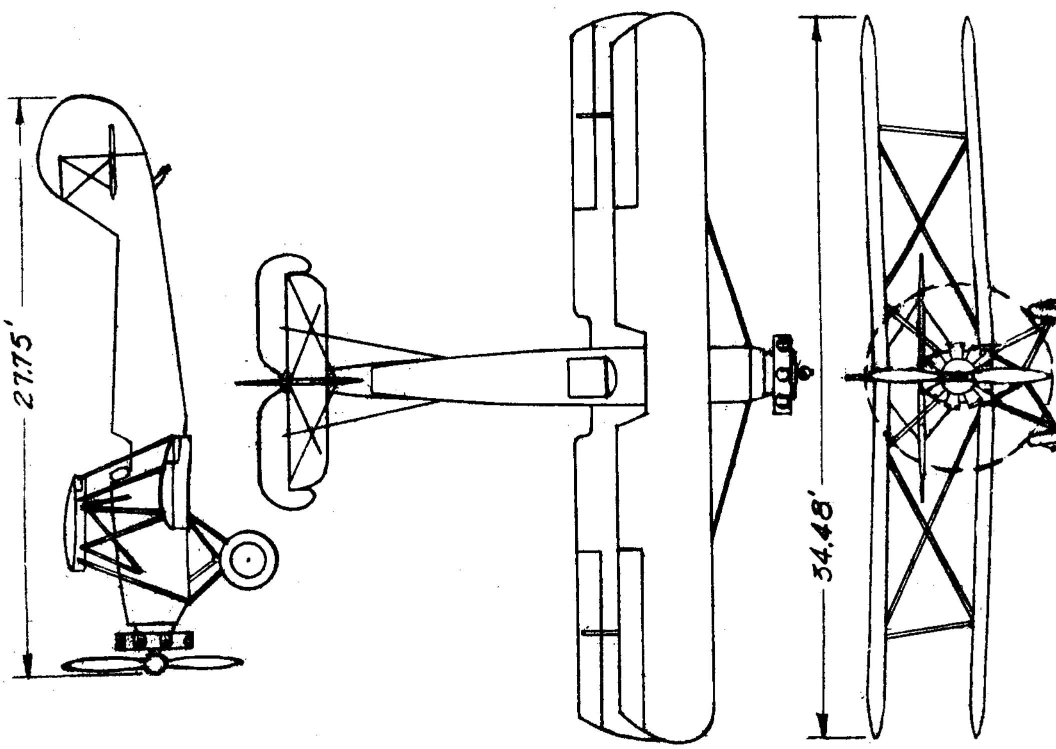

Figure 31.- The NY-1 airplane.
Test weight, 2622 pounds.

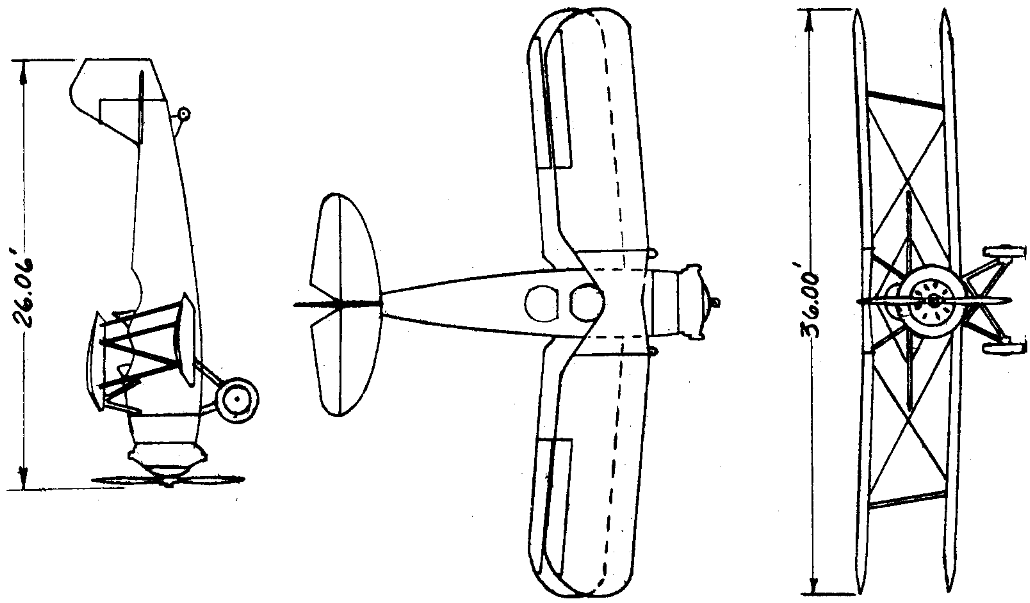

Figure 34.- The O3U-1 airplane.
Test weight, 4057 pounds.

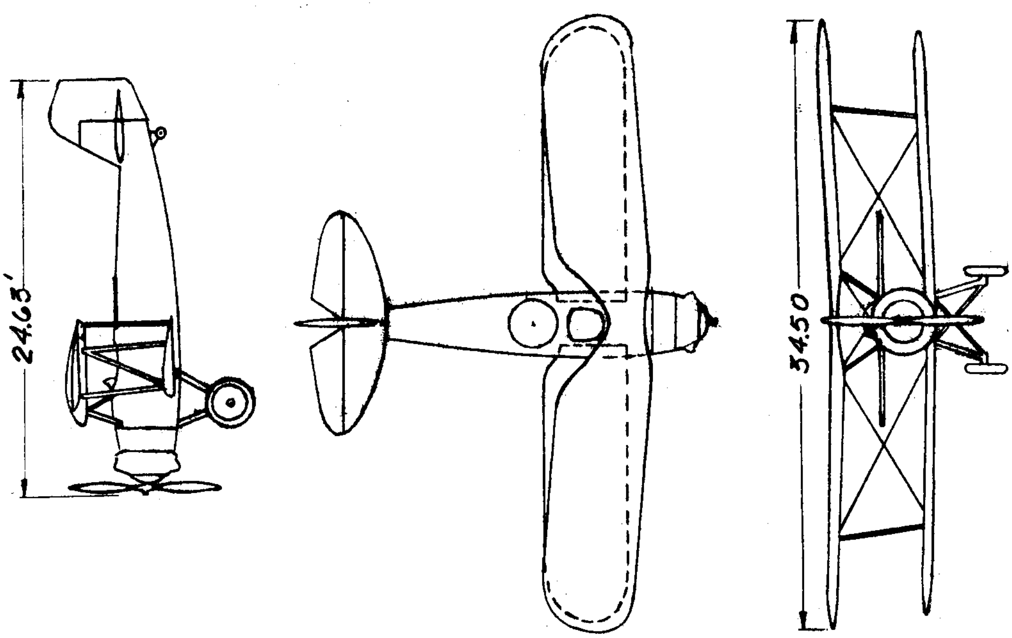

Figure 33.- The O2U-3 airplane.
Test weight, 3550 pounds.

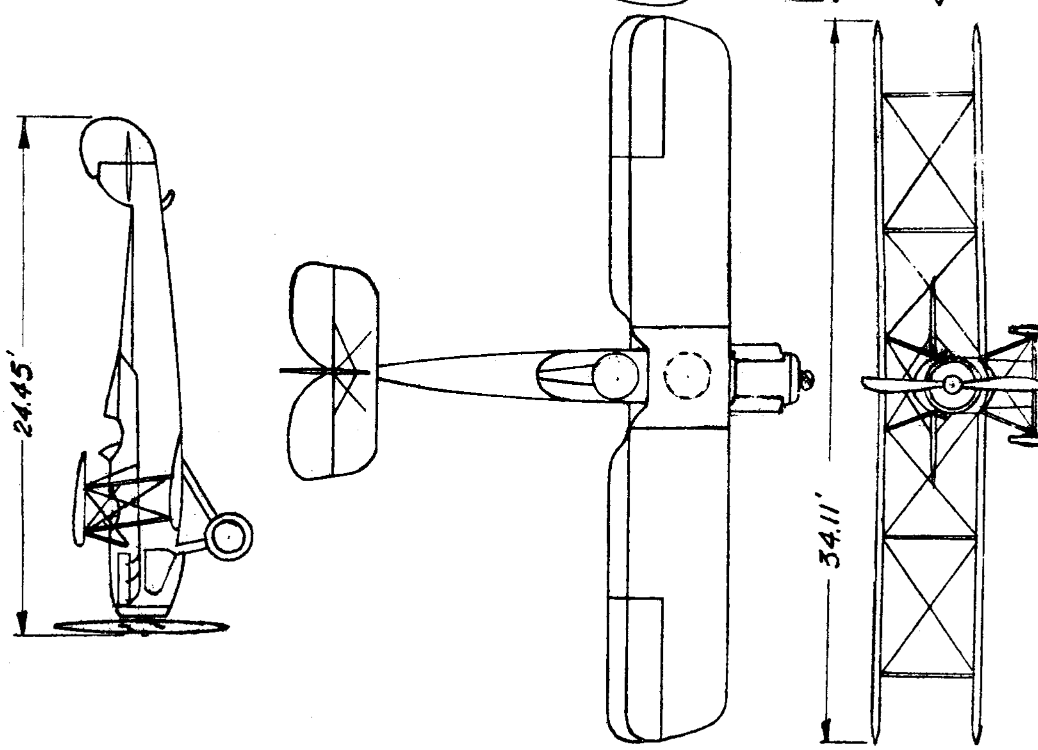

Figure 35. - The VE-7 airplane.
Test weight, 2208 pounds.

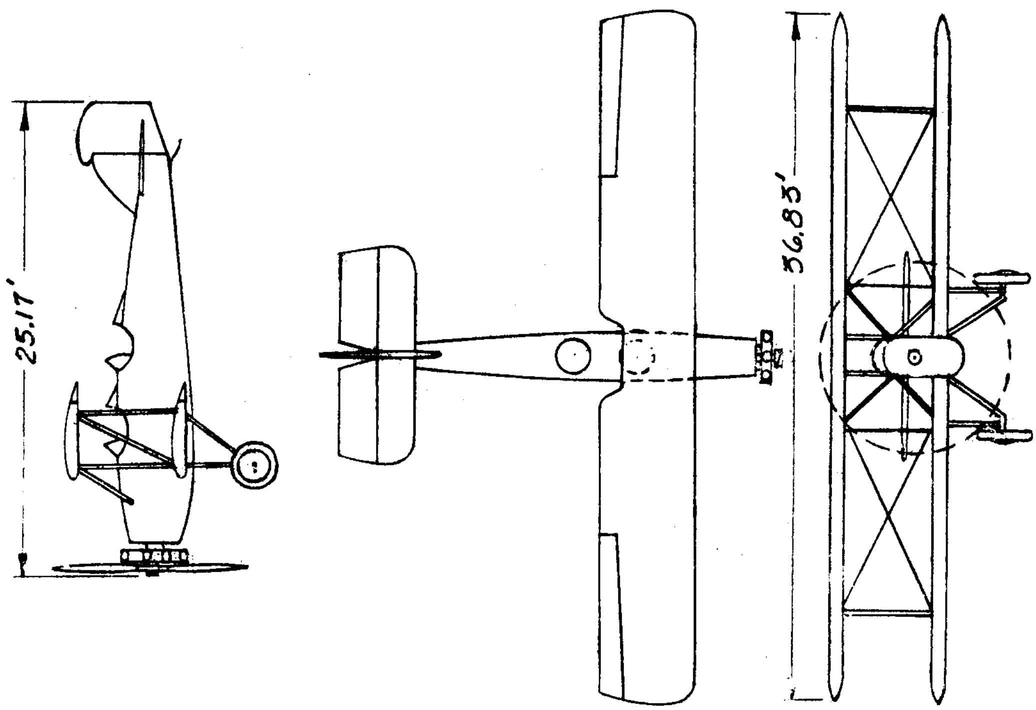

Figure 36. - The NB-1 airplane.
Test weight, 2544 pounds.