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CHARACTERISTICS OF CLARK Y AIRFOILS OF SMALL ASPECT RATIOS

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AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Symbol	Unit	Symbol
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of one kilogram	kg	weight of one pound	lb.
Power	P	kg/m/s		horsepower	hp
Speed		km/h	k. p. h.	mi./hr.	m. p. h.
		m/s	m. p. s.	ft./sec.	f. p. s.

2. GENERAL SYMBOLS, ETC.

W, Weight $=mg$

g, Standard acceleration of gravity $=9.80665$ m/s^2 $=32.1740$ ft./sec.^{2}

m, Mass $=\frac{W}{g}$

ρ, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg-m^{-4} s^2) at 15° C. and 760 mm $=0.002378$ (lb.-ft.$^{-4}$ sec.^{2}).

Specific weight of "standard" air, 1.2255 kg/m^3 $=0.07651$ lb./ft.^{3}.

mk^2, Moment of inertia (indicate axis of the radius of gyration k, by proper subscript).

S, Area.

S_w, Wing area, etc.

G, Gap.

b, Span.

c, Chord.

$\frac{b^2}{S}$, Aspect ratio.

μ, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

V, True air speed.

q, Dynamic (or impact) pressure $=\frac{1}{2}\rho V^2$.

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_o}=\frac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force.

i_w, Angle of setting of wings (relative to thrust line).

i_t, Angle of stabilizer setting (relative to thrust line).

Q, Resultant moment.

Ω, Resultant angular velocity.

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, at 15° C., the corresponding number is 234,000;

or for a model of 10 cm chord 40 m/s, the corresponding number is 274,000.

C_p, Center of pressure coefficient (ratio of distance of c. p. from leading edge to chord length).

α, Angle of attack.

ϵ, Angle of downwash.

α_o, Angle of attack, infinite aspect ratio.

α_i, Angle of attack, induced.

α_a, Angle of attack, absolute. (Measured from zero lift position.)

γ Flight path angle.

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REPORT No. 431

CHARACTERISTICS OF CLARK Y AIRFOILS OF SMALL ASPECT RATIOS

By C. H. Zimmerman

SUMMARY

This report presents the results of a series of wind-tunnel tests showing the force, moment, and autorotational characteristics of Clark Y airfoils having aspect ratios varying from 0.5 to 3.

An airfoil of rectangular plan form was tested with rectangular tips, faired tips, and semicircular tips. Tests were also made on one airfoil of circular plan form and two airfoils of elliptical plan form.

The tests revealed a marked delay of the stall and a decided increase in values of maximum lift coefficient and maximum resultant force coefficient for aspect ratios of the order of 1 as compared with the values for aspect ratios of 2 and 3. The largest value of $C_{R max}$ was 2.17 with a wing of circular plan form and an aspect ratio of 1.27. The same wing gave a $C_{L max}$ of 1.85 and an L/D ratio of 1.63 at 45° angle of attack.

Wings having aspect ratios of about 1 were found to have moment characteristics more favorable to stability than those having larger aspect ratios. Decreasing the aspect ratio greatly reduced ranges and rates of autorotation based on a given span and air speed. Results, when reduced to infinite aspect ratio by conventional formulas, indicate that such formulas are not applicable for aspect ratios less than 1.5. It is apparent that the plan form and tip shape of the wing are of major importance among the factors affecting airfoil characteristics at aspect ratios of 1.5 and smaller.

INTRODUCTION

In recent years there has been an increasing demand for an airplane suited to the needs of the private owner. Without going into a discussion of the problem it may be said that such an airplane should be capable of descending along a steep path at such a low rate of speed that it will be unnecessary for the pilot to alter the direction of the flight path or the speed when near the ground in order to make a satisfactory landing.

The present tests were suggested by a study of means of obtaining such characteristics. It was immediately apparent that it is necessary to secure a high resultant force coefficient and a low lift/drag ratio at maximum resultant force coefficient. It was thought possible to derive benefit from the large induced drag of small aspect-ratio wings when at large values of lift coefficient. A survey of the results of previous investigations of the effects of varying the aspect ratio (references 1 and 2) and of the unpublished results of tests of flat plates with aspect ratios of 1 and 2 in the original atmospheric wind tunnel of the National Advisory Committee for Aeronautics revealed a scarcity of data for aspect ratios less than 3, and suggested the possibility of obtaining high maximum lift coefficients with wings having aspect ratios of the order of 1.

In order to determine characteristics of small aspect-ratio wings within the usable range, force tests were made on Clark Y airfoils with aspect ratios varying from 3 to 0.5. The airfoils were tested with rectangular, faired, and semicircular tips in order to determine the effect of tip shape upon the characteristics. In addition to force tests at 0° yaw, autorotational tests at 0° yaw and force tests at 20° yaw were made upon those airfoils having aspect ratios of approximately 3 and approximately 1 to investigate the stability characteristics of low aspect-ratio wings. Test data on a rectangular Clark Y airfoil of an aspect ratio of 6 were included for comparison.

MODELS AND APPARATUS

The models were constructed of laminated mahogany; dimensions were held within 0.01 inch of those specified. The ordinates for the Clark Y airfoil section are given in Table I. An airfoil which was rectangular in plan form with a 42.43-inch span and a 14.14-inch chord was used as the basic model. Changes in aspect ratio were effected by cutting off the ends of the basic model.

The faired-tip models were evolved from the basic model by attaching the faired tips to it. (Fig. 1.) The section of the faired tip is a semicircle when taken on the plane perpendicular to the mean camber line of the tip section of the basic model.

The semicircular tips are also shown in Figure 1. The Clark Y profile was preserved from the section having a chord of 14.14 inches to the section having a chord of 5 inches. The remainder of the tip was faired. Points on the upper surface of the airfoil at the maximum thickness of the section were kept in a plane parallel to the plane of the bottom surface of the basic model. The chord of each section of the tip was kept parallel to the chord of the basic model. Since the smallest aspect ratio possible with semicircular tips (circular plan form) is 1.27, elliptical wings were made up with aspect ratios of 1 and of 0.75. Each half of the elliptical wings differed from the semicircular tips in plan form only.

All tests were made in the N. A. C. A. 7 by 10 foot tunnel on the 6-component balance and the rotation apparatus described in reference 3.

TESTS

The force tests were made at a dynamic pressure corresponding to an air speed of 80 miles per hour under standard atmospheric conditions, giving a Reynolds Number of approximately 860,000 based on the chord of 14.14 inches.

Rotation tests were made at the same air speed with the exception of tests of the airfoil with circular plan form, which, because of its small span, had a rotational velocity exceeding the capacity of the apparatus at the standard air speed.

Lift, drag, and pitching moment were measured on each airfoil at 0° yaw. The aspect ratios of the airfoils tested were as follows: With the rectangular tips 3, 2, 1.5, 1.25, 1.0, 0.75, and 0.5; for the faired tips 3.15, 2.15, 1.65, 1.40, 1.15, 0.90, and 0.65; for the semicircular tips 3.23, 2.24, 1.74, 1.51, and 1.27; for the elliptical airfoils 1 and 0.75. All six components were measured at 20° yaw for the airfoils having the following aspect ratios: For the rectangular tips 3 and 1; for the faired tips 3.15 and 0.90; for the semicircular tips 3.23 and 1.27.

All airfoils given force tests with 20° yaw and the elliptical airfoil with an aspect ratio of 1 were tested for autorotation with 0° yaw.

RESULTS

Results are presented in the form of absolute coefficients. Pitching-moment coefficients are based on the central chord and refer to its quarter-chord point. Angles of attack and values of drag have been corrected for tunnel-wall effect by the method given in reference 4.

For tests at 0° yaw values of C_L and C_D are plotted against angle of attack in Figures 2, 3, and 4, and values of C_p and L/D are plotted against angle of attack in Figures 5, 6, and 7. A summary of values of C_{Lmax}, C_{Dmin} the ratio C_{Lmax} to C_{Dmin}, C_{Rmax}, L/D ratio, and L/D at C_{Rmax} plotted against aspect ratio is given in Figures 8 and 9 and Table IX. Values of C_m are plotted against C_L in Figure 10.

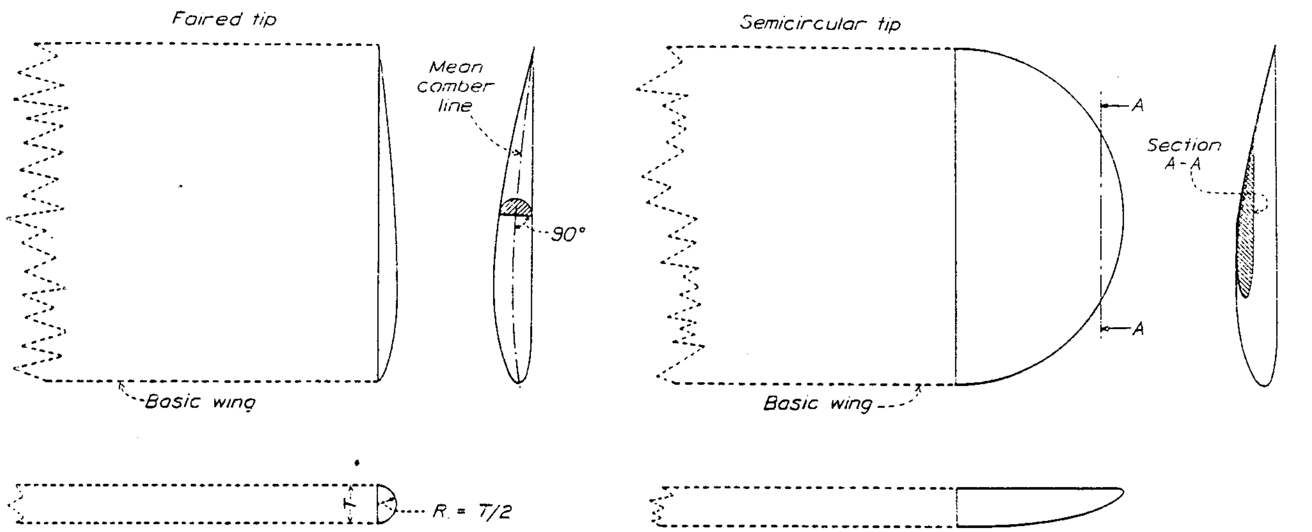

FIGURE 1.—Tip details of small aspect-ratio airfoils of Clark Y section

All six components for the wings tested at 20° yaw are given in Tables II to VIII, inclusive. Values of C_m, C_l, and C_n are plotted against C_L. (Figs. 11 and 12.) The moments refer to chord axes.

The rates of stable autorotation are expressed in terms of $\frac{p'b}{2V}$ and plotted against angle of attack in Figure 13. The symbol p' refers to angular velocity about the longitudinal axis of the tunnel.

Profile drag and the angle of attack for infinite aspect ratio were calculated by the formulas given in reference 5. The correction constants for rectangular wings were used in making the computations for both rectangular airfoils and faired-tip airfoils. The constants for the very small aspect ratios were determined by extrapolation of the curves given in reference 5. The uncorrected formulas for elliptical wings were used in the calculations for the wings with semicircular tips. The values calculated are plotted against C_L in Figures 14, 15, and 16.

The probable errors in measurements are as given in reference 3.

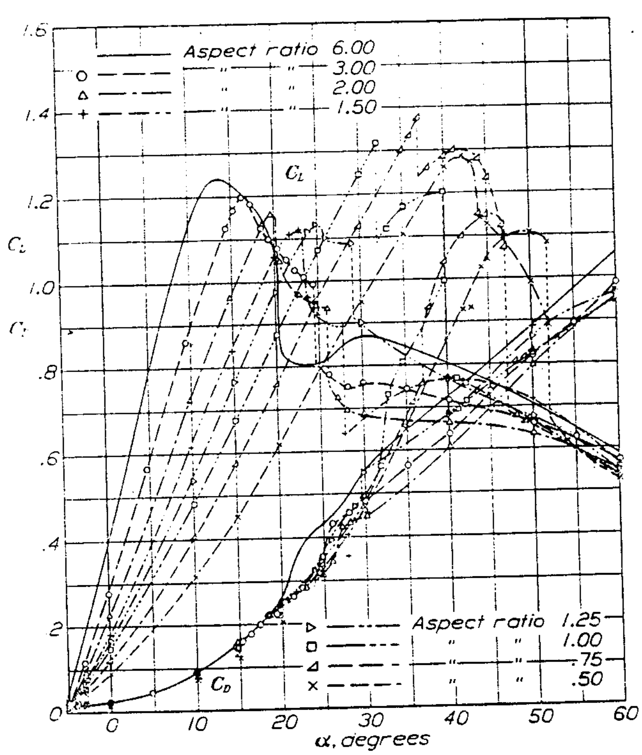

FIGURE 2.—Variations of lift and drag coefficients with angle of attack. Rectangular tips. 0° yaw

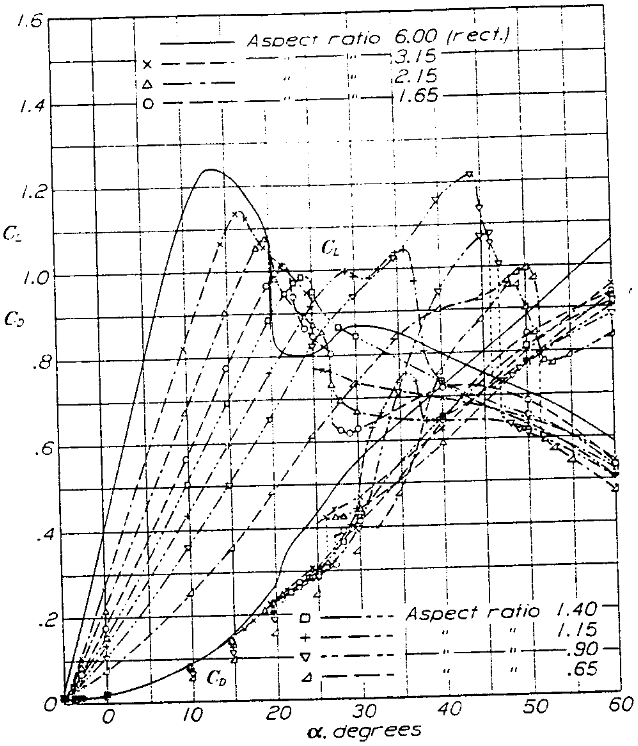

FIGURE 3.—Variations of lift and drag coefficients with angle of attack. Faired tips. 0° yaw

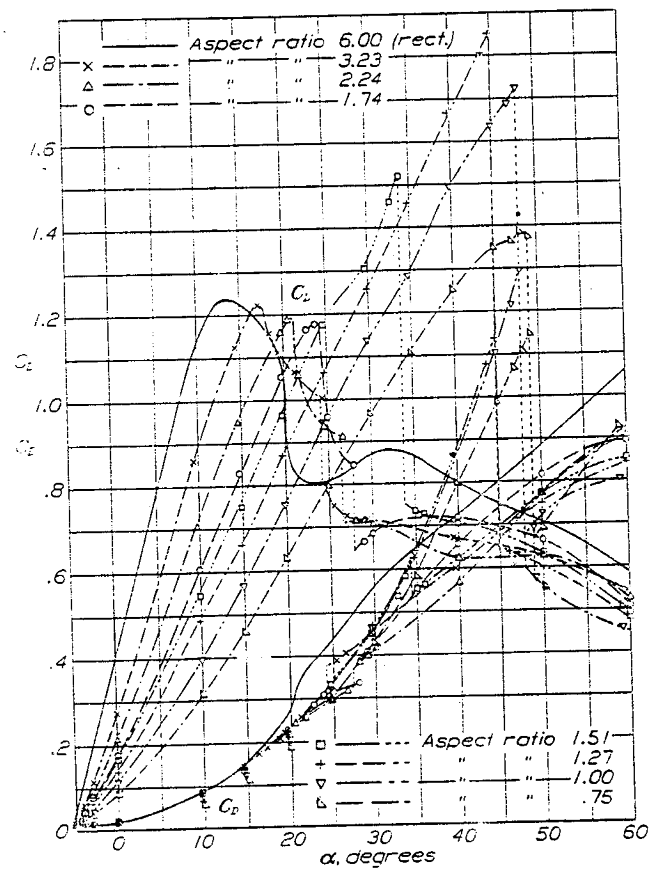

FIGURE 4.—Variations of lift and drag coefficients with angle of attack. Semicircular tips. 0° yaw

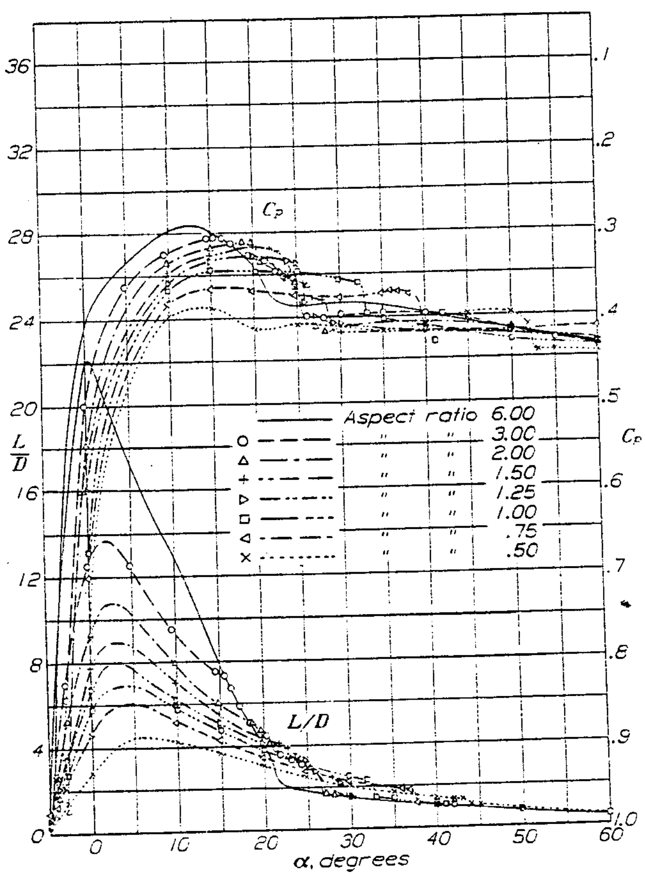

FIGURE 5.—Variations of L/D ratio and center of pressure location with angle of attack. Rectangular tips. 0° yaw

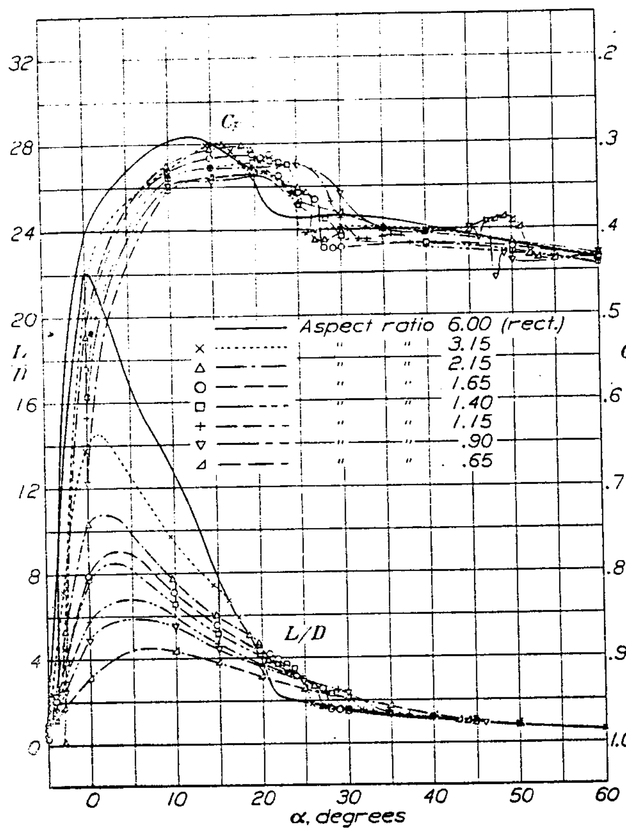

FIGURE 6.—Variations of L/D ratio and center of pressure location with angle of attack. Faired tips. 0° yaw

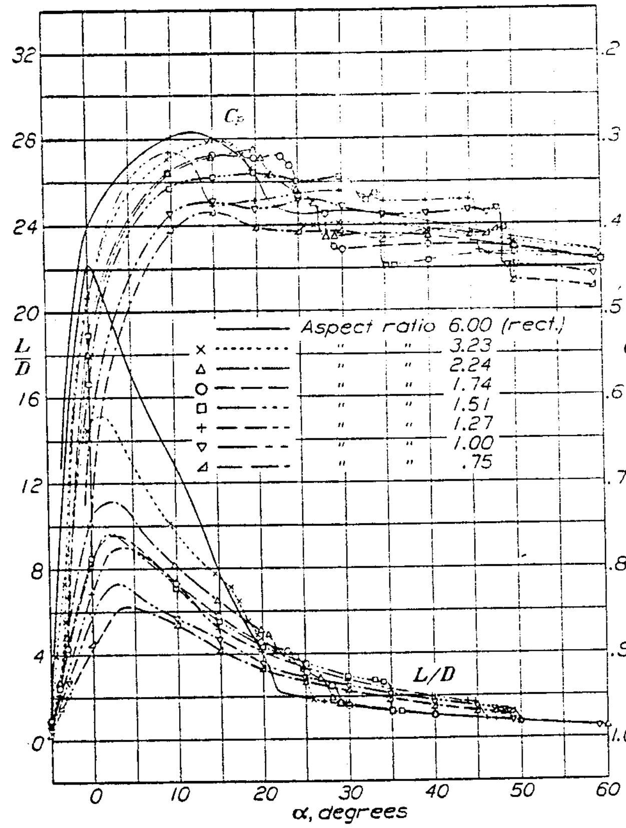

FIGURE 7.—Variations of L/D ratio and center of pressure location with angle of attack. Semicircular tips. 0° yaw

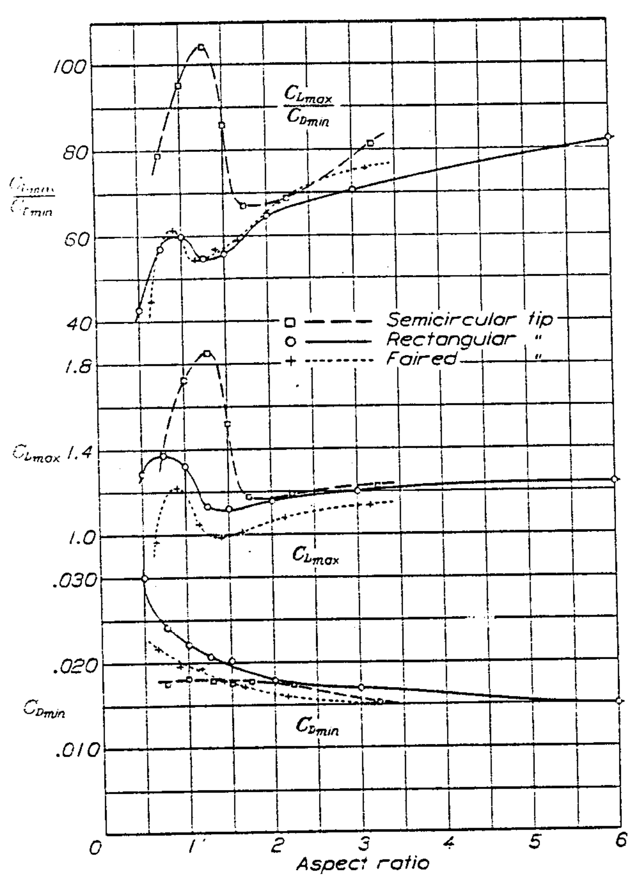

FIGURE 8.—Variations of maximum lift coefficient, minimum drag coefficient, and the ratio CL_{max}/CD_{min} with aspect ratio. 0° yaw

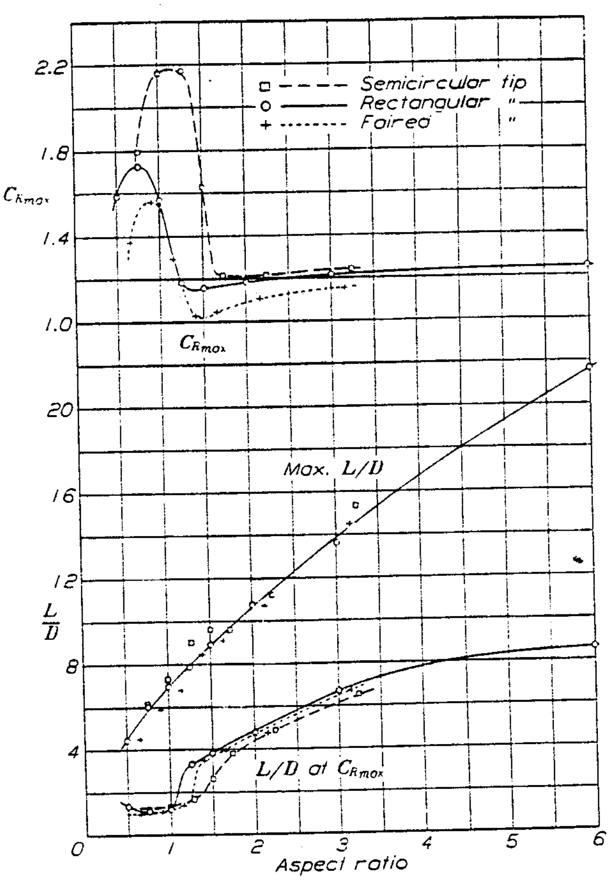

FIGURE 9.—Variations of maximum resultant force coefficients, maximum L/D ratio, and L/D ratio at maximum resultant force with aspect ratio. 0° yaw

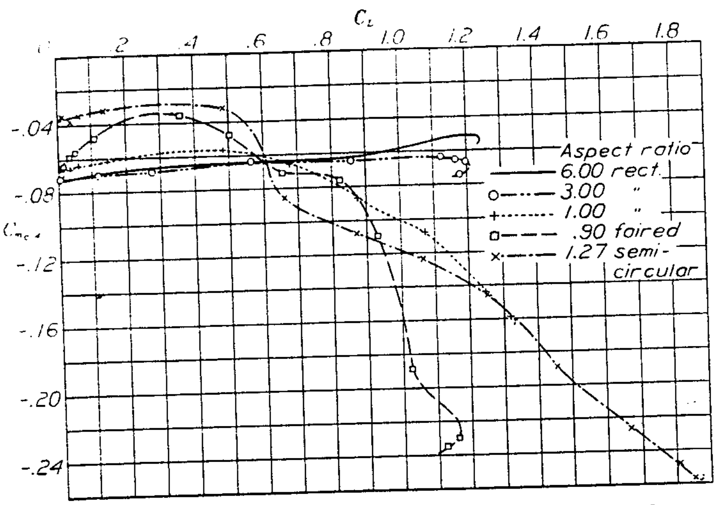

FIGURE 10.—Variations of pitching-moment coefficient with lift coefficient. 0° yaw

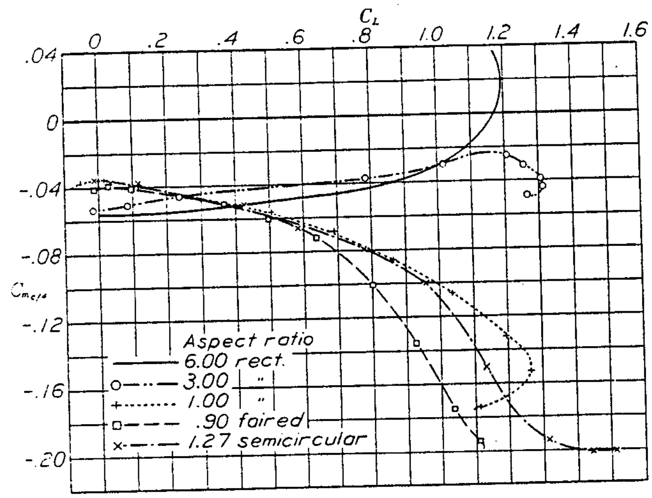

FIGURE 11.—Variations of pitching-moment coefficient with lift coefficient. —20° yaw

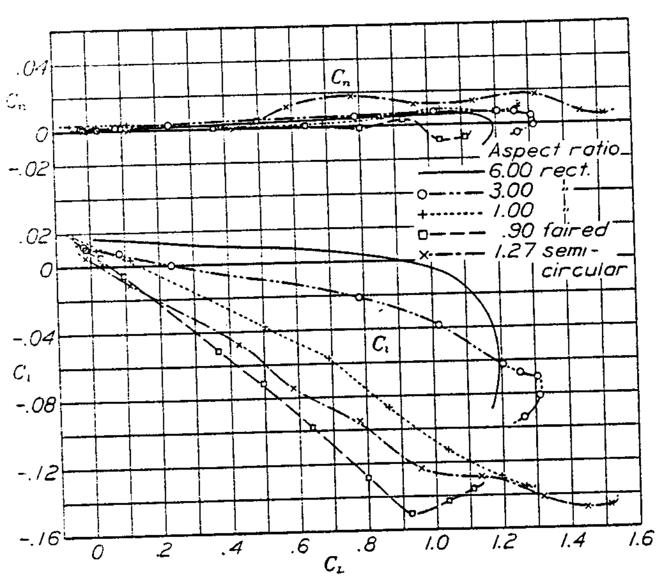

FIGURE 12.—Variations of rolling-moment and yawing-moment coefficients with lift coefficient. —20° yaw

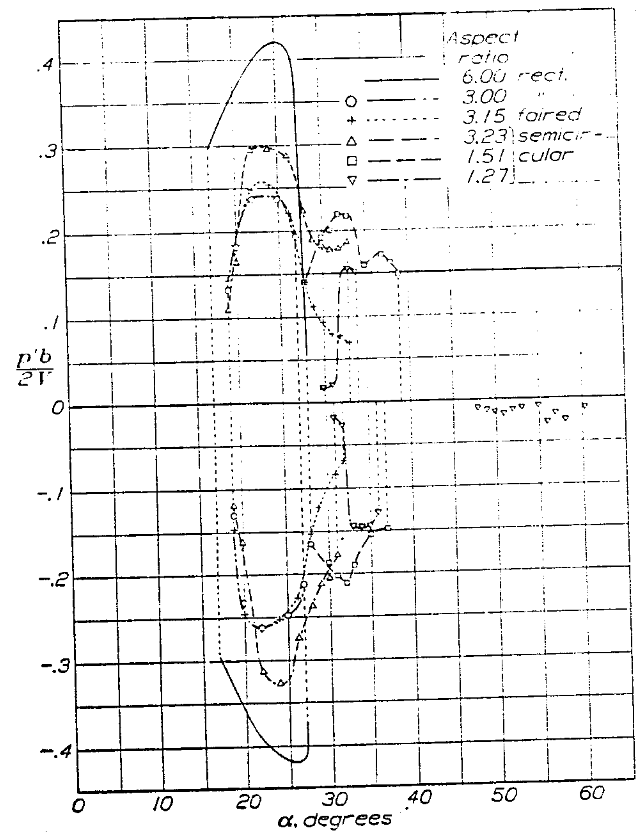

FIGURE 13.—Variations of $p'b/2V$ with angle of attack. 0° yaw

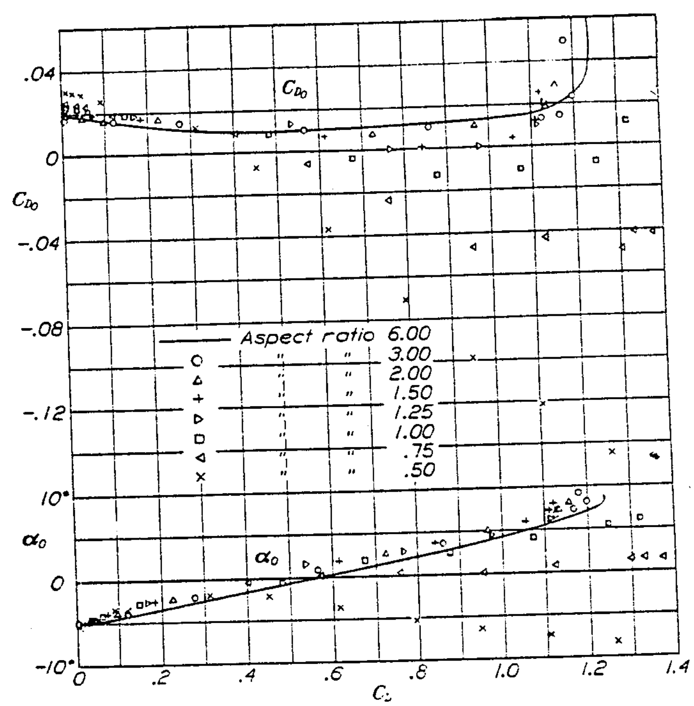

FIGURE 14. Variations of angle of attack for infinite aspect ratio and profile drag with lift coefficient. Rectangular tips. 0° yaw

DISCUSSION

The reader should bear in mind that results herein discussed apply to airfoils alone and that in making performance calculations the characteristics of the airplane as a whole must be very carefully taken into account.

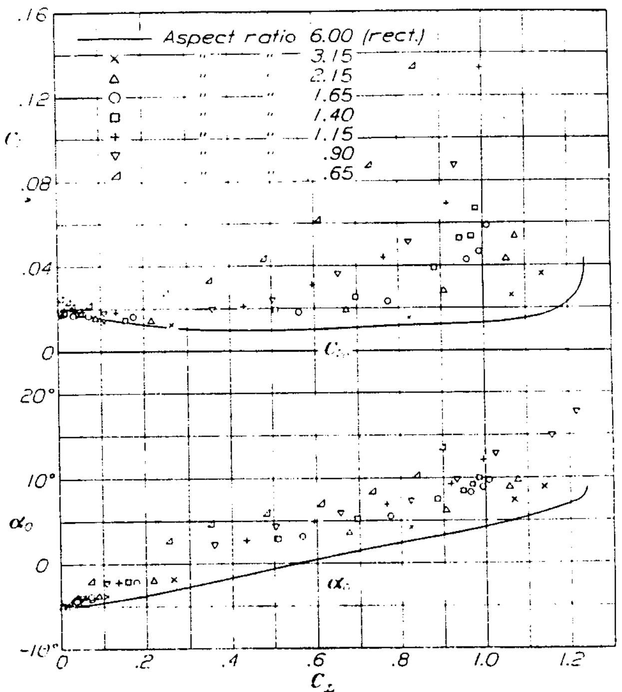

FIGURE 15.—Variations of angle of attack for infinite aspect ratio and profile drag with lift coefficient. Faired tips. 0° yaw

In this discussion, effects of tip form and of aspect ratio will be considered concurrently to avoid repetition. The Clark Y airfoil, having a rectangular plan form and an aspect ratio of 6, will be referred to as a standard wing in making comparisons between the standard wing and the airfoils with small aspect ratios.

The tests upon the standard wing were made at a Reynolds Number of approximately 609,000 (80 miles per hour, 10-inch chord). It is thought that the difference between this value and the value for the small aspect-ratio wings (860,000) is not enough to affect seriously the comparison given in this report. However, since nothing is known of the effect of an increase to a Reynolds Number of the order of those common in flight upon the characteristics of small aspect-ratio airfoils, too much dependence should not be placed on the comparison herein given in making performance calculations.

Slope of lift curve.—A survey of the curves of lift coefficient plotted against angle of attack (figs. 2, 3, and 4) reveals that the slope of the lift curve decreases with decrease of aspect ratio much as would be predicted by theory. The discrepancies between results observed and theory are discussed later in the section on reduction to infinite aspect ratio.

Maximum lift coefficient.—Previous tests have shown decreases of maximum lift coefficient with decreasing aspect ratios for aspect ratios in the range from 8 to 2 (reference 1) and the present tests give similar results. The airfoils having rectangular tips revealed a decrease of maximum lift coefficient continuing to an aspect ratio of 1.5; similar decreases for the semicircular and the faired tips continued to aspect ratios of 1.74 and 1.40, respectively. The minimum values reached were 93 per cent for the rectangular tips, 79 per cent for the faired tips, and 95 per cent for the semicircular tips, based on the value for the standard wing. (See fig. 8.)

Further decreases in aspect ratio gave increases in maximum lift coefficient. The maximum values reached were 111 per cent, 98 per cent, and 149 per cent, based on the value for the standard wing, at aspect ratios of 0.75 for the rectangular tips, 0.90 for the faired tips, and 1.27 for the semicircular tips, respectively. These increases are the results of the delay of the burble, caused probably by end flow. It is apparent that tip form plays an important part in this phenomenon.

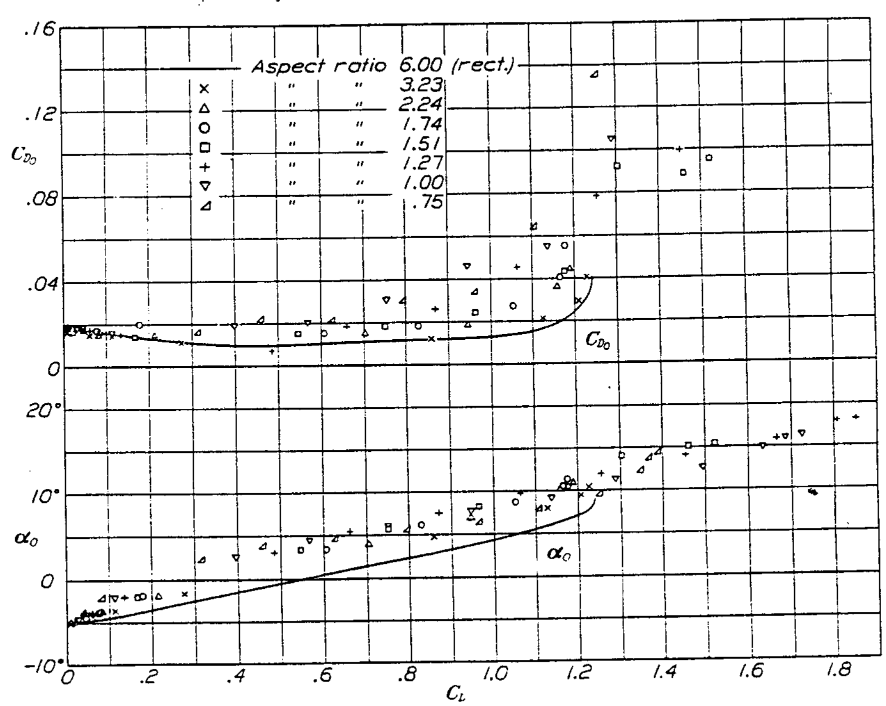

FIGURE 16.—Variations of angle of attack for infinite aspect ratio and profile drag with lift coefficient. Semicircular tips. 0° yaw

Maximum resultant force coefficient.—Curves of $C_{R_{max}}$ plotted against aspect ratio (fig. 9) have the same general shape as curves of $C_{L_{max}}$. Minimums of 93 per cent, 82 per cent, and 98 per cent, based on the value for the standard wing, occur at aspect ratios of 1.5, 1.40, and 1.74 for the rectangular tips, the faired tips, and the semicircular tips, respectively. Maximums of 138 per cent, 125 per cent, and 174 per cent occur at aspect ratios of 0.75, 0.90, and 1.27, respectively. The differences between the shapes of the C_{Rmax} curves and the C_{Lmax} curves are due to the greater induced drag of the low aspect-ratio airfoils. The value of maximum resultant force coefficient is more relevant to landing characteristics for steep glide landings than the value of maximum lift coefficient. In such landings the resultant force opposes the weight of the airplane, and the coefficient is therefore a criterion of the landing speed.

Drag coefficients.—Curves of drag coefficient plotted against angle of attack fall within a fairly wide band at angles of attack less than 20° for all wings tested. The points for the lower aspect ratios fall near the top of the band at angles of attack below 0° and near the lower border of the band at angles of attack above 0°. It is of interest to note that the airfoils of aspect ratios of approximately 1.5 and smaller show a sharp decrease of drag coefficient immediately after passing the burble point.

Minimum drag coefficients.—Curves of C_{Dmin} plotted against aspect ratio are much as would be expected for the rectangular and faired-tip airfoils. (Fig. 8.) For each of these types the minimum drag coefficient increases with decrease of aspect ratio because of the increasing importance of tip drag. The faired-tip wings were found to have the lesser drag of the two for a given aspect ratio. The airfoil with semicircular tips gave a minimum drag coefficient of 0.0151 at an aspect ratio of 3.23, the same as did the faired-tip airfoil at an aspect ratio of 3.15 and the rectangular-tip airfoil at an aspect ratio of 6. From this point the minimum drag coefficient increased to 0.0177 at an aspect ratio of 1.74 and remained practically the same for all lower aspect ratios. This strange behavior is probably due to the fact that the semicircular tips are examples of exaggerated taper and have comparatively small losses from tip effect.

Lift/drag ratio.—The curves of L/D ratio plotted against angle of attack are nearly alike for all wings above 25° angle of attack. (Figs. 5, 6, and 7.) The curves between zero lift and 25° angle of attack become flatter with decrease in aspect ratio. This characteristic indicates that the smaller aspect ratios would require larger horsepower to give a fixed weight-velocity product than would the larger aspect ratios, that minimum gliding angles would increase with decrease in aspect ratio, and that high rates of climb would be very difficult to obtain with small aspect-ratio wings. Definite statements as to the amounts of these effects would be misleading, unless differences of structural weight, parasite drag, and effects of Reynolds Number are taken into account very carefully.

A curve of maximum values of L/D plotted against aspect ratio is given in Figure 9 and has a decided positive slope, indicating much larger minimum gliding angles for the wings alone for the small aspect ratios as compared to those for the larger aspect ratios. Tip effects upon this curve are slight, the semicircular tips being slightly better than the rectangular tips at aspect ratios less than 1.5 and the faired tips being slightly worse over most of the range.

Maximum-lift/minimum-drag ratio.—Curves of C_{Lmax}/C_{Dmin} plotted against aspect ratio (fig. 8) have the same general characteristics as curves of C_{Lmax} plotted against aspect ratio for the same tip forms. However, the curves for the rectangular and faired tip wings fall off much more markedly than their respective C_{Lmax} curves, because of the great increase in minimum drag with decrease in aspect ratio for these airfoils. Since the minimum drag coefficient for the semicircular-tip wings remains practically constant for aspect ratios less than 1.74, the curve of C_{Lmax}/C_{Dmin} has a decided peak, reaching a maximum value of 104 for the aspect ratio of 1.27, as compared with the value of 82.1 for the standard wing.

Lift/drag ratio at maximum resultant force.—Another characteristic which is of importance in the study of steep-glide landings is the lift/drag ratio at maximum resultant force coefficient for the complete airplane; this ratio is the cotangent of the gliding angle and therefore must be small for a steep glide. Values of this ratio decrease with decrease of aspect ratio from 8.59 for the standard wing to values of about 1.2 for the aspect ratios of about 1, corresponding to gliding angles for the wings alone of 8.5° and 39.8°, respectively. The curves for the three tip shapes are similar, the semicircular-tip wing giving somewhat lower values than the others, especially for aspect ratios of 1.27 and 1.5.

Center of pressure coefficient.—The center-of-pressure-coefficient curves reveal unstable characteristics similar to those of the standard wing at low angles of attack, but the center of pressure does not travel as far forward for the lower aspect ratios as for the higher ones. (See figs. 5, 6, and 7.) It is of interest to note that the C_p curves of all the airfoils move forward rapidly to an angle corresponding to the stall of the standard wing beyond which point the center of pressure, in general, remains practically stationary until a stall of a particular wing is reached, at which point it moves back markedly. The curve for the wing of circular plan form (aspect ratio 1.27) is an exception to these general statements, the center of pressure moving much farther forward than it does for other aspect ratios of nearly the same value.

Pitching-moment coefficient.—Curves of $C_{m\ c/4}$ plotted against C_L for representative airfoils of the series tested with zero sideslip (fig. 10) show the smaller aspect ratios to have slightly smaller diving-moment coefficients at low values of lift coefficeint than do the higher aspect ratios. They also have negative slopes for values of C_L of 0.5 and above for the extremely small aspect ratios. This characteristic of longitudinal stability of the very small aspect ratios is first noticeable in the rectangular airfoil at an aspect ratio of 1, the semicircular-tip airfoil at an aspect ratio of 1.5, and the faired-tip airfoil at an aspect ratio of 0.65.

The pitching-moment-coefficient curves for the airfoils when yawed 20° (fig. 11) differ somewhat from those for zero sideslip. The slope of the curve for the standard wing is positive, especially at lift coefficients near C_{Lmax}. The slope of the curve for the rectangular wing of aspect ratio 3 is positive at small values of C_L but becomes negative before C_{Lmax} is reached. The slopes of the curves for the airfoils of aspect ratios of about 1 were more decidedly negative at all values of C_L below the stall with the exception of the curve for the airfoil with circular plan form. This latter curve has a steep negative slope except near the stall where the slope becomes practically zero.

Rolling and yawing moment coefficients.—Since the measurements were made with a negative angle of yaw, positive yawing moments and negative rolling moments are such as to restore the wing to a condition of no yaw. All the yawing-moment coefficients are small in comparison with pitching and rolling moment coefficients for the same airfoils, but are of the order of magnitude of yawing-moment coefficients given by the rudder of a conventional airplane. (Reference 6.) All the airfoils gave positive moments through practically the entire range. Note that all moments refer to chord axes.

All the airfoils exhibit unstable rolling moments at zero lift. (Fig. 12.) The rolling-moment coefficient for the standard wing did not become negative until a lift coefficient of 0.9 was reached. The airfoils with smaller aspect ratios gave negative rolling moments at much lower values of lift coefficient. They also gave values of rolling-moment coefficient much greater in the negative direction than did the standard wing at all values of lift coefficient greater than zero.

Autorotational characteristics.—Curves of $\frac{p'b}{2V}$ plotted against angle of attack reveal a decided decrease in range and rate of autorotation based on a given span and air speed with decrease in aspect ratio. (Fig. 13.) The rectangular wing of aspect ratio 1, the faired-tip wing of aspect ratio 0.90, and the elliptical wing of aspect ratio 1 would not autorotate. It seems probable, considering the tendency toward decreased range of autorotation and value of $p'\ b/2\ V$ with decrease of aspect ratio, that airfoils having still smaller aspect ratios would also not autorotate.

The airfoil of circular plan form, aspect ratio 1.27, showed autorotation over a small range of angles of attack between 30° and 36°. It also autorotated very slowly at angles of attack between 48° and 60°. Reference to Figure 4 reveals that the range of autorotation between 30° and 36° occurs at a point of decided positive slope of the curve of lift coefficient well below the angle of attack for the stall, and it would seem that autorotation for this wing is a result of some characteristic of the 3-dimensional flow rather than the result of the reversal of slope of the normal-force curve.

Reduction to infinite aspect ratio.—Curves of angle of attack at infinite aspect ratio and of profile drag plotted against C_L (figs. 14, 15, and 16) show that the theoretical correction factors can not be applied with satisfactory results for the wings of aspect ratios of 1.5 or less. For low aspect ratios the correction factors are too large in the case of the rectangular wings and too small in the cases of the faired and semicircular-tip wings. The correction factors for aspect ratios less than 1.5 are much more nearly true for the semicircular-tip wings than for the rectangular and faired tip wings.

CONCLUSIONS

1. There is a range of aspect ratios extending approximately from 0.75 to 1.50 wherein end flow causes a marked delay in the breakdown of the longitudinal flow as the angle of attack of an airfoil is increased.

2. It is possible within this range to obtain maximum lift coefficients considerably higher than can be obtained for an airfoil of this same section having an aspect ratio of 6. The highest maximum lift coefficient obtained in this series of tests was 1.85 at 45° angle of attack for the wing with circular plan form as compared with 1.24 at 14° angle of attack for the rectangular airfoil having an aspect ratio of 6.

3. The tip shape is of paramount importance among the factors affecting the force and moment characteristics within this range. The airfoils with semicircular tips were found to be much superior to those having rectangular or faired tips.

4. Airfoils within this range have the characteristics desirable for steep glide landings; namely, large values of maximum resultant force coefficient and small values L/D ratio at maximum resultant force coefficient. In this series of tests a value of C_{Rmax} of 2.16 with and L/D ratio of 1.33 was obtained for the elliptical wing having an aspect ratio of 1 as compared with a value of C_{Rmax} of 1.25 with an L/D ratio of 8.59 for the rectangular airfoil having an aspect ratio of 6.

5. Decreasing the aspect ratio decreases the range and rate of autorotation based on a given span and air speed. The elliptical airfoil of aspect ratio 1, the rectangular airfoil of aspect ratio 1, and the faired-tip airfoil of aspect ratio 0.9 would not autorotate.

6. Autorotation at angles of attack below the stall was found to exist in the case of the airfoil of circular plan form, aspect ratio 1.27.

7. Decreasing the aspect ratio increases rolling-moment coefficients in the stable sense when yawed.

8. Decreasing the aspect ratio tends to improve static longitudinal stability characteristics of an airfoil when not yawed and also when yawed 20°.

9. Additional experiments should be carried out to determine the effects of Reynolds Number, airfoil section, plan form, and tip shape within the range of aspect ratios from 0.75 to 1.50.

10. A theoretical study should be made of the 3-dimensional air flow about airfoils having small aspect ratios.

National Advisory Committee for Aeronautics,
Langley Memorial Aeronautical Laboratory,
Langley Field, Va., *May 5, 1932.*

REFERENCES

1. Prandtl, L.: Applications of Modern Hydrodynamics to Aeronautics. T. R. No. 116, N. A. C. A., 1921.
2. Eiffel, G.: The Resistance of the Air and Aviation. Houghton, Mifflin and Co., 1913.
3. Harris, Thomas A.: The 7 by 10 Foot Wind Tunnel of the National Advisory Committee for Aeronautics. T. R. No. 412, N. A. C. A., 1931.
4. Theodorsen, Theodore: The Theory of Wind-Tunnel Wall Interference. T. R. No. 410, N. A. C. A., 1931.
5. Jacobs, Eastman N., and Anderson, Raymond F.: Large-Scale Aerodynamic Characteristis of Airfoils as Tested in the Variable Density Wind Tunnel. T. R. No. 352, N. A. C. A., 1930.
6. Weick, Fred E., and Wenzinger, Carl J.: Wind-Tunnel Research Comparing Lateral Control Devices, Particularly at High Angles of Attack. I—Ordinary Ailerons on Rectangular Wings. T. R. No. 419, N. A. C. A., 1932.

TABLE I

ORDINATES OF CLARK Y SECTION IN PER CENT OF CHORD

Rad. L. E. 1.50 Rad. T. E. 0.06

Distance from L. E.	Upper	Lower
0	3.50	3.50
1.25	5.45	1.93
2.5	6.50	1.47
5	7.90	.93
7.5	8.85	.63
10	9.60	.42
15	10.69	.15
20	11.36	.03
30	11.70	.00
40	11.40	.00
50	10.52	.00
60	9.15	.00
70	7.35	.00
80	5.22	.00
90	2.80	.00
95	1.49	.00
100	.12	.00

TABLE II

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=6 Rectangular tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	0.010	0.016	0.000	0.0167	−0.056	0.0002
−3.1	.138	.017	.001	.0154	−.056	.0010
−0.2	.333	.021	.000	.0111	−.051	.0019
4.6	.654	.043	−.005	.0076	−.047	.0039
9.4	.943	.079	−.014	−.0028	−.034	.0052
11.3	1.040	.094	−.016	−.0084	−.027	.0060
12.3	1.082	.103	−.019	−.0125	−.020	.0058
13.3	1.110	.112	−.021	−.0187	−.012	.0061
14.3	1.153	.123	−.024	−.0351	−.005	.0045
15.2	1.164	.132	−.027	−.0363	.003	.0031
16.2	1.194	.146	−.032	−.0493	.012	.0002
19.2	1.176	.217	−.056	−.0866	.035	−.0109
25	.888	.416	−.146	−.1219	.024	−.0113
30	.872	.507	−.183	−.1173	−.004	−.0061
40	.796	.666	−.234	−.0552	−.073	.0139
50	.734	.866	−.300	−.0435	−.118	.0255
60	.616	1.035	−.358	−.0459	−.157	.0382

TABLE III

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=3.0 Rectangular tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	−0.017	0.021	0.005	0.0120	−0.053	0.0014
−3	.084	.020	.005	.0074	−.051	.0020
−0.2	.238	.023	.003	.0000	−.046	.0027
9.5	.786	.082	−.015	−.0215	−.037	.0059
14.3	1.019	.136	−.029	−.0387	−.030	.0070
19.2	1.207	.207	−.055	−.0623	−.025	.0067
20.2	1.256	.228	−.066	−.0675	−.031	.0064
21.2	1.306	.251	−.080	−.0702	−.039	.0040
22.2	1.312	.282	−.095	−.0813	−.044	−.0018
23.2	1.265	.309	−.108	−.0941	−.049	−.0067
25	1.020	.437	−.166	−.0990	−.077	−.0082
26	1.008	.458	−.178	−.0995	−.078	−.0055
30	.937	.531	−.201	−.0797	−.100	.0077
40	.789	.652	−.237	−.0328	−.127	.0187
50	.703	.815	−.290	−.0243	−.158	.0296
60	.595	.957	−.341	−.0228	−.190	.0383

TABLE IV

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=3.15 Faired tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	−0.006	0.017	0.000	0.0159	−0.057	0.0007
−3.1	.089	.017	.001	.0112	−.053	.0014
−0.2	.231	.019	−.002	.0045	−.048	.0022
9.5	.734	.075	−.020	−.0257	−.024	.0039
14.4	.952	.123	−.035	−.0411	−.021	.0060
19.2	1.128	.188	−.062	−.0596	−.019	.0062
21.2	1.225	.230	−.088	−.0640	−.036	.0041
25	.978	.410	−.169	−.0891	−.071	−.0089
26	.966	.432	−.182	−.0903	−.075	−.0062
30	.903	.512	−.231	−.0865	−.099	−.0079
40	.778	.653	−.241	−.0352	−.127	.0173
50	.686	.797	−.282	−.0222	−.154	.0273
60	.582	.938	−.335	−.0232	−.184	.0377

TABLE V

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=3.23 — Semicircular tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	0.003	0.017	0.000	0.0138	−0.057	0.0020
−3.1	.093	.016	.000	.0101	−.054	.0021
−0.1	.235	.019	.000	.0035	−.051	.0026
9.5	.751	.075	−.020	−.0204	−.045	.0049
14.4	.978	.128	−.037	−.0388	−.030	.0061
19.3	1.179	.189	−.045	−.0575	−.027	.0072
21.2	1.269	.231	−.079	−.0678	−.035	.0047
*25	1.022	.418	−.162	−.0890	−.082	−.0047
26	.996	.436	−.181	−.0925	−.087	−.0063
30	.910	.504	−.193	−.1004	−.099	.0018
40	.711	.606	−.201	−.0329	−.123	.0211
50	.640	.748	−.252	−.0221	−.145	.0206
60	.554	.896	−.302	−.0336	−.171	.0340

TABLE VI

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=1.0 — Rectangular tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	−0.051	0.037	0.015	0.0175	−0.037	0.0023
−3	.018	.036	.015	.0101	−.035	.0020
0	.117	.036	.015	.0034	−.039	.0038
9.9	.504	.094	−.003	−.0379	−.056	.0050
14.9	.690	.151	−.024	−.0576	−.068	.0017
19.8	.864	.234	−.048	−.0867	−.086	.0037
24.8	1.038	.351	−.084	−.1128	−.106	.0094
29.7	1.194	.468	−.123	−.1282	−.132	.0067
32.7	1.266	.548	−.156	−.1354	−.153	.0092
35	1.113	.694	−.255	−.0891	−.174	.0105
40	.984	.751	−.273	−.0548	−.171	.0106
50	.651	.761	−.270	−.0348	−.180	.0121
60	.555	.893	−.324	−.0406	−.205	.0111

TABLE VII

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=0.90 — Faired tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	−0.014	0.029	0.000	0.0092	−0.041	0.0010
−3	.029	.025	.000	.0048	−.039	.0015
0	.097	.025	.000	−.0060	−.041	.0018
9.9	.368	.066	−.018	−.0514	−.051	−.0001
14.9	.497	.109	−.040	−.0717	−.060	.0044
19.9	.638	.178	−.068	−.0979	−.072	.0001
24.9	.800	.268	−.105	−.1289	−.101	−.0016
19.8	.926	.387	−.148	−.1501	−.136	.0029
34.8	1.038	.607	−.325	−.1427	−.175	−.0096
39.8	1.110	.800	−.368	−.1360	−.194	−.0082
46	.825	.796	−.285	−.0784	−.186	.0027
48	.616	.690	−.227	−.0578	−.166	.0095
50	.591	.694	−.231	−.0598	−.160	.0110
60	.476	.788	−.263	−.0627	−.177	.0040

TABLE VIII

LIFT, DRAG, CROSS-WIND FORCE, ROLLING-MOMENT, PITCHING-MOMENT, AND YAWING-MOMENT COEFFICIENTS

−20° Yaw

Aspect ratio=1.27 — Semicircular tips

α Deg.	C_L	C_D	C_C	C_l	C_m	C_n
−5	−0.011	0.027	0.004	0.0059	−0.035	0.0011
−3	.034	.023	.000	.0014	−.037	.0000
0	.114	.023	.004	−.0120	−.038	.0000
9.9	.427	.070	−.008	−.0478	−.051	−.0002
14.9	.584	.115	−.027	−.0749	−.066	.0119
19.9	.778	.188	−.053	−.0948	−.079	.0172
24.8	.958	.287	−.095	−.1237	−.100	.0121
29.8	1.133	.397	−.153	−.1287	−.150	.0139
34.8	1.314	.557	−.199	−.1411	−.194	.0177
38.8	1.443	.709	−.229	−.1478	−.210	.0076
39.7	1.512	.781	−.233	−.1468	−.209	.0059
42	.698	.602	−.145	−.0632	−.168	.0302
50	.538	.643	−.179	−.0618	−.164	.0197
60	.450	.746	−.222	−.0714	−.179	.0126

TABLE IX

SUMMARY OF FORCE CHARACTERISTICS

Tip shape	$\frac{b^2}{S}$	$C_{L_{max}}$	$C_{D_{min}}$	$\frac{C_{L_{max}}}{C_{D_{min}}}$	Max. L/D	$C_{R_{max}}$	L/D at $C_{R_{max}}$	α at $C_{R_{max}}$
Rect	6.00	1.240	0.0151	82.1	21.64	1.245	8.59	15
Do	3.00	1.198	.0170	70.4	13.60	1.210	6.69	17
Do	2.00	1.156	.0179	64.5	10.77	1.181	4.75	20
Do	1.50	1.120	.0201	55.7	8.93	1.158	3.82	23
Do	1.25	1.130	.0207	54.6	7.84	1.180	3.27	25
Do	1.00	1.320	.0221	59.7	6.97	1.564	1.20	40
Do	.75	1.372	.0241	56.9	6.00	1.722	1.11	44
Do	.50	1.281	.0301	42.7	4.42	1.582	1.37	43
Faired	3.15	1.139	.0151	75.4	14.50	1.150	6.70	17
Do	2.15	1.078	.0160	67.4	10.68	1.102	4.73	20
Do	1.65	1.010	.0172	58.7	9.06	1.041	3.95	22
Do	1.40	.984	.0178	55.3	8.46	1.024	3.43	24
Do	1.15	1.045	.0193	54.1	6.78	1.290	1.38	36
Do	.90	1.218	.0196	62.1	5.87	1.558	1.06	45
Do	.65	.968	.0217	44.6	4.47	1.370	.97	49
Semicircular	3.23	1.225	.0151	81.1	15.31	1.239	6.52	18
Do	2.24	1.188	.0173	68.6	11.20	1.212	4.85	21
Do	1.74	1.176	.0177	66.4	9.60	1.215	3.81	24
Do	1.51	1.520	.0177	85.8	9.60	1.626	2.61	34
Do	1.27	1.850	.0178	104.0	9.00	2.170	1.63	45
Elliptical	1.00	1.722	.0181	95.1	7.31	2.158	1.33	48
Do	.75	1.388	.0176	78.8	6.13	1.791	1.20	49

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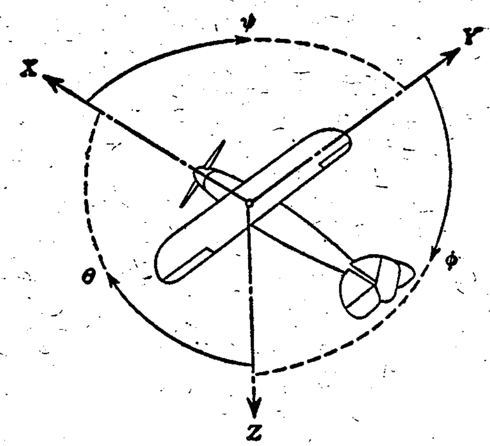

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	ϕ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{qbS} \qquad C_m = \frac{M}{qcS} \qquad C_n = \frac{N}{qbS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.
p, Geometric pitch.
p/D, Pitch ratio.
V', Inflow velocity.
V_s, Slipstream velocity.
T, Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$
Q, Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$
P, Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$.
C_S, Speed power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$.
η, Efficiency.
n, Revolutions per second, r. p. s.
Φ, Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.
1 kg/m/s = 0.01315 hp
1 mi./hr. = 0.44704 m/s
1 m/s = 2.23693 mi./hr.
1 lb. = 0.4535924277 kg.
1 kg = 2.2046224 lb.
1 mi. = 1609.35 m = 5280 ft.
1 m = 3.2808333 ft.