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A PROBABILITY ANALYSIS OF THE METEOROLOGICAL FACTORS CONDUCIVE TO AIRCRAFT ICING IN THE UNITED STATES

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SUMMARY

Meteorological icing data obtained in flight in the United States are analyzed statistically and methods are developed for the determination of: (1) the various simultaneous combinations of the three basic icing parameters (liquid-water content, drop diameter, and temperature) which would have equal probability of being exceeded in flight in any random icing encounter; and (2) the probability of exceeding any specified group of values of liquid-water content associated simultaneously with temperature and drop-diameter values lying within specified ranges. The methods are particularly useful in the design of anti-icing equipment intended to operate through the United States, to define simultaneous combinations of the meteorological variables which could be encountered, and to ascertain the effectiveness of the equipment in withstanding the natural icing conditions to which it may be subjected. In addition, a mathematical basis is provided for the future statistical analysis of meteorological icing data that might be obtained throughout the world.

INTRODUCTION

The program of research in aircraft ice prevention which has been conducted by the NACA during the past several years has been directed primarily toward the development of practical methods for the design of thermal ice-prevention equipment for various airplane components. Since a rational ice-prevention design requires a knowledge of the physical characteristics of icing conditions, an important phase of the research program has been an investigation of the meteorological conditions conducive to icing.

The severity of an encounter with icing conditions is determined principally by four factors; namely, the liquid-water content, the

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diameter of the drops, the temperature, and the horizontal extent of the conditions. The meteorological investigation has, therefore, been concerned with obtaining measurements of these quantities in a wide variety of natural icing conditions in order to establish the range and relative frequency of occurrence of various values, and combinations of values, of these quantities.

During the period 1945 through 1948, a considerable amount of data was collected by the Ames and Lewis Laboratories. These results, and discussions of various phases of the investigation, have been reported in references 1 through 4. All of these results, together with similar data from observations made by other organizations, were used as a basis for a listing of estimated maximum icing conditions recommended for consideration in the design of anti-icing equipment (reference 5). The probable maximum values listed in reference 5 were estimated on the basis of what was considered a reasonable extrapolation from a limited amount of data, and the concept of the probability of encountering various simultaneous combinations of values entered only in a subjective and qualitative way.

In this report, the available data obtained in icing flights in the United States are analyzed statistically and the results are presented graphically in two forms. In the first form, the results are plotted to show the various combinations of liquid-water content, drop diameter, and ambient-air temperature which have equal probability of being exceeded. In the second form, plots are presented to determine the probability of exceeding any specified value of liquid-water content under the condition that the value is associated simultaneously with values of temperature and drop diameter lying within specified intervals. Both plots are based on preselected values of horizontal extent; however, methods for estimating values applicable to other horizontal extents are also presented.

Although the results of this report are directly applicable only to the United States, an estimation of the conditions prevailing in other localities of similar climatic conditions can be made based upon the results presented herein. As meteorological icing data are accumulated for regions outside the United States, the methods of this report should provide a framework for placing the data on a statistical basis that is not limited in scope to the United States.

SYMBOLS

The following symbols are used throughout this report:

A,B,C designation applied to three events occurring simultaneously but not necessarily independently

c	chord length, feet
D	mean-effective drop diameter, microns
D_{cr}	the smallest drop diameter for which drops will impinge upon a thermal element
ΔD	interval of drop diameter, microns
e	Naperian base of logarithms
E_m	collection efficiency of an airfoil section, percent
F	factor by which liquid-water-content values are multiplied to make them applicable to other distances of cloud horizontal extent, dimensionless
M_t	weight rate of water intercepted, pounds per hour per foot of airfoil span
n	an integer
O	an icing condition considered to be defined by specifications of the three variables, liquid-water content, temperature depression below freezing, and drop diameter
P	probability of overloading any given element of a thermal anti-icing system
P_A	probability of the occurrence of event A
P_{ABC}	probability of the simultaneous occurrence of three events, A, B, and C
$P_B(A)$	probability of the occurrence of event B, under the condition that event A will occur
$P_C(A,B)$	probability of the occurrence of event C under the condition that both events A and B will occur
P_e	probability that any random icing encounter will be characterized by a combination of values of the variables (liquid-water content, temperature depression below freezing, and drop diameter), in which each variable must, respectively, equal or exceed a specified value of that variable
P_G	probability defined by the Gumbel probability distribution and expressed by equation (A1)

ΔP_i — probability that, for a random icing encounter, the following three conditions will simultaneously be fulfilled: (1) liquid-water content exceeding a specified value; (2) temperature within a specified interval; and (3) drop diameter within a specified interval

$P_{\Delta D}$ — probability that drop diameter will have a value lying within a specified range (ΔD)

$P_{\Delta T}$ — probability that temperature will have a value lying within a specified range (ΔT)

$P_{W_i}(\Delta T)$ — probability that liquid-water content will be equal to or greater than W_i under the condition that the temperature lies within a specified range (ΔT)

Q — probability defined by the probability function $Q(T')$ for a fixed value of T'

$Q(T')$ — probability function, determined by the variation with temperature depression below freezing for the most severe icing conditions, of the percent of icing encounters in which the temperature depression below freezing is less than certain specified values

R — probability that the maximum liquid-water content in an icing encounter is equal to or greater than W, under the condition that the temperature depression below freezing is equal to or greater than T'

$R(W,T')$ — probability function expressing R in terms of W and T'

R' — probability that the liquid-water content associated with the maximum value of drop diameter in an icing encounter is equal to or greater than W, under the conditions that the temperature depression below freezing is equal to or greater than T' and the maximum value of drop diameter is equal to or greater than D

$R'(W,D,T')$ — probability function expressing R' in terms of W, D, and T'

S — probability that the drop diameter is equal to or greater than D, under the conditions that the temperature depression below freezing is equal to or greater than T' and the liquid-water content is equal to or greater than W

$S(D,W,T')$ — probability function expressing S in terms of D, W, and T'

S' — probability that the maximum value of drop diameter in an icing encounter is equal to or greater than D, under the condition that the temperature depression below freezing is equal to or greater than T'

$S'(D,T')$ probability function expressing S' in terms of D and T'

$S''(D)$ composite distribution function consisting of parts of each of the functions $S(D,0,0')$ and $S'(D,0)$

t airfoil thickness, percent of chord length

T temperature existing in an icing condition, $^{\circ}F$

T' temperature depression below freezing, equal to 32-T, F°

T_{cr}' temperature depression below freezing at which the heat output of the thermal element can bring the surface temperature of the element just to freezing temperature, F°

ΔT interval of temperature, $^{\circ}F$

$\Delta T'$ interval of temperature depression below freezing, F°

u a constant which denotes the mode in the Gumbel probability equation

V airspeed, miles per hour

W liquid-water content, grams per cubic meter

$W_{0.05}$ value of liquid-water content corresponding to a probability value of 0.05 defined by the Gumbel probability equation

$W_{0.63}$ value of liquid-water content corresponding to a probability value of 0.63 defined by the Gumbel probability equation

ΔW interval of liquid-water content, grams per cubic meter

x independent variable in the Gumbel probability equation

α a parameter which depicts the concentration of a frequency distribution about the mode in the Gumbel distribution equation

Subscripts

1 any selected icing condition

i a representative value of a variable

ANALYSIS

The data used in this analysis consist of 1038 sets of values of liquid-water content, drop diameter, and temperature obtained during 252 encounters with icing conditions. Values of liquid-water content and drop diameter were measured by means of the rotating multicylinder method. All data available at the time of preparation of this report were used in the analysis, including (1) data obtained by the Ames and Lewis Laboratories and published in references 1, 2, 3, 4, and 6; (2) data contained in the monthly reports of the Air Materiel Command Aeronautical Ice Research Laboratory, the major portion of which is presented in reference 7; and (3) data obtained by United Air Lines (reference 8) and American Airlines (reference 9) during icing flights with Douglas DC-6 type airplanes.

For the purposes of this analysis, the basic unit of data is the icing encounter which consists of all measurements made during flight through a single area of continuous or intermittent icing conditions. This area was separated from other icing conditions[3] by relatively large areas in which no icing was observed. The problem considered herein is to determine the probability that an icing encounter chosen at random will include conditions of a given severity averaged over a certain distance. Since most of the rotating-cylinder observations represent averages over distances of about 3 miles in cumulus clouds and 10 miles in layer clouds, these distances are regarded for purposes of data reduction as standard values of horizontal extent, applicable to data from the two principal cloud types. The probabilities obtained on this basis may be applied to other distances by using the data concerning the relation between horizontal extent and average liquid-water content obtained from continuous records of the rotating-disk icing-rate meter. (See reference 3.) It has been assumed that the greatest values of liquid-water content and drop diameter measured by the rotating cylinders during an icing encounter represent maximum values averaged over the standard distances of 3 or 10 miles. This assumption is probably approximately true, since a particular effort was made to obtain rotating-cylinder data during the periods of most rapid ice formation.

Since the variations of temperature during a single icing encounter are usually not very great, the most severe icing conditions measured during a particular icing encounter then will be either the greatest value of liquid-water content and the corresponding value of drop diameter, or possibly, the greatest value of drop diameter and the corresponding value of liquid-water content. Both of these possibilities for a severe

[3]The term "icing condition" as used herein denotes a state of the atmosphere defined by a set of values of temperature, liquid-water content, drop diameter, and pressure altitude in which the temperature is below freezing and the liquid-water content is greater than zero.

icing condition are considered in the probability analysis which, for the sake of convenience, is developed in three phases.

The first phase of the analysis is the classification of the available data according to cloud type and geographical location; the second phase is the determination of the particular combinations of liquid-water content, drop diameter, and ambient-air temperature, for a given horizontal cloud extent, that have the same probability of being simultaneously equaled or exceeded in a random icing encounter; and the third phase is the determination of the probability of exceeding any specified group of values of liquid-water content associated simultaneously with temperature and drop-diameter values lying within specified ranges.

Classification of Data According to Cloud Type and Geographical Location

For classification purposes in this report, clouds will be divided into two classes, cumulus clouds and layer clouds; and the area of the United States will be divided into three regions, the Pacific coast region, the plateau region, and eastern United States with boundaries as indicated in figure 1. This system of classification divides the observations into six cases as indicated in table I, which includes information relating to the number of icing encounters and the number of individual measurements of liquid-water content and drop diameter for each group. The two cases for which the greatest amount of data are available, eastern layer clouds (case 5) and Pacific coast cumulus clouds (case 2), are treated in considerable detail. The other cases are treated in as much detail as the data permit, with the exception of case 6 (cumulus clouds in eastern United States) which is omitted because of insufficient data.

Determination of Particular Combinations of Liquid-Water Content, Drop Diameter, and Temperature Depression Having the Same Probability of Being Simultaneously Equaled or Exceeded in a Random Icing Encounter

Exceedance probability.- The three fundamental icing variables used to define an arbitrary icing condition are liquid-water content W, drop diameter D, and temperature T. For purposes of probability analysis, however, it is more convenient to refer temperature to the freezing point and to use temperature depression below freezing T' as a variable to substitute for the temperature T. Thus, any particular icing condition can be represented by selected values of the variables W_1, D_1, and T_1'. The probability that any random icing encounter of standard extent will provide simultaneous values of each of the icing variables equal to or greater than each of the selected values (i.e., $W \geq W_1$, $D \geq D_1$, and $T' \geq T_1'$) will be termed the "exceedance probability" P_e corresponding to the icing condition W_1, D_1, and T_1'.

The exceedance probability represents the probability of the simultaneous occurrence of three events which are not necessarily independent. In order to evaluate P_e, use is made of the theorem of compound probability (reference 10) which states that the probability of the simultaneous occurrence of three events, A, B, and C, is equal to the probability of the occurrence of A times the conditioned probability of the occurrence of B, under the condition that A will occur, times the conditioned probability of the occurrence of C, under the conditions that both A and B will occur. This theorem can be expressed symbolically as follows:

$$P_{ABC} = P_A \, P_B(A) \, P_C(A,B) \tag{1}$$

If it is considered that the probability of exceeding certain values of T', W, and D is analogous to the probability of the occurrence of events A, B, and C expressed in equation (1), then the exceedance probability P_e may be represented by the product of three probability factors Q, R, and S, according to the equation

$$P_e = QRS \tag{2}$$

In applying equation (2) to the meteorological variables, the probability Q may be considered as a function of T' alone, R a function of W and T', and S a function of D, T', and W. The functional relations between Q, R, and S, and D, T', and W are denoted, respectively, by the expressions $Q(T')$, $R(W,T')$ and $S(D,T',W)$. The meaning of each expression is as follows:

$Q(T')$ a relation that defines the probability Q in terms of T' and expresses the probability for a given value of T' of a random icing encounter having a temperature depression below freezing equal to or greater than T'

$R(W,T')$ a relation that defines the probability R in terms of W when T' is specified and expresses the probability for a given value of W of a random icing encounter having a liquid-water content equal to or greater than W, under the condition that the temperature depression below freezing is equal to or greater than T'

$S(D,W,T')$ a relation that defines the probability S, in terms of D when W and T' are specified and expresses the probability for a given value of D of a random icing encounter having a value of drop diameter equal to or greater than D, under the condition that the liquid-water content is equal to or greater than W and the temperature depression below freezing is equal to or greater than T'

The physical significance of the term exceedance probability is best illustrated by graphical means. Figure 2 presents a three-dimensional plot with the variables W, D, and T' as the axes. Any icing condition of standard duration defined by selected values of these three variables, such as W_1, D_1, and T_1', appears as a single point O_1 on the plot. Corresponding to the point O_1, there is an exceedance probability P_{e_1} which expresses the probability that a random icing encounter will provide simultaneous values of each of the icing variables equal to or greater than each of the selected values, W_1, D_1, and T_1'. According to equation (2), the value of P_{e_1} would be given by the equation

$$P_{e_1} = Q_1 R_1 S_1$$

Thus, when P_e is regarded as a constant, P_{e_1}, equation (2) defines a surface passing through O_1, specified by all the combinations of W, T', and D which have the same probability of being simultaneously equaled or exceeded. This surface is shown as the equiprobability surface in figure 2, and represents the locus of all icing conditions which have the same probability of being equaled or exceeded. The quantitative value of exceedance probability applying to individual points lying on the surface is expressed by equation (2).

It should be noted that since Q, R, and S represent probabilities, they cannot be greater than one nor less than zero. Moreover, Q is equal to one when T' equals zero and continuously decreases as T' increases; R is equal to one when W is zero; and for any constant value of T', R is a continuously decreasing function of W. Finally, S is equal to one when D is zero and, for any particular pair of values of T' and W, S is a continuously decreasing function of D. The point at which the equiprobability surface intersects the T' axis is determined by the value of T' for which $Q = P_e$, since R and S are both equal to one. Similarly, the surface intersects the W and D axes at values of W and D which make R and S, respectively, equal to P_e. A constant-temperature contour passing through the point O_1 on the equiprobability surface would be defined by the condition $T' = T_1'$, from which it follows that $Q = Q_1$, and hence $RS = P_e/Q_1$. These values are depicted in figure 2.

The preceding theory concerning exceedance probability must be modified slightly to make it applicable to the particular problem under consideration, in which the probabilities are to apply to the most severe icing conditions in an icing encounter. In most cases, the most severe icing condition is represented by the maximum value of liquid-water content and the corresponding (simultaneously observed) value of drop diameter. For the portion of the equiprobability surface which represents low values of liquid-water concentration and large values of drop diameter, on the other hand, the critical condition is represented by the maximum value of drop size and the corresponding value of liquid-water content. To take account of the first of these possible combinations, the function $R(W,T')$ was determined from the greatest values of liquid-water content for each icing encounter and the function $S(D,W,T')$ was obtained from the corresponding values of drop diameter; and to take account of the second possible combinations, another function $S'(D,T')$ was determined from the greatest values of drop diameter and a corresponding function $R'(W,D,T')$ was obtained from the corresponding values of liquid-water content. The functions $R(W,T')$ and $S(D,T')$ were then used to determine the main portion of the equiprobability surfaces, while the functions $R'(W,D,T')$ and $S'(D,T')$ were used for the portion representing large values of D and small values of W.

Construction of the equiprobability surfaces.- The procedure used in the construction of equiprobability surfaces from a particular set of data was essentially that of determining graphically the relations describing the functions $Q(T')$, $R(W,T')$, $S(D,W,T')$, $R'(W,D,T')$, and $S'(D,T')$. This procedure is illustrated in appendix A.

The mathematical functions used to represent the various distributions were chosen mainly on the basis of the empirical criterion of goodness of fit, although theoretical considerations also were given considerable weight, particularly in the selection of a function to represent the distribution of maximum values in those cases in which each item in the distribution is the maximum of a group of observed values. The function chosen for representing such distributions is one originated by Dr. E. J. Gumbel (reference 11) to describe a distribution of extreme values. This function will be referred to hereinafter as "Gumbel's distribution."

Because of an almost complete lack of observational data in flight concerning icing conditions at very low temperatures, reliance has been placed on the results of laboratory experiments on the formation of ice crystals to aid further in the construction of the equiprobability charts. The experiments relied upon (summarized in reference 12) indicate that a critical temperature exists in the neighborhood of -40° F, below which ice crystals are formed in very great numbers in atmospheric air whenever conditions of saturation with respect to liquid water occur. On the basis of this fact, it is inferred that clouds at temperatures below -40° F are composed either entirely or almost entirely of ice crystals; hence the liquid-water content is assumed to be zero at all values of T' greater than 72° F.

The methods outlined in the preceding paragraphs have been used to prepare constant-temperature contours defining the equiprobability surfaces for values of P_e equal to 0.1, 0.01, and 0.001 for cases 1 through 5 (table I). These curves are presented in figures 3 through 7. Details of the probability calculations for cases 2 and 5 are given as examples in appendix A.

The equiprobability surfaces defined by the contours presented in figures 3 through 7 are constructed only for three exceedance probabilities, but it may be desired to ascertain a point which lies between the surfaces defined for the three different exceedance probabilities. If such a point is desired, it can be identified in T'-D-W space by logarithmic interpolation between the equiprobability surfaces, since the probability functions decrease approximately exponentially with increasing values of T', W, and D.

The inclusion of horizontal extent as an additional variable.- Experience has shown that the more localized a cloud formation, the higher will be its liquid-water content. The results of the foregoing analysis (figs. 3 through 7), however, are directly applicable only to conditions not differing greatly in horizontal extent from the average distance over which the measurements were made, namely, 3 miles in cumulus clouds and 10 miles in layer clouds. The results may be made applicable, though, to other values of horizontal extent by making use of the data contained in reference 3 concerning the relation between maximum average liquid-water content and distance along the flight path.

In order to modify values of liquid-water content obtained for one horizontal extent to be applicable to another, it has been found necessary to make three assumptions. These assumptions are that (1) the variables, drop diameter, and temperature are assumed not to be significantly related to horizontal extent; (2) the variation of average liquid-water content with horizontal extent is assumed to be independent of drop size and temperature; and (3) the results from reference 3 are assumed to be representative of conditions as they would be encountered in normal flight operations. This last assumption is uncertain because of the limited number of flights analyzed in reference 3 (11 flights in layer-type clouds and 26 in cumulus clouds) and because the flight paths were frequently chosen with the aim of prolonging and maximizing the exposure to icing conditions. On the basis of these assumptions, equiprobability surfaces applicable to other values of horizontal extent may be constructed by multiplying all values of liquid-water content from the probability curves of figure 3 to 7 inclusive by the variable factor F presented in figure 8.

From figure 8, it can be seen that the horizontal extent selected as being applicable to a particular icing encounter can have a considerable effect on the factor F used to modify liquid-water-content values. This effect is particularly pronounced in the case of layer clouds where the horizontal extent can be very large. For example, if an icing encounter in layer clouds is considered to extend over 100 miles instead of 10, the factor to use to modify liquid-water-content values is about 0.4 instead of 1.0. Consequently, it is necessary to consider the appropriateness for design purposes of the standard values of cloud horizontal extent (3 miles for cumulus and 10 miles for layer clouds). For the "instantaneous" and "intermittent" classifications of reference 5 (classes I and II as defined therein), the horizontal extents of 1/2 and 3 miles, respectively, as assigned in reference 5, are considered to be reasonable values for design purposes. For the "continuous" classification (class III), a horizontal extent of 20 miles appears appropriate for design. Although it is realized that continuous icing might well extend beyond 20 miles, reduction of the liquid-water content by a factor less than 0.8 (an approximate value of F corresponding to 20 miles horizontal extent) does not appear justified for design purposes. It should be noted that these three design values of horizontal cloud extent, shown in figure 8 by broken lines, are suggested values only and do not preclude the use of the curves for the estimation of the factor F by selecting other values of horizontal extent.

The Determination of the Probability of Exceeding Any Specified Group of Values of Liquid-Water Content Associated Simultaneously With Temperature and Drop-Diameter Values Lying Within Specified Ranges

The equiprobability surfaces discussed in the preceding section provide information concerning the simultaneous combinations of liquid-water content, drop diameter, and temperature having the same probability of occurrence. Another useful expedient for the designer, however, would be a means for determining the probability of encountering any icing condition not included by the values of liquid-water content, drop diameter, and temperature specified as design criteria for a particular component of a thermal system. The basis for such a technique lies in determining the probability of equaling or exceeding certain values of liquid-water content specified for a number of icing conditions.

Probability of encountering icing conditions which exceed certain specified values.- If, for a particular element of a thermal system and a chosen value of horizontal cloud extent, combination values of liquid-water content, drop diameter, and temperature are specified which the element can just tolerate, the values would define a surface in T'-D-W space. Such a surface can be viewed as representing the locus of liquid-water-content values to which the element is critical over specified ranges of temperatures and drop diameters. Individual values on such a surface would be obtained through knowledge of the area, rate, and distribution of water-drop impingement for the particular element.

For purposes of explanation, figure 9 is presented to show the locus of critical values of liquid-water content for an assumed element of a thermal system. The locus of liquid-water-content values will be termed a "marginal" surface to indicate that values of W lying above the surface cannot be tolerated by the element of the thermal system under consideration. In general, the shape of the marginal surface in the W-D plane for the case of a wing element having a given heat output may be assumed to have a shape resembling a hyperbola. A hyperbolic shape is appropriate because for drop diameters less than the critical diameter[4] for the element under consideration, an infinitely large amount of liquid water is tolerable; and, as drop diameter progressively increases, decreasing amounts of liquid water are tolerable. In any W-T' plane, however, W will be assumed to vary from the values of W at $T' = 0$ to $W = 0$ at some value of T_i', say T_{cr}', where the heating capacity of the element can bring the surface temperature of the element just up to freezing temperature (a clear-air condition).

For convenience, the base of the marginal surface (the T'-D plane at $W = 0$) is divided into n rectangles of dimension ΔD and $\Delta T'$ such that variations are small in values of liquid-water content included by the projection of each rectangle on the marginal surface. The value of liquid-water content considered to be representative for each rectangle is designated as W_i in figure 9. The values of T', T, and D corresponding to W_i shall be called T_i', T_i, and D_i, respectively. For every set of W_i-D_i-T_i' values, there exists a probability function ΔP_i which expresses the probability that, for a random icing encounter, the liquid-water content will exceed W_i and the drop diameter and temperature depression will be within the ranges ΔD and $\Delta T'$, respectively.

The n values of ΔP_i corresponding to the n rectangles in the T'-D plane represent the probabilities of the occurrence of an event (the overloading of the anti-icing element) in n different, mutually exclusive ways. The probability that the event will occur is the sum of the probabilities that it will occur in any number of different ways provided the different possibilities are mutually exclusive. (See reference 10.) Hence, the probability of overloading the given anti-icing element is approximately equal to the sum of the partial probabilities for the n rectangles according to the equation

$$P = \sum_{1}^{n} \Delta P_i \tag{3}$$

In order to evaluate ΔP_i for each of the n rectangles, it would appear that a separate determination of the relationship between W and ΔP_i for each rectangle would be necessary. This procedure is not practicable, however, because if the number of rectangles is chosen large enough to make variations in W small for each rectangle, then the amount of data available to define ΔP_i as a function of W is too small to yield statistically reliable results for the individual rectangles. Moreover, the presentation of the results would require separate curves or tables for each rectangle. It is possible to overcome these difficulties by utilizing the fact that the

[4]The critical diameter may be defined as the smallest diameter for which drops will impinge on the element. For some airfoils, the critical size can be estimated by a technique outlined in NACA TN 2476 (reference 13).

distribution of drop diameter is approximately independent of temperature and liquid-water content. Although there is a slight tendency toward higher frequencies of large values of drop diameter when the liquid-water content is low, this effect is too small to have an important influence in the determination of the total marginal probability. By making the assumption that the distribution of drop diameter is independent of temperature and liquid-water content, it is possible to determine values of ΔP_i for different combinations of temperature depression below freezing, liquid-water content, drop diameter, and horizontal extent.

When, for any given cloud horizontal extent, drop-diameter distribution is assumed to be independent of temperature and liquid water, the probability equation

$$\Delta P_i = P_{\Delta D} \, P_{\Delta T} \, P_{W_i}(\Delta T) \tag{4}$$

can be written from analogy with equation (1). The interpretation of equation (4) is that the partial probability ΔP_i applicable to a chosen point on the marginal surface is the product of three probabilities: the probability, $P_{\Delta D}$, that the drop diameter will be within the interval ΔD; the probability, $P_{\Delta T}$, that the temperature will lie within the interval ΔT; and the conditioned probability, $P_{W_i}(\Delta T)$, that the liquid-water content will be equal to or greater than W_i under the condition that the temperature lies within the interval ΔT.

To evaluate equation (4), values of $P_{\Delta D}$ were obtained from the frequency distribution of all observations of drop diameter; values of $P_{\Delta T}$ were obtained from the frequency distribution of the temperature of the most severe[5] icing observations per icing encounter; and values of $P_{W_i}(\Delta T)$ were obtained from the frequency distributions of maximum liquid-water content per encounter for the various temperature intervals. Because of the assumption that drop size is independent of temperature and liquid-water content, it was not possible to provide exactly for the case in which the maximum observed drop diameter determines the most severe icing condition in an icing encounter. An approximate allowance was made for this factor, however, by substituting the frequency distribution of the greatest value of drop diameter per encounter over a limited range of large drop diameters.

Equation (4) was evaluated for different horizontal extents, and to do this the frequency distribution for maximum liquid-water content per encounter was altered according to a relation from reference 3 between maximum average liquid-water content per encounter and the distance along the flight path.

[5] For the purposes of this report, the most severe icing condition is considered as the one having the largest liquid-water content.

The partial probability charts.- In evaluating equation (4) for any one value of cloud horizontal extent, numerous values of ΔP_i are possible from the meteorological data for different combinations of $P_{\Delta D}$, $P_{\Delta T}$, and $P_{W_i}(\Delta T)$. The values of ΔP_i, however, can be conveniently summarized in the form of partial probability charts. Such charts are presented in figure 10 for cases 1 through 5 of table I.

As is evident from figure 10, seven intervals of drop diameter and six intervals of temperature were used in the construction of the charts. These intervals were chosen so as to avoid large variations of W encompassed by any particular ΔD-$\Delta T'$ rectangle projected on the marginal surface. In all conceivable combinations, the temperature and drop-diameter intervals provide 42 different values of ΔP_i which must be summed (in accordance with equation (3)) to obtain the probability, P, of overloading the particular element.

To obtain a specific value of ΔP_i from a chart of figure 10, the following procedure is used: Choose a value of liquid-water content W_i and follow a vertical line until it intersects with the desired horizontal-extent line; from this intersection, follow horizontally until an intersection with the appropriate temperature-interval curve is obtained; from the point of this intersection, follow a vertical line downward to obtain an intersection with a chosen drop-diameter curve; a horizontal line through the latter intersection yields the desired value of ΔP_i on the ordinate scale. For ease in observing this procedure, each chart in figure 10 has a system of arrows leading from some arbitrary value of W_i through the horizontal-extent, temperature, and drop-diameter curves to a corresponding value of ΔP_i.

It should be noted that partial probabilities presented in figure 10 can be determined, for values of horizontal extent not specifically represented in the charts, by interpolation between the horizontal-extent curves. It is not permissible, however, to interpolate between the curves representing temperature and drop diameter since the partial probabilities are dependent upon the size of the intervals for which the temperature and diameter curves are constructed. The approximate median values of temperature and drop diameter, which correspond to the intervals of T_i and D_i used in the charts of figure 10, are listed in table II for reference. Median values are used because the median has the advantage that its value is not greatly influenced by the chance variations of extreme items. A detailed description of the techniques used in constructing the partial probability charts, with application to selected cases, is presented in appendix B.

DISCUSSION

Reliability of the Equiprobability and the Partial Probability Charts

The reliability of the probability analysis reported herein is affected chiefly by four factors; namely, (1) errors of measurement, (2) amount of data available, (3) representativeness of the data, and (4) limitations with respect to climate and altitude. The possible effect of these factors is discussed in the following paragraphs.

Errors of measurement.- As noted in reference 14, the measurement of liquid-water content and drop diameter by the multicylinder method is subject to error. The following factors may be listed as possible sources of error: (1) runoff, (2) bounce-off, (3) evaporation, (4) variations in drop concentration in the neighborhood of the cylinders due to disturbing influences of the airplane, (5) collection of snow flakes as well as liquid drops in mixed clouds, (6) inaccuracies due to the theoretical drop-size distributions differing from those assumed in the calculations, (7) failure to fulfill, with the apparatus used, the theoretical assumption of two-dimensional flow, (8) irregularities in the surfaces of the ice-covered cylinders, (9) errors in weighing, (10) irregular breaking of the ice when separating cylinders, (11) errors in measuring the final diameter of the ice-covered cylinders or in calculating the average diameter from an assumed value of ice density, (12) errors in the measurement of airspeed and exposure time, and (13) errors in matching the experimental data with the theoretical collection-efficiency curves.

A detailed discussion of the possible effects of each of these factors on the accuracy of the probability analysis presented herein is beyond the scope of this report. The problem of runoff, though, is worthy of discussion since this factor imposes an upper limit to the range of liquid-water-content values that can be measured. The results of heat-transfer calculations reported in reference 15 indicate that the maximum value of liquid-water content that can be measured with rotating cylinders as they are normally used varies approximately linearly with temperature depression below freezing, reaching about 2 grams per cubic meter at 5° F for an airspeed of about 200 miles per hour. An examination of the data used in the present analysis indicates that not more than about 5 percent of the icing encounters in any of the cases analyzed contained observed values of liquid-water content which were close enough to the theoretical maximums to be regarded as likely to be influenced by runoff. In order to estimate the effect of a small percentage of errors of this type on the results of the probability analysis, it is necessary to consider the manner in which the liquid-water-content distribution functions were determined from the experimental data. In this process, the cumulative frequency distribution of maximum liquid-water content per icing encounter was plotted on Gumbel's distribution paper, and the straight line of best fit was drawn to represent the entire distribution;

thus errors in the upper 5 percent of the distribution had only a minor effect in the final selection of the line used to represent the distribution function. Moreover, it was noted that in nearly all cases the entire distribution followed the straight line with minor and apparently random variations and without any consistent or marked tendency for large negative departures at the upper end of the distribution. It may be inferred, therefore, that the limitation on the measurement of liquid-water content discussed in reference 15 does not have an important influence on the results of this analysis.

In view of the number and complexity of the possible sources of error, it is not possible at this time to make a reliable estimate of the total accuracy of the measurement of liquid-water content. In a statistical analysis, however, reasonably reliable results may be obtained as long as the errors are small compared to the real variations of the quantity measured and are distributed approximately at random. It is believed that these conditions are approximately fulfilled for the values of liquid-water content measured by the multicylinder technique.

Multicylinder measurements of drop diameter, on the other hand, while quite accurate and reliable for small drops, become increasingly inaccurate as the drop size increases. Moreover, the errors are not normally distributed, since large positive errors are more probable than large negative errors, especially at large values of drop diameter (see reference 14). The result of these errors is an increase in the dispersion of the frequency distribution and an exaggeration of the probability of occurrence of large values of drop diameter. This effect is probably not important for observations in the eastern United States because in this case the drop size rarely exceeds the range of reliable measurement. In the Pacific coast area, on the other hand, where large drops occur with much greater frequency, it is quite likely that the extreme values of drop diameter are influenced considerably by this factor.

Amount of available data.- Some concept of the accuracy of the probability analyses for the various cases may be obtained by reference to the number of icing encounters for each geographical area and each cloud-type classification. These data are presented in table I. It is noted that the greatest amount of data for any one case is 110 encounters in layer clouds in the eastern United States. A sample of this size is sufficient to establish the probabilities with considerable confidence at the 0.01 probability level, and the extrapolation to a probability of 0.001 is not likely to introduce serious errors. In the remaining cases, the sample size ranges from 25 to 44 encounters. In these cases, the extrapolation to a probability of 0.001 may introduce considerable uncertainty.

Representativeness of the data.- Another factor, which probably has as great an influence on the reliability and applicability of the analysis as either errors of measurement or sample size, is the systematic bias introduced by the fact that all the test data were obtained during flights in which icing conditions were deliberately sought instead of being accidentally encountered. It is difficult to estimate the effect of this factor since measurements of liquid-water content and cloud-drop size encountered during ordinary transport operations are not yet available. It would appear that the principal difference between transport experience and icing research flights would be the greater frequency of encounters with icing conditions in the latter. This difference alone, however, would not affect the validity of the analysis since the icing encounter is taken as the unit of experience upon which the analysis is based. The question to be considered is whether the icing encounters which occurred during the research flights represent an unbiased sample of the encounters which would occur during normal flight operations.

The differences in severity and extent between icing conditions experienced during research flights and the conditions which would have been encountered during an ordinary flight in the same general area at the same time are of two types. The first type occurred in cases in which no icing would have been encountered on a normal flight through the area but, due to the existence of clouds in thin layers or localized areas, it was possible to experience icing with the test airplane. Encounters of this type would be expected to be less severe and extensive, on the average, than typical encounters during normal operations. Differences of the second type occurred in cases in which icing conditions would have been encountered on a normal flight but, due to a conscious effort on the part of the pilot, the conditions experienced by the research airplane were more severe and extensive. It is evident, therefore, that the special procedures followed during research flights have resulted in the inclusion in the test results of data from icing encounters which were both more and less severe than those which would have been experienced under normal operating conditions. It is not possible to state with certainty what is the net effect of these two types of differences, but it is believed that differences of the second type predominate, with the result that the data are somewhat, though not greatly, biased by the inclusion of an abnormally high frequency of severe icing conditions.

Effect of climate and altitude.- Another factor having an important bearing on the applicability of the results is the effect of climate and flight altitude on the temperature of icing conditions. It is likely that the distribution of temperatures of icing conditions encountered by the test airplanes closely resembles that which would be found in normal operations of unpressurized airplanes during winter and spring in the areas studied. Different temperature distributions would be expected, however, with high-altitude airplanes or with conventional airplanes in other climates and seasons. For example, if an airplane is expected to cruise at altitudes of from 18,000 to 28,000 feet, where the temperature is normally in the range between 0 and -40° F, a much higher frequency of icing at low temperatures is to be expected and the results of an analysis of icing conditions occurring mostly at higher temperatures would be applicable only to a limited extent.

Comparison of Results of the Equiprobability Analysis With Results of NACA TN 1855

Certain values of liquid-water content, drop diameter, and temperature were presented in NACA TN 1855 (reference 5) for consideration in the design of anti-icing equipment. The combinations of meteorological variables recommended, however, were lacking in one respect, namely, the probability of occurrence of these particular combinations. Fortunately, with the aid of the equiprobability curves, some indication of these probabilities can be obtained when corresponding meteorological conditions are compared.

Four classes of icing conditions are presented in NACA TN 1855: I, instantaneous; II, intermittent; III, continuous; and IV, freezing rain. All classes, except IV, are subdivided into two types of icing conditions, maximum and normal, and are confined to icing conditions associated with definite cloud formations of characteristic horizontal extents. Hence, classes I, II, and III are ideal for comparison with the results which can be obtained from the equiprobability charts (figs. 3 to 7) and the relations for horizontal extent (fig. 8).

Two groups of icing conditions are chosen as a basis for comparison between the analysis of this report and NACA TN 1855. The first group consists of icing conditions selected from the probability analysis for the case of layer-type clouds with an exceedance probability of $P_e = 0.001$. The second group consists of icing conditions for the case of Pacific coast cumulus, also with an exceedance probability of 0.001. These two groups are compared, respectively, with classes III-M and II-M from NACA TN 1855. The comparisons are made by determining values of liquid-water content from figure 4 and figures 3, 5, and 7 for the various combinations of temperature and drop diameter given in table I of NACA TN 1855 for class II-M and III-M conditions, respectively. The three values of liquid-water content obtained from figures 3, 5, and 7 corresponding to class III-M were averaged by weighting the values according to the horizontal extent of the region to which they apply. The three regions (fig. 1) have area ratios of about 1/8, 2/8, and 5/8 for the Pacific coast, plateau, and eastern regions, respectively; and these area ratios were used to weight the values of liquid-water content from the equiprobability charts so that the resulting average values would be on a comparable basis with the values expressed in NACA TN 1855. The weighted average values of liquid-water content were corrected by figure 8 to make them applicable to a horizontal extent of 20 miles, a figure reasonably applicable to the layer-cloud data of NACA TN 1855. For class II-M conditions, no correction for horizontal extent was required because the values in NACA TN 1855 and figure 4 both apply to 3 miles horizontal-cloud extent.

The results of the comparisons between NACA TN 1855 and the probability analysis are shown in tables III and IV for classes II-M and III-M, respectively. In general, the liquid-water-content values agree quite closely, except at temperatures of -4° F and below. In this low temperature range, the values of liquid-water content derived from the equiprobability surfaces are considerably lower than values given in NACA TN 1855. The reason is that the probability analysis entails some extrapolation in this temperature range, and so may not provide exactly the correct value of liquid-water content. On the other hand, the values listed in NACA TN 1855 are not restricted to actual measurements, and therefore may not be of precise magnitude. For design purposes, however, the values given by the probability analysis should be of proper magnitude. In this regard, it should be noted that the majority of the data utilized in the probability analysis was taken at comparatively low altitude (13,000 feet), whereas the temperature range between -4° F and -40° F represents considerably higher altitudes (18,000 to 28,000 feet). This difference in altitude could have some bearing on the accuracy of the probability data, particularly at very high altitudes.

By a procedure similar to that used in the comparisons for the two example cases, the correspondence between classes of icing conditions represented by the equiprobability charts and other classes of conditions presented in NACA TN 1855 could be ascertained approximately. The correspondence for these classes of conditions, and also for the two example cases, are shown in table V.

An inspection of table V reveals a consistency between the classes of design values recommended in NACA TN 1855 and specific values of exceedance probabilities. Only the instantaneous maximum icing condition ($P_e = 0.0001$) appears to be incongruous with the maximum conditions in classes II-M and III-M ($P_e = 0.001$). This apparent incongruity is directly attributable to the fact that the instantaneous maximum condition presented in NACA TN 1855 was calculated for tall tropical cumulus clouds. Such extremely severe icing conditions would, of course, be exceeded infrequently as is borne out by the probability analysis. Another point of interest in regard to table V is the general order of magnitude of the exceedance probabilities which are found to apply to the various classes of icing conditions specified in NACA TN 1855. For example, any one of the icing conditions listed under class II-M (intermittent maximum) or class III-M (continuous maximum) will be exceeded once in only about a thousand encounters with icing conditions. It is believed that the icing conditions specified by classes II-M and III-M do not impose too severe design requirements and, therefore, if the ice-prevention system will cope with these conditions, the results of the probability analysis indicate that satisfactory ice protection would be provided for the vast majority of icing encounters.

USE OF THE EQUIPROBABILITY AND PARTIAL PROBABILITY CHARTS AS APPLIED TO THE DESIGN OF THERMAL ICE-PREVENTION EQUIPMENT

Heretofore, no information has been available regarding the probability of encountering icing conditions which exceed certain specified values, so that some discussion of probability as applied to the design of thermal anti-icing equipment seems worthwhile. In general, two design problems exist: one problem is to establish consistent simultaneous combinations of liquid-water content, drop diameter, and temperature which can occur in nature, and the other problem is to determine how effectively a given thermal element of an anti-icing system will cope with the entire array of icing conditions existing in nature. It is the purpose of the equiprobability charts to aid in the solution of the first problem, while the partial probability charts are intended to aid in the solution of the second problem.

The Equiprobability Charts

The equiprobability charts essentially are a means to determine quickly consistent sets of liquid-water content, temperature, and drop-diameter values which have the same probability of being exceeded during icing encounters. Sets of such values obtained from the equiprobability charts can be used as criteria for the design of thermal anti-icing equipment. No need to use these charts exists if it is found that table V will define a class of conditions from NACA TN 1855 suitable for the needs of the particular design problem. However, should it be desired to design, say, a jet-engine inlet duct for an exceedance probability of 0.001 (only one chance in a thousand of exceeding a specified amount of liquid water), table V would not be of help because corresponding icing conditions from NACA TN 1855 are not defined for this particular case. Hence, reliance would have to be placed on the equiprobability charts in order to establish consistent sets of values for a chosen value of P_e.

General procedure for using the equiprobability charts.- The following procedure for the use of the equiprobability charts is suggested:

1. Determine for what class of icing condition (I, II, or III of NACA TN 1855) a design is to be made by considering the nature of the component to be protected. (See table V.)

2. Establish the severity of icing condition within the class by selecting the exceedance probability for which the component is to be designed. (See table V.)

3. With the aid of table V and steps 1 and 2, determine what equiprobability chart to use in defining values of W consistent with different sets of T and D. The selection of charts can be made by noting the type of cloud formation applicable to the class (i.e., I, II, or III) and severity of icing condition established in steps 1 and 2. For cumulus clouds, it is suggested that the charts for the Pacific coast area be used for design purposes rather than the charts for the plateau area. The reason is that, until more data are available for the eastern United States, Pacific coast cumulus-cloud data should provide liquid-water-content values that are conservative for the remainder of the United States. For layer clouds, however, it is suggested that the charts for all three geographical areas be used for design purposes by using a weighted average of liquid-water-content values from the three

charts. The justification for using a weighted average is that layer clouds can be regarded as being structurally continuous over exceedingly large geographical areas; consequently, only average values of the parameters defining an icing condition would seem to have much significance in the design of an anti-icing component for layer clouds existing throughout the United States.

4. From the appropriate equiprobability chart, choose values of W corresponding to a variety of sets of T and D. In the case of layer clouds, where more than one chart may be employed to determine values of W, a suitable weighted average for the entire United States can be obtained by weighting the liquid-water-content values for the Pacific coast, plateau, and east coast regions, in the ratios of 1/8, 2/8 and 5/8, respectively. These weight ratios, 1/8, 2/8, and 5/8, are proportional to the three geographical areas and may be considered as approximately accounting for the length of time that an airplane flying transcontinentally would be in each area.

5. Multiply the values of W determined in step 4 by a factor, chosen from figure 8, for including the effect of horizontal extent of cloud formation. The factor chosen should lie within the range of horizontal extent indicated in figure 8 for the particular class icing condition established in step 1; it should correspond also to the curve for the exceedance probability decided upon in step 2. Recommended values of the factor correspond to values of horizontal extent identified in figure 8 as design values. These design values are for cloud horizontal extents of 1/2, 3, and 20 miles and correspond to icing condition classes I, II, and III, respectively.

<u>Example showing the use of the equiprobability charts</u>.- For this example, a design of a jet-engine inlet-guide vane will be assumed in which it is desirable to protect against those icing conditions having a probability of being exceeded once in a thousand icing encounters ($P_e = 0.001$). The steps in determining sets of values of W, T, and D are:

1. From table V, jet-engine intakes are recommended to be designed for class I icing conditions.

2. The exceedance probability for which the design is to be made is $P_e = 0.001$.

3. In table V, Pacific coast cumulus clouds are considered typical for class I icing conditions. Hence, equiprobability charts for Pacific coast cumulus (fig. 4(c)) are chosen for obtaining values of liquid-water content corresponding to various combinations of temperature and drop diameter.

4. Values of W selected from figure 4(c) for different arbitrarily chosen sets of T and D are presented in the following table:

Instantaneous Conditions Corresponding to
$P_e = 0.001$
[Horizontal extent not considered]

W	T	D
2.65	32	
2.39	10	
1.91	0	15
1.20	-10	
.55	-20	
1.50	32	
1.02	10	
.90	0	30
.49	-10	
.18	-20	
.57	32	
.49	10	
.32	0	45
.13	-10	
.01	-20	

5. From figure 8, it is determined that the factor by which to multiply the liquid-water-content values presented in step 4 is 1.3. This particular value corresponds to the design value of cloud horizontal extent suggested in figure 8 for class I (instantaneous) icing conditions with P_e taken as 0.001. A tabulation of the final liquid-water-content values suitable for design purposes is presented in the following table:

Instantaneous Conditions Corresponding to
$P_e = 0.001$
[Horizontal extent considered]

W	T	D
3.44	32	
3.11	10	
2.48	0	15
1.56	-10	
.71	-20	
1.95	32	
1.32	10	
1.17	0	30
.63	-10	
.23	-20	
.74	32	
.63	10	
.41	0	45
.17	-10	
.01	-20	

In the above table, care should be taken to note that the exceedance probability, $P_e = 0.001$, applies to each icing condition listed and not to the class in general.

The Partial Probability Charts

The partial probability charts essentially provide a rapid means of determining the approximate probability of exceeding any specified group of values of liquid-water content associated simultaneously with temperature and drop-diameter values lying within specified ranges. Thus, if an anti-icing component (e.g., unit span of a jet-inlet guide vane) is designed by the method of reference 16 to withstand a constant value[6] of weight rate of water-drop impingement M_t, as computed from a particular set of values of liquid-water content, temperature, and drop diameter, and if other combinations of these same variables are found which correspond to the same weight rate of drop impingement, then the probability of encountering conditions under which the component will not perform satisfactorily can be determined approximately. The calculation of this probability can be performed with the aid of the partial probability charts (fig. 10).

General procedure for using the partial probability charts.- To use the partial probability charts, the following procedure is suggested:

1. Establish values of liquid-water content, W_i, for various combinations of drop diameter and temperature, which are regarded as critical (a marginal surface) for a given component of a thermal system. Values of W_i can be obtained by computing values of liquid-water content required to provide a constant weight rate of water impingement on the component. The equation

$$W_i = \frac{M_t}{0.33E_m Vtc} \tag{5}$$

[6]The choice of a constant value of weight rate of water-drop impingement rather than, for example, the use of the amount of heat to raise the surface temperature to 32° F, provides, according to reference 16, a reasonable basis for estimating the probability of encountering an icing condition too severe for the ice-protection equipment. The amount of heat supplied for the particular weight rate of water impingement selected depends upon the type of protection required for the component, that is, whether, for example, all the water impinging should be evaporated instead of having the surface temperature merely brought to 32° F. Since for various water-drop sizes and temperatures the amount of heat required for protection is not unique for a given weight rate of water-drop impingement, the marginal surface will not represent the same degree of protection for all combinations of drop size and temperature. However, it will very nearly do so. As a result, the probability value computed from use of the marginal surface is not exactly correct.

derived in reference 16, can be used to calculate the values of W_i as a function of drop diameter and temperature, since E_m is a function of these latter two variables. The particular value of M_t used to calculate values of W_i is obtained by inserting into equation (5) initial values of W_i, E_m, V, t, and c corresponding to a selected meteorological design condition and airfoil section. The appropriate values of E_m to employ in equation (5) for the chosen values of V, t, and c must be determined by using some type of water-drop-trajectory calculation, such as presented in reference 13. When using equation (5) to obtain values of W_i, the computations should be made for all the possible combinations of T_i and D_i listed in table II. In total, there are 42 such combinations.

2. Determine the probability, ΔP_i, of exceeding each of the values of liquid-water content, W_i, determined in step 1. To determine a specific value of ΔP_i from a particular partial probability chart presented in figure 10, the procedure is as follows: Follow a vertical line representing the liquid-water content until the line intersects a horizontal-extent line. From the horizontal-extent line, follow horizontally until an intersection with a temperature-interval contour is obtained. From this intersection, follow a vertical line downward to obtain an intersection with a diameter-interval line. A horizontal line through this latter intersection yields the value of ΔP_i on the ordinate scale. When it happens that a horizontal line drawn from the horizontal-extent curve will not intersect the desired temperature-interval line, all that can be said about the value of ΔP_i for a given diameter interval is that it is smaller than the value the horizontal line would indicate when projected to the extreme left side of the partial probability chart. Such values are usually small enough to be neglected.

3. Establish the probability, P, of exceeding all the values of liquid-water content defined in step 1. This probability may be obtained by summing all the individual values of ΔP_i, determined in step 2, in accordance with equation (3). The number of ΔP_i values which must be summed is 42, since this number represents all combinations of temperature and drop diameter possible by using the charts of figure 10.

Example showing the use of the partial probability charts.- The example of the jet-engine inlet-guide vane will be continued to demonstrate the use of the partial probability charts for a vane section assumed to be heated to 100-percent chord. The starting point of this example will be taken with established sets of values of liquid-water content, drop diameter, and temperature that define a marginal surface for a unit span of the vane. These sets of values, which are presented in table VI, will be presumed to have been obtained by calculating values of W_i from equation (5), using a particular design icing condition from the equiprobability charts. The particular icing condition presumed to have been selected for the computations is one taken from the example use of the equiprobability charts and defined by the following values: $W = 2.48$ grams per cubic meter; $T = 0^\circ$ F; $D = 15$ microns; and cloud horizontal extent, 0.5 mile. With these available data, the steps employed to use the partial probability charts are:

1. Values of ΔP_i are selected from the marginal probability charts for Pacific coast cumulus (fig. 10(b)) to correspond with the conditions chosen from the equiprobability charts for a horizontal extent of 0.5 mile. The results are tabulated in table VI for the 42 different possible combinations of T_i and D_i.

2. The probability of encountering an icing condition lying outside the surface defined in W-T'-D space by the W_i-T_i-D_i values listed in table VI is obtained by adding the partial probabilities found for the various conditions presented in the table. In the summation, values entered in table VI as being less than a certain specified amount are neglected because the values are exceedingly small and not exactly determined in magnitude. These values arise from the fact that the charts are not constructed to encompass quite small values of ΔP_i. For the example presented in table VI, the value of the probability, P, was found to be 0.0657. The value of 0.0657 can be interpreted to mean that in approximately 93 out of 100 icing encounters the values of liquid-water content, for all combinations of drop diameter, will lie below the marginal surface and hence be encompassed by the design.

CONCLUDING REMARKS

The equiprobability surfaces and marginal probability calculation charts presented herein provide, for the United States, a representation of the data now available on the meteorological factors responsible for aircraft icing, expressed in terms of the probability of exceeding various combinations of values of these factors. The usefulness of the results is limited somewhat by the amount of data available; errors of measurement, especially at large values of drop diameter; nonrepresentativeness of the flight procedures during research flights; and limited altitude and geographical extent of the research flight program.

In spite of these limitations, the probability analysis presented does provide an indication, heretofore unavailable, of the combinations of icing conditions having equal probability of being exceeded in the United States, and also the probability of exceeding a special set of icing conditions. Also, a procedure has been established for the statistical analysis of future icing meteorological data obtained on a world-wide basis.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, California, April 18, 1952

APPENDIX A

DETAILS OF DETERMINING THE EQUIPROBABILITY SURFACE CONTOURS

The distribution functions used in the equiprobability study are the normal and the Gumbel probability distributions. Graphical methods were used to evaluate the distributions by using normal-probability graph paper for the normal distributions and a specially constructed paper for the Gumbel distributions. These special types of graph paper have the property that the cumulative distribution curves are represented by straight lines.

The Gumbel distribution, taken from reference 11, is given by the equation

$$P_G = 1-e^{-e^{-\alpha(x-u)}} \tag{A1}$$

In equation (A1), the constant u is the mode and α is a quantity which measures the concentration of the frequency distribution about the mode. Since on the specially constructed Gumbel distribution graph paper equation (A1) will appear as a straight line, it is desirable to define the line by specifying two points on the curve. For the purposes of this analysis, the points are chosen at which the probability P_G has the values of 0.63 and 0.05. The value of $P_G = 0.63$ may be found by placing $x = u$ and the value of $P_G = 0.05$ may be found by placing $\alpha(x-u) = 3.0$.

When a straight-line fit could not be obtained from the data by using Gumbel distribution paper, an arbitrary smooth curve on normal probability paper was used to represent the distribution function. This procedure conserved the essential form of the observed distribution, smoothed out the irregularities, and provided a reasonable basis for limited extrapolation.

Cases 2 and 5 (table I) are used to demonstrate how the normal and Gumbel probability distributions were applied to the data since these cases were the ones in which the largest number of encounters with icing were recorded. As mentioned in the text, the general procedure is to evaluate the functions $Q(T')$, $R(W,T')$, $S(D,W,T')$, $R'(W,D,T')$ and $S'(D,T')$.

Case 2, Cumulus Clouds in the Pacific Coast Area

The distribution functions.- The distribution of the temperature depression below freezing of the most severe icing condition in each icing encounter for this case is shown in figure 11 plotted on Gumbel's distribution paper. In this figure the broken line represents the observed values and the straight line is the estimated line of best fit. This line defines the function $Q(T')$.

The distribution of maximum liquid-water content per encounter was plotted for all values of T' equal to or greater than each of the following values: 0, 8, 12, 17, 22, 27, 28, 32, and 37. The plots for $T' \geq 0$, and $T' \geq 27$ are presented in figure 12. The values of the distribution parameters, $W_{0.63}$ and $W_{0.05}$, obtained from these distributions and plotted as functions of T', are shown in figure 13. The smoothed curves in figure 13 define the liquid-water-content distribution function, $R(W,T')$. These curves were drawn to the spontaneous freezing temperature, $T' = 72\ F^{o}$.

The distribution, $S(D,W,T')$ of drop size to be used with the distribution of maximum liquid-water content per encounter is to be determined for all observations with W and T' greater than certain values. Small values of W need not be considered in this case since conditions of small W and large D are to be expressed in terms of S' and R'. The distribution, $S(D,W,T')$, of drop diameter for all values of $T' \geq 0$ and $W \geq 0.5$ is shown in figure 14 plotted on normal probability paper. Similar plots were made for $W \geq 0.5$, $T' \geq 22$, and $W \geq 0.8$, $T' \geq 0$; and since these distributions were very similar to the one for $W \geq 0.5$, $T' \geq 0$, a statistical test, the chi-square test (reference 17) was applied to test the significance of the differences. The results of this test showed that the differences between these distributions were no greater than would be expected as a result of random sampling; hence it was concluded that any dependence of S upon W and T' for values of $W \geq 0.5$ was too small to be reliably indicated by the data. Accordingly, the distribution $S(D,W,T')$ shown in figure 14 was used for all values of $W \geq 0.5$.

In order to define the equiprobability surfaces in the region where W is small and D is large, the distributions $S'(D,T')$ and $R'(W,D,T')$ must be determined. The distribution, $S'(D,T')$, of the maximum value of D per icing encounter was examined for $T' \geq 0$, $T' \geq 22$, and $T' \geq 32$; and the application of the chi-square test to these distributions indicated that there was no significant detectable dependence of S' on temperature. The distribution, $S'(D,T')$, for $T' \geq 0$ is shown in figure 15.

In order to determine $R'(W,D,T')$, the distribution of W for large values of D, this distribution was plotted for the following cases: $D \geq 20$, $T' \geq 0$; $D \geq 30$, $T' \geq 0$; $D \geq 44$, $T' \geq 0$; $D \geq 20$, $T' \geq 22$; and $D \geq 30$, $T' \geq 22$. A study of these distributions indicated that there was apparently no effect of T' on the variation of R' with W. There was, however, a significant effect of D on the variation of R' with W. Figure 16 shows the distributions for $T' \geq 0$ and $D \geq 20$, 30, and 44. Figure 17 shows the parameters defining the R' distributions for $T' \geq 0$ plotted as a function of D. It is likely that the failure of the data to show for large values of D a decrease of W with increasing T', as was the case in figure 13, is due to the fact that insufficient data are available for large values of D to define the function R' when T' is greater than 22, since it is only in this range that the function R becomes strongly dependent upon T'. In the absence of more complete information, it has been assumed that when $T' \geq 22$, the dependence of R' on T' is similar to the dependence of R on T'. Accordingly, the values of the R' distribution parameters (the mode $W_{0.63}$ and the fifth percentile $W_{0.05}$) shown in figure 17 are altered to obtain values of the distribution parameters which are considered applicable when $T' > 22$. The factor used to alter any particular value of W presented in figure 17 is established from figure 13 by forming the ratio of the value of W at a selected value of T' to the value of W applicable to values of $T' \leq 22$. The values of $W_{0.63}$ and $W_{0.05}$ which define the function R' were obtained in this manner, and are presented in table VII.

Construction of the equiprobability contours.- After the distribution functions have been established, the equiprobability contours for various temperature depressions can be determined. The following examples illustrate the use of the probabilities Q, R, S, R', and S' in determining the equiprobability surface for Pacific coast cumulus clouds corresponding to an exceedance probability of 0.01. For a value of $P_e = 0.01$, the equation

$$QRS = 0.01 \tag{A2}$$

applies to large values of W. Similarly, the equation

$$QR'S' = 0.01 \tag{A3}$$

applies to small values of W. In evaluating equations (A2) and (A3), two equiprobability surface contours are selected. These contours are for $T' = 0$ and $T' = 32$.

Large values of W: Table VIII shows the calculation of W, using equation (A2) for various arbitrarily chosen values of D. Computations are shown when $T' = 0$ and when $T' = 32$.

When $T' = 0$, it should be noted that $Q = 1.0$. Thus, equation (A2) becomes

$$R = 0.01/S \tag{A4}$$

The values of S necessary in equation (A4) to calculate values of R for various values of D are obtained from figure 14. Values of R calculated from equation (A4) are used to define the desired values of W on the curve depicting the distribution function $R(W,T')$ for a value of $T' = 0$. This function is defined as a straight line on Gumbel distribution paper by establishing the values of W for the mode and at the fifth percentile. The values, $W_{0.63}$ and $W_{0.05}$, used in defining the straight-line distribution of $R(W,T')$ are obtained from figure 13 corresponding to a value of $T' = 0$. For this particular case, the result would be the same as presented in figure 12. The desired values of W chosen from the straight-line distribution for the calculated values of R are presented in table VIII, part (a).

When $T' = 32$, Q is found from figure 11 to have the value of 0.176. Hence, the equation expressing values of R is

$$R = \frac{0.01}{0.176S} \tag{A5}$$

Values of S required to solve equation (A5) are obtained again from figure 14 for various arbitrarily selected drop diameters. To find values of W to which the calculated values of R correspond, it is again necessary to establish $R(W,T')$ as a straight-line function of W on Gumbel distribution paper. The two points required to establish the straight-line variation are the values of W at the mode $(W_{0.63})$ and at the fifth percentile $(W_{0.05})$. These values, for a value of $T' = 32$, are found from figure 13 to be $W_{0.63} = 0.28$ and $W_{0.05} = 1.26$. The values of W found from the straight-line variation of W with $R(D,T')$ are tabulated in table VIII, part (b), along with the corresponding values of D, S, and R.

Small values of W: Sections of the equiprobability contours corresponding to small values of W are calculated in identically the same manner as for large values of W. In both the case of $T' = 0$ and of $T' = 32$, values of S' were obtained from figure 15 for calculating values of R' from the equation

$$R' = P/QS'$$

The values of W at the mode and the fifth percentile for defining W as a function of $R'(W,D,T')$ were obtained from table VII. The calculations for $T' = 0$ are presented in table IX, part (a), and the calculations for $T' = 32$ are presented in table IX, part (b).

Intermediate values of W: From tables VIII and IX, equiprobability curves can be plotted for both high and low values of W, as is shown in figure 18. For the intermediate values of W, however, the final equiprobability contours must be obtained by joining the upper and the lower portions by a smooth connecting line, shown by a broken line in figure 18. The upper portions of the final contours represent maximum values of liquid-water content per icing encounter and the corresponding drop diameter; the lower portions represent maximum drop size per encounter and the corresponding liquid-water content; and the intermediate portions may represent either one of these combinations.

The equiprobability contours presented in figure 18 are an incomplete set of those presented in figure 4(b).

Case 5, Layer Clouds in Eastern United States

The distribution functions.- The distribution of the temperature of the most severe icing observation in each icing encounter for this case is shown in figure 19. Since a satisfactory straight-line representation could not be found for this frequency distribution, the smooth curve on normal probability paper given in figure 19 was used to define the function $Q(T')$.

The distribution of maximum liquid-water content per icing encounter, $R(W,T')$, was plotted for values of T' equal to or greater than 0, 7, 12, 17, 22, and 27. The curves for $T' \geq 0$ and $T' \geq 17$ are presented in figure 20. The values of $W_{0.63}$ and $W_{0.05}$ obtained from each of the six distribution curves were plotted as functions of T' (fig. 21), and smooth curves were drawn to represent the distribution parameters which define the function $R(W,T')$.

The distribution of drop diameter for values of $W \geq 0.2$ and $T' \geq 0$ is shown in figure 22. As in case 2, this distribution was found to be approximately independent of W and T' for values of W greater than 0.2. The curve in figure 22 was, therefore, used to represent the function $S(D,W,T')$.

The distribution of the largest value of D, $S'(D,T)$ observed in each icing encounter is shown in figure 23. This distribution curve, which was found to be approximately independent of T', was used to define the function $S'(D,T')$.

In order to determine $R'(W,D,T')$, the distribution of W corresponding to large values of D, the distribution of W was plotted for all values of T' and for values of D equal to or greater than 15, 17, 19, 21, 23, and 25. Values of $W_{0.63}$ and $W_{0.05}$ for each of these distributions were plotted against D as shown in figure 24. As a result of the small amount of data available for large values of D, there is a considerable scatter in these distribution parameters. The smooth curves shown in figure 24 were drawn to represent approximately the dependence of R' upon D. To determine the dependence of R' upon T', cases of $D \geq 17$ were chosen since this was the largest value of D for which sufficient data were available to provide a reasonably reliable sample. Distributions of W for $D \geq 17$ and $T' \geq 0$, 12, and 17 were plotted and the resulting values of $W_{0.63}$ and $W_{0.05}$ from these distributions were used with the data from figure 24 to provide the basis for the construction of a set of curves defining the parameters of the distribution, $R'(W,D,T')$, (fig. 25). Curves representing values of $W_{0.63}$ and $W_{0.05}$ for $D \geq 17$ were drawn through the data points (fig. 25), using the shape of the curves of figure 21 (the distribution parameters for R) as a guide. The curves for other values of D were then drawn from the points determined by the values for $T' = 0$ (obtained from fig. 24) in such a way as to conform to the pattern established by the curves for $D \geq 17$. These curves define the function $R'(W,D,T')$.

Values of the probability terms, Q, R, S, R', and S', were obtained with the aid of figures 19, 20, 22, 25, and 23, respectively, and were used to construct the equiprobability surfaces for values of P_e from 0.1 to 0.001, which are presented in figure 7.

Cases 1, 3, and 4

By following procedures similar to those described in the preceding examples, equiprobability surfaces also could be determined for the following cases: (1) layer clouds in the Pacific coast area (fig. 3); (2) layer clouds in the plateau area (fig. 5); and (3) cumulus clouds in the plateau area (fig. 6). The case of cumulus clouds in the eastern United States was omitted because of insufficient data.

APPENDIX B

DETAILS OF THE PREPARATION OF THE PARTIAL PROBABILITY CHARTS

The same cases as were used in describing the construction of the equiprobability charts will be used to describe in detail the construction of the partial probability charts. The general procedure in constructing the charts is first to evaluate the equation

$$\Delta P_i = P_{\Delta T} \, P_{\Delta D} \, P_{W_i}(\Delta T)$$

The resulting values of ΔP_i for different combinations of liquid-water content and intervals of drop diameter and temperature are then arranged into a graphical form for convenient use.

Evaluation of ΔP_i

Case 5, layer clouds in eastern United States.- Values of $P_{\Delta T}$ for this case were obtained from figure 19 which provides information as to the probability that the temperature depression will be lower than any particular value. To find a value of $P_{\Delta T}$ (the probability that the temperature will lie within the interval ΔT) from figure 19, it is only necessary to take the difference between the probabilities indicated for the two chosen limits of ΔT. Values of $P_{\Delta T}$ determined from figure 19 are presented in table X, part (a), for selected temperature intervals.

Values of $P_{\Delta D}$ (the probability that a drop diameter will lie within the interval ΔD) were obtained in a manner similar to that used in obtaining values of $P_{\Delta T}$. In this case, however, a composite drop-diameter distribution curve was used in order to allow approximately for the occurrence of icing encounters in which the conditions of maximum severity are determined primarily by drop size rather than liquid-water content. The composite distribution curve is presented in figure 26 where the probability values in this special case are called S''. As can be seen from figure 26, the composite distribution curve follows the distribution of all observed values of drop diameter for the lowest 90 percent of the distribution and the distribution of maximum observed value per encounter (fig. 23) for the highest 1 percent of the cases. The intermediate 9 percent of the cases is represented by a straight line connecting the two distributions. Values of $P_{\Delta D}$ determined through use of figure 26 are presented in table X, part (b), for selected drop-diameter intervals.

In order to determine values of $P_{W_i}(\Delta T)$, first the distributions of maximum liquid-water content per encounter were plotted on Gumbel distribution graph paper for each of the chosen temperature intervals above -9.5° F in a manner similar to that done in figure 20 for two ranges of values of T'. Values of $W_{0.63}$ and $W_{0.05}$ thus could be obtained; and these values were plotted as a function of temperature and extrapolated to obtain values of $W_{0.63}$ and $W_{0.05}$ applicable to temperature intervals below -9.5° F. The resulting values of $W_{0.63}$ and $W_{0.05}$ for all temperature intervals are presented in table X, part (c). In addition, there are presented values of W which were chosen arbitrarily from the straight-line distribution established by the two points, $W_{0.63}$ and $W_{0.05}$. Corresponding to the selected values of W exist values of $P_{W_i}(\Delta T)$ which also can be read from the straight-line distribution defined by the points $W_{0.63}$ and $W_{0.05}$. These values of $P_{W_i}(\Delta T)$ are also presented in table X, part (c).

After the values of $P_{\Delta D}$, $P_{\Delta T}$, and $P_{W_i}(\Delta T)$ are determined, values of ΔP_i can be determined simply by multiplying the constituent probabilities together for various combinations of liquid-water content and intervals of temperature and drop diameter. Such computations are summarized in table XI for two temperature intervals (32.0° to 20.5° F and 20.5° to 10.5° F), two drop-size intervals (0 to 9.5 microns and 9.5 to 12.5 microns) and a range in liquid-water-content values. The values of ΔP_i listed in the table are ones which can be obtained through use of the partial probability charts.

Other cases.- The quantities used in calculating values of ΔP_i for the remaining cases studied were obtained in the same manner as case 5, with one exception. In case 2, the distribution of the greatest drop diameter per encounter was used for the upper 5 percent of the diameter distribution instead of the upper 1 percent. The reason for this deviation was because of the greater frequency of encounters in which the most severe icing condition was determined by the maximum value of drop size.

Example Construction of the Partial Probability Charts

Rather than compute values of ΔP_i every time they are required, use of the data presented in tables X and XI in chart form is a great convenience. In general, the technique of constructing a suitable chart for evaluating values of ΔP_i utilizes the relation

$$\log \Delta P_i = \log P_{\Delta D} + \log P_{\Delta T} + \log P_{W_i}(\Delta T) \tag{B1}$$

in such a manner that the chart performs the function of adding the right-hand terms in the equation.

The various steps in the construction of a chart to solve equation (B1) (derived from equation (4)) are shown in figure 27. Figure 27(a) establishes the liquid-water content as a function of the sum of the logarithms of $P_{\Delta T}$ and $P_{W_i}(\Delta T)$. Since the value of $P_{\Delta T}$ corresponding to a given temperature interval is a constant, the liquid-water-content distribution curve is displaced horizontally from the origin by the amount of $\log P_{\Delta T}$. The values of $P_{\Delta T}$, $P_{W_i}(\Delta T)$ and W_i used in preparing figure 27(a) were obtained from table X, parts (a) and (c), for a temperature interval of 32° to 20.5° F.

The next step in the construction is shown in figure 27(b) where the liquid-water-content scale on the right is transformed into a corresponding scale at the top by use of a line drawn at an angle of 45° on the grid system. This line may be labeled the 10-mile horizontal-extent line because the ratio of any value on the top scale to the corresponding value on the right-side scale is unity. For example, in the figure a value of $W_i = 0.5$ on the top scale leads to a value of $W_i = 0.5$ on the vertical scale when the 10-mile horizontal-extent line is used. For other horizontal extents, however, a factor must be applied to the liquid-water content for 10-mile horizontal extent to obtain the value appropriate to the extent under consideration. Accordingly, another line (derived from reference 3) has been drawn on figure 27(b) to incorporate automatically the factor which should be applied to a 50-mile horizontal extent at different values of $P_{\Delta T}$ and $P_{W_i}(\Delta T)$. Thus, a value of $W_i = 0.5$ on the top scale would be transformed into a value of $W_i = 0.82$ on the right-hand scale for a horizontal extent of 50 miles. The point on the liquid-water distribution curve to which the value of $W_i = 0.82$ applies is labeled point A in figure 27(b).

The only other factor which must be included in the partial probability charts is the probability factor $P_{\Delta D}$. A method of incorporating the term is by using a scale-transformation reference line (similar in operation to the horizontal-extent lines). Figure 27(c), the top part of which is the same as figure 27(b), is presented to show the function of the scale-transformation reference line in determining the total sum of $\log P_{\Delta T}$, $\log P_{W_i}(\Delta T)$, and $\log P_{\Delta D}$. As an example, a value of $W_i = 0.5$ is taken for which it is desired to find the sum of the logs of the three probabilities, $P_{\Delta T}$, $P_{W_i}(\Delta T)$, and $P_{\Delta D}$ applicable to a 10-mile horizontal extent, a temperature interval of 32° to 20.5° F and a drop-diameter interval of 0 to 9.5 microns. Accordingly, a value of $W_i = 0.5$ is selected on the liquid-water-content scale and is followed downward, vertically, until an intersection with the 10-mile horizontal-extent line is obtained (point 1). From point 1, a line is followed horizontally until an intersection with the liquid-water-content distribution curve for the temperature interval of 32° to 20.5° F (point 2). The horizontal distance that point 2 is away from the log reference line is a measure of the sum of $\log P_{\Delta T}$ and $\log P_{W_i}(\Delta T)$. If a vertical line from point 2 is followed downward until an intersection with the scale transformation reference line is reached (point 3), the absolute value of the sum of $\log P_{\Delta T}$ and $\log P_{W_i}(\Delta T)$ can be read on the scales either to the left or to the right of the point. However, if the vertical line followed from point 2 is continued downward until the intersection with the drop-diameter interval line is reached (point 4), the sum of the logs of $P_{\Delta T}$, $P_{W_i}(\Delta T)$ and $P_{\Delta D}$ can be read from the left-hand scale for log of ΔP_i. The reason why the total can be read is that the drop-diameter interval line is displaced vertically downward from the scale transformation reference line by the amount of $\log P_{\Delta D}$ for the diameter interval of 0 to 9.5 microns. The actual value of $P_{\Delta D}$ used in the construction was obtained from table X, part (b).

The final value of $\log \Delta P_i$ read from the partial probability chart in figure 27(c) is the result sought, namely, a solution of equation (B1). For most rapid use of the chart, however, it is desirable to have values of ΔP_i obtainable directly. This added convenience can be incorporated in a chart by making each integer of the logarithmic scales in figure 27(c) correspond to a cycle on semilogarithmic graph paper. The partial probability charts presented in figure 10 have been constructed to include this feature, and hence will yield values of ΔP_i directly.

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TABLE I.- CLASSIFICATION OF ICING CONDITIONS
AND AMOUNT OF DATA AVAILABLE FOR EACH CASE

Case	Geographical area	Cloud type	Number of icing encounters	Number of observations of liquid-water content and drop diameter
1	Pacific coast	Layer	39	171
2	Pacific coast	Cumulus	44	227
3	Plateau	Layer	30	119
4	Plateau	Cumulus	25	91
5	Eastern U.S.	Layer	110	404
6	Eastern U.S.	Cumulus	4	26

TABLE II.- INTERVALS OF TEMPERATURE AND DROP DIAMETER USED IN THE DETERMINATION OF MARGINAL PROBABILITIES, AND MEDIAN VALUES IN EACH INTERVAL TO BE USED IN CALCULATING THE MARGINAL LIQUID-WATER CONTENT

Intervals of temperature, ΔT (°F)			Intervals of drop diameter, ΔD (microns)		
Lower limit	Upper limit	Median of interval, T_i	Lower limit	Upper limit	Median of interval, D_i
20.5	32.0	24.0	0	9.5	8
10.5	20.5	15.5	9.5	12.5	11
.5	10.5	6.0	12.5	15.5	14
-9.5	.5	-4.0	15.5	19.5	17
-19.5	-9.5	-13.0	19.5	29.5	24
-40.0	-19.5	-25.0	29.5	49.5	37
			49.5	∞	60

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TABLE III.- COMPARISON OF VALUES OF LIQUID-WATER CONTENT FOR PACIFIC COAST CUMULUS CLOUDS, $P_e = 0.001$, WITH CORRESPONDING VALUES FOR INTERMITTENT MAXIMUM CONDITIONS LISTED IN NACA TN 1855

Temperature, T (°F)	Drop diameter, D (microns)	Liquid water content, W (g/m^3)	
		TN 1855	Probability analysis
32	20	2.5	2.46
32	30	1.3	1.50
32	50	.4	.42
14	20	2.2	2.30
14	30	1.0	1.35
14	50	.3	.37
-4	20	1.7	1.45
-4	30	.8	.70
-4	50	.2	.15
-22	20	1.0	.37
-22	30	.5	.15
-22	50	.1	0
-40	20	.2	0
-40	30	.1	0
-40	50	<.1	0

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TABLE IV.- COMPARISON OF WEIGHTED-AVERAGE VALUES OF LIQUID WATER CONTENT FOR LAYER CLOUDS, $P_e = 0.001$, WITH CORRESPONDING VALUES FOR CONTINUOUS MAXIMUM CONDITIONS LISTED IN NACA TN 1855

Temperature, T (^{o}F)	Drop diameter, D (microns)	Liquid-water content, W (g/m^3)	
		TN 1855	Probability analysis
32	15	0.80	0.77
32	25	.50	.49
32	40	.15	.20
14	15	.60	.50
14	25	.30	.28
14	40	.10	.08
-4	15	.30	.17
-4	25	.15	.09
-4	40	.06	.02
-22	15	.20	.02
-22	25	.10	.01
-22	40	.04	.003
-40	15	.05	0
-40	25	.03	0
-40	40	.01	0

TABLE V.- CORRESPONDENCE BETWEEN THE EQUIPROBABILITY ANALYSIS AND THE ANALYSIS IN NACA TN 1855

Type of cloud formation	Exceedance probability, P_e	Approximate corresponding class icing condition as defined by TN 1855	Equipment to which condition is applicable
Tropical cumulus	[a]0.0001	Instantaneous maximum (I-M)	Any part of airplane, such as guide vanes, inlet ducts, where a sudden large mass of supercooled water would be critical, even for short duration
Pacific coast cumulus	.001	None	
Do - - - -	.01	None	
Do - - - -	.1	Instantaneous normal (I-N)	
Pacific coast cumulus	.001	Intermittent maximum (II-M)	Any critical component of the airplane, where ice accretions, even though slight and of short duration, could not be tolerated
Do - - - -	.01	None	
Do - - - -	.1	Intermittent normal (II-N)	
Layer clouds occurring anywhere in the U.S. (Pacific coast, plateau and east coast regions)	.001	Continuous maximum (III-M)	All components of the airplane; that is, every part of the airplane should be examined with the question in mind, "Will this part be affected seriously by accretions during continuous flight in icing conditions?"
	.01	None	
	.1	Continuous normal (III-N)	

[a]Extrapolated from Pacific coast cumulus data.

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TABLE VI.- EXAMPLE PARTIAL PROBABILITY CALCULATIONS FOR PACIFIC COAST CUMULUS CLOUDS

Medium drop diameter, D_i (microns)	Medium temperature, T_i (°F)	24.0		15.5		6.0		-4.0		-13.0		-25.0	
	Diam. interval, ΔD (microns) / Temperature interval, ΔT (°F)	32.0 to 20.5		20.5 to 10.5		10.5 to 0.5		0.5 to -9.5		-9.5 to -19.5		-19.5 to -40	
		W_i	ΔP_i	W_i	ΔP_i	W_i	ΔP_i	W_i	ΔP_i	W_i	ΔP_i	W_i	ΔP_i
8	0 to 9.5	5.00	0.00001*	4.50	0.00001*	3.93	0.00001*	3.35	0.00001	2.80	0.00001*	2.08	0.00001*
11	9.5 to 12.5	4.10	.00001*	3.70	.00001*	3.23	.00009	2.75	.00007	2.30	.00001*	1.70	.00001*
14	12.5 to 15.5	3.63	.00002*	3.26	.00003	2.86	.00045	2.42	.00030	2.03	.00002*	1.51	.00002*
17	15.5 to 19.5	3.25	.00004*	2.92	.00014	2.55	.00130	2.16	.00080	1.82	.00004*	1.36	.00004*
24	19.5 to 29.5	2.62	.00007*	2.35	.00140	2.05	.00800	1.75	.00380	1.46	.00007*	1.08	.00007*
37	29.5 to 49.5	1.75	.00004	1.56	.00320	1.37	.00700	1.16	.00230	.97	.00015	.73	.00001*
60	49.5 to ∞	.85	.00120	.75	.01600	.66	.01400	.55	.00430	.47	.00100	.35	.00013
TOTALS		----	.00124	----	.02077	----	.03084	----	.01158	----	.00115	----	.00013

$$P = \sum_{1}^{n} \Delta P_i = 0.0012 + 0.0208 + 0.0308 + 0.0116 + 0.0012 + 0.0001 = 0.0657$$

NOTE: Values marked with an asterisk, *, denote that the value is actually smaller than the value stated. Since such values arise because of termination of the partial probability charts, they are usually so small that the values can be neglected in the summation for total probability.

TABLE VII.- PARAMETERS DEFINING THE FUNCTION, R',
FOR PACIFIC COAST CUMULUS CLOUDS (CASE 2)

T (°F)	32 to 10		0		-10		-20		-30		-40	
T', (F°)	0 to 22		32		42		52		62		72	
D (microns)	Liquid-water content, W (g/m^3)											
	$W_{0.05}$	$W_{0.63}$	$W_{0.05}$	$W_{0.63}$	$W_{0.05}$	$W_{0.63}$	$W_{0.05}$	$W_{0.63}$	$W_{0.05}$	$W_{0.63}$	$W_{0.05}$	$W_{0.63}$
30	0.70	0.20	0.62	0.12	0.48	0.07	0.32	0.035	0.16	0.02	0	0
35	.60	.16	.53	.10	.41	.055	.27	.03	.13	.015	---	---
40	.51	.12	.45	.075	.35	.04	.23	.02	---	---	---	---
45	.42	.09	.37	.055	.29	.03	.19	.015	---	---	---	---
50	.35	.06	.31	.04	.24	.02	---	---	---	---	---	---
55	.28	.045	.25	.03	.19	.015	---	---	---	---	---	---
60	.22	.03	.19	.02	---	---	---	---	---	---	---	---
65	.17	.02	---	---	---	---	---	---	---	---	---	---
70	.14	.015	---	---	---	---	---	---	---	---	---	---
75	.11	.01	---	---	---	---	---	---	---	---	---	---

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TABLE VIII.- EXAMPLE CALCULATIONS OF EQUIPROBABILITY CONTOURS FOR SMALL VALUES OF DROP DIAMETER FOR PACIFIC COAST CUMULUS CLOUDS (CASE 2)

(a) $P_e = 0.01$; $T' = 0$; $Q = 1.00$

Drop diameter, D (microns)	Probability values		Liquid-water content, W (g/m^3)
	S	R	
0	1.00	0.0100	1.96
10	.96	.0104	1.94
15	.78	.0128	1.88
20	.415	.024	1.67
25	.116	.086	1.24
30	.015	.67	.40

(b) $P_e = 0.01$; $T' = 32$; $Q = 0.176$

Drop diameter, D (microns)	Probability values		Liquid-water content, W (g/m^3)
	S	R	
0	1.00	0.057	1.22
10	.96	.059	1.20
15	.78	.073	1.14
20	.415	.137	.91
25	.116	.490	.40

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TABLE IX.- EXAMPLE CALCULATIONS OF EQUIPROBABILITY CONTOURS FOR LARGE VALUES OF DROP DIAMETER FOR PACIFIC COAST CUMULUS CLOUDS (CASE 2)

(a) $P_e = 0.01$; $T' = 0$; $Q = 1.00$

Drop diameter, D (microns)	Probability values		Liquid-water content, W (g/m^3)		
	S'	R'	$W_{0.05}$	$W_{0.63}$	W
30	0.33	0.030	0.70	0.20	0.79
35	.20	.050	.60	.16	.60
40	.12	.083	.51	.12	.43
45	.07	.143	.42	.09	.30
50	.04	.250	.35	.06	.18
55	.023	.43	.28	.045	.08
62	.010	1.00	---	---	0

(b) $P_e = 0.01$; $T' = 32$; $Q = 0.176$

Drop diameter, D (microns)	Probability values		Liquid-water content, W (g/m^3)		
	S'	R'	$W_{0.05}$	$W_{0.63}$	W
30	0.33	0.172	0.62	0.12	0.40
35	.20	.284	.53	.10	.26
40	.12	.47	.45	.075	.13
47	.057	1.00	---	---	0

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TABLE X.- VALUES OF THE FACTORS USED IN THE DETERMINATION OF PARTIAL PROBABILITIES APPLICABLE TO EASTERN LAYER CLOUDS (CASE 5)

(a) Temperature

Temperature interval, ΔT (°F)	Upper limit of temperature interval		Lower limit of temperature interval		Probability, $P_{\Delta T}$, that temperature lies within the interval ΔT
	Value of T' (F°)	Probability, Q of exceeding T'	Value of T' (F°)	Probability, Q of exceeding T'	
32 to 20.5	0	1.000	11.5	0.520	0.480
20.5 to 10.5	11.5	.520	21.5	.175	.345
10.5 to 0.5	21.5	.175	31.5	.063	.112
0.5 to -9.5	31.5	.063	41.5	.016	.047
-9.5 to -19.5	41.5	.016	51.5	.003	.013
-19.5 to -40.0	51.5	.003	72.0	0	.003

(b) Drop diameter

Drop-diameter interval, ΔD, (microns)	Lower limit of diameter interval		Upper limit of diameter interval		Probability, $P_{\Delta D}$ that drop diameter lies within the interval ΔD
	Value of D (microns)	Probability, S'', of exceeding D	Value of D, (microns)	Probability, S'', of exceeding D	
0 to 9.5	0	1.000	9.5	0.780	0.220
9.5 to 12.5	9.5	.780	12.5	.480	.300
12.5 to 15.5	12.5	.480	15.5	.230	.250
15.5 to 19.5	15.5	.230	19.5	.090	.140
19.5 to 29.5	19.5	.090	29.5	.022	.068
29.5 to 49.5	29.5	.022	49.5	.0006	.0214
49.5 to ∞	49.5	.0006	∞	0	.0006

TABLE X.- CONCLUDED

(c) Liquid-water content

Temperature interval, ΔT (^{o}F)	Liquid-water-content distribution parameters		Liquid-water content, W_i (g/m^3)	$P_{W_i}(\Delta T)$
	$W_{0.63}$	$W_{0.05}$		
32 to 20.5	0.23	0.64	0.23	0.63
Do - - -	-do-	-do-	.50	.13
Do - - -	-do-	-do-	.75	.024
Do - - -	-do-	-do-	1.00	.004
20.5 to 10.5	.24	.72	.24	.63
Do - - -	-do-	-do-	.50	.18
Do - - -	-do-	-do-	.75	.04
Do - - -	-do-	-do-	1.00	.009
10.5 to 0.5	.14	.37	.14	.63
Do - - -	-do-	-do-	.25	.20
Do - - -	-do-	-do-	.37	.05
Do - - -	-do-	-do-	.50	.009
0.5 to -9.5	.10	.24	.10	.63
Do - - -	-do-	-do-	.20	.11
Do - - -	-do-	-do-	.30	.014
Do - - -	-do-	-do-	.40	.002
-9.5 to -19.5	.07	.17	.07	.63
Do - - -	-do-	-do-	.20	.028
Do - - -	-do-	-do-	.30	.0017
-19.5 to -40.0	.04	.10	.04	.63
Do - - -	-do-	-do-	.10	.05
Do - - -	-do-	-do-	.15	.004

TABLE XI.- PARTIAL PROBABILITY VALUES FOR EASTERN LAYER CLOUDS (CASE 5) FROM DATA PRESENTED IN TABLE X

Temperature interval, ΔT (^oF)	Diameter interval, ΔD (microns)	Liquid-water content, W_i, for ΔT and ΔD (g/m^3)	$P_{\Delta T}$	$P_{\Delta D}$	$PW_i(\Delta T)$	ΔP_i
32.0 to 20.5	0 to 9.5	0.23	0.48	0.22	0.63	0.0665
		.50			.13	.0137
		.75			.024	.0025
		1.00			.004	.0004
		1.30			.0005	.00005
20.5 to 10.5	0 to 9.5	.24	.345	.22	.63	.0478
		.50			.18	.0136
		.75			.04	.0030
		1.00			.009	.0007
32.0 to 20.5	9.5 to 12.5	.23	.48	.30	.63	.0906
		.50			.13	.0187
		.75			.024	.0034
		1.00			.004	.0006
20.5 to 10.5	9.5 to 12.5	.24	.345	.30	.63	.0653
		.50			.18	.0186
		.75			.04	.0041
		1.00			.009	.0009

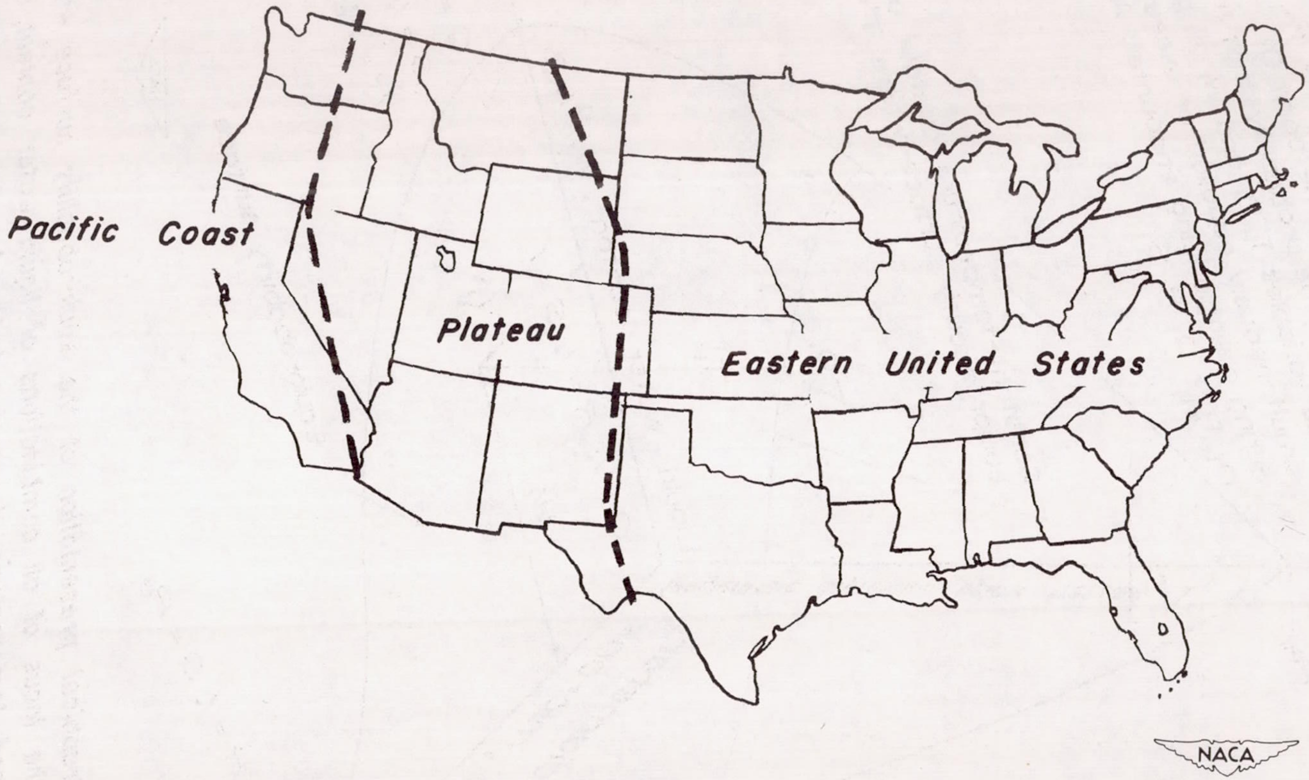


Figure 1.– Map of the United States showing approximate boundaries of areas used in the geographical classification of icing data.

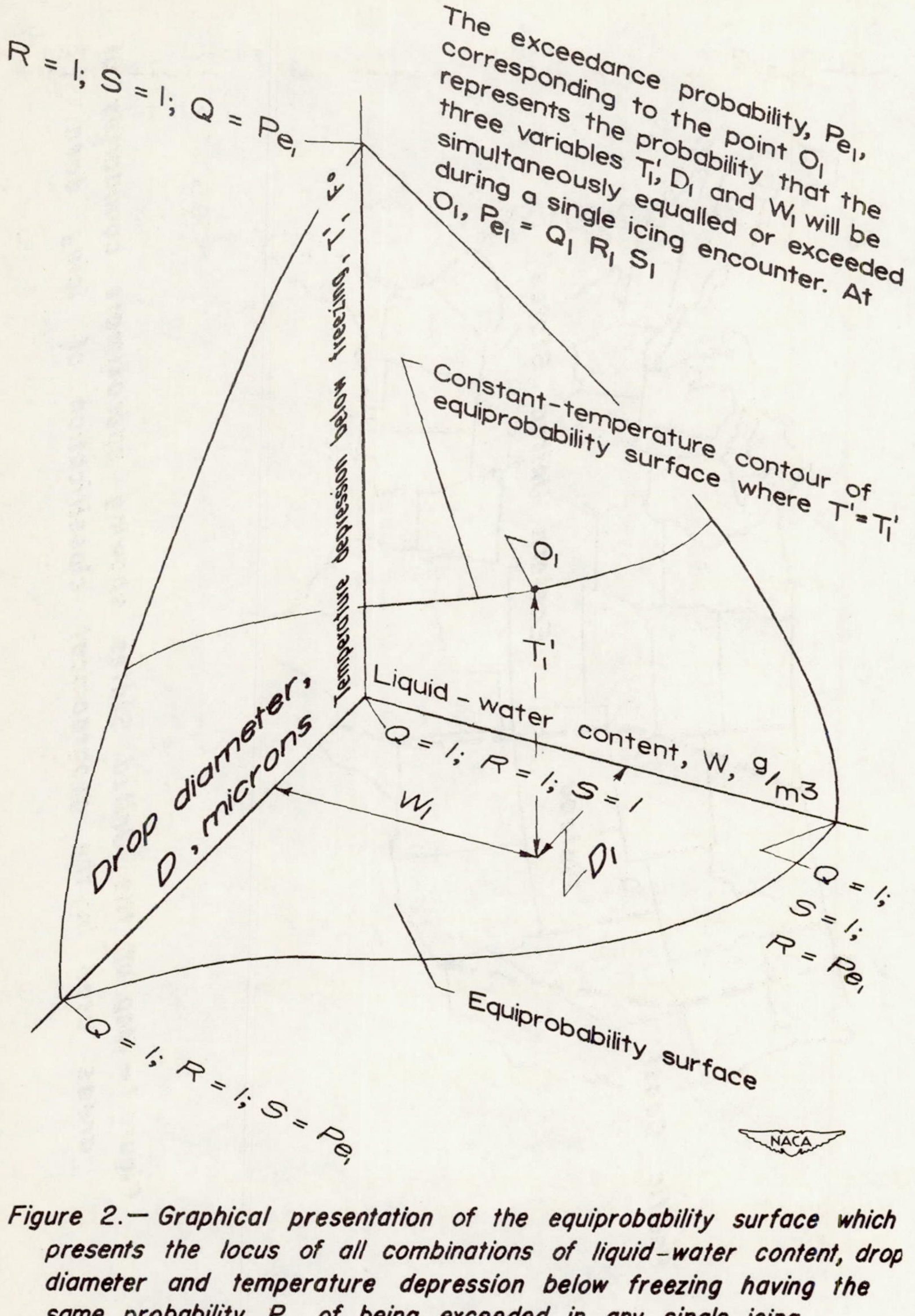
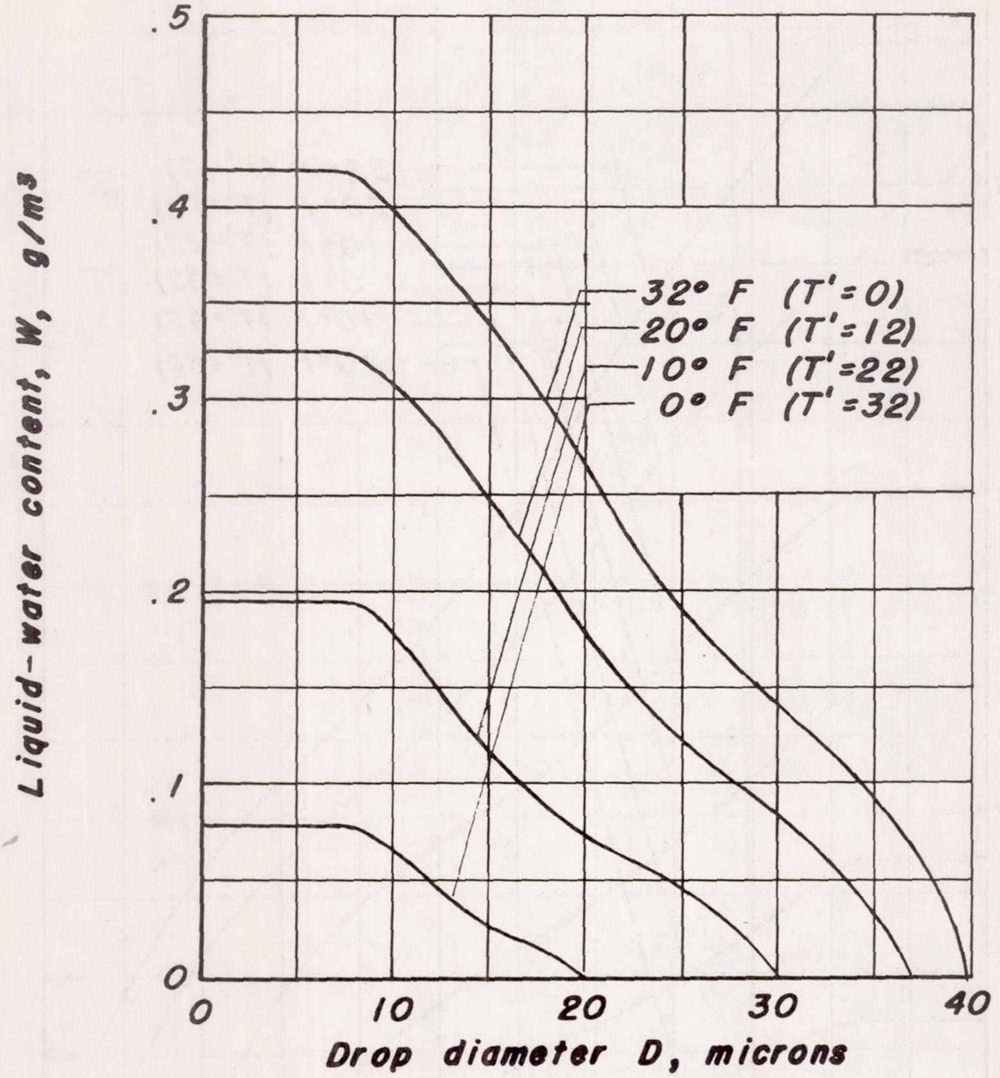
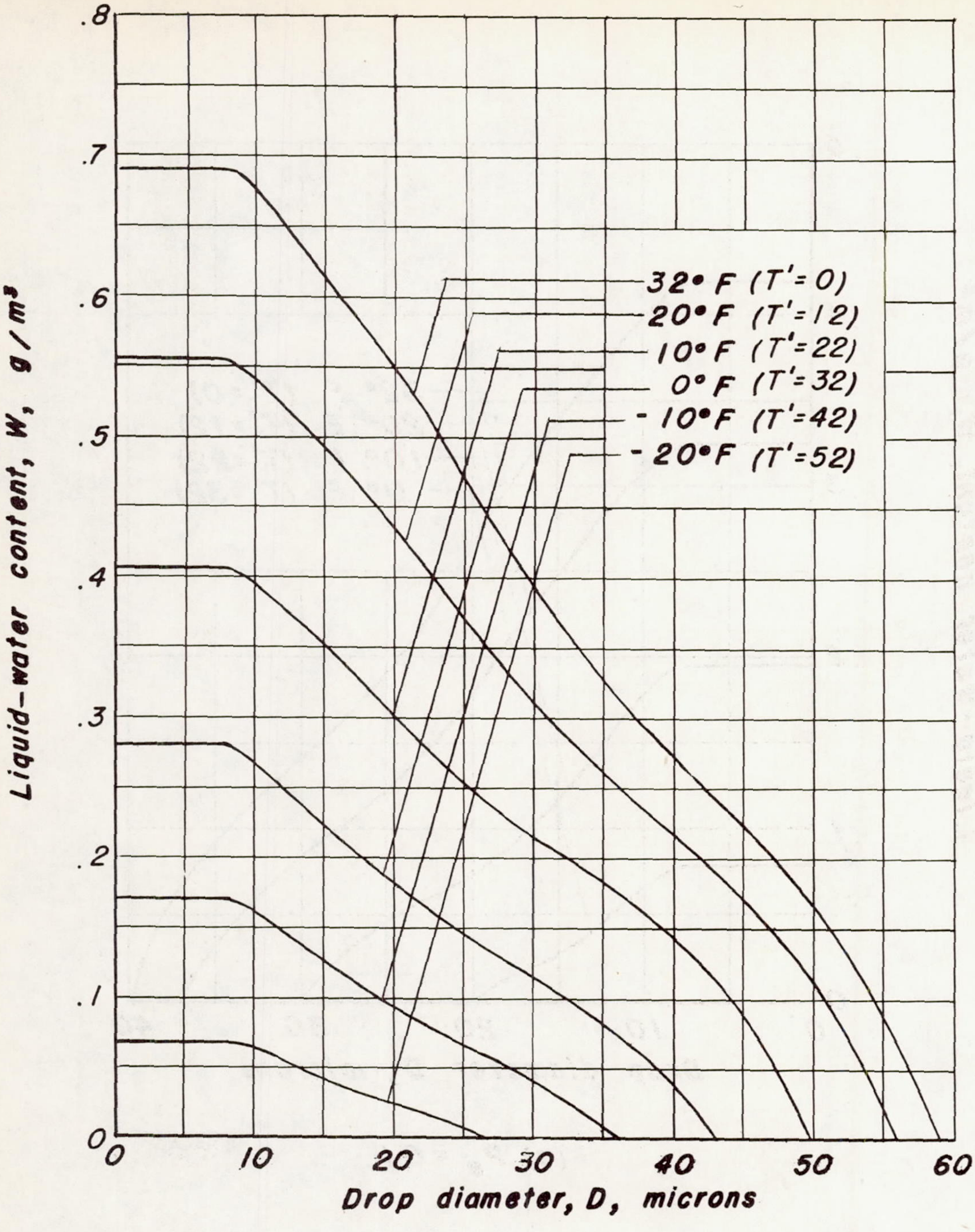


Figure 2.— Graphical presentation of the equiprobability surface which presents the locus of all combinations of liquid-water content, drop diameter and temperature depression below freezing having the same probability, P_{e_1}, of being exceeded in any single icing encounter.



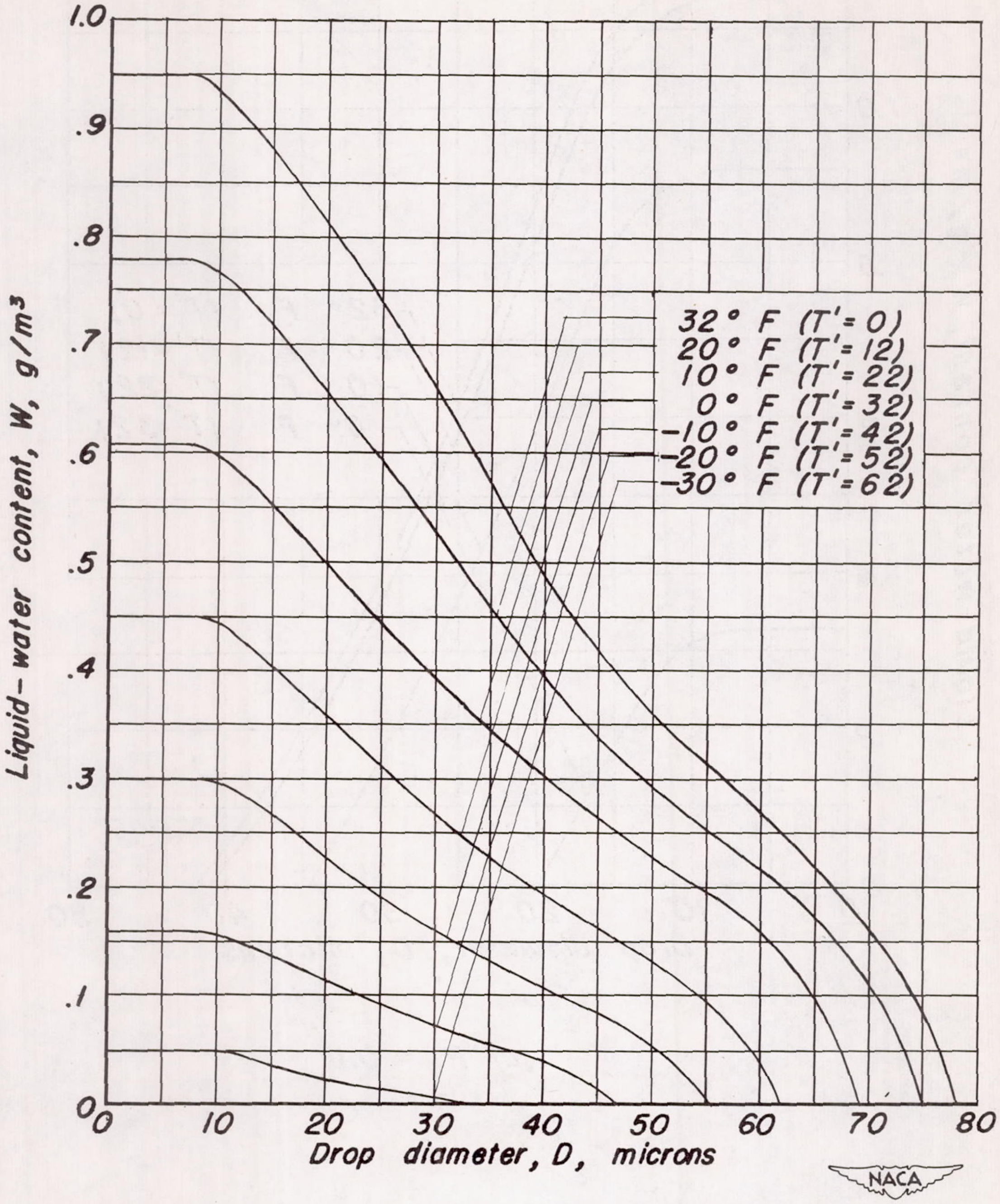
(a) $P_e = 0.1$

Figure 3. — Constant-temperature contours defining the equiprobability surface for Case I. (Layer clouds, Pacific coast).



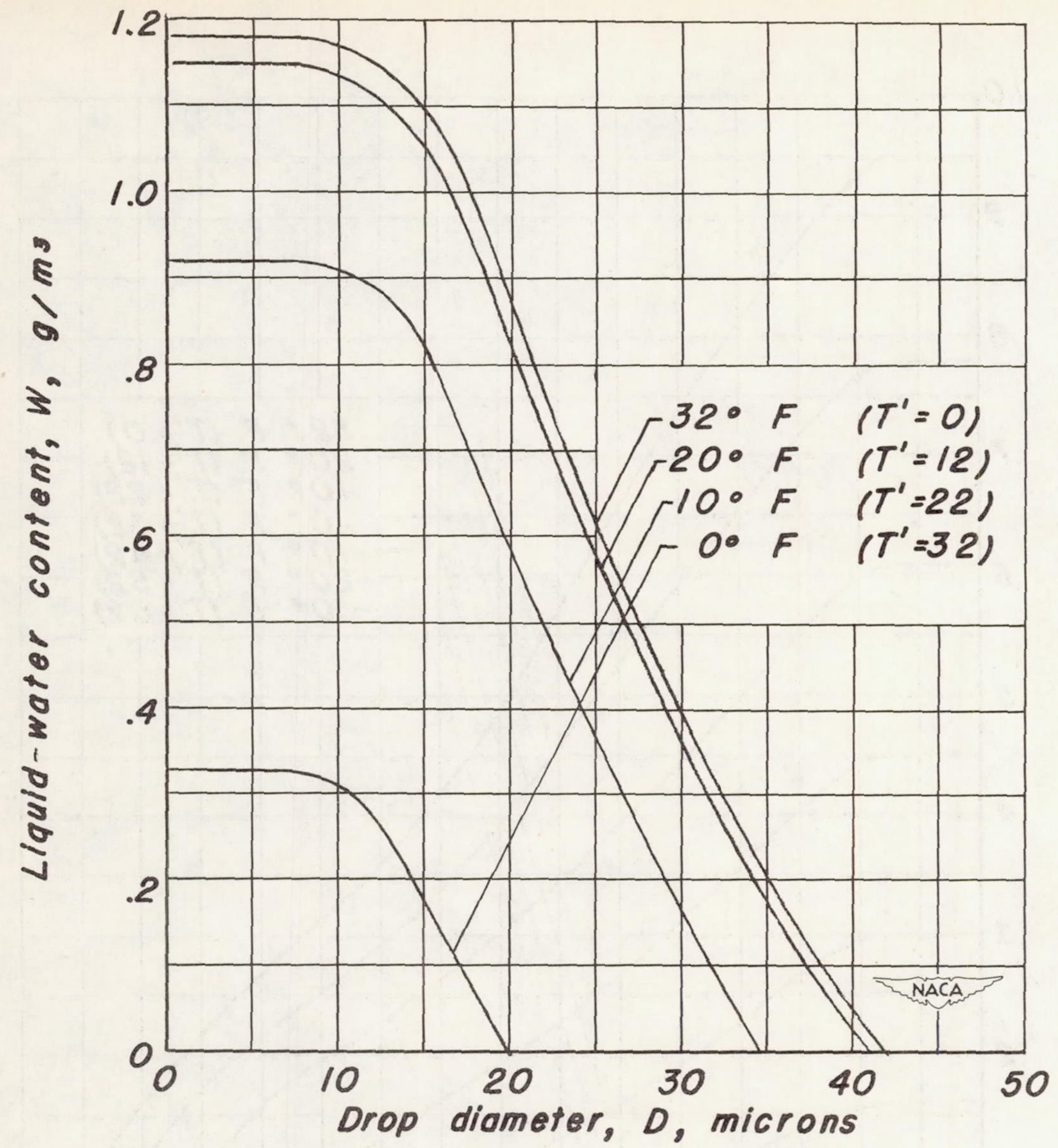
(b) $P_e = 0.01$

Figure 3.– Continued.



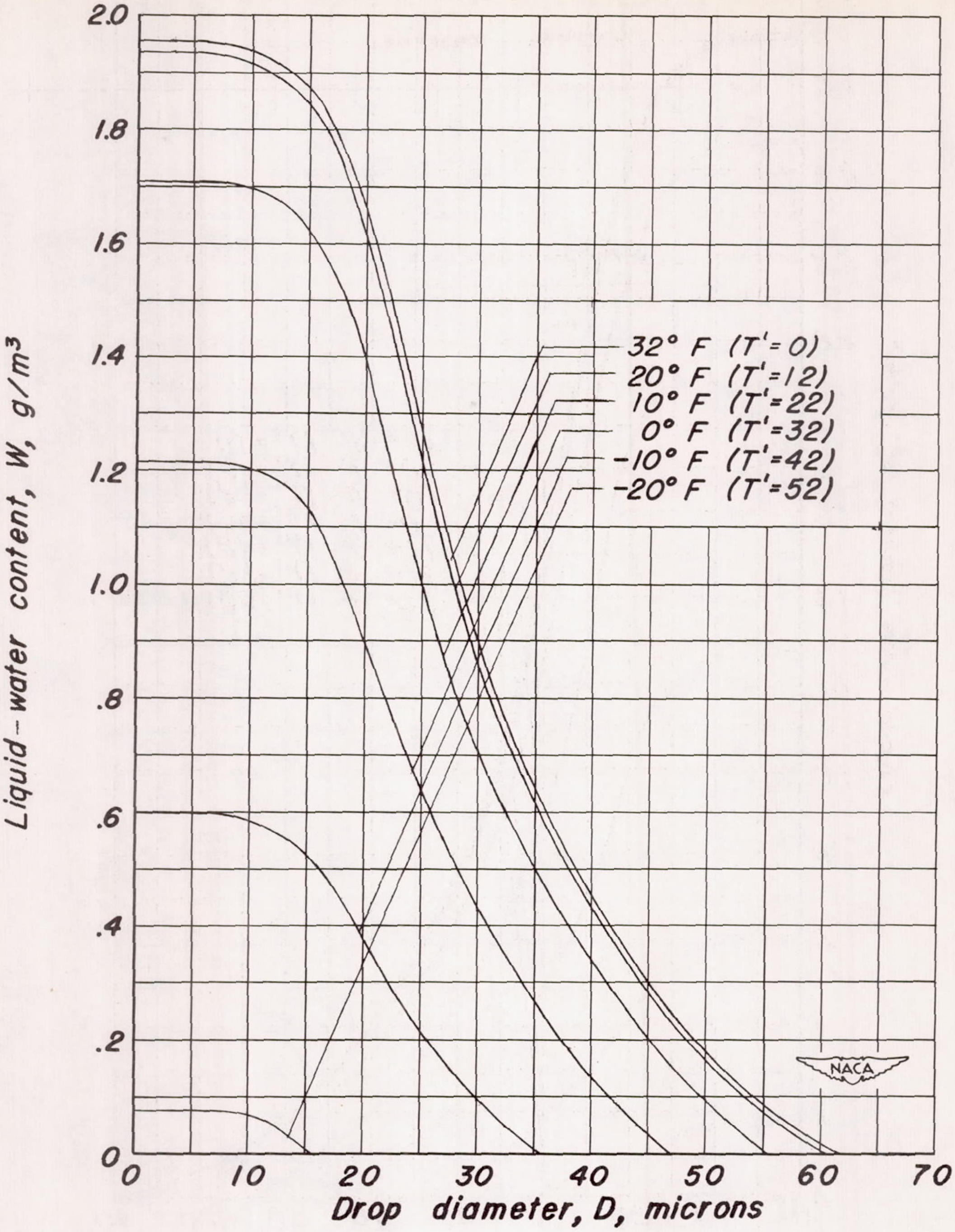
(c) $P_e = 0.001$

Figure 3.– Concluded.



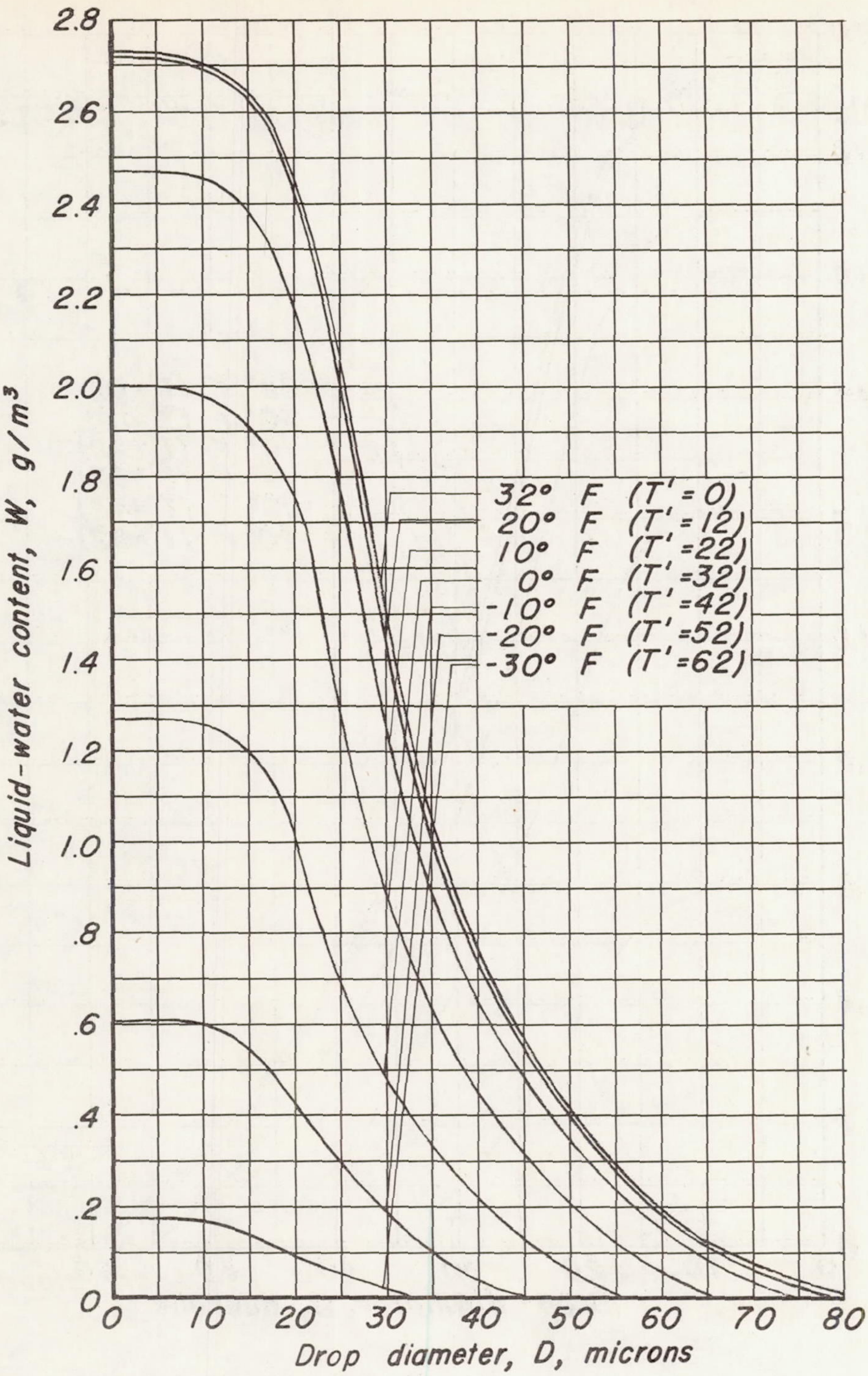
(a) P_e = 0.1

Figure 4.— Constant-temperature contours defining the equiprobability surface for Case 2. (Cumulus clouds, Pacific coast).



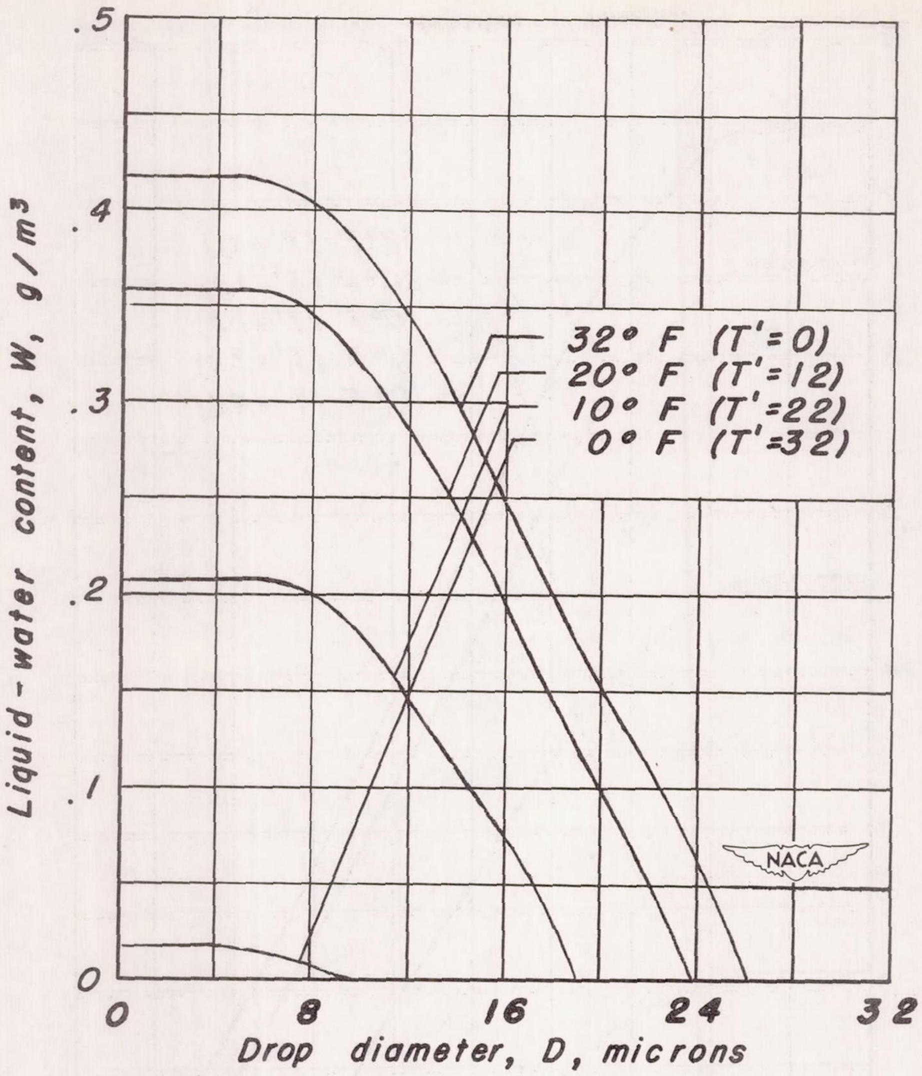
(b) $P_e = 0.01$

Figure 4. - Continued.



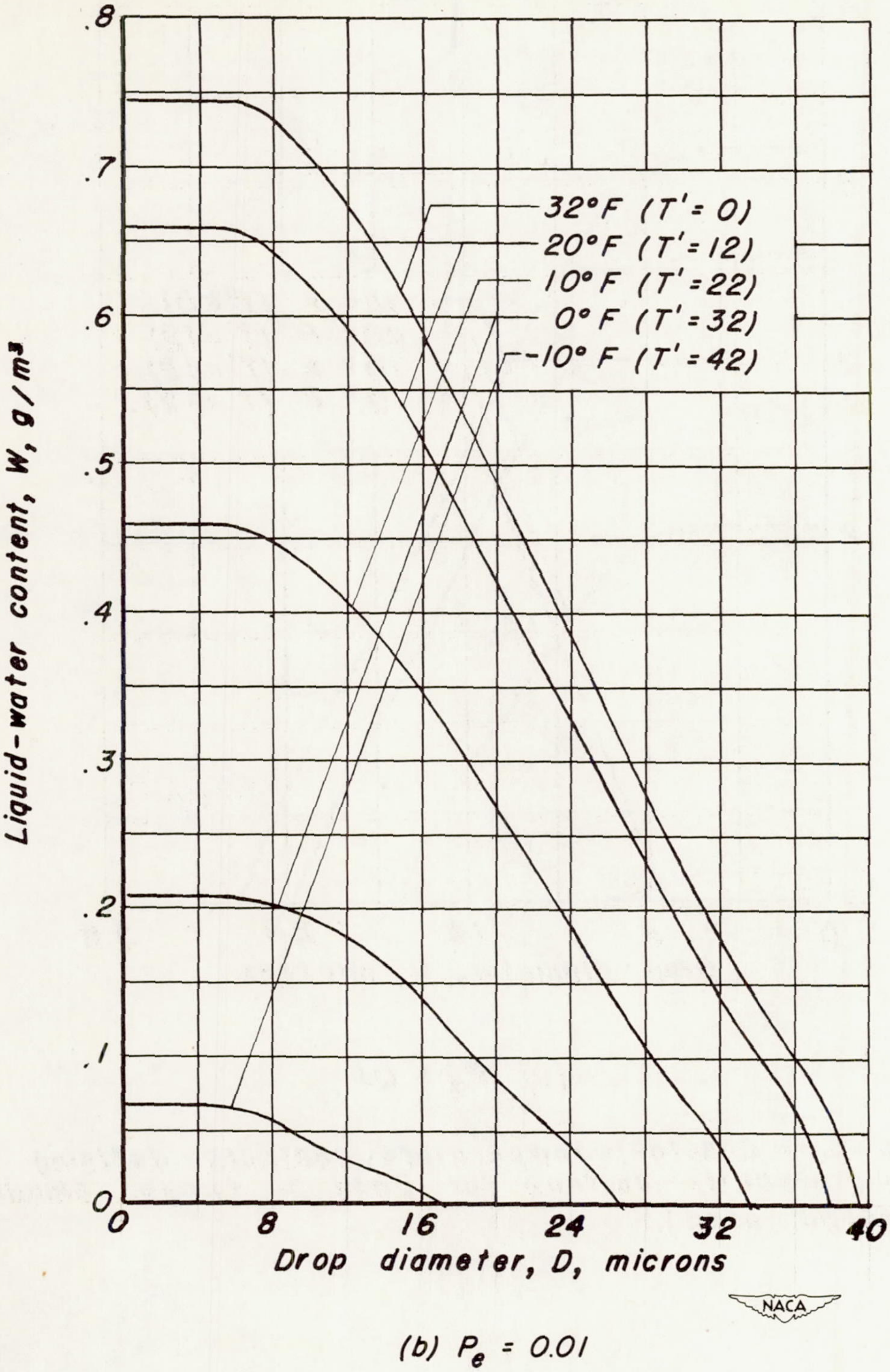
(c) $P_e = 0.001$

Figure 4. - Concluded.



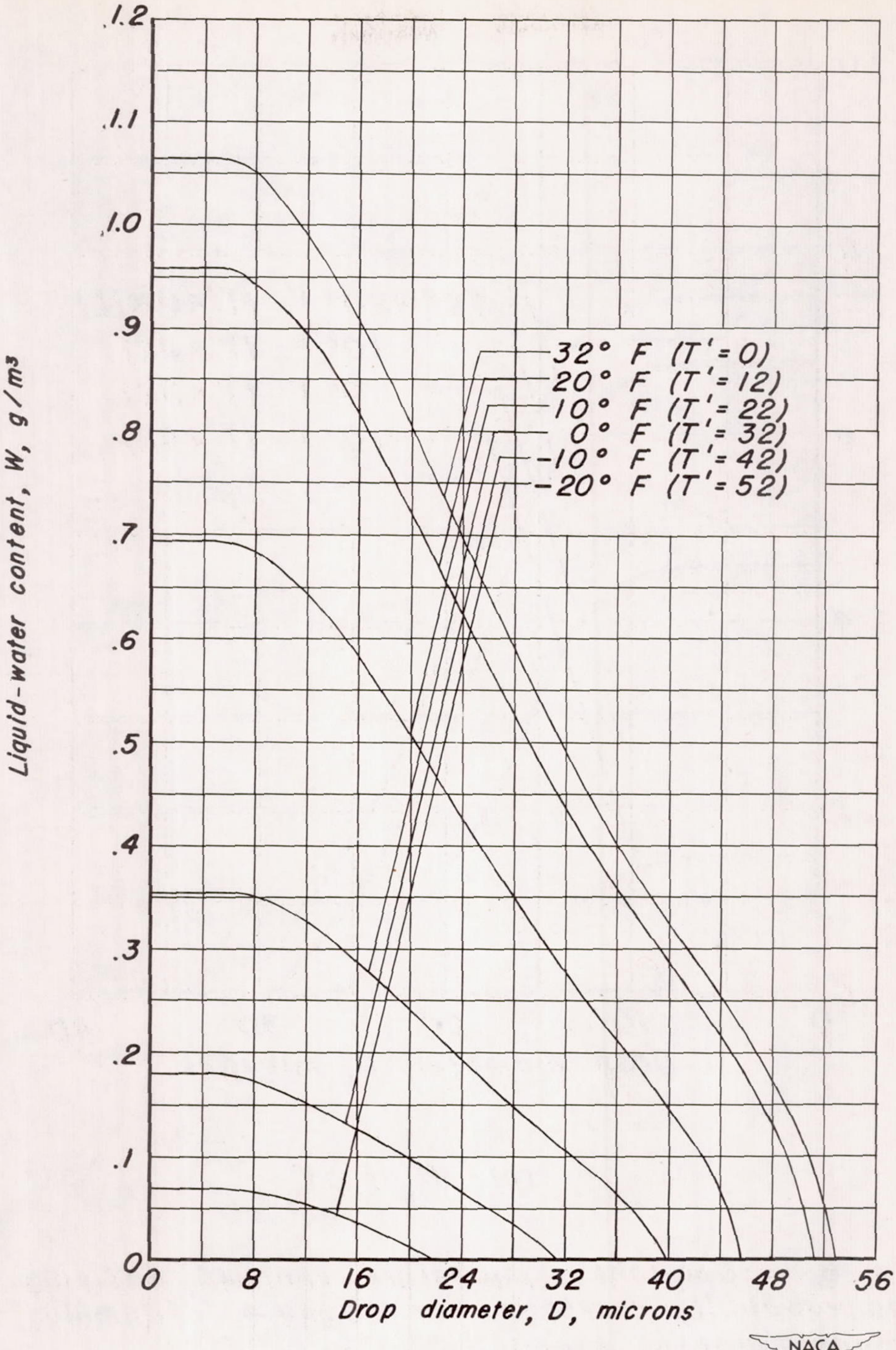
(a) P_e = 0.1

Figure 5.— Constant-temperature contours defining the equiprobability surface for Case 3. (Layer clouds, Plateau area).



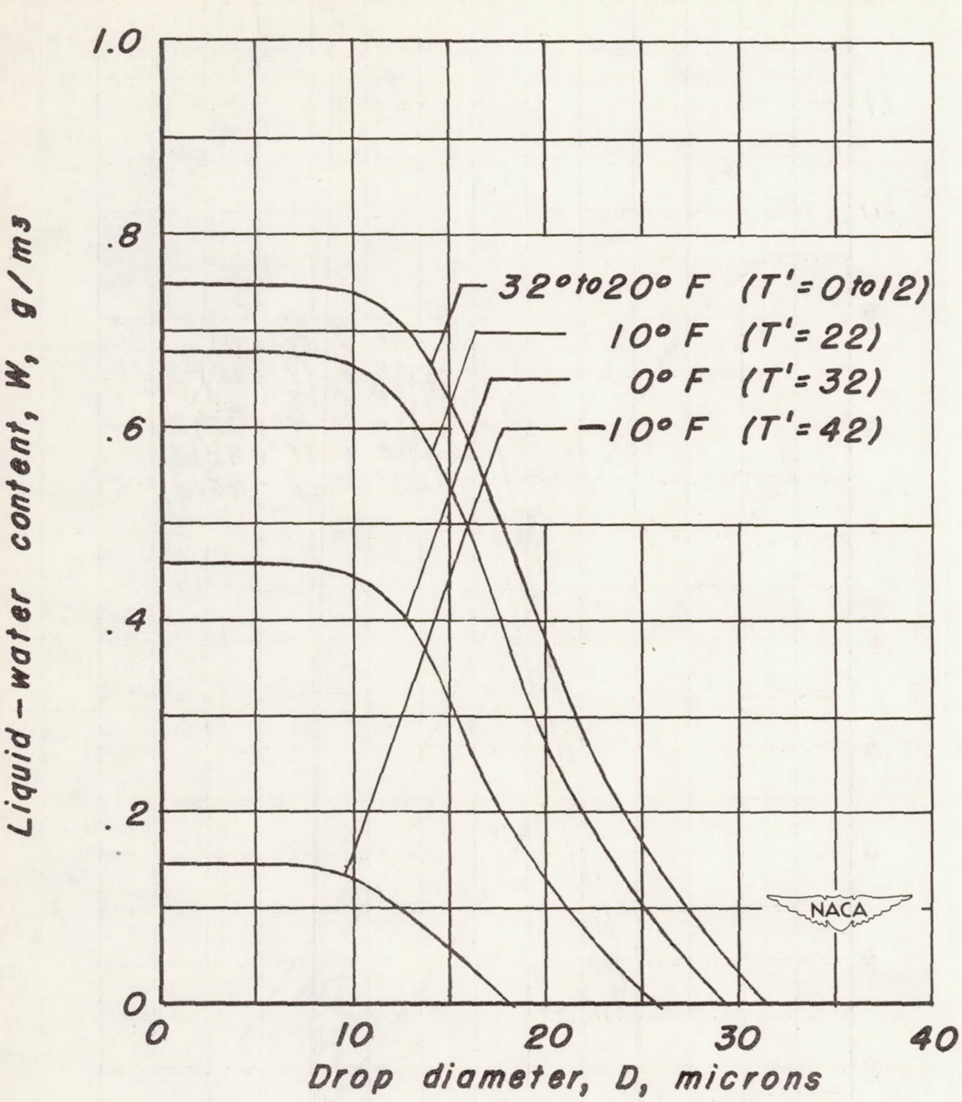
(b) P_e = 0.01

Figure 5. — Continued.



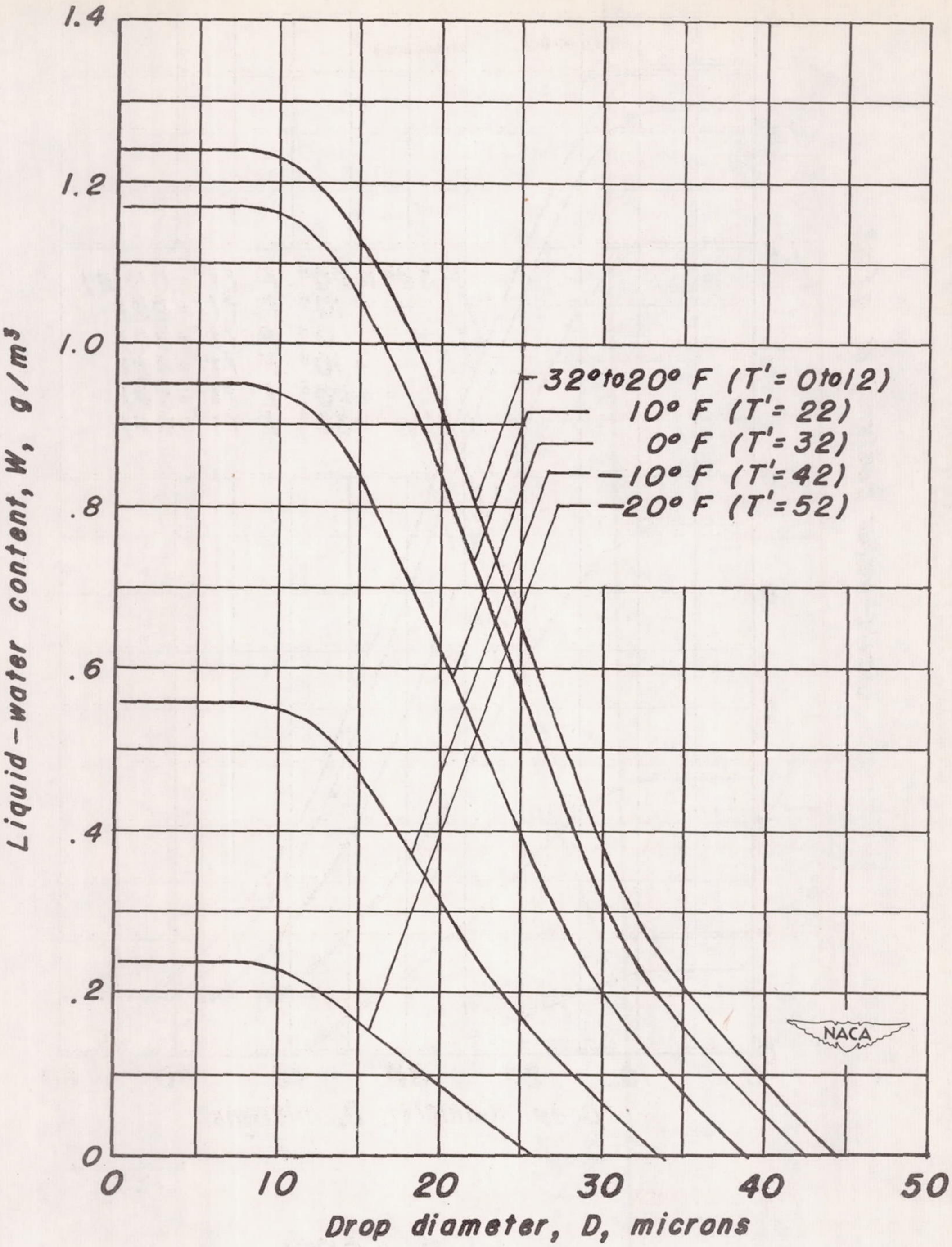
(c) $P_e = 0.001$

Figure 5. - Concluded.



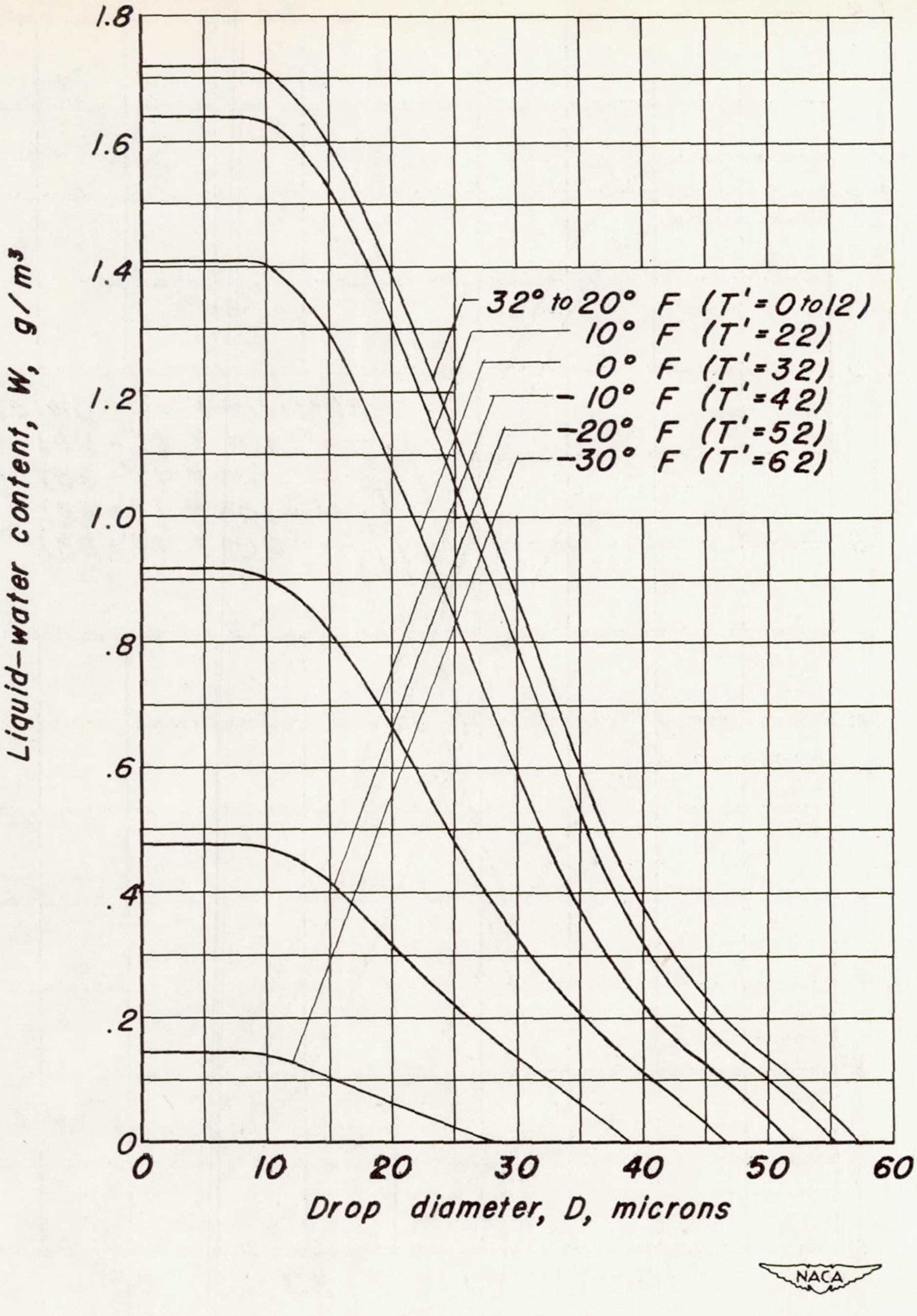
(a) $P_e = 0.1$

Figure 6. — Constant-temperature contours defining the equiprobability surface for Case 4. (Cumulus clouds, Plateau area).



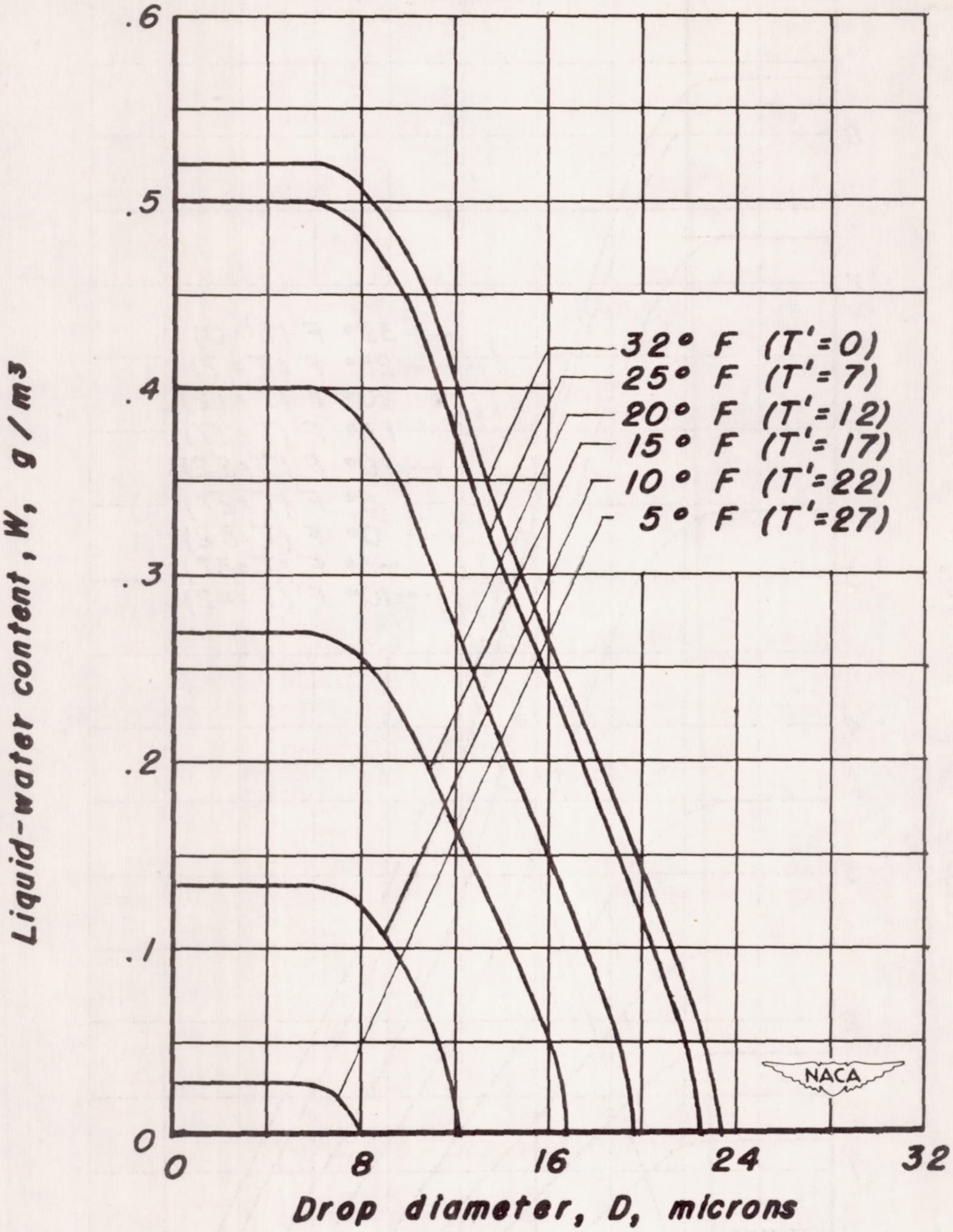
(b) P_e = 0.01

Figure 6. - Continued.



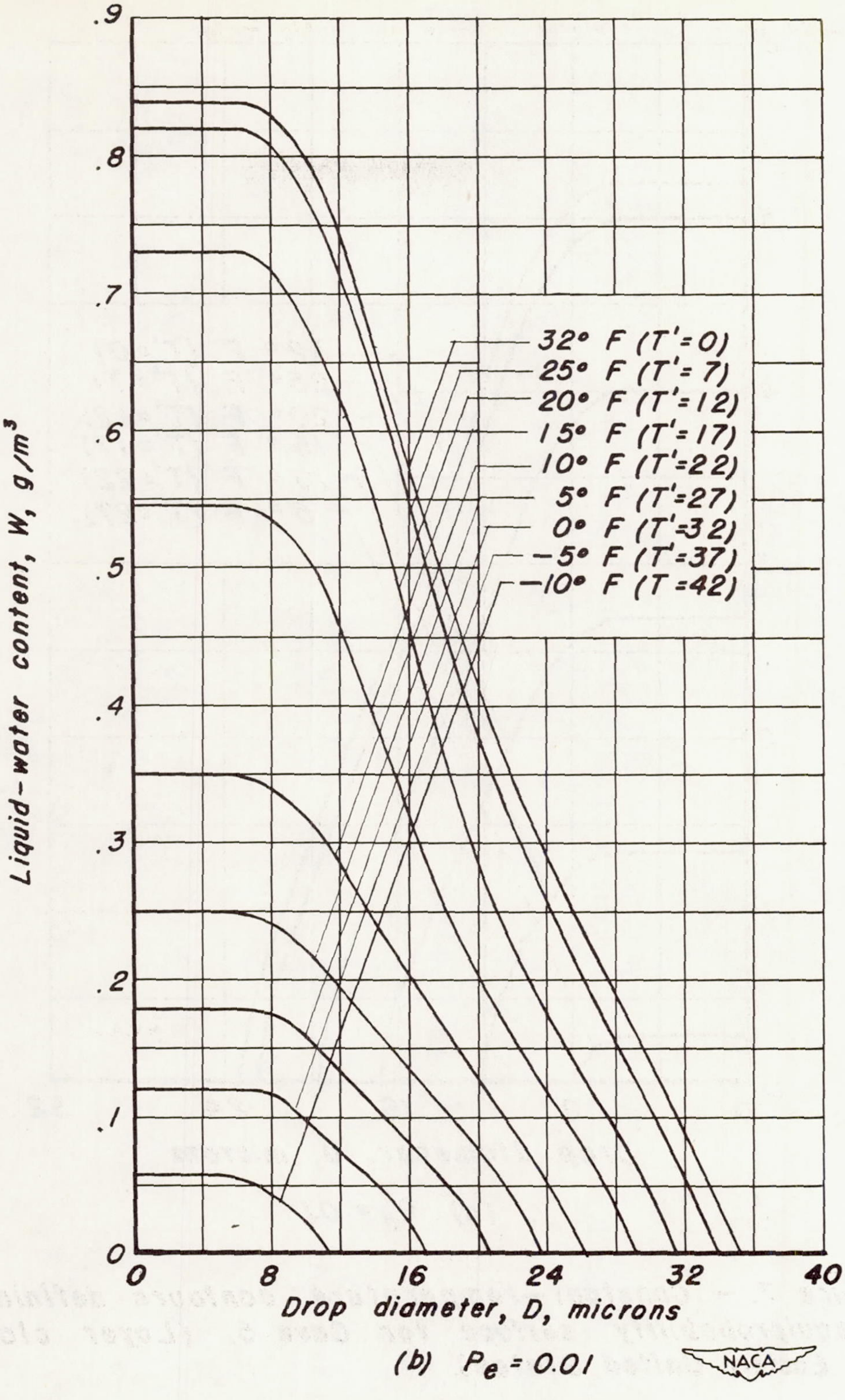
(c) $P_e = 0.001$

Figure 6. — Concluded.



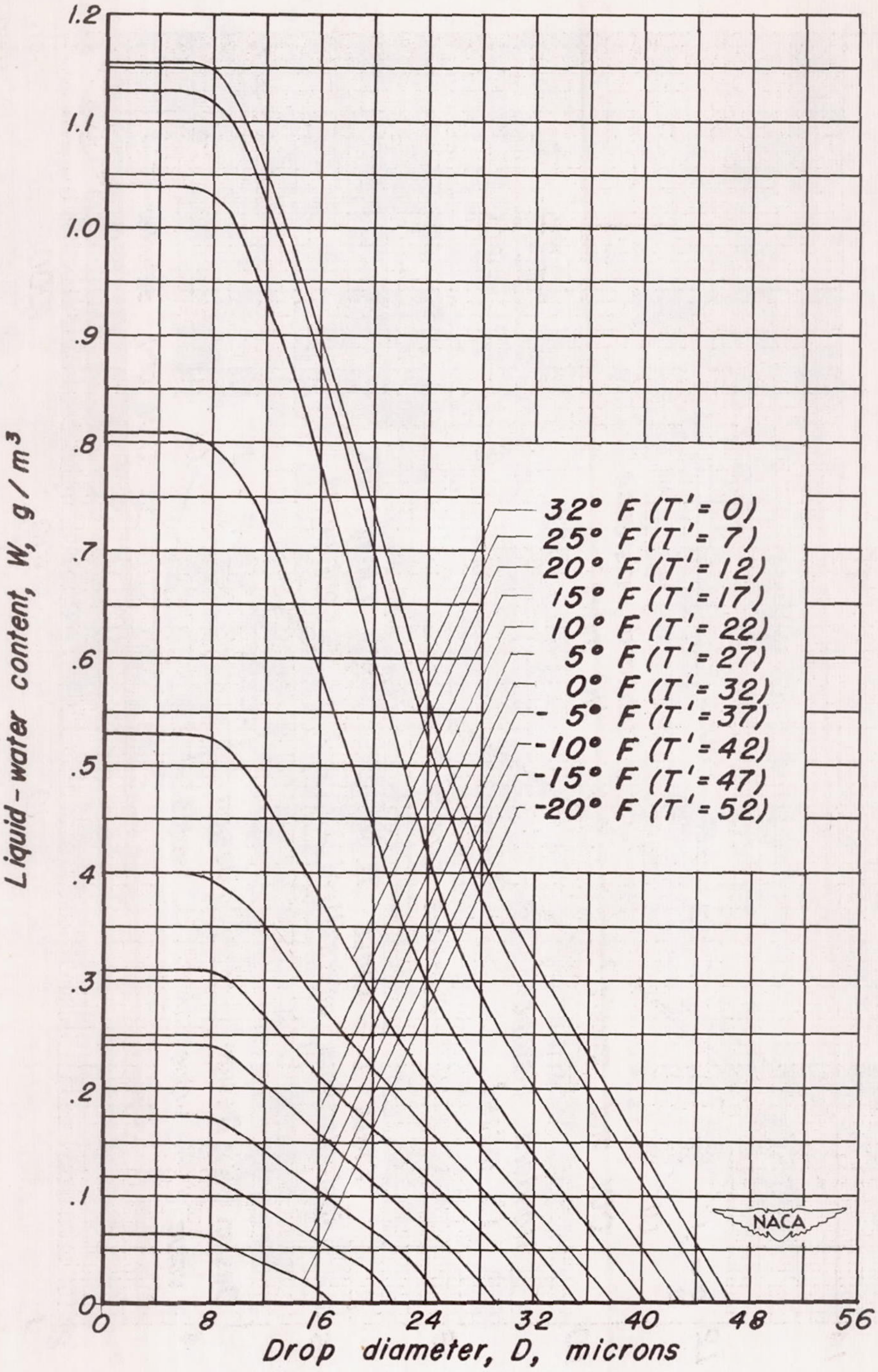
(a) $P_e = 0.1$

Figure 7. — Constant–temperature contours defining the equiprobability surface for Case 5. (Layer clouds, Eastern United States).



(b) $P_e = 0.01$

Figure 7. – Continued.



(c) $P_e = 0.001$

Figure 7. - Concluded.

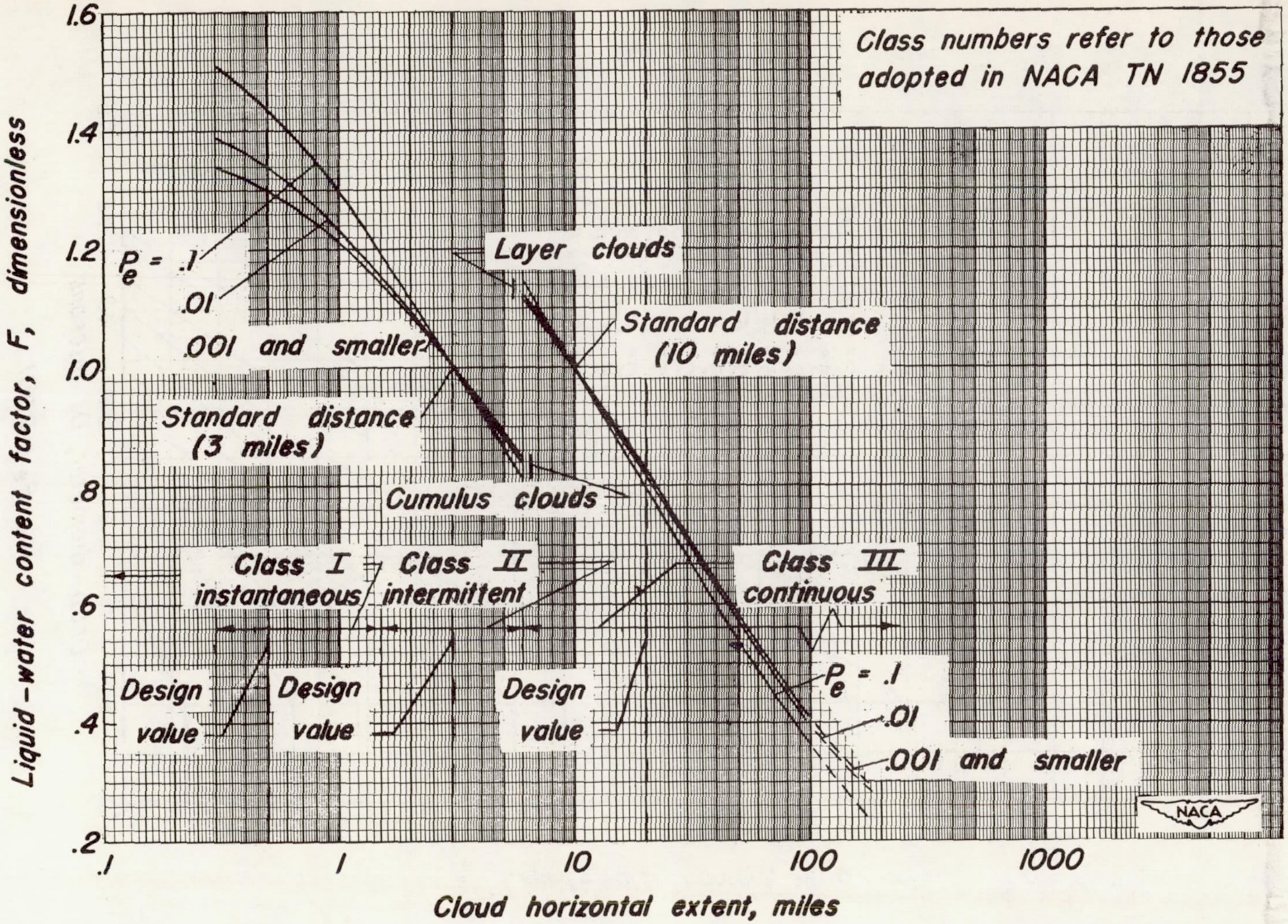


Figure 8.- Variation, with horizontal extent, of the factor by which values of liquid-water content from the equiprobability charts should be multiplied to include the effect of horizontal cloud extent for different exceedance probabilities.

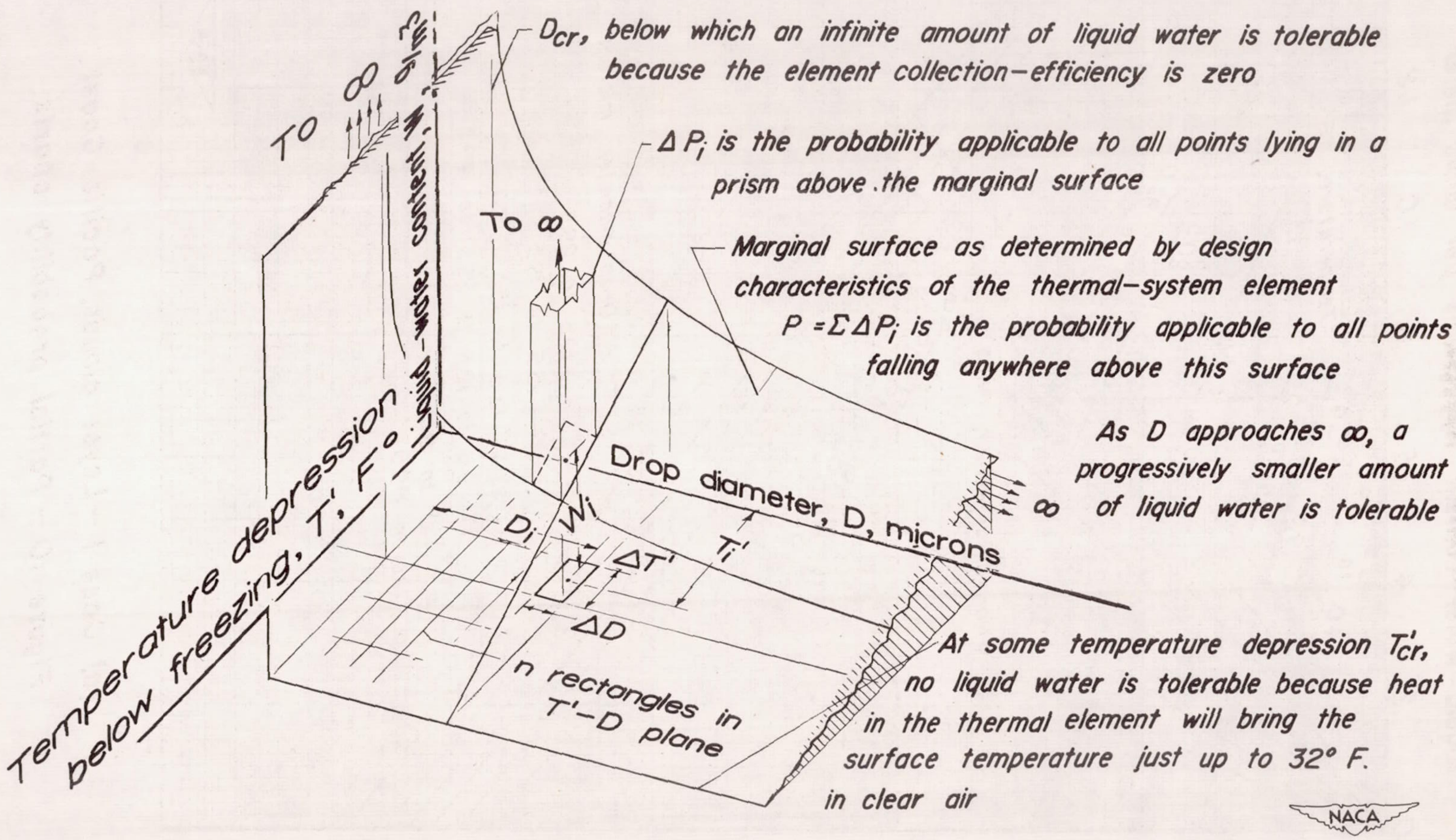
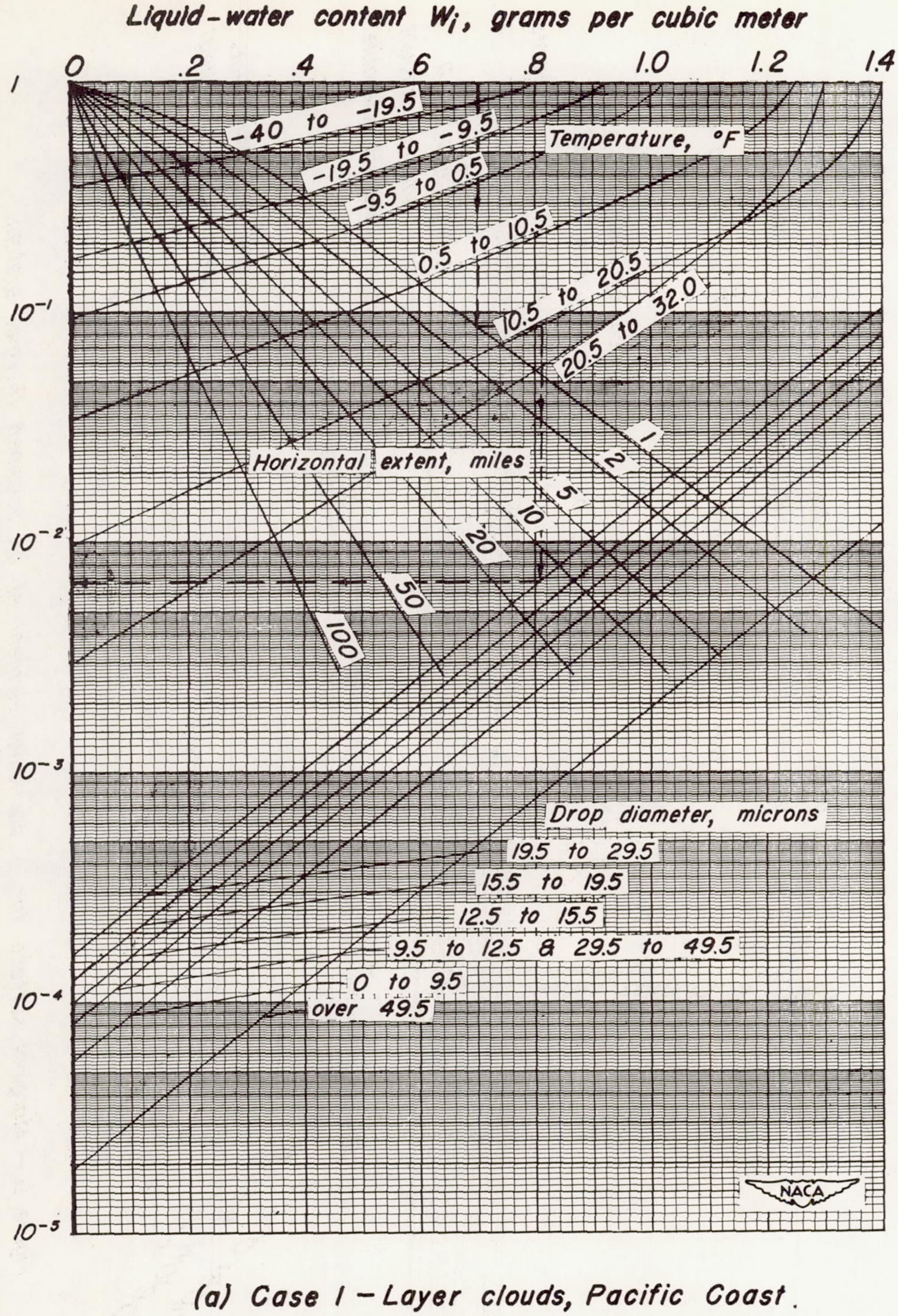
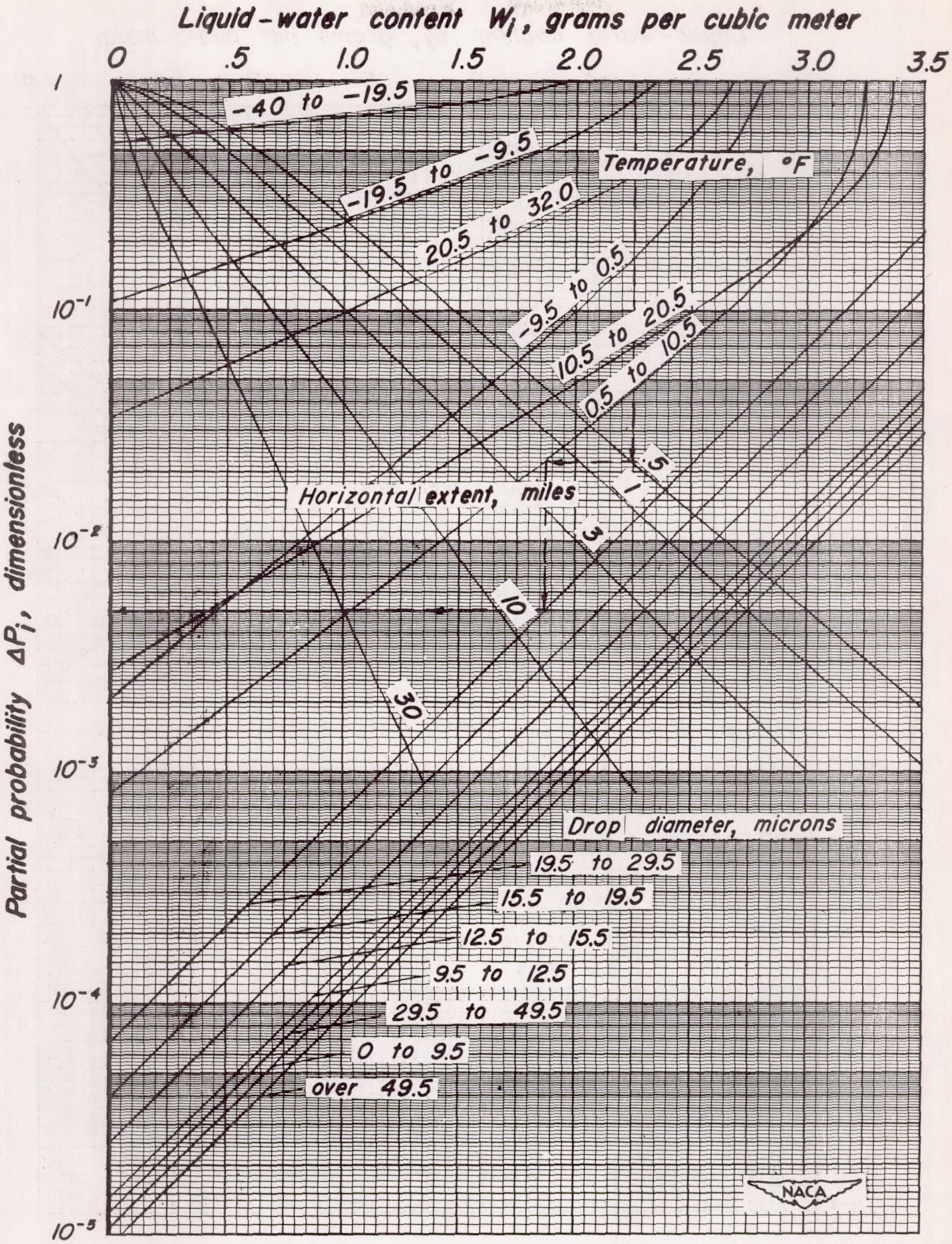


Figure 9.— Marginal surface for a particular element of an assumed thermal system.



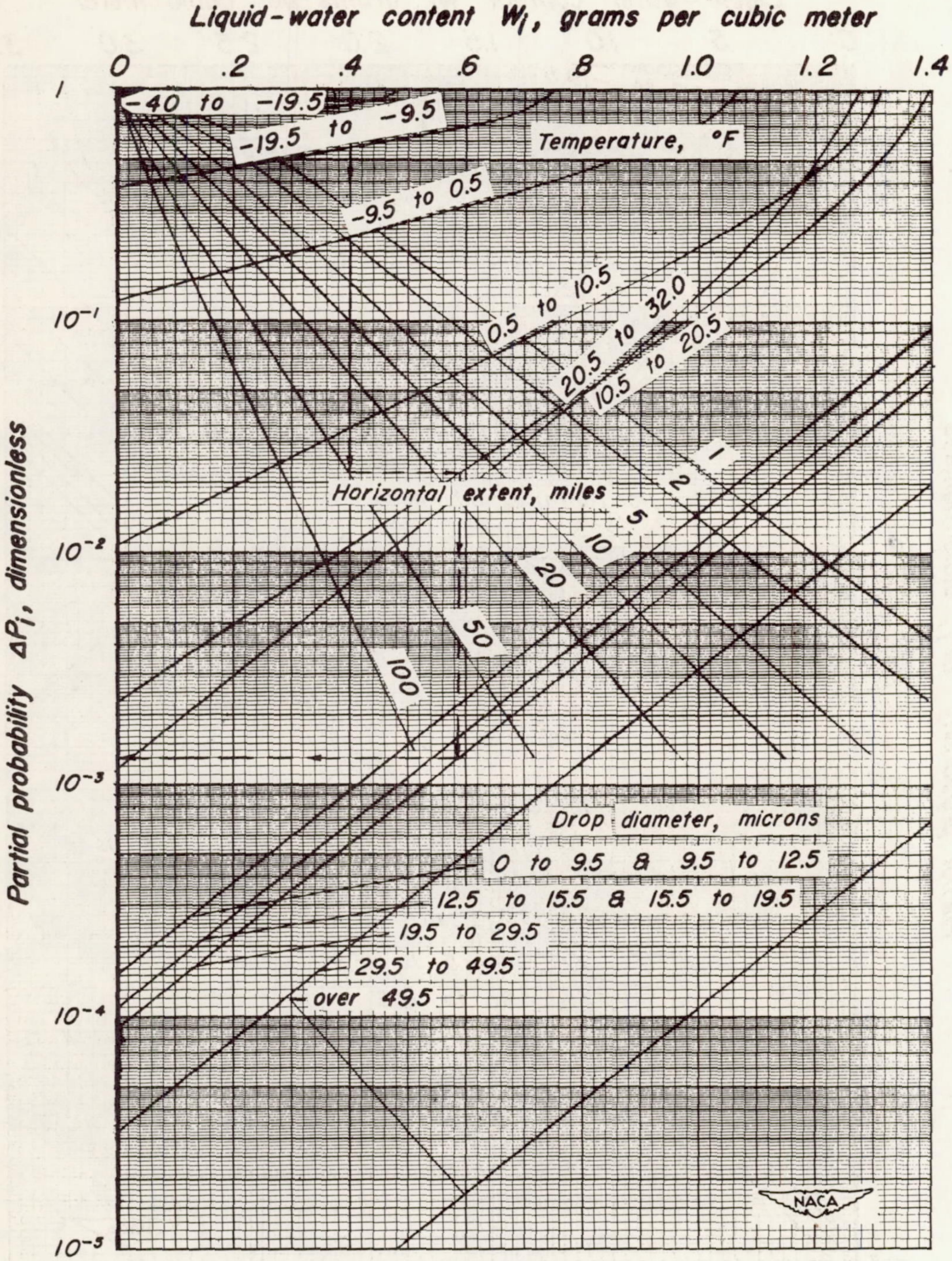
(a) Case I – Layer clouds, Pacific Coast.

Figure 10.– Partial probability charts.



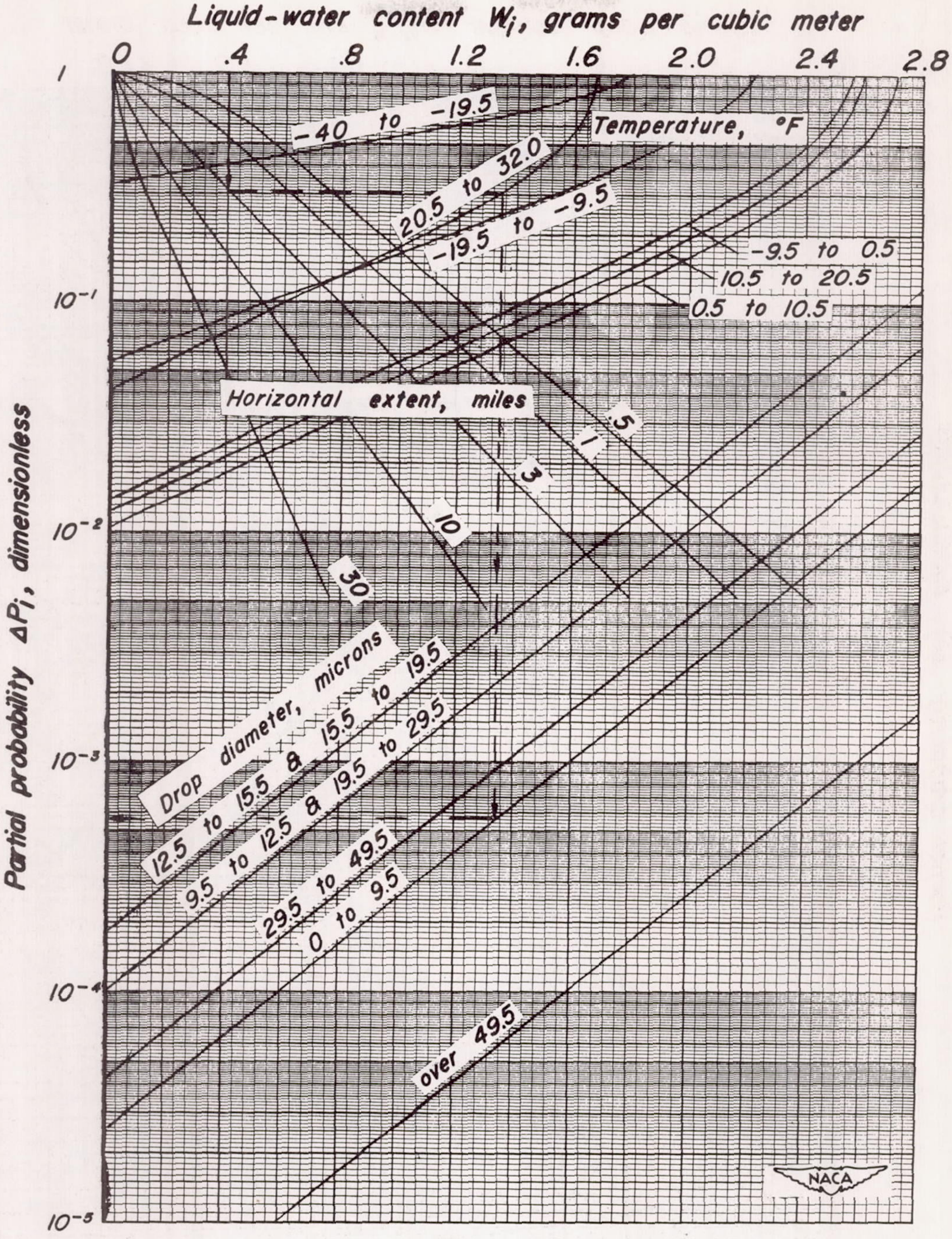
(b) Case 2 - Cumulus clouds, Pacific Coast.

Figure 10. - Continued.



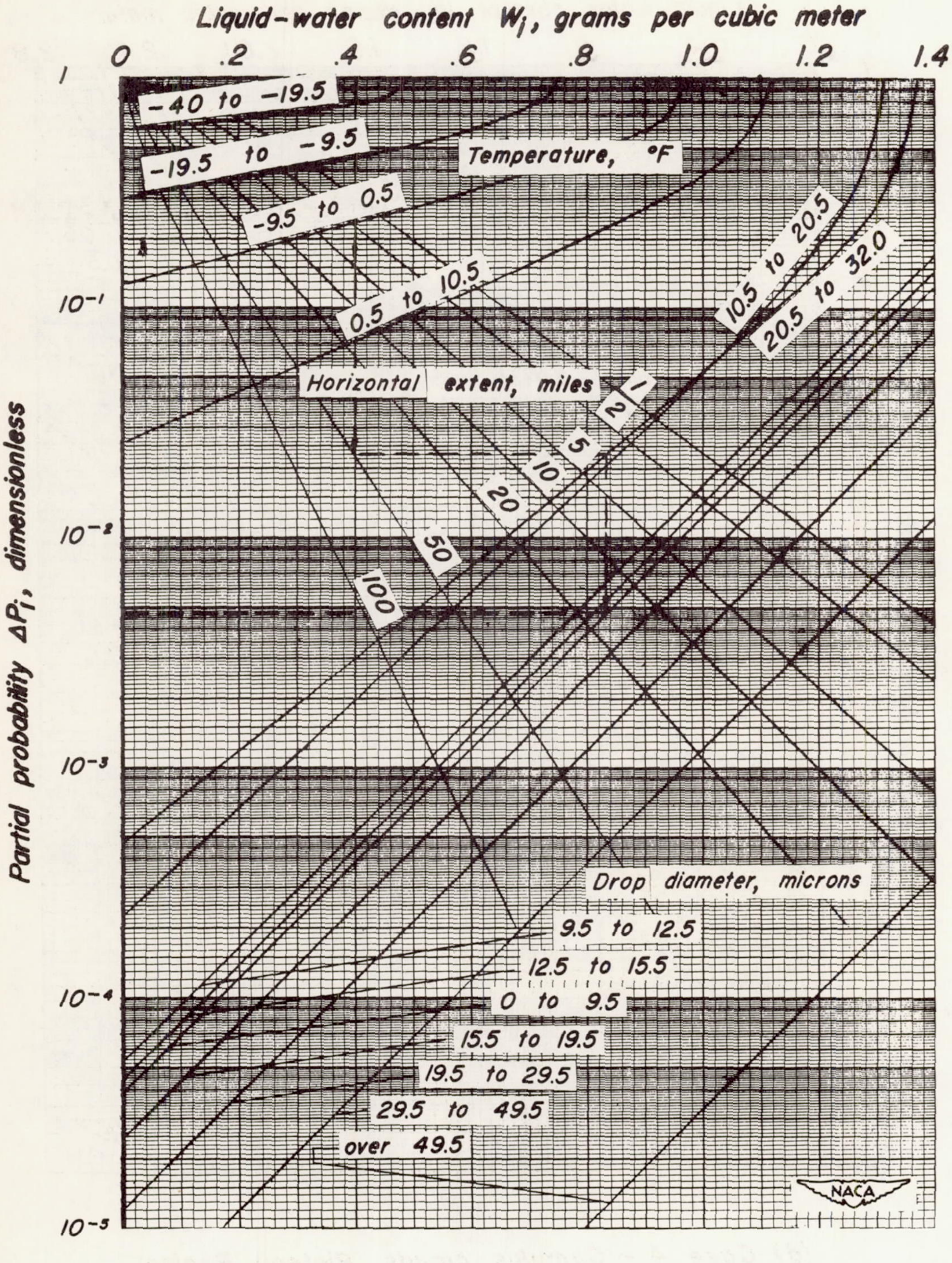
(c) Case 3 — Layer clouds, Plateau Region.

Figure 10.—Continued.



(d) Case 4 – Cumulus clouds, Plateau Region.

Figure 10.– Continued.



(e) Case 5 — Layer clouds, Eastern United States.

Figure 10.— Concluded.

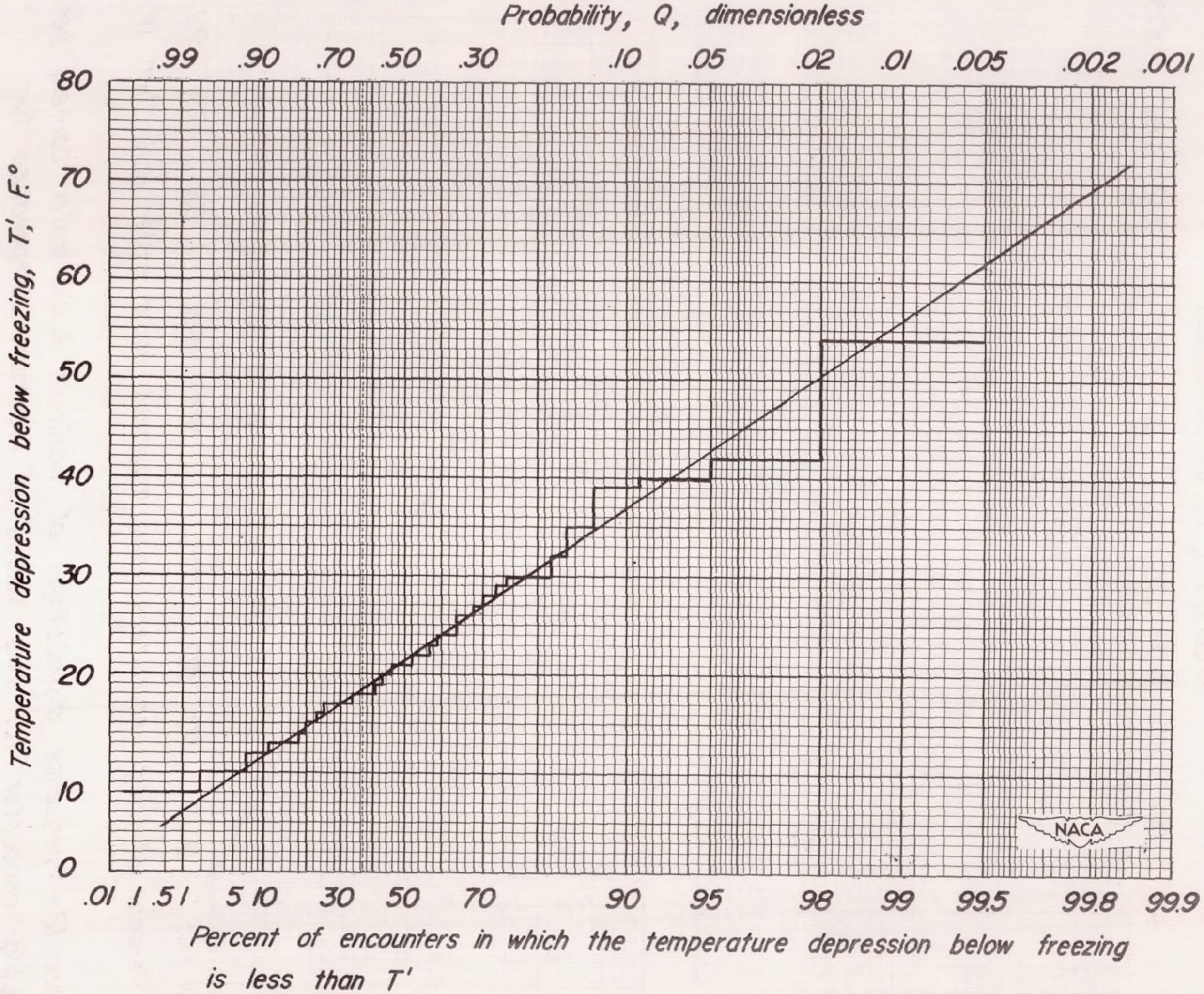


Figure 11.– Frequency distribution of the temperature depression below freezing of the most severe icing condition per icing encounter for Pacific Coast cumulus clouds.

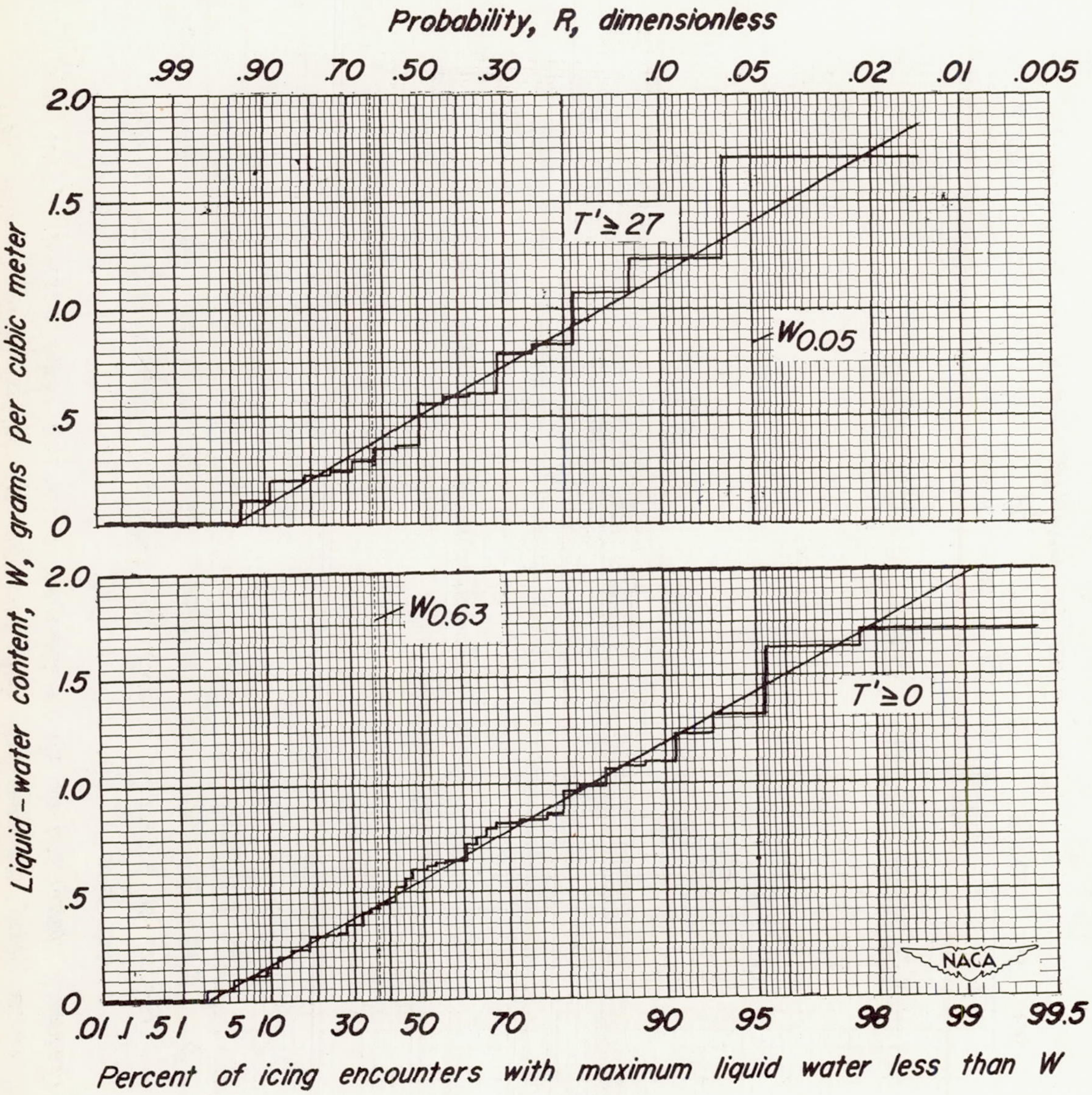


Figure 12.– Frequency distributions of maximum liquid-water content per icing encounter for two values of maximum temperature for Pacific Coast cumulus clouds.

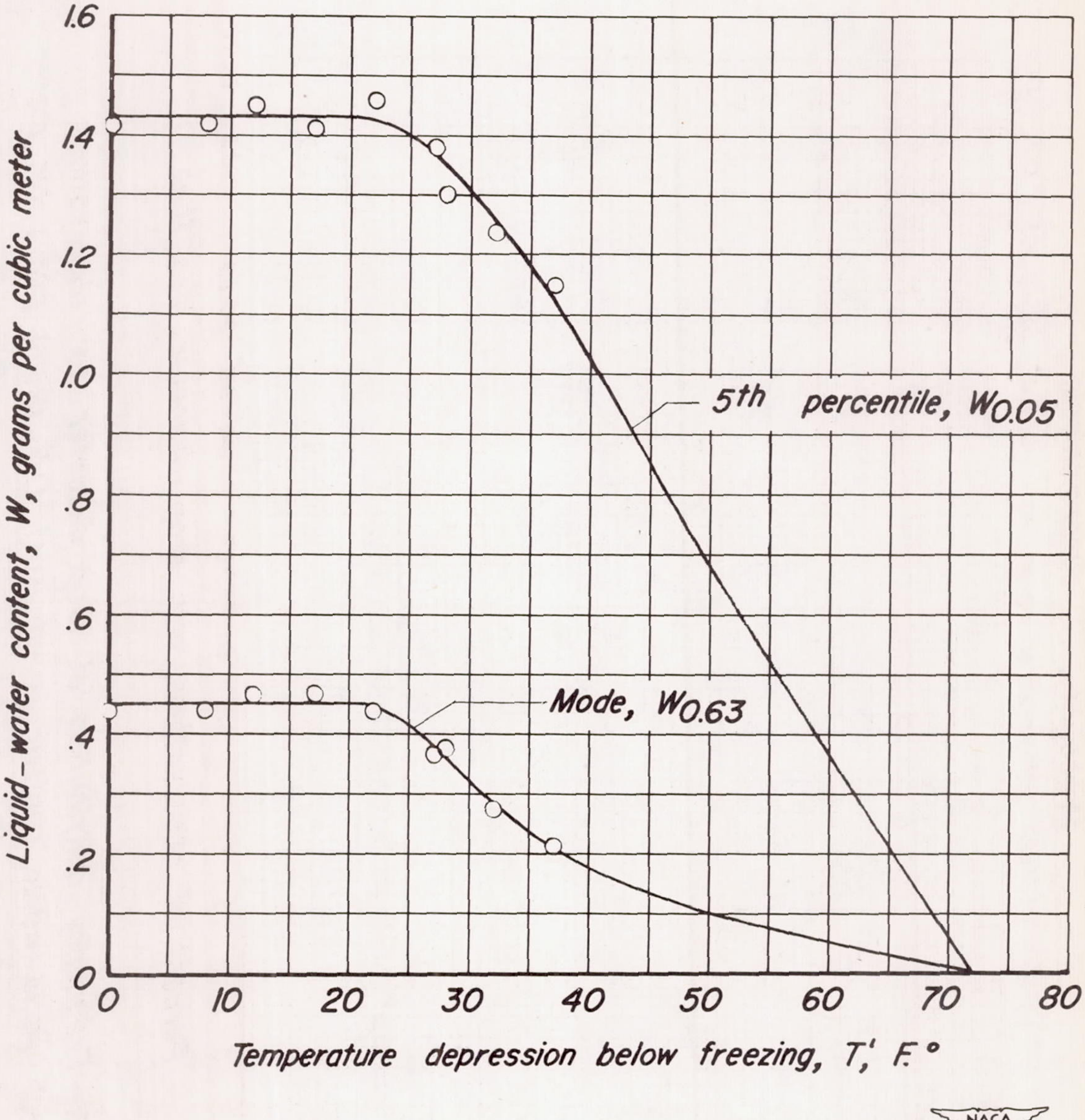


Figure 13.— Distribution parameters which define the probability function R(W,T') for Pacific Coast cumulus clouds.

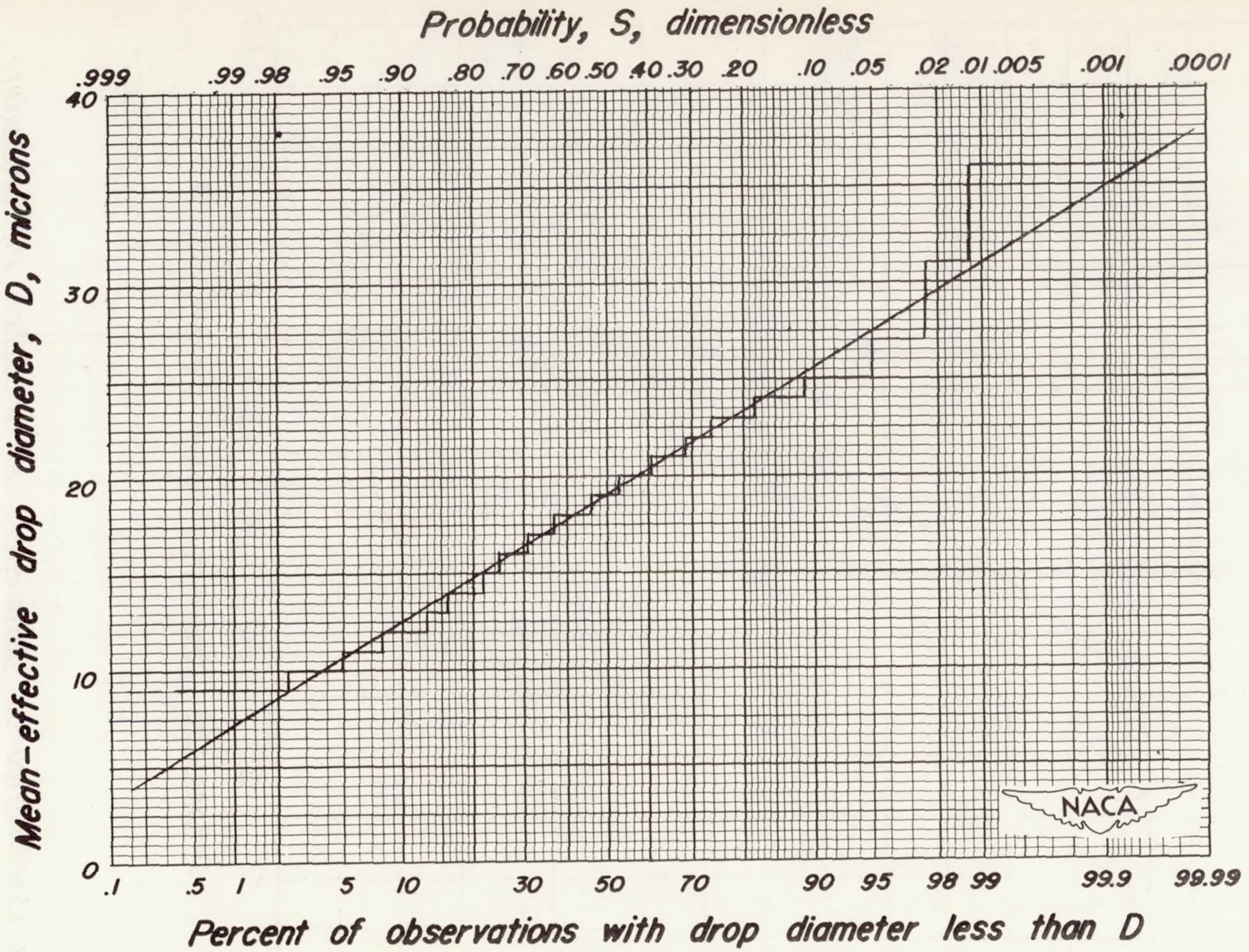


Figure 14.– Frequency distribution of drop diameter for observations in which liquid-water content is at least 0.5 grams per cubic meter for Pacific Coast cumulus clouds; $T' \geq 0$.

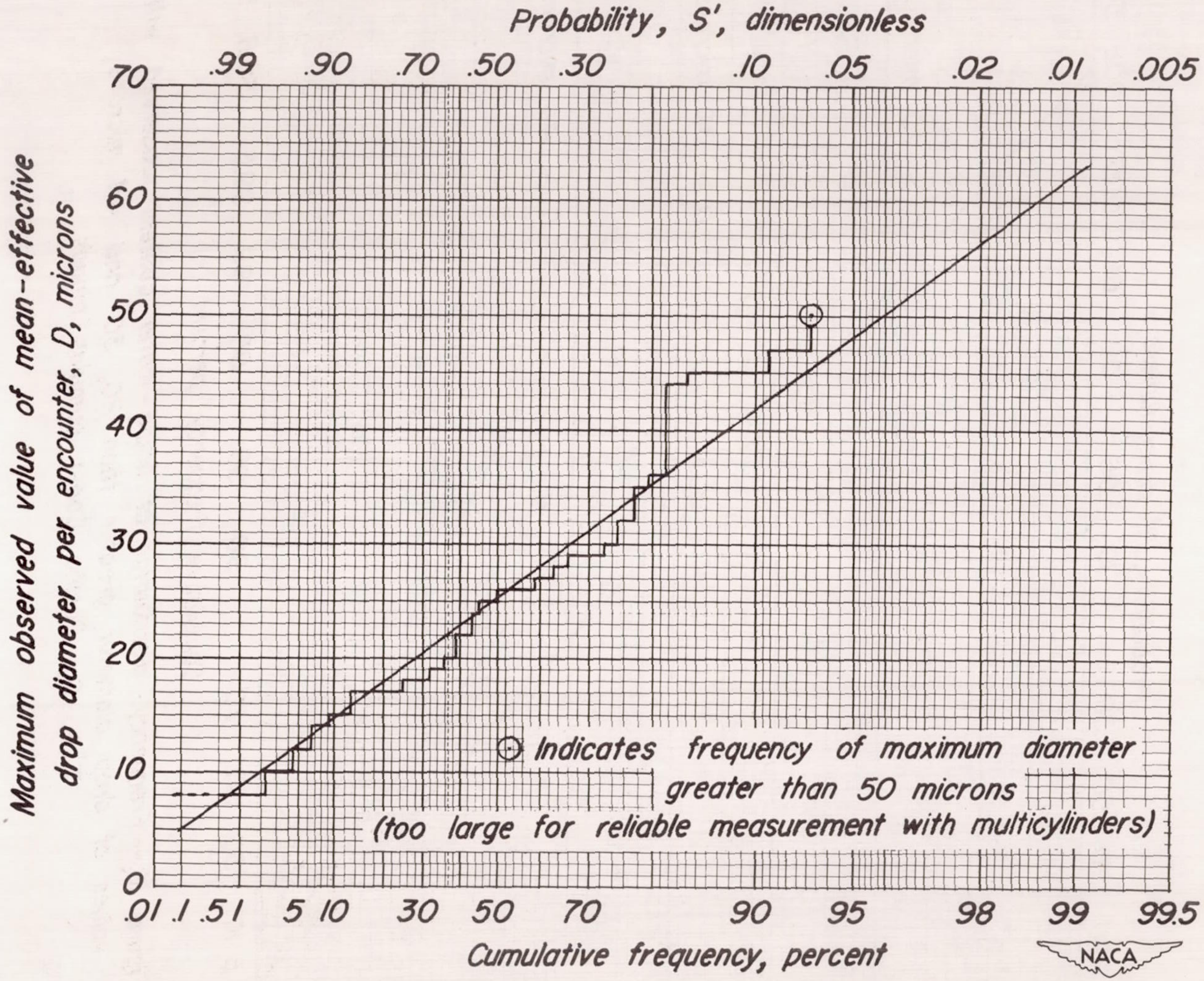


Figure 15. - Frequency distribution of maximum value of mean-effective drop diameter observed in each icing encounter for Pacific Coast cumulus clouds, $T' \geq 0$.

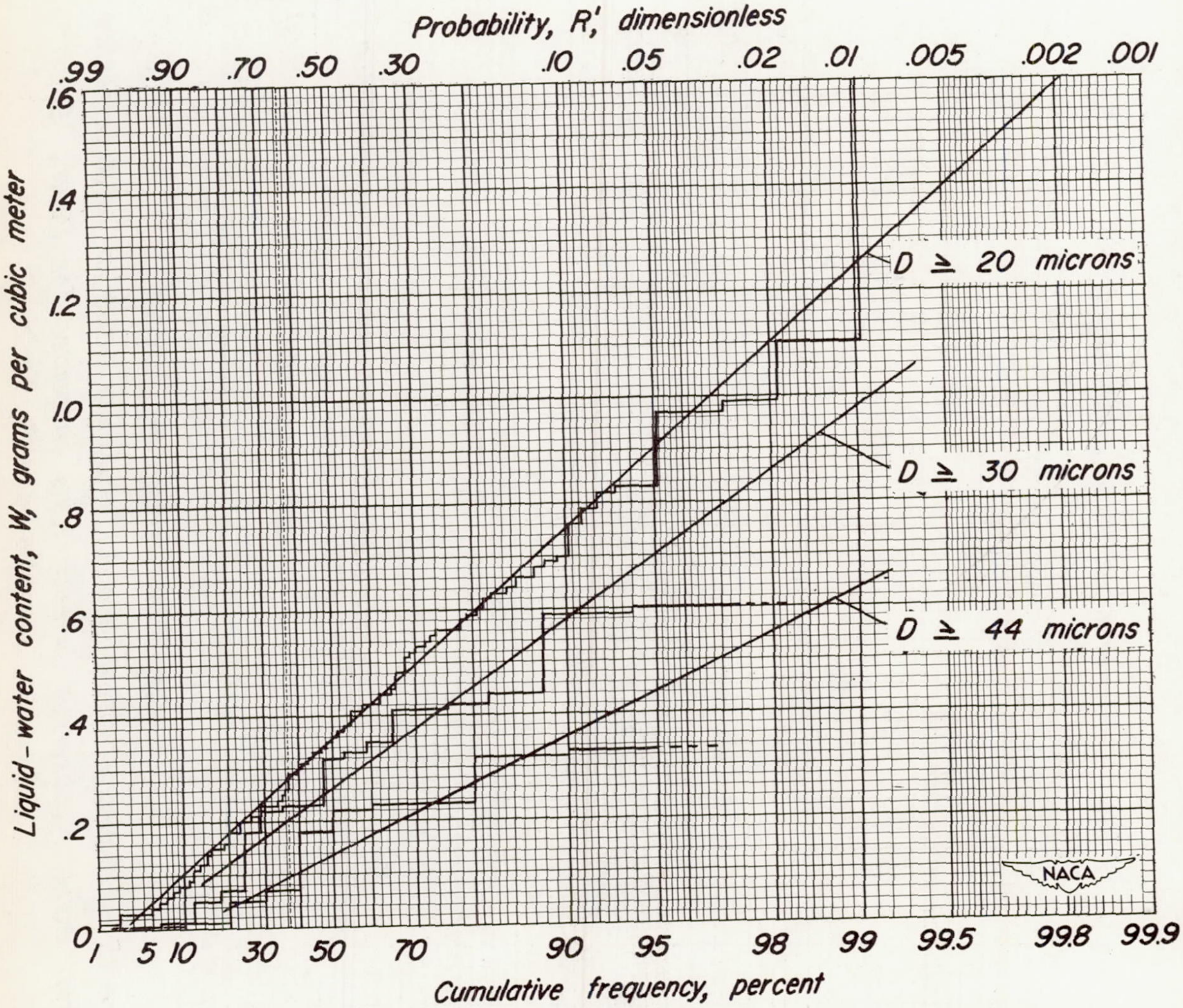


Figure 16.— Frequency distribution of liquid-water content observed with values of drop diameter greater than 20, 30 and 44 microns; $T' \geq 0$; Pacific Coast cumulus clouds.

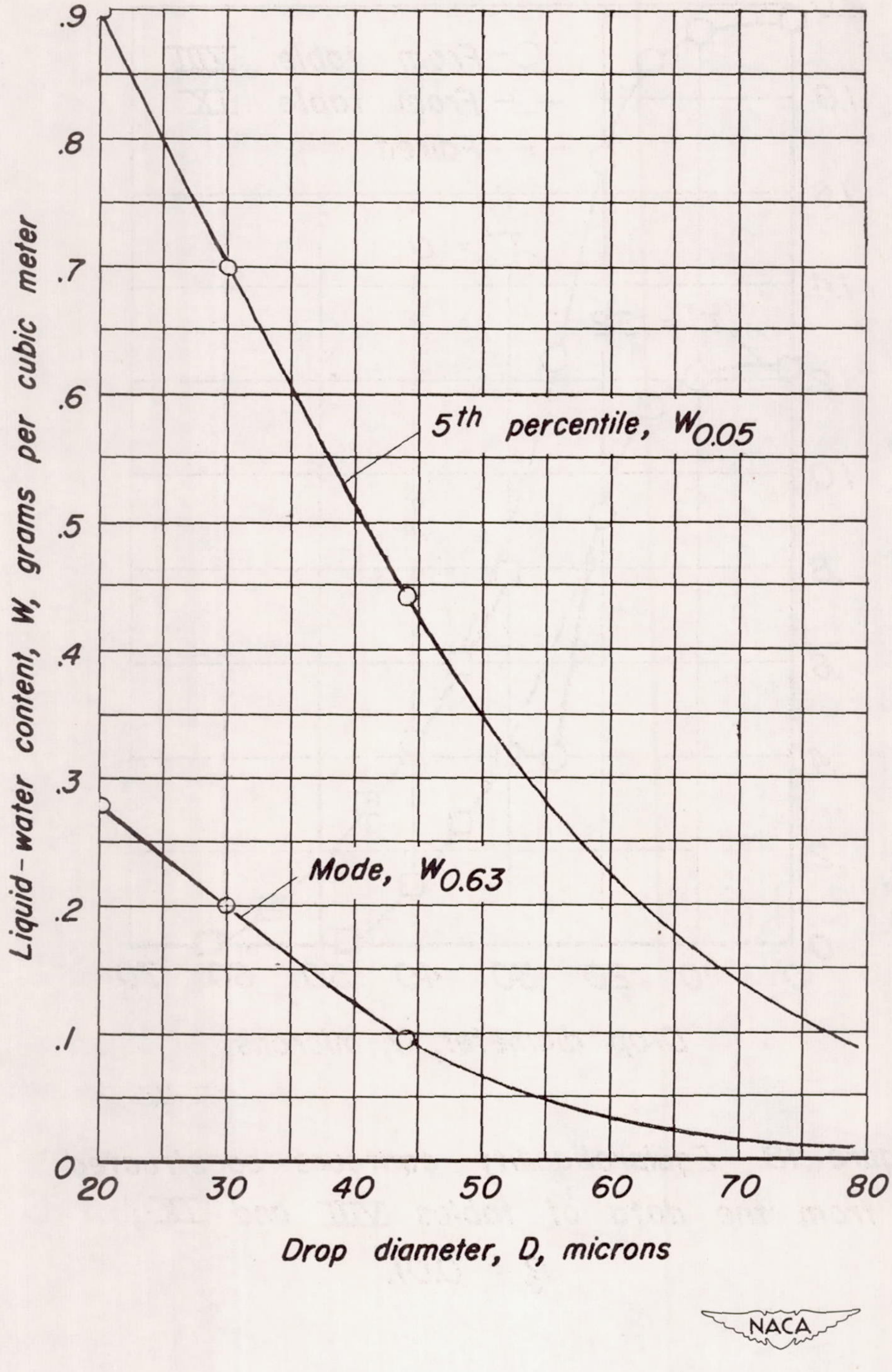


Figure 17.—Parameters defining the distribution of liquid water content for values of drop diameter greater than D for Pacific Coast cumulus clouds; $T' \geq 0$.

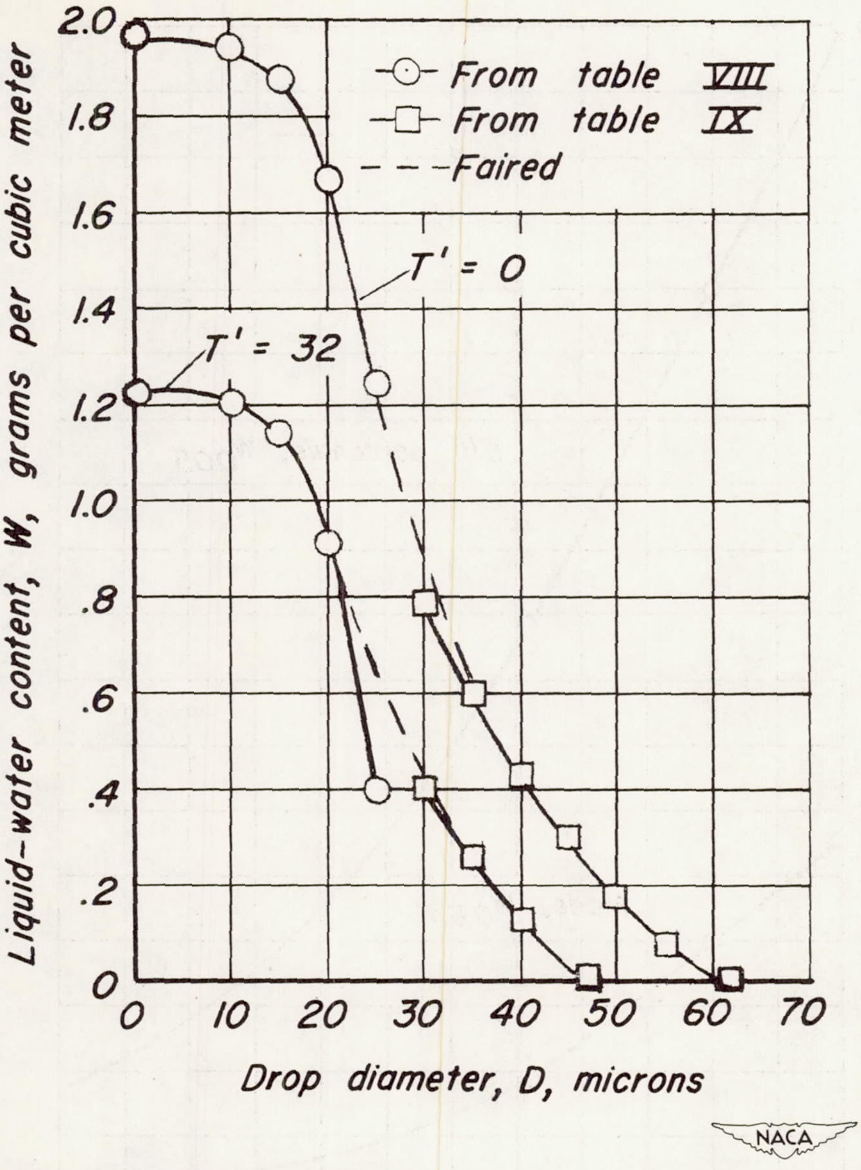


Figure 18.—Equiprobability contours constructed from the data of tables VIII and IX; P_e = 0.01.

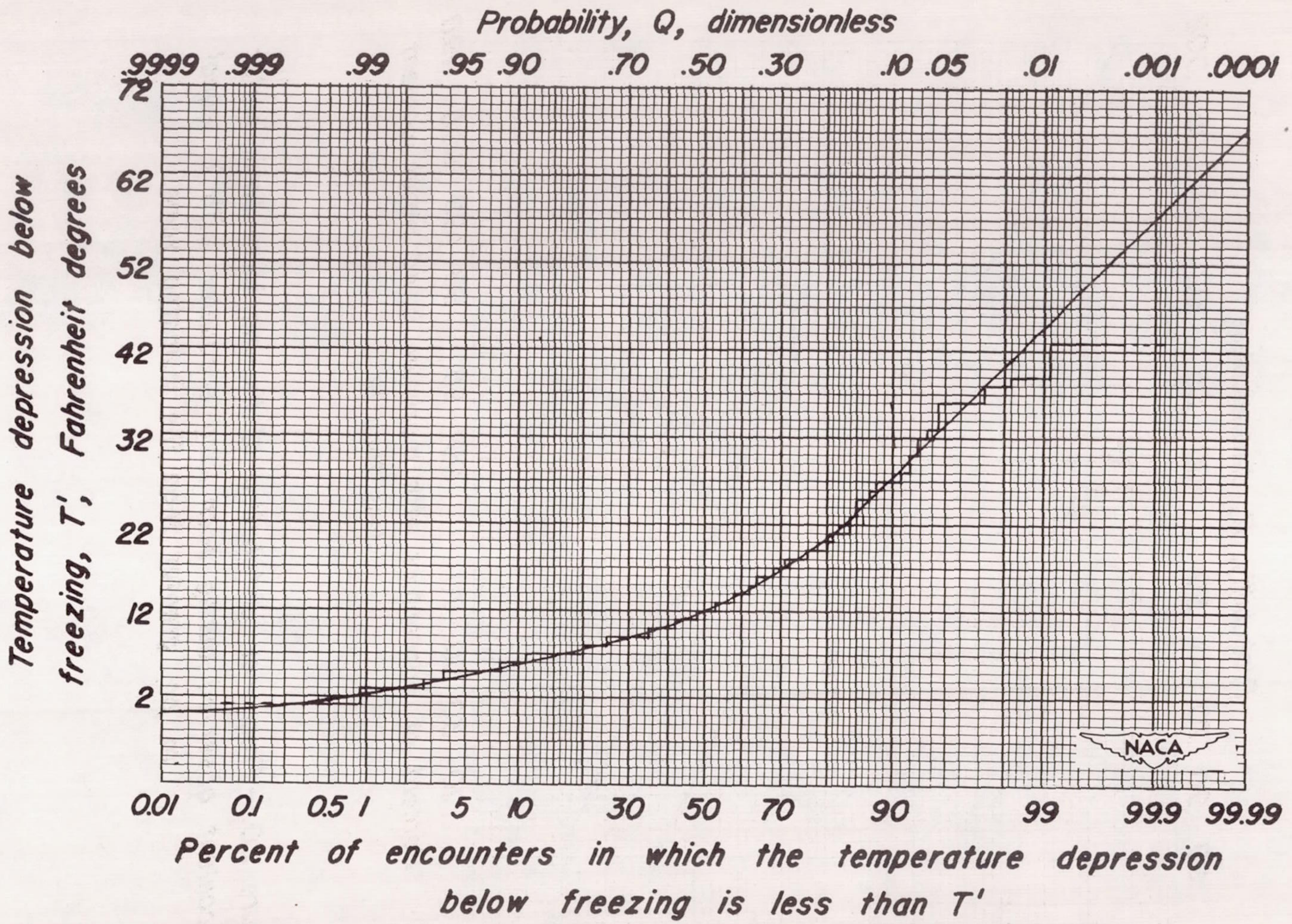


Figure 19.— Frequency distribution of the temperature depression below freezing of the most severe icing condition per icing encounter for Eastern U. S. layer clouds.

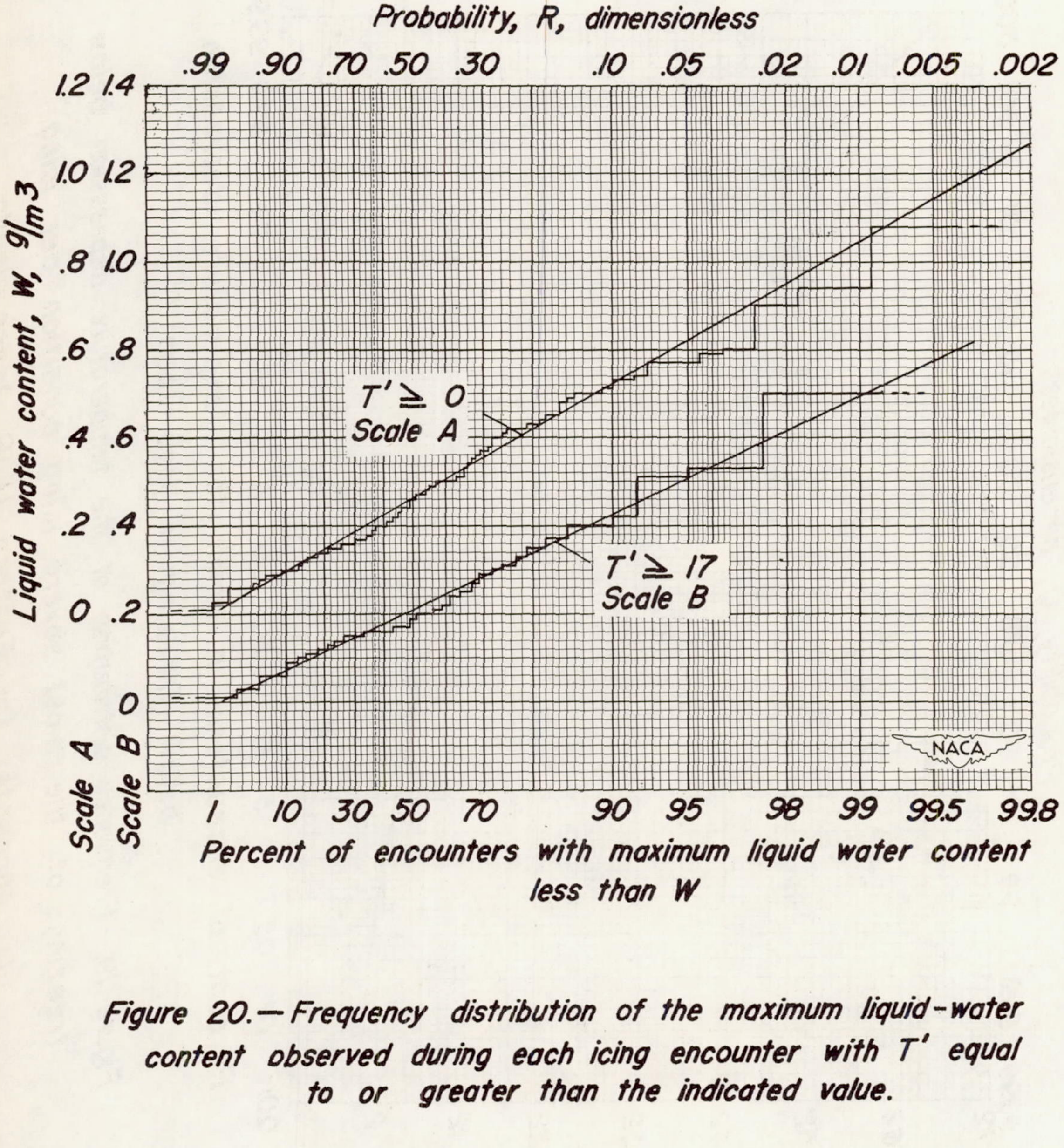


Figure 20.— Frequency distribution of the maximum liquid-water content observed during each icing encounter with T' equal to or greater than the indicated value.

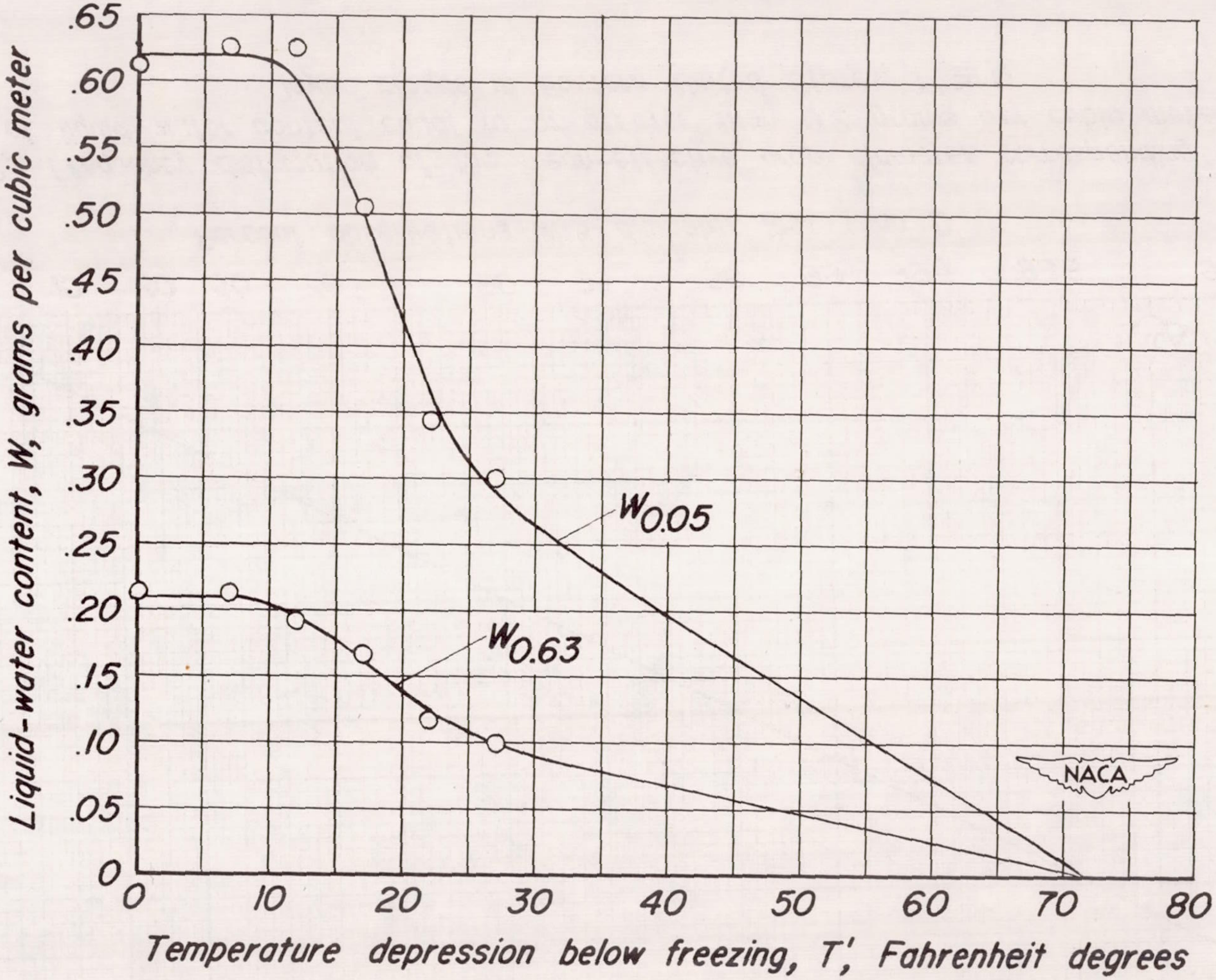


Figure 21.—Distribution parameters which define the probability function R(W,T') for layer clouds, Eastern United States.

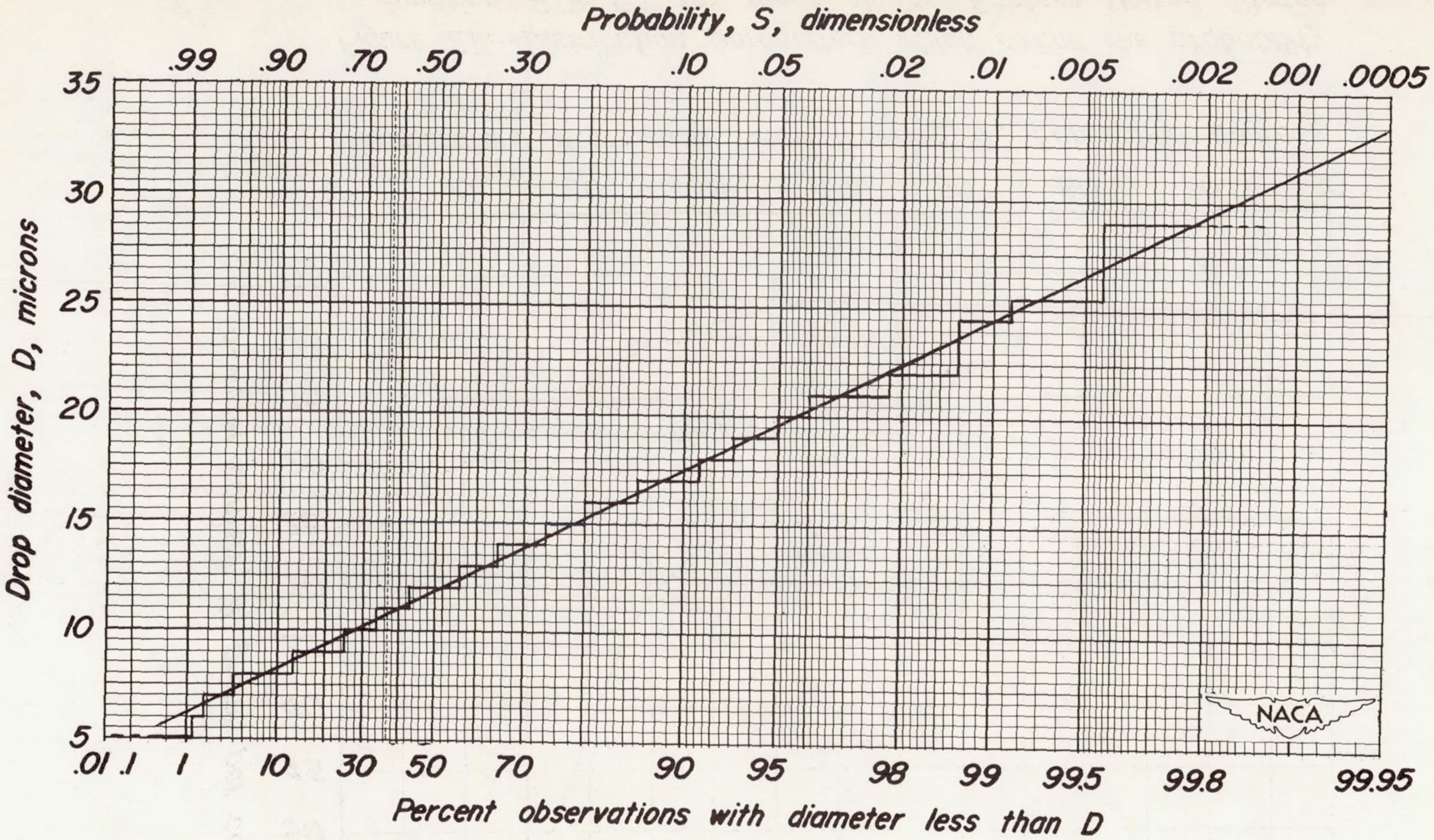


Figure 22.— Frequency distribution of the mean-effective drop diameter corresponding to values of liquid-water content equal to or greater than 0.2 grams per cubic meter; layer clouds in Eastern United States; $T' \geq 0$.

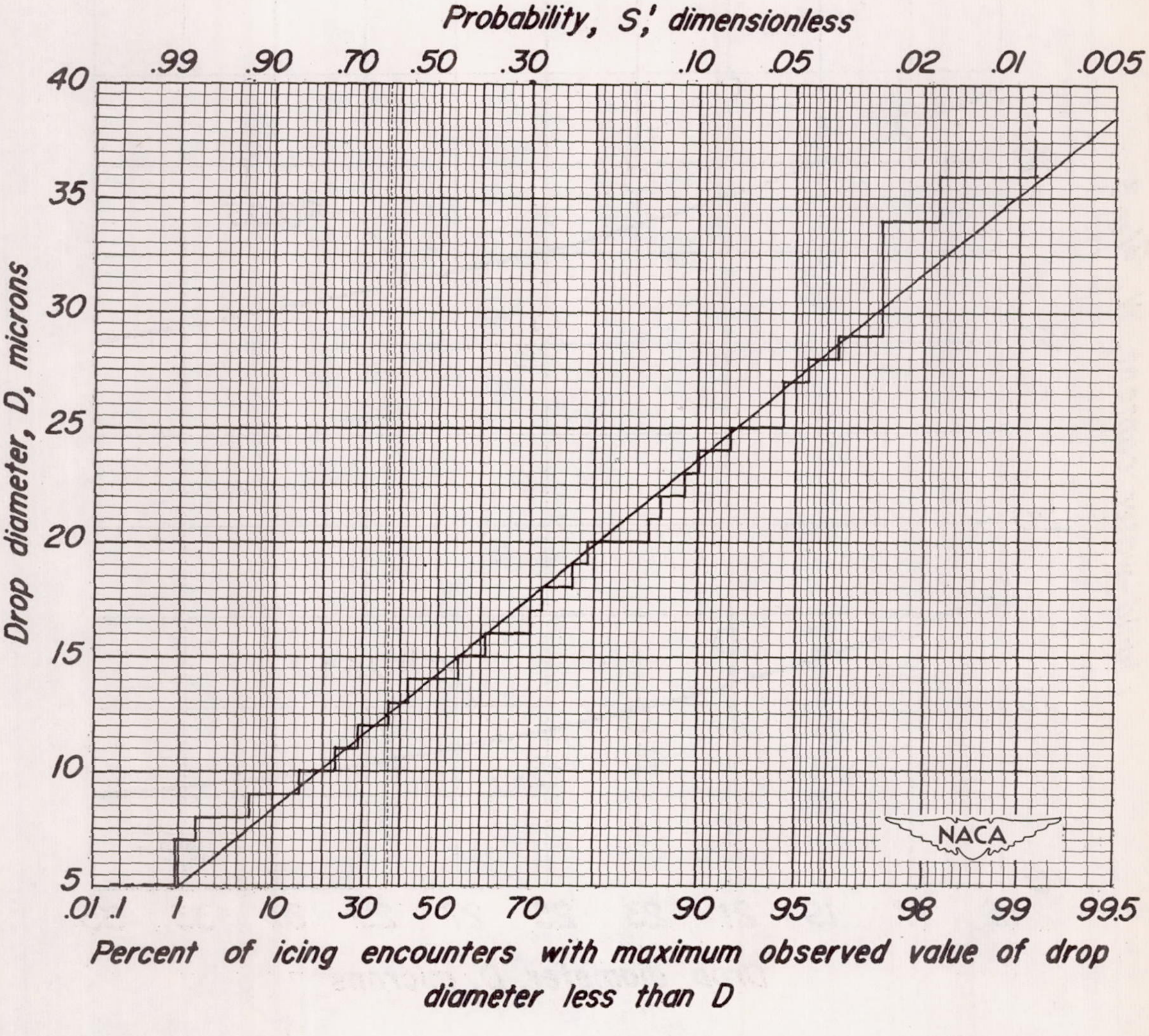


Figure 23.—Frequency distribution of the largest value of drop diameter observed during each icing encounter; layer clouds, Eastern United States.

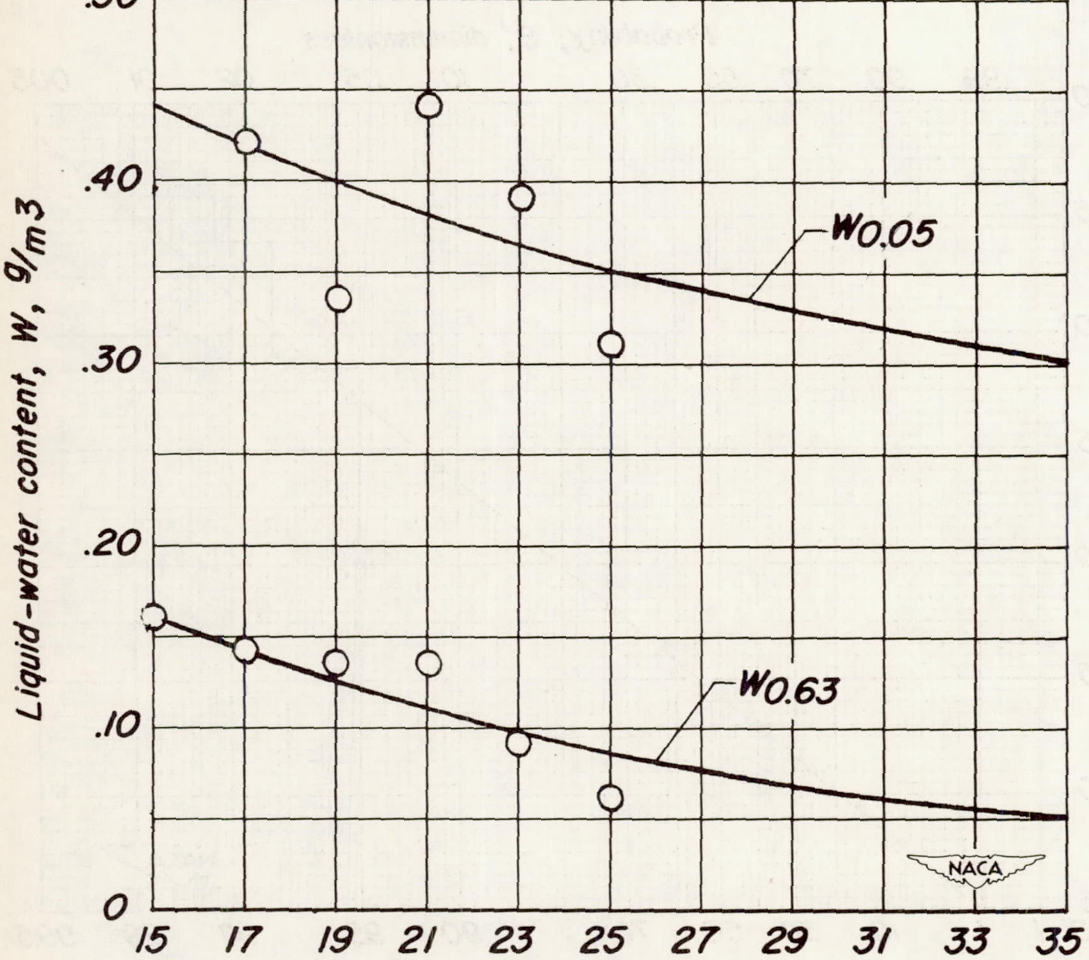


Figure 24.– Distribution parameters defining the function R′(W,D) for the case of T′ ≥ 0; layer clouds, Eastern United States.

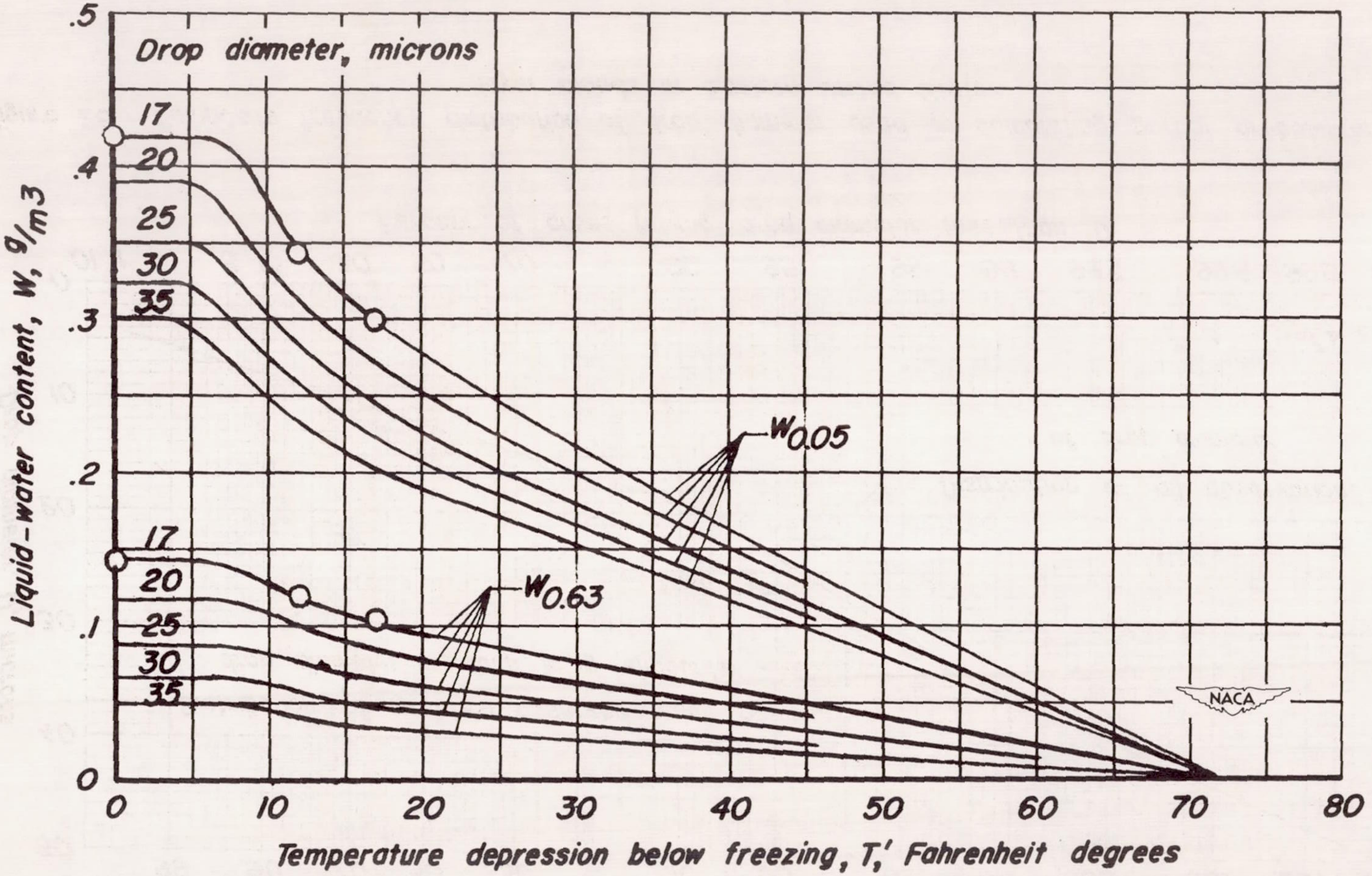


Figure 25.— Distribution parameters defining the function $R'(W,D,T')$; layer clouds, Eastern United States.

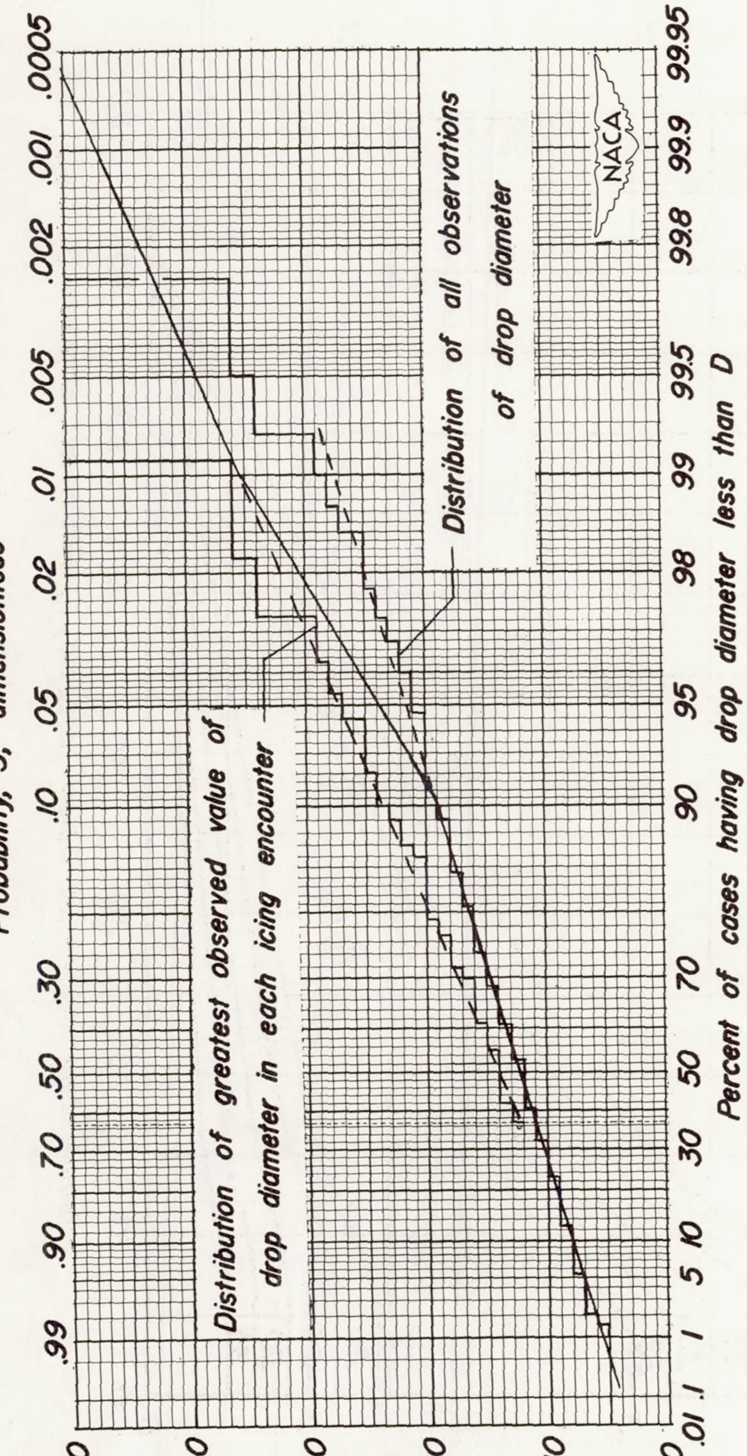
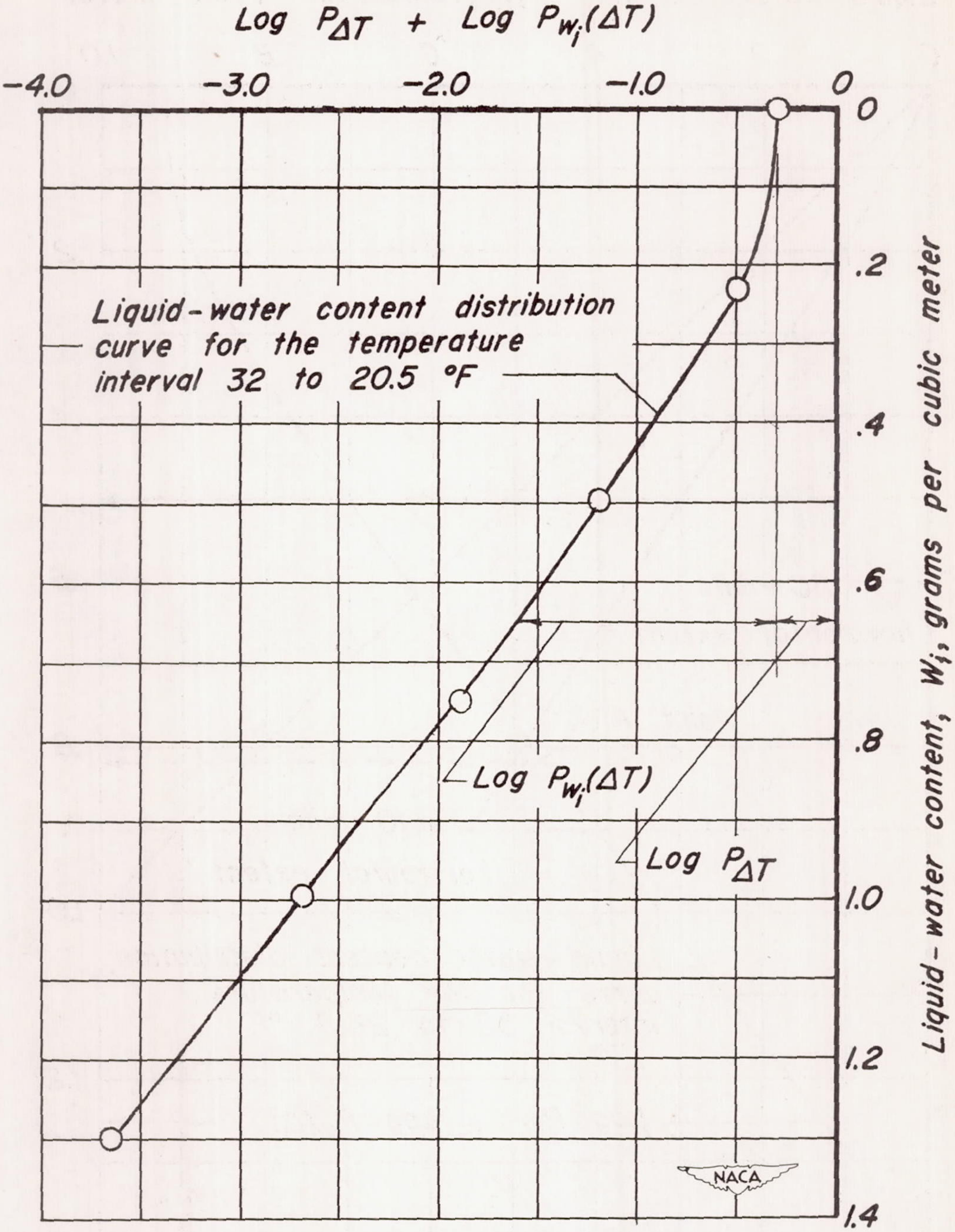
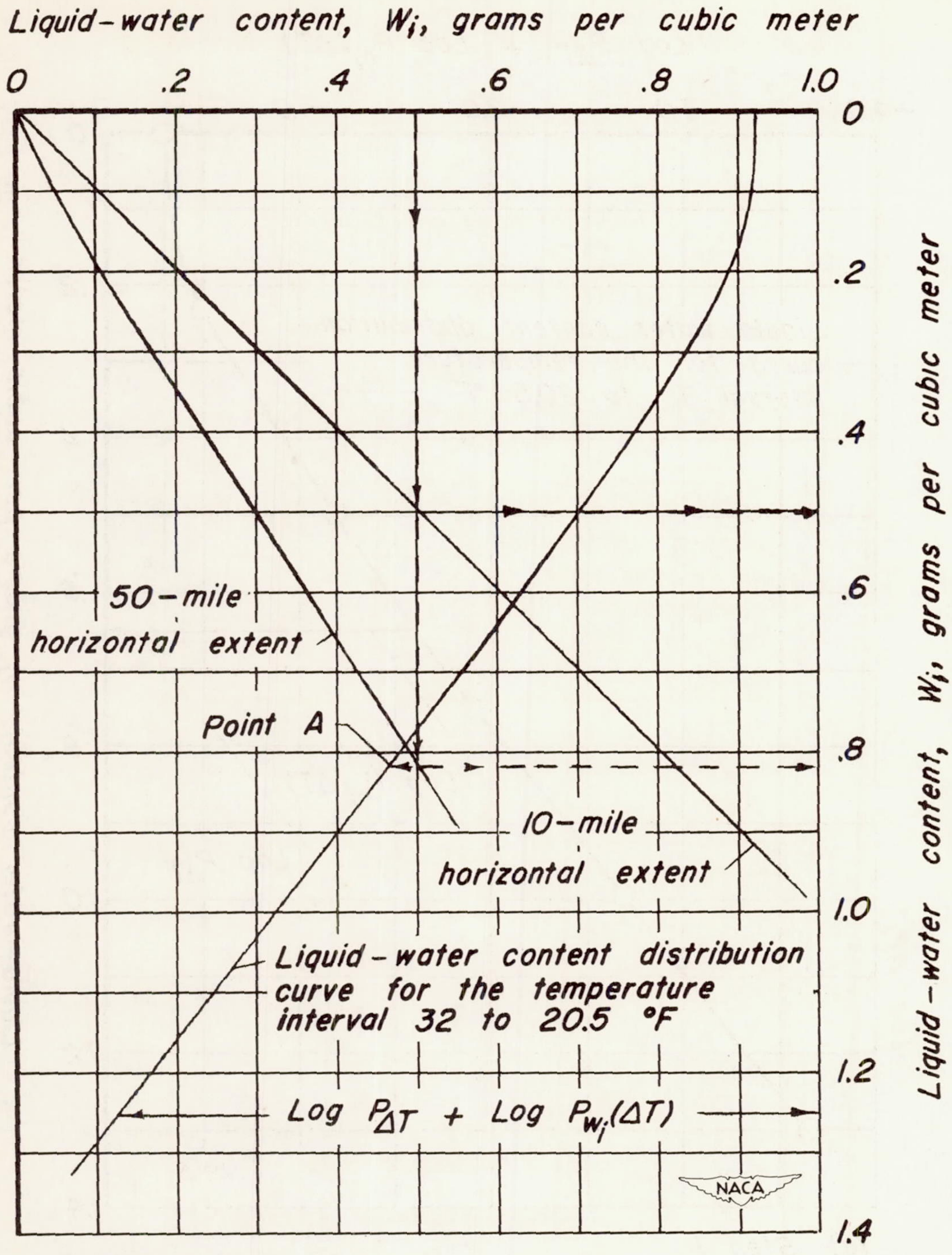


Figure 26.—Composite frequency distribution of drop diameter used in calculating partial probabilities for layer clouds in Eastern United States.



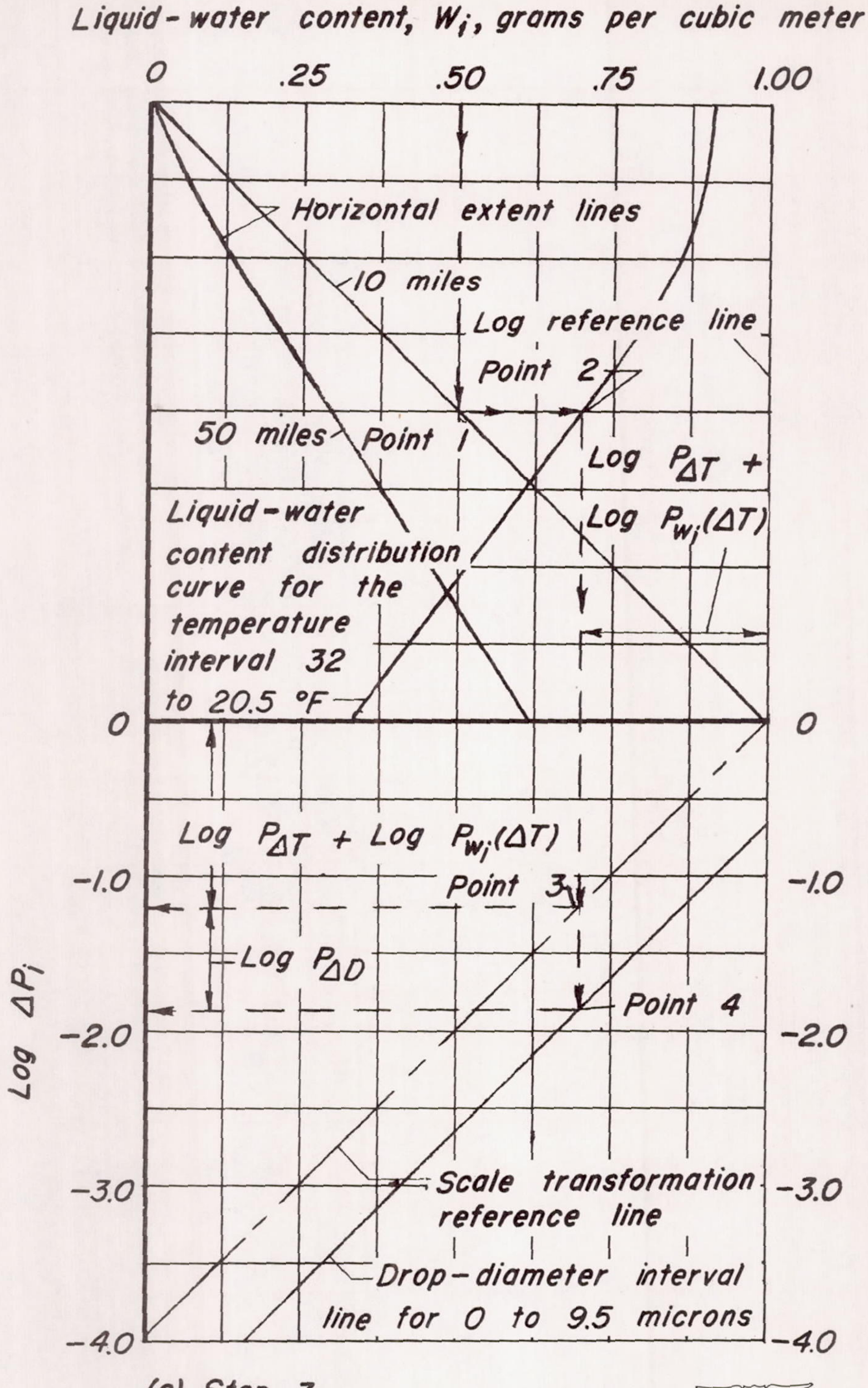
(a) Step 1

Figure 27. – Example construction of a partial probability chart.



(b) Step 2

Figure 27. – Continued.



(c) Step 3

Figure 27. — Concluded.

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