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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 572

DETERMINATION OF THE CHARACTERISTICS OF TAPERED WINGS

By RAYMOND F. ANDERSON

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft (or mi)
Time	t	second	s	second (or hour)	sec (or hr)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb
Power	P	horsepower (metric)		horsepower	hp
Speed	V	kilometers per hour	kph	miles per hour	mph
		meters per second	mps	feet per second	fps

2. GENERAL SYMBOLS

W Weight $=mg$

g Standard acceleration of gravity $=9.80665$ m/s^2 or 32.1740 ft/sec^2

m Mass $=\frac{W}{g}$

I Moment of inertia $=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ Coefficient of viscosity

ν Kinematic viscosity

ρ Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C and 760 mm; or 0.002378 lb-ft^{-4} sec^2

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb/cu ft

3. AERODYNAMIC SYMBOLS

S Area

S_w Area of wing

G Gap

b Span

c Chord

A Aspect ratio, $\frac{b^2}{S}$

V True air speed

q Dynamic pressure, $\frac{1}{2}\rho V^2$

L Lift, absolute coefficient $C_L=\frac{L}{qS}$

D Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0 Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

i_w Angle of setting of wings (relative to thrust line)

i_t Angle of stabilizer setting (relative to thrust line)

Q Resultant moment

Ω Resultant angular velocity

R Reynolds number, $\rho\frac{Vl}{\mu}$ where l is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)

α Angle of attack

ϵ Angle of downwash

α_0 Angle of attack, infinite aspect ratio

α_i Angle of attack, induced

α_a Angle of attack, absolute (measured from zero-lift position)

γ Flight-path angle

2626°

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By RAYMOND F. ANDERSON
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REPORT No. 572

DETERMINATION OF THE CHARACTERISTICS OF TAPERED WINGS

By Raymond F. Anderson

SUMMARY

Tables and charts for use in determining the characteristics of tapered wings are presented. Theoretical factors are given from which the following characteristics of tapered wings may be found: The span lift distribution, the induced-angle-of-attack distribution, the lift-curve slope, the angle of zero lift, the induced drag, the aerodynamic-center position, and the pitching moment about the aerodynamic center.

The wings considered cover the complete range of taper ratios and a range of aspect ratios from 2 to 20. The factors given include the effects of sweepback and twist and apply to wings having a straight taper plan form with rounded tips and an elliptical plan form. The general formulas of the usual wing theory are also given from which the characteristics of a wing of any form may be calculated when the section characteristics are known from experiment.

In addition to the tables and charts, test results are given for nine tapered wings, including wings with sweepback and twist. The test results verify the values computed by the methods presented in the first part of the report. A final section is given outlining a method for estimating the lift coefficient at which a tapered wing begins to stall. This method, which should be useful for estimating the maximum lift coefficient of tapered wings, is applied to one of the wings tested.

INTRODUCTION

A large amount of work has been done on the determination of tapered-wing characteristics from airfoil theory. Glauert has given some of the characteristics of wings with straight taper for a limited range of aspect ratios (references 1 and 2). Hueber has given other characteristics of wings with straight taper for a large range of aspect ratios (reference 3). Several other papers have given various characteristics of tapered wings. The data of all the papers, however, have been limited by one or more of the following factors: Range of aspect ratio and taper ratio, number of characteristics given, and omission of data on wings with sweepback and twist. In order to provide more complete information, data are given in this report for a large range of aspect ratios, for the complete range of taper ratios, and for wings with sweepback and twist. As airplane wings are usually rounded at the tips, the data are given for wings with rounded tips.

In addition to the theoretical characteristics, the results of tests of nine tapered wings, including wings with sweepback and twist, and a comparison of some of the test results with theoretical values are presented.

The characteristics are given for wings having a straight taper and rounded tips and for wings having an elliptical plan form, with an aspect-ratio range from 2 to 20. For these wings, formulas are given using factors that are presented in tables and charts. From the formulas and factors the following characteristics of tapered wings may be determined: Span lift distribution, induced-angle-of-attack distribution, lift-curve slope, angle of zero lift, induced drag, aerodynamic-center position, and pitching moment about the aerodynamic center.

METHOD OF OBTAINING DATA

BASIC CONCEPTS

When obtaining the data used to determine the characteristics of wings, a tapered wing is considered to consist of a series of airfoil sections that may vary in shape, chord length, and in angle of attack from root to tip. Each airfoil section is assumed to have an aerodynamic center through which the lift and drag act and about which the pitching moment is constant.

With the section characteristics as a basis, characteristics of the entire wing are obtained by integration across the span. Formulas for the integrations will first be given for a wing of any shape and zero dihedral; that is, the aerodynamic centers of all the sections along the span lie in a plane which passes through the root chord and which is perpendicular to the plane of symmetry. Wings of particular shape will be considered later and a method for including the effect of dihedral will be given.

For any tapered wing the span lift distribution may be considered to consist of two parts. One part, which will be called the "basic distribution," is the distribution that depends principally on the twist of the wing and occurs when the total lift of the wing is zero; it does not change with the angle of attack of the wing.

The second part of the span lift distribution, which will be called the "additional distribution," is the lift due to change of the wing angle of attack; it is independent of the wing twist and maintains the same form throughout the reasonably straight part of the lift curve.

In the designation of the characteristics of a wing, lower-case letters will be used for section characteristics and upper-case letters for the characteristics of the entire wing. The basic and additional section lift coefficients are then c_{l_b} and c_{l_a}. A complete list of symbols follows. It is convenient to find the additional lift coefficient for a wing C_L of 1 and it is then designated $c_{l_{a1}}$. The two coefficients are related by $c_{l_a}=C_Lc_{l_{a1}}$. The total lift coefficient at any section is found from the basic and additional coefficients from

$$c_{l_0}=c_{l_b}+C_Lc_{l_{a1}}$$

where c_{l_0} is the lift coefficient perpendicular to the local relative wind at any section as distinguished from c_l, which is perpendicular to the relative wind at a distance. For convenience, however, c_l will be used and may be considered equal to c_{l_0}.

SYMBOLS

- A, aspect ratio, b^2/S.
- b, span.
- c, chord at any section along the span.
- c_t, tip chord (for rounded tips, c_t is the fictitious chord obtained by extending the leading and trailing edges to the extreme tip).
- c_s, chord at root of wing or plane of symmetry.
- S, wing area.
- β, angle of sweepback, measured between the lateral axis and a line through the aerodynamic centers of the wing sections. (See fig. 1.)
- ϵ, aerodynamic twist in degrees from root to tip, measured between the zero-lift directions of the center and tip sections, positive for washin.
- x, longitudinal coordinate, parallel to the root chord.
- y, lateral coordinate, perpendicular to plane of symmetry.
- z, vertical coordinate in the plane of symmetry, perpendicular to the root chord.
- $x_{a.c.}$, x coordinate of wing aerodynamic center.
- a, wing lift-curve slope, per degree.
- a_0, wing section lift-curve slope, per degree.
- m, wing lift-curve slope, per radian.
- m_0, wing section lift-curve slope, per radian.
- α, angle of attack at any section along the span.
- α_s, wing angle of attack measured from the chord of the root section.
- α_{a_s}, absolute wing angle of attack measured from the zero-lift direction of the root section.
- $\alpha_{l_{0_s}}$, angle of zero lift of the root section.
- $\alpha_{l_{0_t}}$, angle of zero lift of the tip section.
- $\alpha_{s_{(L=0)}}$, wing angle of attack for zero lift.
- α_i, section induced angle of attack.
- c_l, section lift coefficient perpendicular to the distant relative wind.
  - Subscripts for c_l:
    - 0, refers to section lift coefficient perpendicular to the local relative wind.
    - b, refers to basic lift ($C_L=0$).
    - a, refers to additional lift (any C_L).
    - $a1$, refers to additional lift ($C_L=1$).
- c_{d_i}, section induced-drag coefficient.
- c_{d_0}, section profile-drag coefficient.
- $c_{m_{a.c.}}$, section pitching-moment coefficient about section aerodynamic center.
- l, section lift.
- m_{l_a}, section pitching moment due to additional lift forces.
- M_{l_a}, wing pitching moment due to additional lift forces.
- $C_{m_{l_a}}$, wing pitching-moment coefficient due to additional lift forces.
- $C_{m_{l_b}}$, wing pitching-moment coefficient due to basic lift forces.
- C_{m_s}, wing pitching-moment coefficient due to the pitching moments of the wing sections.
- $C_{m_{a.c.}}$, wing pitching-moment coefficient about its aerodynamic center.
- C_L, wing lift coefficient.
- C_{D_i}, wing induced-drag coefficient.

GENERAL FORMULAS

Formulas in terms of the section characteristics.—The induced angle of attack at any section is obtained from c_l by

$$\alpha_i=\alpha-\frac{c_l}{m_0}$$

The section induced-drag coefficient is obtained from α_i and c_l from

$$c_{d_i}=\alpha_ic_l$$

and the induced-drag coefficient for the entire wing may be obtained by integration across the semispan from the section values:

$$C_{D_i}=\frac{2}{S}\int_0^{b/2}\alpha_ic_lcdy \qquad (1)$$

In order to obtain the aerodynamic center and the pitching moment of the wings, a system of reference axes was used; the origin was at the aerodynamic center of the root section and the axes were as shown in figure 1. The x axis (fig. 1 (a)) is parallel to the root chord, and the y axis (fig. 1 (b)) is perpendicular to the plane of symmetry with positive directions following the vectors. The wing axis is the locus of the aerodynamic centers of the sections and lies in the x–y plane. The lift l and the coefficient c_l of any section along the span are represented in figure 1.

A typical section with the aerodynamic center located at a distance x from the y axis has a moment arm of

$$x \cos \alpha_s$$

and a pitching moment about the lateral axis (fig. 1) due to the additional lift force of

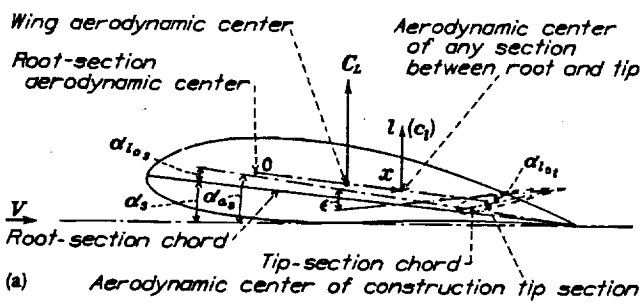
$$m_{l_a} = -x \cos \alpha_s l_a$$

but the lift increment of any section is

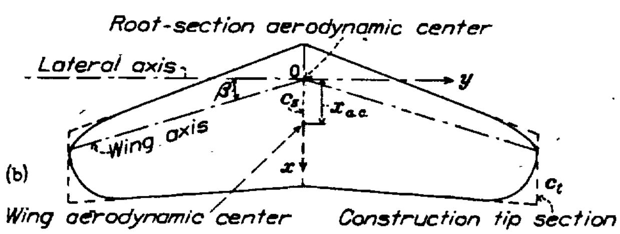
$$l_a = c_{l_a} q c$$

and the pitching moment for the entire wing is obtained from

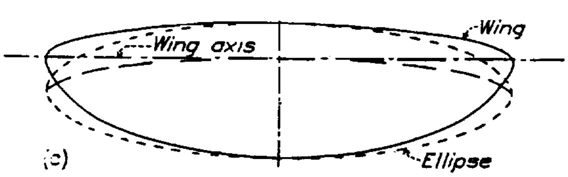
$$M_{l_a} = -2q \cos \alpha_s \int_0^{b/2} c_{l_a} cx dy$$



(a) Determination of twist.



(b) Straight-taper wing with rounded tips.



(c) Distorted elliptical wing.

FIGURE 1.—Form of wings.

Pitching-moment coefficients for the entire wing will be based on a chord length of S/b so that

$$C_m = \frac{Mb}{qS^2}$$

The pitching-moment coefficient due to the additional lift forces then becomes

$$C_{m_{l_a}} = -\frac{2b}{S^2} \cos \alpha_s \int_0^{b/2} c_{l_a} cx dy$$

The additional lift forces have a centroid through which the lift may be considered to act. This point is the aerodynamic center of the wing and its x coordinate will be designated $x_{a.c.}$. (See fig. 1.) This distance corresponds to d in reference 4. The term $C_{m_{l_a}}$ then may also be expressed

$$C_{m_{l_a}} = -(x_{a.c.} \cos \alpha_s) \frac{b}{S} C_L$$

If the previous expression for $C_{m_{l_a}}$ is used, $x_{a.c.}$ is obtained as a fraction of S/b by

$$\frac{x_{a.c.}}{S/b} = \frac{\frac{2b}{S^2} \int_0^{b/2} c_{l_a} cx dy}{C_L} \tag{2}$$

The moment due to the drag forces has been omitted because it is relatively small, except for wings with large amounts of sweepback or dihedral.

The pitching moment of the basic lift forces is a couple and is therefore independent of the axis about which it is determined. The lateral axis was used to facilitate computation but, when the pitching moment is used, it is convenient to consider it constant about an axis through the aerodynamic center. According to the method previously used, the pitching-moment coefficient due to the basic lift forces is

$$C_{m_{l_b}} = \pm \frac{2}{S^2} b \int_0^{b/2} c_{l_b} cx dy \tag{3}$$

The cos $\alpha_{s_{(L=0)}}$ (the cosine of the angle of zero lift of the wing measured from the root chord) has been omitted because it is practically equal to unity.

In addition to the basic lift forces, the pitching moment of each section also contributes to the pitching moment of the wing, which is obtained by

$$C_{m_s} = \frac{2b}{S^2} \int_0^{b/2} c_{m_{a.c.}} c^2 dy \tag{4}$$

The total moment about the aerodynamic center is then the sum of the two foregoing parts

$$C_{m_{a.c.}} = C_{m_{l_b}} + C_{m_s}$$

Formulas in terms of the coefficients of the Fourier series.—In order to obtain data from the foregoing formulas, the spanwise distribution of the lift coefficient (following Glauert) was expressed as the Fourier series:

$$c_l = \frac{4b}{c} \Sigma A_n \sin n\theta$$

where θ is related to the distance along the span (fig. 1) by $y = (-b/2) \cos \theta$ and only odd values of n are used. When c_l is expressed in the foregoing manner, it is possible to obtain the induced angle of attack in the form

$$\alpha_i = \Sigma n A_n \frac{\sin n\theta}{\sin \theta}$$

Also the coefficients A_n may be expressed in the form

$$A_n = B_n \alpha_{a_s} + C_n \epsilon$$

where α_{a_s} is the absolute angle of attack of the root section; that is, the angle of attack of the root section, measured from its direction of zero lift, and ϵ is the wing twist measured between the zero-lift directions of the root and tip sections.

When the preceding expressions for c_l and α_l are substituted in the foregoing formulas, the characteristics are obtained in terms of the coefficients B_n and C_n, which in turn are grouped into factors.

From (1) the induced-drag coefficient may be obtained in the form:

$$C_{D_i}=\frac{C_L^2}{\pi A u}+C_L\epsilon a_0 v+(\epsilon a_0)^2 w$$

where A is the aspect ratio, and

$$\frac{1}{u}=\frac{1}{B_1^2}\left[\sum_{n=3,5,7\ldots\infty} nB_n^2\right]+1$$

$$v=\frac{2}{m_0 B_1}\left[\sum_{n=3,5,7\ldots\infty} nB_n\left(C_n-\frac{C_1}{B_1}B_n\right)\right]$$

$$w=\frac{\pi A}{m_0^2}\left[\sum_{n=3,5,7\ldots\infty} n\left(C_n-\frac{C_1}{B_1}B_n\right)^2\right]$$

In the determination of the aerodynamic-center position, the wing axis is considered to be a straight line and the angle of sweepback is β (fig. 1), then

$$x=|y|\tan\beta$$

and from (2) the x coordinate of the aerodynamic center is obtained as

$$\frac{x_{a.c.}}{S/b}=HA\tan\beta$$

where

$$H=\frac{2}{\pi B_1}\left(\frac{B_1}{3}+\frac{B_3}{5}-\frac{B_5}{21}+\frac{B_7}{45}+\ldots\right.$$
$$\left.\frac{B_n}{4}\left\{\frac{\sin[(n-2)\pi/2]}{(n-2)}-\frac{\sin[(n+2)\pi/2]}{(n+2)}\right\}\right)$$

From (3) the moment due to the basic lift forces becomes

$$C_{m_{l_b}}=-G\epsilon a_0 A\tan\beta$$

where a_0 is the section lift-curve slope for the wing and

$$G=\frac{2A}{m_0}\left[\left(\frac{C_3}{5}-\frac{C_5}{21}+\frac{C_7}{45}\ldots\right)-\frac{C_1}{B_1}\left(\frac{B_3}{5}-\frac{B_5}{21}+\frac{B_7}{45}\ldots\right)\right]$$

(The term $C_{m_{l_b}}$ is equal to C_{m_T} in reference 4.)

Also from equation (4) the pitching moment of the wing due to the pitching moments of the sections is expressed as

$$C_{m_s}=Ec_{m_{a.c.}}$$

where $c_{m_{a.c.}}$ is constant across the span and

$$E=\frac{2b}{S^2}\int_0^{b/2}c^2dy$$

In addition to the foregoing formulas, the following formulas were obtained in terms of B_n and C_n for other characteristics. The basic and additional lifts at any point along the span were expressed by the dimensionless quantities

$$L_b=\frac{4A}{m_0}\left[\sum_{n=3,5,7\ldots\infty}\left(C_n-\frac{C_1}{B_1}B_n\right)\sin n\theta\right]$$

and

$$L_a=\frac{4}{\pi}\left[\sum_{n=1,3,5,7\ldots\infty}\frac{B_n}{B_1}\sin n\theta\right]$$

so that

$$c_{l_b}=\frac{\epsilon a_0 S}{cb}L_b$$

and

$$c_{l_{a1}}=\frac{S}{cb}L_a$$

The lift-curve slope was obtained in the form

$$a=\frac{\pi A B_1}{57.3}$$

By the introduction of the slope for an elliptical wing, a may be expressed

$$a=f\frac{a_0}{1+\frac{57.3a_0}{\pi A}}$$

where

$$f=\frac{a}{a_0}\left(1+\frac{57.3a_0}{\pi A}\right)$$

The angle of zero lift was obtained in the form

$$\frac{\alpha_{a_s}}{\epsilon}=-\frac{C_1}{B_1}=J$$

The angle of attack of a wing may then by given by

$$\alpha_s=\frac{C_L}{a}+\alpha_{l_{0_s}}+J\epsilon$$

where α_s is the angle of attack measured from the chord of the root section, and $\alpha_{l_{0_s}}$ is the angle of zero lift of the root section.

The general formulas and the factors used with them have now been outlined. The manner of obtaining the data will be completed by explaining the method of finding the coefficients B_n and C_n used in computing the factors.

Determination of the coefficients of the Fourier series.—The coefficients B_n and C_n depend on the shape of the wing. The two wing shapes used are shown on figure 1. Wing (b) has a straight taper plan form with rounded tips and (c) an elliptical plan form. The tapered wing is shown with sweepback and the elliptical wing without, but either wing may or may not have sweepback. The rounded tip of the tapered wing is formed within a trapezoidal tip of length c_t, and the taper of the wing is determined by the tip to root chord ratio c_t/c_s. The aerodynamic centers of the airfoil sections lie on a straight line across the semispan and form the wing axis. The elliptical wing is formed by distorting an ellipse until the wing axis becomes straight. In order to determine the wing axis, the aerodynamic centers of the airfoil sections were taken at the quarter-chord point. The straight wing axis may then be given sweepback with each chord moving parallel to its original position. The same process would be used to change the sweepback of the tapered wing.

For the wings considered, the twist varies linearly from root to tip and the total angle of twist is ϵ. As shown in figure 1, ϵ is the twist measured between the zero-lift directions of the root and tip sections.

Tapered wing.—For the tapered wing the coefficients B_n and C_n were determined from the equation

$$\alpha_a=\Sigma A_n \sin n\,\theta\left(\frac{4b}{m_0c}+\frac{n}{\sin\theta}\right) \tag{5}$$

where α_a is the absolute angle of attack at any section; that is, the angle of attack measured from the zero-lift direction for the section. The coefficients B_n and C_n are related to A_n by

$$A_n=B_n\alpha_{a_s}+C_n\epsilon$$

where α_{a_s} is the absolute angle of attack of the root section. The value of m_0 used in the preceding equation was 5.79 per radian, which approximates the lift-curve slope of good airfoil sections. For the linear taper α_a becomes

$$\alpha_a=\alpha_{a_s}+\epsilon\cos\theta$$

For a wing of any particular aspect ratio and taper ratio, equation (5) was satisfied at four points along the semispan by the usual method (except for $c_t/c_s=0$ for which six points were necessary to obtain sufficient accuracy), and values of B_n and C_n for $n=1$, 3, 5, and 7 were found.

The elliptical wing.—For the elliptical wing the foregoing fundamental equation may be simplified and a new series of coefficients, independent of aspect ratio, may be obtained. The coefficient A_n for $n=3, 5, 7 \ldots \infty$ may be obtained in the form

$$A_n=\frac{k_n\epsilon}{\frac{\pi A}{m_0}+n}$$

where k_n is determined from

$$\cos\theta=k_3\left(1+\frac{\sin 3\theta}{\sin\theta}\right)-k_5\left(1-\frac{\sin 5\theta}{\sin\theta}\right)+k_7\left(1+\frac{\sin 7\theta}{\sin\theta}\right)$$

The factors for the elliptical wing then take the form

$$L_b=4A\left[\sum_{n=3,5,7,\ldots\infty}\frac{k_n}{\pi A+nm_0}\sin n\theta\right]$$

$$L_a=\frac{4}{\pi}\sqrt{1-\left(\frac{y}{b/2}\right)^2}$$

$$a=\frac{a_0}{1+\frac{57.3a_0}{\pi A}}$$

$$f=1$$

$$J=-k_3+k_5-k_7 \ldots$$

$$u=1$$

$$v=0$$

$$w=\frac{\pi A}{m_0^2}\left[\sum_{n=3,5,7,\ldots\infty}\frac{nk_n^2}{\left(\frac{\pi A}{m_0}+n\right)^2}\right]$$

$$H=\frac{2}{3\pi}$$

$$G=\frac{2k_3}{5\pi\left(1+\frac{3m_0}{\pi A}\right)}-\frac{2k_5}{21\pi\left(1+\frac{5m_0}{\pi A}\right)}+\frac{2k_7}{45\pi\left(1+\frac{7m_0}{\pi A}\right)}-\ldots$$

$$E=\frac{32}{3\pi^2}\ (c_{m_{a.c.}}\text{ constant along the span})$$

The foregoing factors were obtained for the elliptical wing and for a straight-taper wing with trapezoidal tips for a range of aspect ratios from 3 to 20 and of taper ratios from 0 to 1. The factors were also obtained for the tapered wing with rounded tips for a sufficient number of aspect ratios and taper ratios so that the complete range could be covered using the factors for the wing with trapezoidal tips as a guide. Cross plots were then made to obtain figures 2 to 9 and the values for wings with rounded tips presented in tables I and II. Although the factors become less reliable as the aspect ratio is decreased, it was considered desirable to extrapolate the curves to an aspect ratio of 2 as the factors in the low-aspect-ratio range may be of use in the absence of other data. Additional spanwise lift-distribution data computed for the elliptical wing are given in table III.

USE OF TABLES AND CHARTS

In order to find the characteristics of a wing having a straight taper and rounded tips or having an elliptical plan form, the tables and charts may be used directly.

The properties of the wing should first be determined; that is, the taper ratio c_t/c_s, aspect ratio A, span b, the area S, the aerodynamic twist ϵ in degrees, the angle of sweepback β, and the average value of section lift-curve slope, as well as the section lift-curve slope a_0, the section pitching-moment coefficient $c_{m_{a.c.}}$, and the chord c at convenient stations along the semispan.

The chord and a_0 should be found at the spanwise stations given in tables I and II to facilitate finding the spanwise lift distribution. Then, for the values of c_t/c_s and A, values of L_b and L_a may be found from tables I and II by interpolation if necessary. The section lift coefficients c_{l_b} and $c_{l_{a1}}$ are then found for each station along the semispan from

$$c_{l_b}=\frac{\epsilon a_0 S}{cb}L_b$$

$$c_{l_{a1}}=\frac{S}{cb}L_a$$

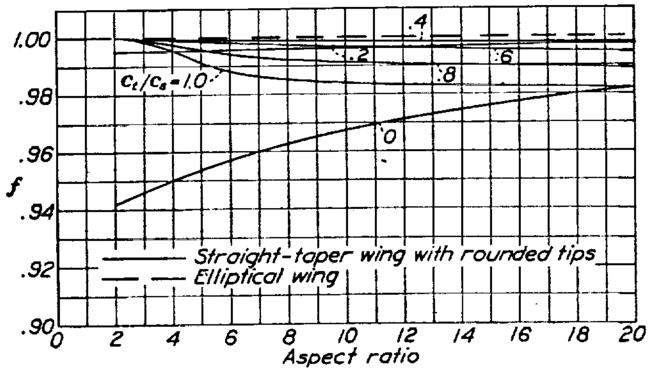


FIGURE 2.—Chart for determining lift-curve slope.

$$a = f\frac{a_0}{1+\frac{57.3\,a_0}{\pi A}} \qquad m = 57.3a$$

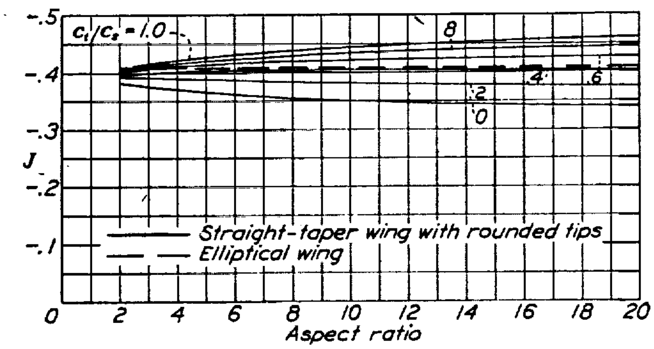


FIGURE 3.—Chart for determining angle of attack.

$$\alpha_s = \frac{C_L}{a} + \alpha_{l_{0_s}} + J\epsilon \qquad \alpha_{s(L=0)} = \alpha_{l_{0_s}} + J\epsilon$$

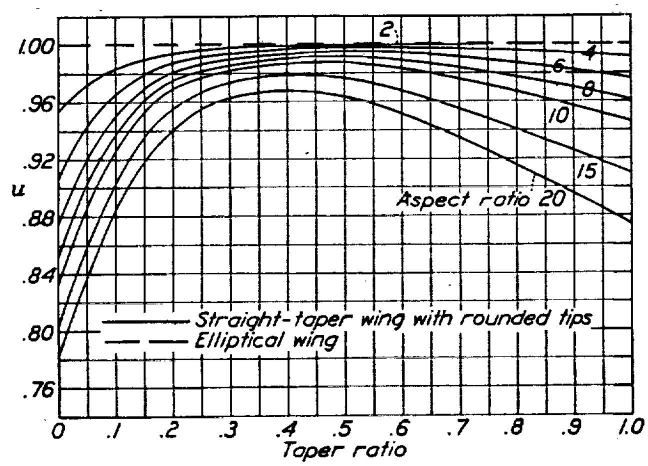


FIGURE 4.—Chart for determining induced-drag factor u.

$$C_{D_i} = \frac{C_L^2}{\pi A u} + C_L \epsilon a_0 v + (\epsilon a_0)^2 w$$

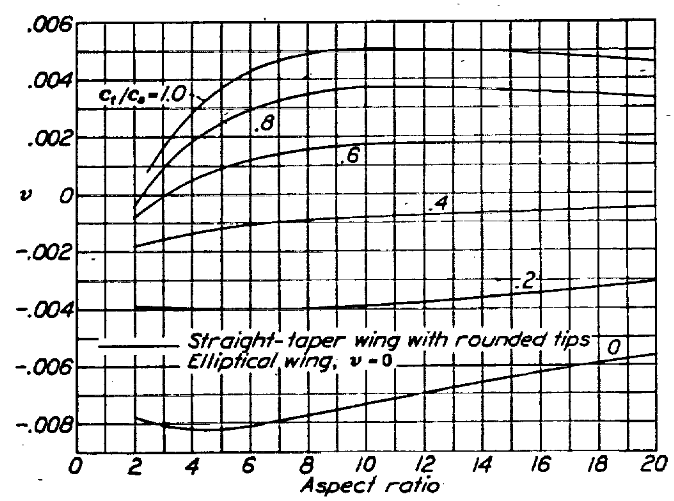


FIGURE 5.—Chart for determining induced-drag factor v.

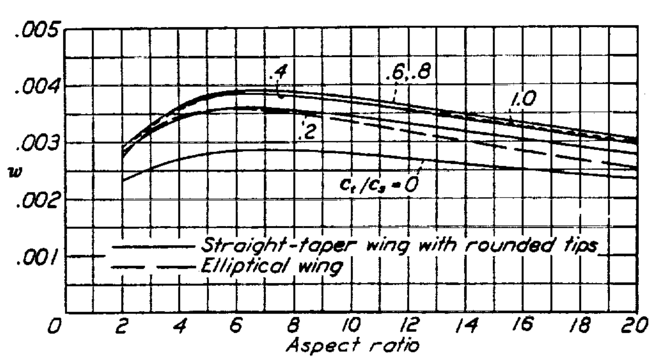


FIGURE 6.—Chart for determining induced-drag factor w.

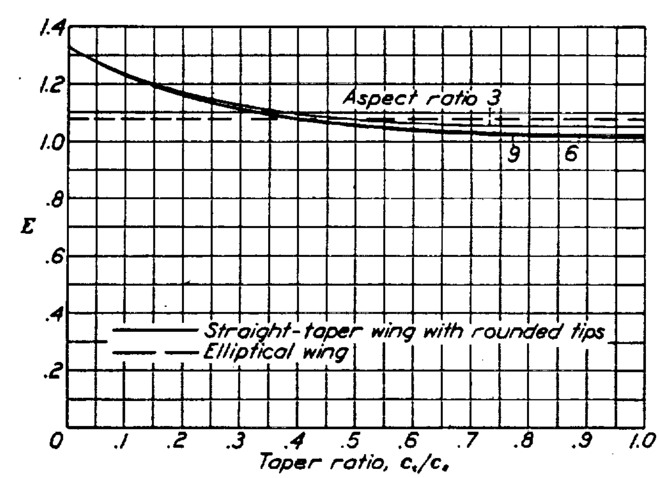


FIGURE 7.—Chart for determining pitching moment due to section moment.

$$C_{m_s} = E c_{m_{a.c.}}$$

For $c_{m_{a.c.}}$ constant across the span.

and c_l for any value of C_L for the wing is obtained from

$$c_l = c_{l_b} + C_L\, c_{l_{a1}}$$

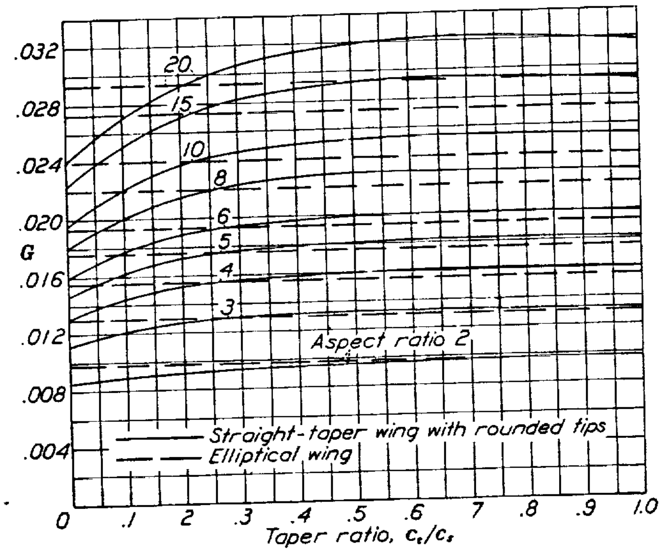


FIGURE 8.—Chart for determining pitching moment due to basic lift forces. $C_{m_{l_b}} = -G\epsilon a_0 A \tan\beta$.

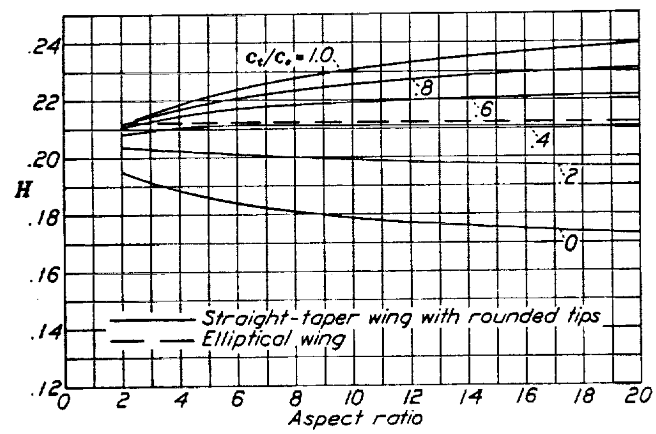


FIGURE 9.—Chart for determining aerodynamic-center position.

$$\frac{x_{a.c.}}{S/b} = HA \tan\beta.$$

The actual basic, additional, and total lifts for any section of the wing may then be obtained from

$$l_b = c_{l_b} qc$$
$$l_a = C_L c_{l_{a1}} qc$$
$$l = c_l qc$$

Values of l may be computed for the various spanwise stations and the curve of the span lift-distribution may be plotted. Typical semispan lift-distribution curves are shown in figure 10.

The semispan induced angle-of-attack distribution may be obtained from

$$\alpha_{i_a} = \alpha_a - \frac{c_l}{a_0}$$

where

$$\alpha_a = \alpha_{a_s} + \frac{y}{b/2}\epsilon$$

$$\alpha_{a_s} = \frac{C_L}{a} + J\epsilon$$

The remaining characteristics are obtained simply by finding the required factor for the desired values of c_t/c_s and A from the charts and by computing the characteristics from the formulas previously given, using the average value of a_0 where a_0 is required. The formulas are summarized here for convenience.

Lift-curve slope:

$$a = f\frac{a_0}{1 + \frac{57.3a_0}{\pi A}}$$

Angle of attack corresponding to any C_L:

$$\alpha_s = \frac{C_L}{a} + \alpha_{l_{0_s}} + J\epsilon$$

Angle of zero lift:

$$\alpha_{s(L=0)} = \alpha_{l_{0_s}} + J\epsilon$$

Induced-drag coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi A u} + C_L \epsilon a_0 v + (\epsilon a_0)^2 w$$

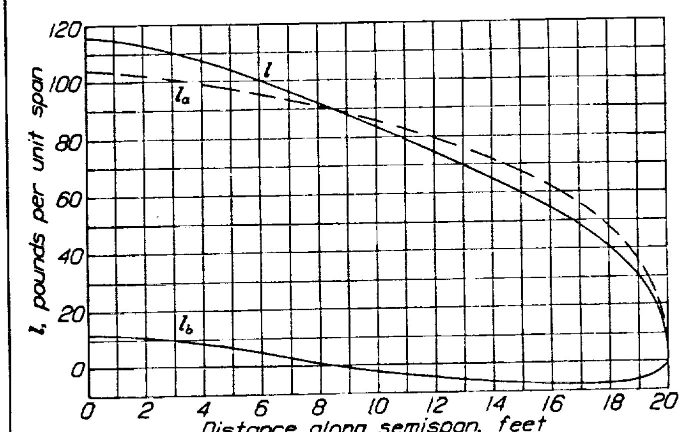


FIGURE 10.—Typical semispan lift distribution. $C_L = 1.2$.

Pitching-moment coefficient about an axis through the aerodynamic center:

$$C_{m_{a.c.}} = C_{m_s} + C_{m_{l_b}}$$
$$C_{m_S} = EC_{m_{a.c.}}$$
$$C_{m_{l_b}} = -G\epsilon a_0 A \tan\beta$$

Aerodynamic-center position (x coordinate):

$$\frac{x_{a.c.}}{S/b} = HA \tan\beta$$

Although C_{m_s} may usually be determined from the foregoing formula, equation (4) should be used if $c_{m_{a.c.}}$ varies considerably across the span.

Illustrative example.—In order to illustrate the method of using the charts, an example will be worked out for a wing with straight taper and rounded tips having the following characteristics:

$$A=6$$
$$c_t/c_s=0.5$$
$$b=40 \text{ feet}$$
$$S=266.7 \text{ sq. ft.}$$
$$\beta=10^\circ$$
$$C_L=1.2$$
$$q=10 \text{ lb./sq. ft.}$$

Root section:	Construction tip section:
N. A. C. A. 4415	N. A. C. A. 2409
$a_{0_s}=$ 0.097	$a_{0_t}=$ 0.099
$\alpha_{l_{0_s}}=-3.8^\circ$	$\alpha_{l_{0_t}}=-1.7^\circ$
$c_{m_{a.c._s}}=-0.083$	$c_{m_{a.c._t}}=-0.044$

The angle of twist measured between the chords of the root and construction tip sections is -5° (washout). Then, by the use of the angles of zero lift of the root and tip sections and by reference to figure 1, the angle of aerodynamic twist is determined to be -7.1°.

The chord at several stations along the semispan and the calculation of the lift distribution are given in table IV. In the table, a_0 and $c_{m_{a.c.}}$ are assumed to have a linear variation along the semispan. Values of L_b and L_a were obtained from tables I and II for an aspect ratio of 6 and a taper ratio of 0.5 and the basic, additional, and total lift distributions were computed and plotted in figure 10. The pitching-moment coefficient $c_{m_{a.c.}}$ varies so much along the semispan that C_{m_s} cannot be found by use of the factor E but must be found from (4). Accordingly, $c_{m_{a.c.}}c^2$ is plotted against y in figure 11 and C_{m_s} is found from the area under the curve to be -0.072.

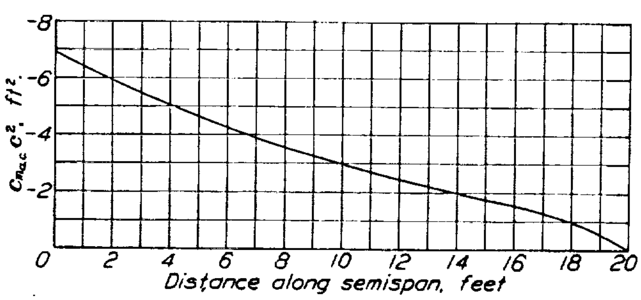


FIGURE 11.—Graphical determination of section pitching moment.
$$C_{m_s}=\frac{2b}{S^2}\int_0^{b/2} c_{m_{a.c.}}c^2dy=-0.072$$

From figures 2 to 9 and the equations on page 7 the remaining factors and characteristics are determined to be

$f=0.998$	$a=0.0755$
$J=-0.408$	$\alpha_s=15.0$
$u=0.995$	$\alpha_{s(L=0)}=-0.9$
$v=0.0001$	$C_{D_i}=0.0786$
$w=0.0039$	
$G=0.0199$	$C_{m_{l_b}}=0.015$
$H=0.214$	$x_{a.c.}=1.51$ ft.

$$C_{m_{a.c.}}=-0.072+0.015=-0.057$$

Method for wing of special form.—If it is desired to find the characteristics of a wing having a chord distribution that lies between the chord distributions of the tapered and elliptical wings, such as a wing with a constant-chord center section, an interpolation may be made between the values for the tapered and elliptical wings to find most of the characteristics.

The lift distribution for such wings may be found by an approximate method that has been tried for a few wings having parallel center sections and has given satisfactory results. The method has been taken from reference 5 with the symbols converted to the notation of this report. Approximate values of L_a, which will be designated L_a', may be calculated from

$$L_a'=\frac{\sqrt{1-\left(\frac{y}{b/2}\right)^2}}{\frac{\sqrt{1-\left(\frac{y}{b/2}\right)^2}}{\frac{m_0c}{b/2}}+\frac{3}{8}}\left(\frac{A}{2}\alpha_a+\frac{1}{\pi}\right)$$

where

$$\alpha_a=\frac{8}{\pi A}\left[\left(\frac{\sqrt{1-\left(\frac{y}{b/2}\right)^2}}{\frac{m_0c}{b/2}}\right)_{\text{mean}}+\frac{1}{8}\right]$$

The procedure is to choose a number of points at convenient intervals along the semispan (12 points should be sufficient for the usual plan forms); then from the values of c at those points the mean value of $\dfrac{\sqrt{1-\left(\frac{y}{b/2}\right)^2}}{\frac{m_0c}{b/2}}$ is calculated. The value of α_a may then be found and from the values of y and c, L_a' at each point along the semispan may be computed. The values of L_a' should correspond to a C_L approximately equal to 1. The actual C_L may be found from

$$C_L=\int_0^1 L_a'd\left(\frac{y}{b/2}\right)$$

and C_L may be conveniently found from the area under a curve of L_a' plotted against $\frac{y}{b/2}$. Finally, L_a may be found from $L_a=L_a'/C_L$. Values of $c_{l_{a1}}$ may then be calculated by the previously indicated method and, if desired, C_{D_i} and $\frac{x_{a.c.}}{S/b}$ may be found from equations (1) and (2)

If a wing has considerable dihedral or a curved wing axis, an integration may be made directly from the section characteristics. For this purpose, the best procedure would be to resolve the section values c_{l_0} and c_{d_0} into components along and parallel to the x and z axes, where the z axis is perpendicular to the x axis and lies in the plane of symmetry. Owing to dihedral, there will be a vertical coordinate of the aerodynamic center and a pitching moment about the aerodynamic center of the force components in the x direction. The coordinates of the aerodynamic center and of the pitching moment about it may be found from integrations like (2) and (3) by substituting the appropriate values of the x and z force components. For example, $x_{a.c.}$ would be found from

$$x_{a.c.}=\frac{\frac{2}{S}\int_0^{b/2} c_{z_a}cx\,dy}{C_{Z_a}}$$

where

$$C_{Z_a}=\frac{2}{S}\int_0^{b/2} c_{z_a}c\,dy$$

The values of $x_{a.c.}$ and C_{Z_a} may be found by plotting $c_{z_a}cx$ and $c_{z_a}c$ against the distance along the semispan and finding the area under the curves.

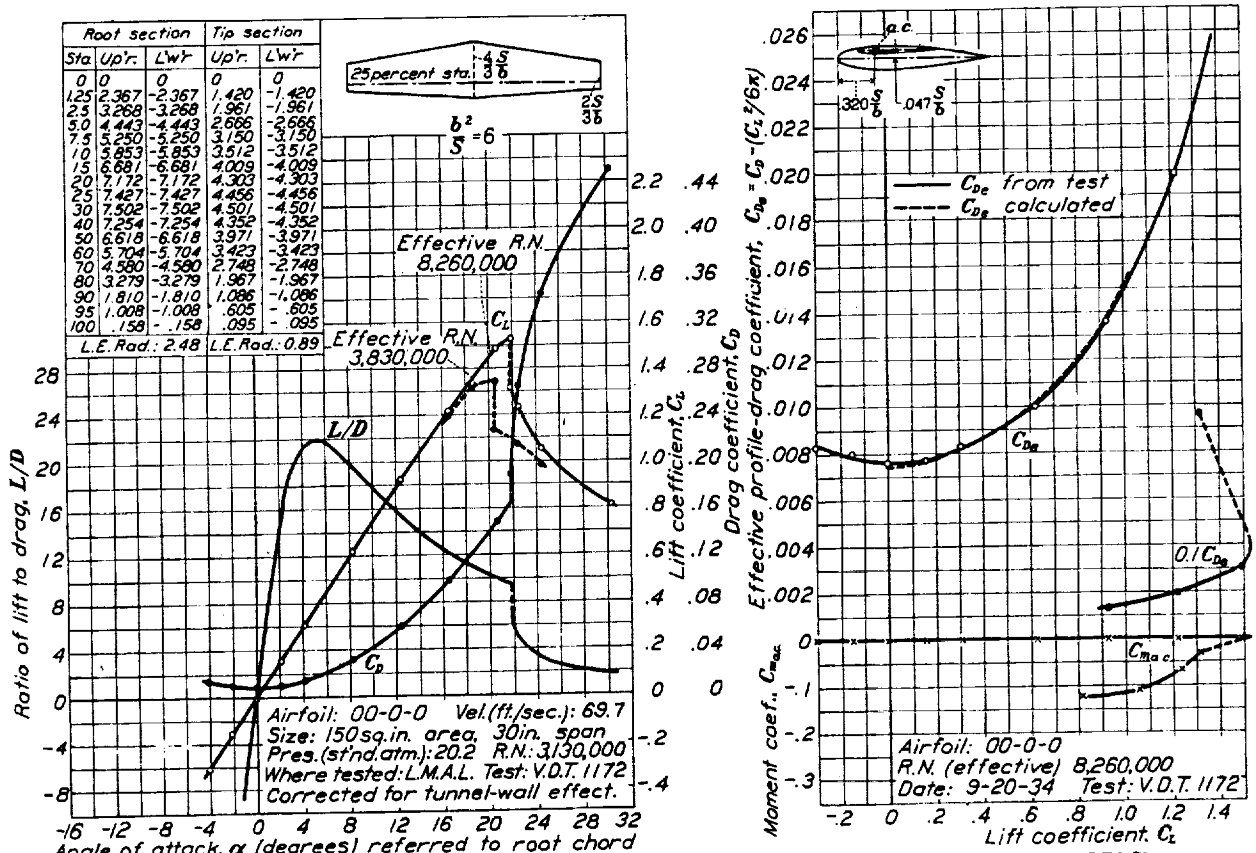
TESTS OF TAPERED WINGS

In order to provide test data on tapered wings, including wings with sweepback and twist, and to provide a check on the previously outlined method of computing characteristics, nine tapered wings were tested. The plan forms and sections of the wings are shown in figures 12 to 20. The aspect ratio of all the wings was 6; the taper ratio of eight of the wings was 0.5 and of one wing was 0.25. For all the wings the thickness ratio of the root section was 15 percent and of the tip sections 9 percent. The tip section was set to the desired angle of twist and the sections between the root and tip were then formed by using straight lines between corresponding stations of the root and tip sections. Formation of the wings in this manner results in a nonlinear distribution of twist along the semispan. In plan view the quarter-chord points of the sections lie on a straight line across the semispan; the sweepback was measured between this line and the lateral axis.

Three different amounts of sweepback, 0°, 15°, and 30°, and three types of airfoil sections, symmetrical, cambered, and reflexed, were used.

As the wings differ primarily in airfoil section, sweepback, and twist, a convenient designating number was used to distinguish the wings, such as 24-30-8.50. In this number 24 designates the N. A. C. A. airfoil mean line, i. e., 2 means 0.2 chord maximum camber and 4 that the maximum camber is at 0.4 chord; 30 gi~ s the sweepback in degrees; and 8.50 gives the washout in degrees.

The wings are listed in table V. The first two wings have no sweepback and no twist and differ only in airfoil section. The next two have increased sweepback. The five remaining wings are examples of various methods of combining sweepback, twist, and airfoil section to obtain wings having a small positive pitching moment; such wings would be suitable for tailless airplanes. The amounts of twist and of



Root section			Tip section	
Sta.	Up'r.	L'w'r.	Up'r.	L'w'r.
0	0	0	0	0
1.25	2.367	-2.367	1.420	-1.420
2.5	3.268	-3.268	1.961	-1.961
5.0	4.443	-4.443	2.666	-2.666
7.5	5.250	-5.250	3.150	-3.150
10	5.853	-5.853	3.512	-3.512
15	6.681	-6.681	4.009	-4.009
20	7.172	-7.172	4.303	-4.303
25	7.427	-7.427	4.456	-4.456
30	7.502	-7.502	4.501	-4.501
40	7.254	-7.254	4.352	-4.352
50	6.618	-6.618	3.971	-3.971
60	5.704	-5.704	3.423	-3.423
70	4.580	-4.580	2.748	-2.748
80	3.279	-3.279	1.967	-1.967
90	1.810	-1.810	1.086	-1.086
95	1.008	-1.008	.605	-.605
100	.158	-.158	.095	-.095
L.E. Rad.: 2.48			L.E. Rad.: 0.89	

FIGURE 12.—Tapered N. A. C. A. 00-0-0 airfoil.

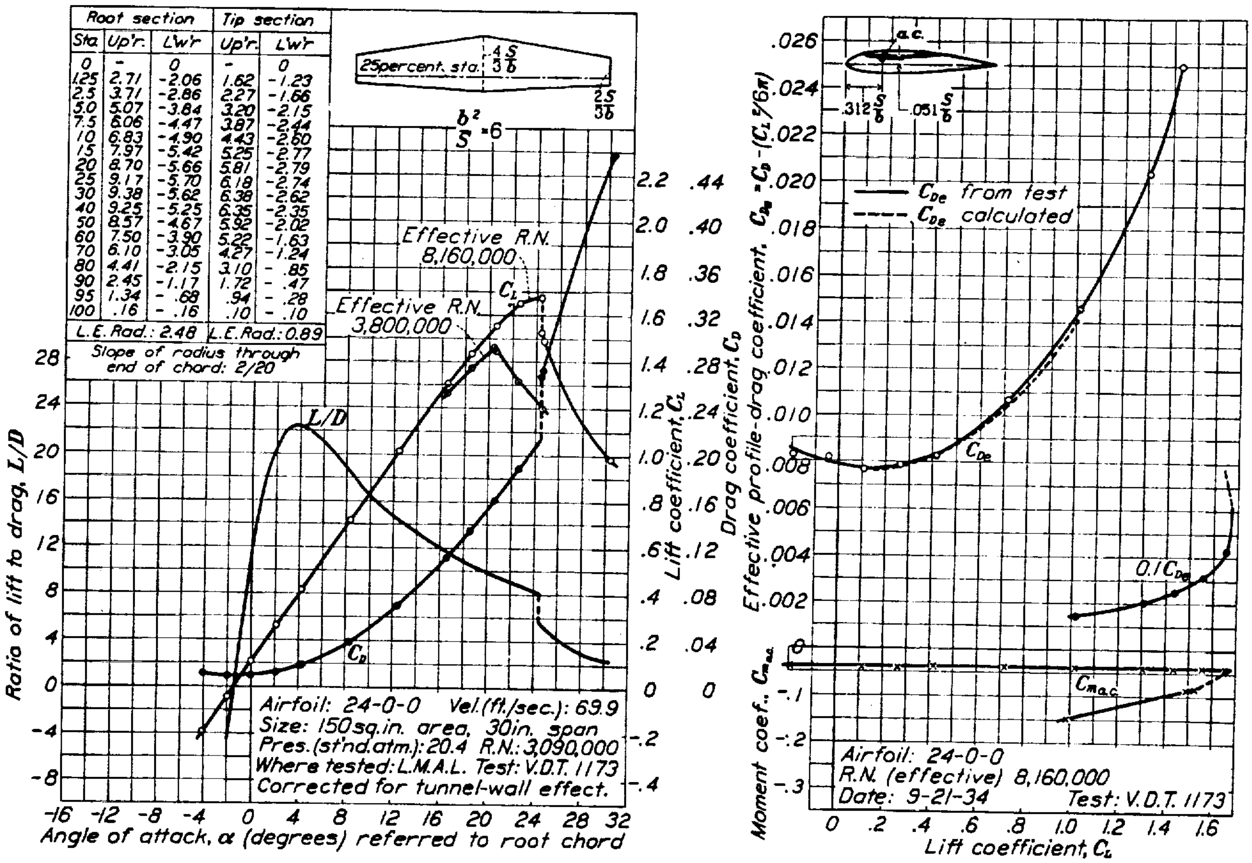


FIGURE 13.—Tapered N. A. C. A. 24-0-0 airfoil.

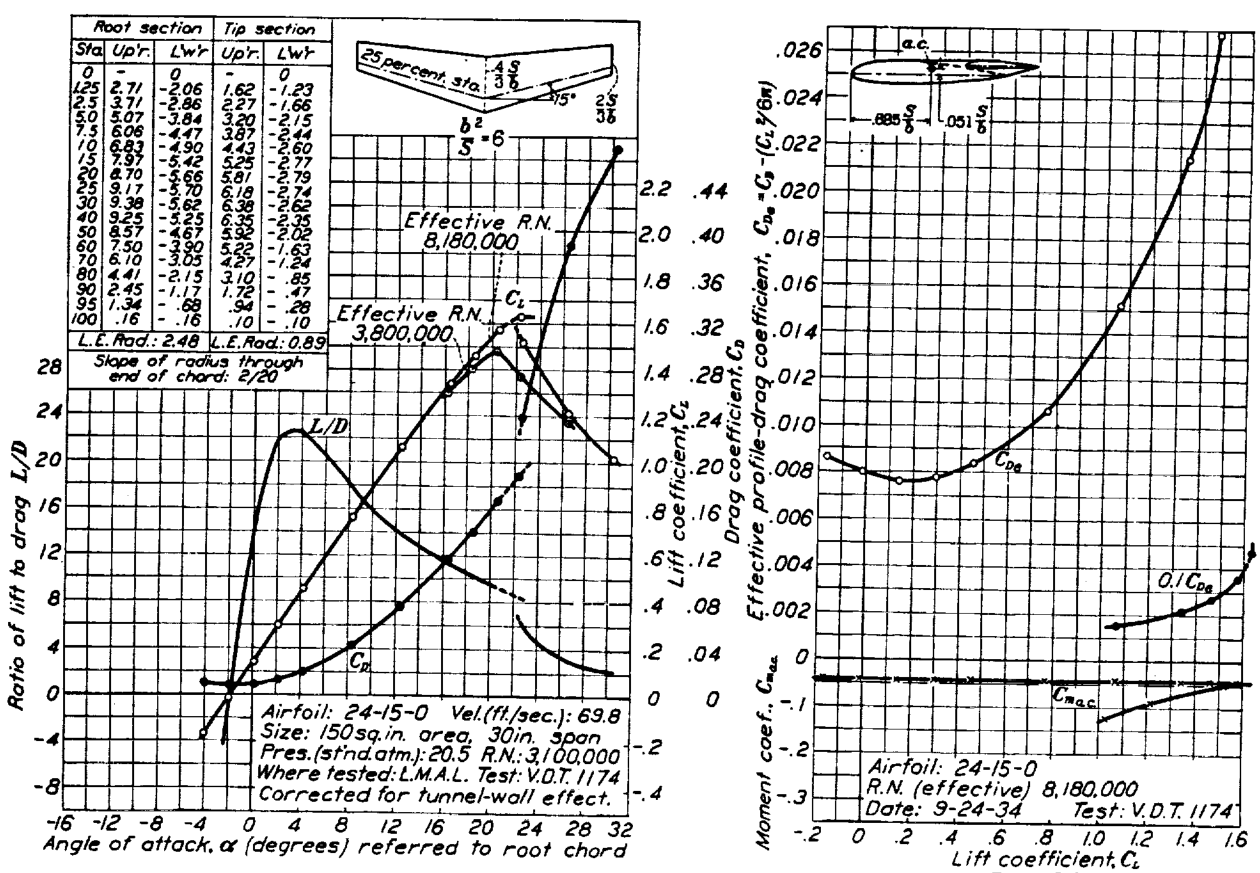
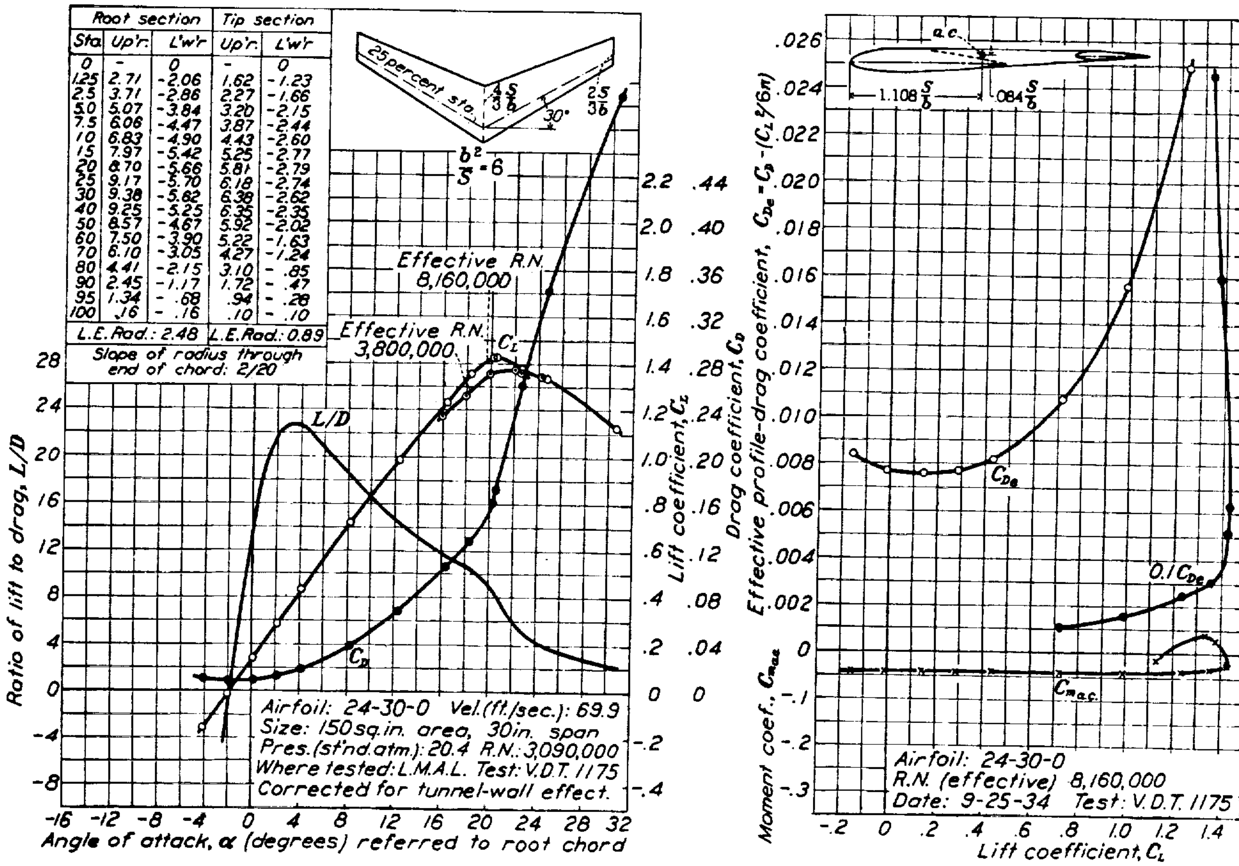
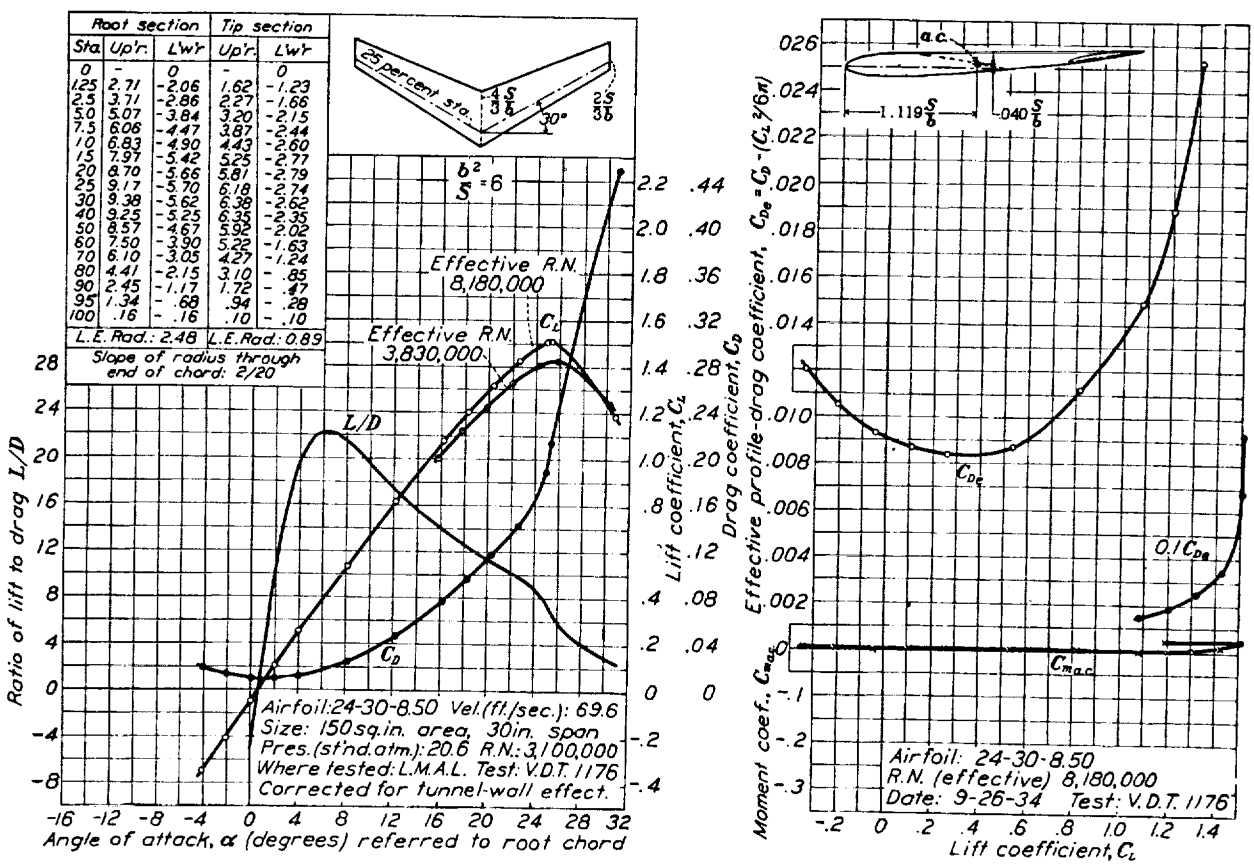


FIGURE 14.—Tapered N. A. C. A. 24-15-0 airfoil.



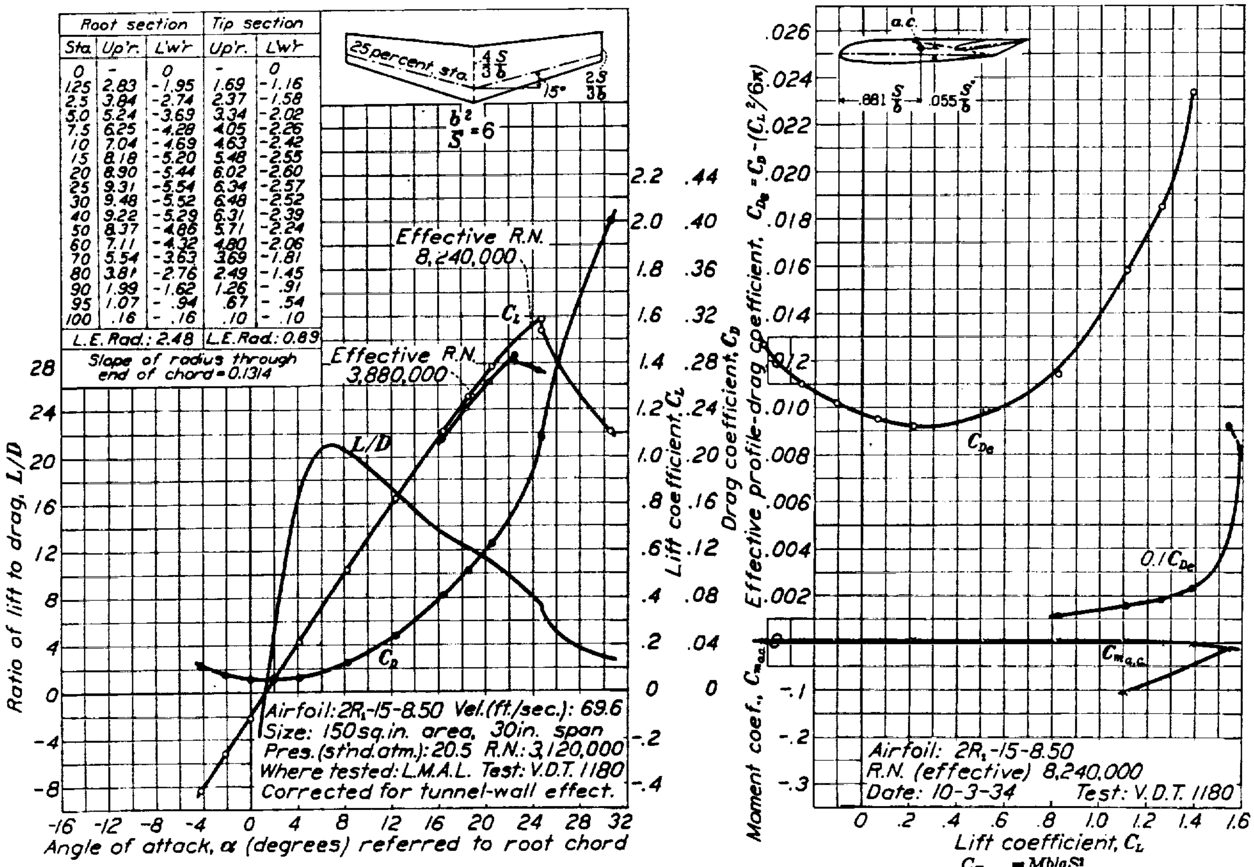
Root section			Tip section	
Sta	Up'r.	L'w'r	Up'r.	L'w'r
0	-	0	-	0
1.25	2.71	-2.06	1.62	-1.23
2.5	3.71	-2.86	2.27	-1.66
5.0	5.07	-3.84	3.20	-2.15
7.5	6.06	-4.47	3.87	-2.44
10	6.83	-4.90	4.43	-2.60
15	7.97	-5.42	5.25	-2.77
20	8.70	-5.66	5.81	-2.79
25	9.17	-5.70	6.18	-2.74
30	9.38	-5.62	6.38	-2.62
40	9.25	-5.25	6.35	-2.35
50	8.57	-4.67	5.92	-2.02
60	7.50	-3.90	5.22	-1.63
70	6.10	-3.05	4.27	-1.24
80	4.41	-2.15	3.10	-.85
90	2.45	-1.17	1.72	-.47
95	1.34	-.68	.94	-.28
100	.16	-.16	.10	-.10
L.E. Rad.: 2.48			L.E. Rad.: 0.89	
Slope of radius through end of chord: 2/20				

FIGURE 15.—Tapered N. A. C. A. 24-30-0 airfoil.



Root section			Tip section	
Sta	Up'r.	L'w'r	Up'r.	L'w'r
0	-	0	-	0
1.25	2.71	-2.06	1.62	-1.23
2.5	3.71	-2.86	2.27	-1.66
5.0	5.07	-3.84	3.20	-2.15
7.5	6.06	-4.47	3.87	-2.44
10	6.83	-4.90	4.43	-2.60
15	7.97	-5.42	5.25	-2.77
20	8.70	-5.66	5.81	-2.79
25	9.17	-5.70	6.18	-2.74
30	9.38	-5.62	6.38	-2.62
40	9.25	-5.25	6.35	-2.35
50	8.57	-4.67	5.92	-2.02
60	7.50	-3.90	5.22	-1.63
70	6.10	-3.05	4.27	-1.24
80	4.41	-2.15	3.10	-.85
90	2.45	-1.17	1.72	-.47
95	1.34	-.68	.94	-.28
100	.16	-.16	.10	-.10
L.E. Rad.: 2.48			L.E. Rad.: 0.89	
Slope of radius through end of chord: 2/20				

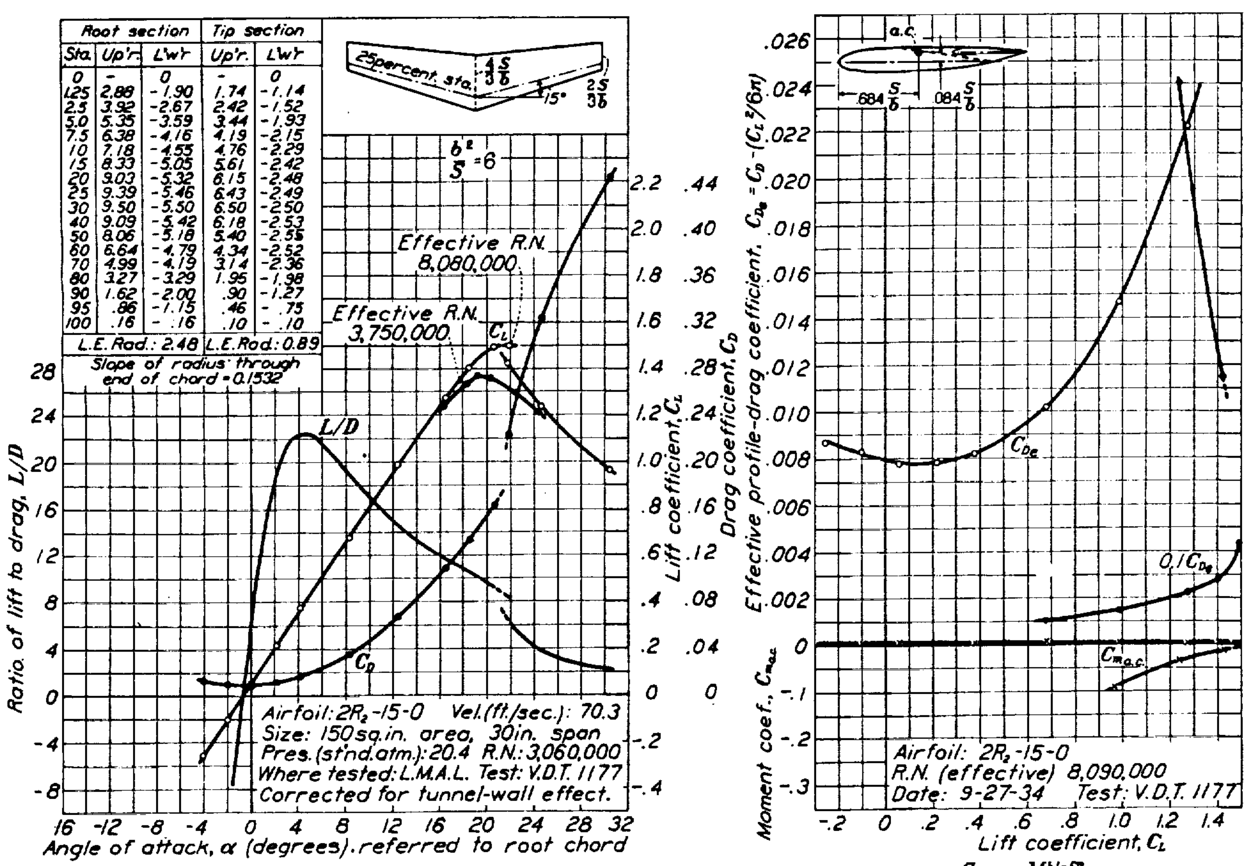
FIGURE 16.—Tapered N. A. C. A. 24-30-8.50 airfoil.



Sta.	Root section Up'r.	Root section L'w'r.	Tip section Up'r.	Tip section L'w'r.
0	-	0	-	0
1.25	2.83	-1.95	1.69	-1.16
2.5	3.84	-2.74	2.37	-1.58
5.0	5.24	-3.69	3.34	-2.02
7.5	6.25	-4.28	4.05	-2.26
10	7.04	-4.69	4.63	-2.42
15	8.18	-5.20	5.48	-2.55
20	8.90	-5.44	6.02	-2.60
25	9.31	-5.54	6.34	-2.57
30	9.48	-5.52	6.48	-2.52
40	9.22	-5.29	6.31	-2.39
50	8.37	-4.86	5.71	-2.24
60	7.11	-4.32	4.80	-2.06
70	5.54	-3.63	3.69	-1.81
80	3.81	-2.76	2.49	-1.45
90	1.99	-1.62	1.26	-.91
95	1.07	-.94	.67	-.54
100	.16	-.16	.10	-.10
	L.E. Rad.: 2.48		L.E. Rad.: 0.89	

Slope of radius through end of chord = 0.1314

FIGURE 17.—Tapered N. A. C. A. 2R₁-15-8.50 airfoil.



Sta.	Root section Up'r.	Root section L'w'r.	Tip section Up'r.	Tip section L'w'r.
0	-	0	-	0
1.25	2.88	-1.90	1.74	-1.14
2.5	3.92	-2.67	2.42	-1.52
5.0	5.35	-3.59	3.44	-1.93
7.5	6.38	-4.16	4.19	-2.15
10	7.18	-4.55	4.76	-2.29
15	8.33	-5.05	5.61	-2.42
20	9.03	-5.32	6.15	-2.48
25	9.39	-5.46	6.43	-2.49
30	9.50	-5.50	6.50	-2.50
40	9.09	-5.42	6.18	-2.53
50	8.06	-5.18	5.40	-2.55
60	6.64	-4.79	4.34	-2.52
70	4.99	-4.19	3.14	-2.36
80	3.27	-3.29	1.95	-1.98
90	1.62	-2.00	.90	-1.27
95	.86	-1.15	.46	-.75
100	.16	-.16	.10	-.10
	L.E. Rad.: 2.48		L.E. Rad.: 0.89	

Slope of radius through end of chord = 0.1532

FIGURE 18.—Tapered N. A. C. A. 2R₂-15-0 airfoil.

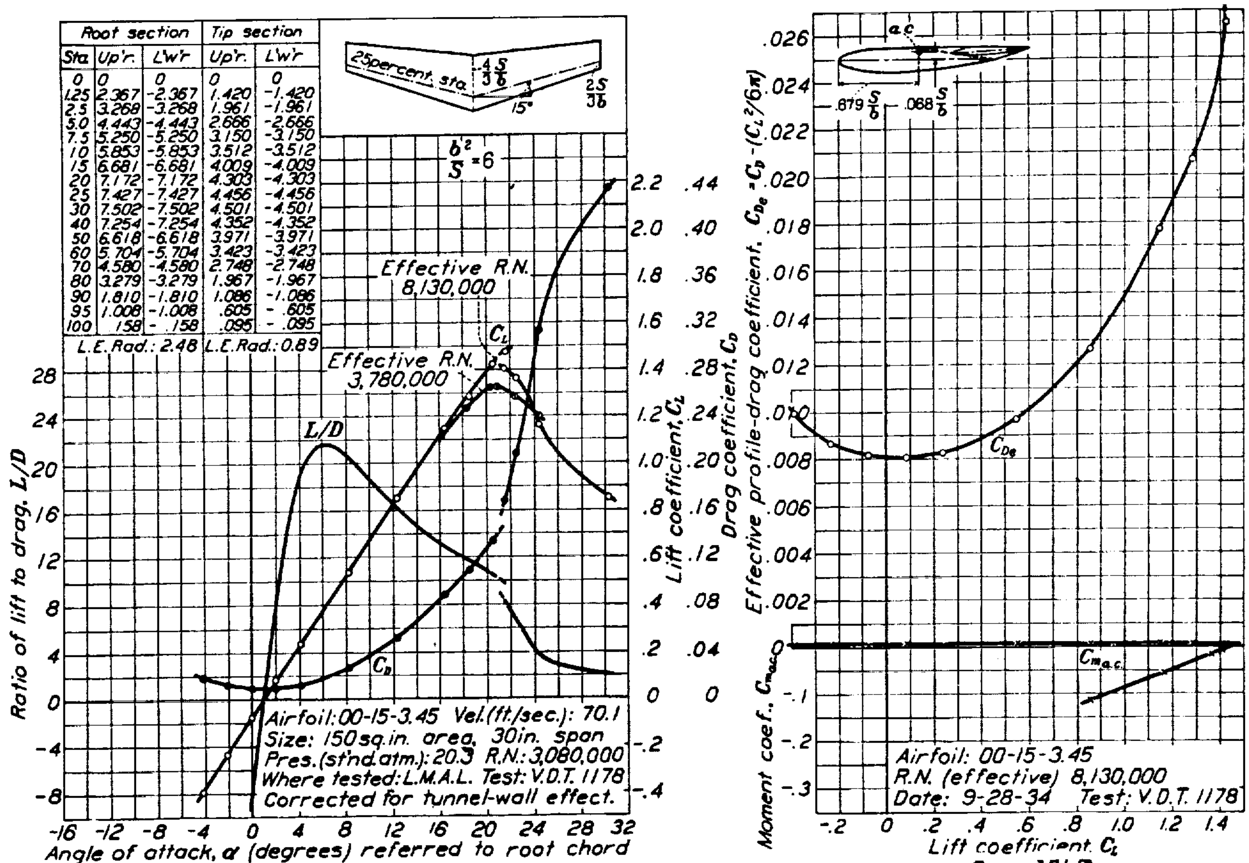


FIGURE 19.—Tapered N. A. C. A. 00-15-3.45 airfoil.

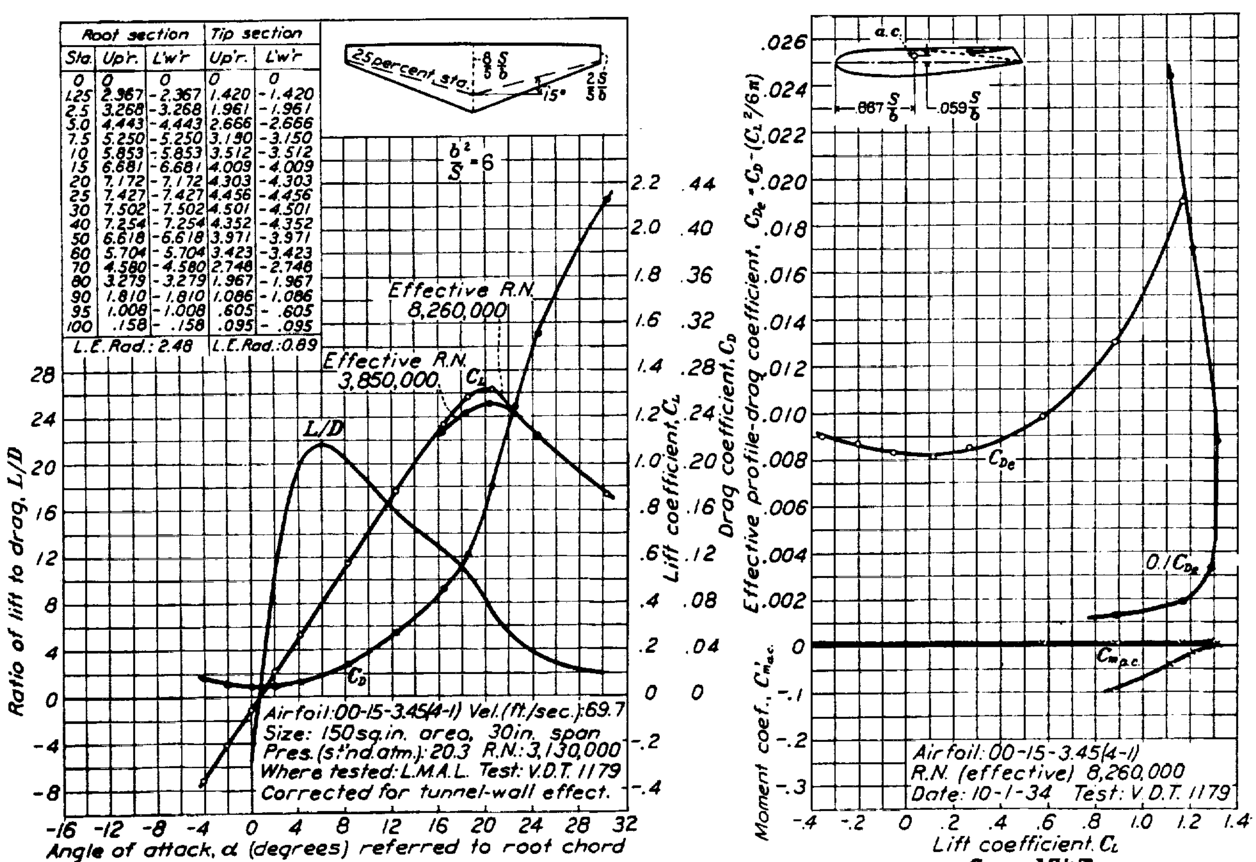


FIGURE 20.—Tapered N. A. C. A. 00-15-3.45 (4:1) airfoil.

sweepback necessary to obtain the desired pitching moment were determined by the method previously given for computing pitching moments, except that data for wings with trapezoidal tips were used. The 24–30–8.50 wing has sufficient twist to obtain the desired pitching moment using a cambered section and 30° sweepback. The $2R_1$–15–8.50 wing has the same twist but half the sweepback and a reflexed airfoil section to obtain a positive pitching moment. The $2R_2$–15–0 airfoil has no twist and increased reflex. A symmetrical section together with twist is used for the 00–15–3.45 wing, while the last wing has the same twist and sweepback as the previous wing but a taper ratio of 0.25.

The variable-density wind tunnel in which the tests were made is described in reference 6 together with the method of making tests. The lift, drag, and pitching moment of the wings were measured at a tank pressure of 20 atmospheres.

The results of the tests, corrected for tunnel-wall effect, are given in the form of dimensionless coefficients and are plotted in figures 12 to 20. The lift-curve peak is given for two values of effective Reynolds Number to indicate the scale effect. The effective Reynolds Number, at which the maximum lift coefficients apply in flight, is the test Reynolds Number multiplied by a turbulence factor, 2.64.

In order to make possible a more accurate reading of drag coefficients than can be made from the plots against angle of attack, a drag coefficient has been plotted against lift coefficient with the induced drag for elliptical span loading deducted; that is

$$C_{D_e}=C_D-\frac{C_L^2}{\pi A}$$

The coefficient C_{D_e} is called the "effective profile-drag coefficient" and is useful for comparing the drag of tapered wings, as it includes with the true profile drag any additional induced drag caused by a departure from the ideal elliptical lift distribution. Notice should be taken that C_{D_e} cannot be used like a profile-drag coefficient to compute the effect of change of aspect ratio but applies only to the particular wings tested. The values of C_{D_e} have been corrected to the effective Reynolds Number (references 7 and 8) by allowing for the reduction in skin-friction drag due to the change from the test to the effective Reynolds Number. The reduction amounted to $C_D=0.0011$.

The pitching-moment coefficients plotted against the lift coefficient are given about an axis through the aerodynamic center of the wings in order to obtain a practically constant value of pitching-moment coefficient. The aerodynamic center was determined from the slope of the test pitching-moment curve. The location of the aerodynamic center is given on the plots by its distance from the leading edge and above the chord of the root section. These distances are given as fractions of the ratio of area to span, S/b.

The shapes of the lift and pitching-moment curves near maximum lift provide information on the nature of the stalling of the wings. The 24–0–0 wing has a sharp drop in lift after the maximum, indicating that stalling occurs almost simultaneously over a considerable portion of the wing. Also the $C_{m_{a.c.}}$ after the stall is like that of normal wings. In contrast to this wing, the 24–30–0 wing, which has the same airfoil sections but 30° sweepback, has a rounded lift-curve peak, indicating that stalling occurs progressively along the span. The pitching-moment coefficient is positive after the stall, which shows that stalling begins at sections behind the aerodynamic center. Washout, as in the case of the 24–30–8.50 wing, reduces the tendency to stall of sections behind the aerodynamic center, which may be verified by reference to the $C_{m_{a.c.}}$ curve. Stalling, however, still begins behind the aerodynamic center, as the $C_{m_{a.c.}}$ is positive after the stall. All the wings, except the 24–30–0 and 24–30–8.50, are stable after the stall.

The important test results for all the wings are summarized in table V. The coordinates of the aerodynamic center are expressed as fractions of S/b. The 24–0–0, 24–15–0, and 24–30–0 wings show a decrease of $C_{L_{max}}$ as the sweepback is increased. For the 24–30–8.50 wing, the effect of sweepback is partly compensated by twist, which reduces the tendency to stall of the low Reynolds Number sections near the tips and therefore increases $C_{L_{max}}$. The drag, however, is also increased. Of the wings designed to have a small positive C_{m_0}, the $2R_2$–15–0 wing has the highest ratio of $C_{L_{max}}/C_{D_{e_{min}}}$.

COMPARISON OF TEST AND CALCULATED RESULTS

Pitching-moment characteristics, lift-curve slope, and drag.—The lift distribution and other theoretical data used to determine the desired pitching-moment coefficient of the wings are now used to predict other characteristics. In addition to C_{m_0}, the aerodynamic-center position, the angle of zero lift, and the lift-curve slope have been calculated. The values of a were calculated from the formula in figure 2. In this formula a value of a_0 corresponding to the a_0 for the N. A. C. A. 0012 and 2412 sections at a Reynolds Number of 3,000,000 was used, inasmuch as the effect of variations of a_0 with section and Reynolds Number is small. As the value of a_0 used in the formula was derived from tests of rectangular wings, a correction for square tips has been applied in order to obtain a better value of the section lift-curve slope. The correction, derived from tests of wings with rounded tips, is given in reference 9.

The calculated values of the pitching-moment coefficient at zero lift, the aerodynamic-center position, the angle of zero lift, and the lift-curve slope are generally in good agreement with the test values (table VI). The agreement of the pitching-moment coefficient at zero lift and the aerodynamic-center position, which are calculated from the basic and additional lift distributions, respectively, indicate that the theoretical lift distributions must also agree reasonably well with the actual distributions.

In addition to the foregoing characteristics, the drag has been calculated for the 00–0–0 and 24–0–0 airfoils. The comparison between calculation and experiment is based on values of the effective profile-drag coefficient. The calculated values were obtained from

$$C_{D_e}=\frac{2}{S}\int_0^{b/2} c_{d_0}c\,dy+C_{D_i}-\frac{C_L^2}{\pi A}$$

In order to find the value of the integral, values of c_{d_0} were determined as follows at several points along the semispan for convenient values of total wing C_L. For each value of C_L the distribution across the semispan of c_l, Reynolds Number, and thickness ratio were calculated. Then, for each point on the semispan, c_{d_0} was found for the appropriate c_l, Reynolds Number,

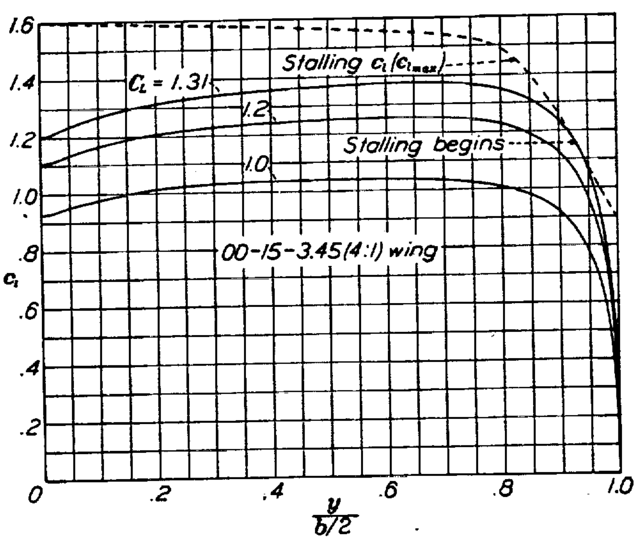


FIGURE 21.—Determination of the C_L at which a tapered wing begins to stall.

and thickness ratio, using data that are expected to be published soon in a report concerning scale effect on airfoils. From the values of c_{d_0}, a curve of $c_{d_0}c$ was plotted against y and the value of the integral was determined from the area under the curve. The value of C_{D_i} was obtained for the formula previously given. The calculated and test values of C_{D_e} are compared in figures 12 and 13. The agreement is considered good.

Estimation of maximum lift coefficient. A final characteristic to be estimated is the maximum lift coefficient, which should be nearly equal to the C_L at which stalling begins. The method of determining the C_L at which stalling begins is demonstrated for the 00–15–3.45 (4:1 taper) wing in figure 21. The lift coefficient at which each section along the semispan stalls (shown by the dashed curve) was obtained by using the maximum lift coefficients of the symmetrical sections given in reference 10 but with the values of $C_{L_{max}}$ increased 3 percent. This correction was made for the same reason that a_0 was corrected; that is, to allow for the effect of square tips and thereby to obtain a closer approach to true section characteristics. Better section characteristics will be obtained as a result of an investigation in progress but the correction used is sufficiently accurate for the present purpose. As the values of $C_{L_{max}}$ given in reference 10 were for a Reynolds Number of 3,000,000, correction increments were applied to correct the values of $C_{L_{max}}$ to the actual Reynolds Number of each section along the span. Correction increments applying to various airfoil sections are expected to be published in the previously mentioned report concerning scale effect on airfoils.

The curves of c_l distribution for several values of wing C_L given in figure 21 were determined by the method previously given for finding c_l distribution. As soon as the c_l curve becomes tangent to the stalling $c_{l_{max}}$ curve, the section at that point reaches its maximum lift coefficient and stalling should soon spread over a considerable part of the wing. Thus, for the 00–15–3.45 (4:1 taper) wing, stalling is indicated as beginning near the tips, at a C_L of 1.31. Stalling, however, is so close to the tip that it may be modified by the tip vortex. The measured $C_{L_{max}}$ is 1.32, but this value is probably low owing to the sweepback of the wing. This method, when applied to several other tapered wings without sweepback but having various taper ratios and aspect ratios, gave a stalling C_L that was within a few percent of the measured $C_{L_{max}}$ for all the wings; therefore, the method should prove useful for estimating the $C_{L_{max}}$ of tapered wings.

The 00–15–3.45 (4:1 taper) wing is an example of the harmful effect of excessive taper on $C_{L_{max}}$. Large taper not only tends to cause a low $C_{L_{max}}$ but also tends to cause stalling near the tips, which results in poor lateral control at low speeds. Improvement could be obtained by using less taper and thicker sections near the tips.

Although all of the characteristics of tapered wings have not yet been satisfactorily calculated, it may be concluded that the following important aerodynamic characteristics—angle of zero lift, the lift-curve slope, the pitching-moment coefficient, the aerodynamic-center position, and the span lift distribution—can be calculated with sufficient accuracy for engineering purposes.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *May 1, 1936.*

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TABLE I.—BASIC SPAN LIFT-DISTRIBUTION DATA

VALUES OF L_b FOR TAPERED WINGS WITH ROUNDED TIPS $c_{l_b}=\frac{ea_0S}{cb}L_b$

A \ c_t/c_s	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	SPANWISE STATION $\frac{y}{b/2}=0$										
2	−0.118	−0.121	−0.122	−0.122	−0.122	−0.121	−0.121	−0.121	−0.120	−0.120	−0.120
3	−.153	−.160	−.162	−.163	−.165	−.164	−.164	−.163	−.162	−.161	−.160
4	−.183	−.192	−.197	−.199	−.199	−.199	−.198	−.197	−.196	−.194	−.192
5	−.211	−.221	−.224	−.226	−.225	−.225	−.224	−.224	−.221	−.219	−.218
6	−.235	−.248	−.253	−.253	−.252	−.252	−.250	−.247	−.244	−.243	−.242
7	−.256	−.269	−.275	−.276	−.274	−.272	−.270	−.268	−.264	−.261	−.258
8	−.274	−.288	−.293	−.293	−.291	−.290	−.288	−.285	−.282	−.279	−.276
10	−.304	−.318	−.322	−.323	−.321	−.320	−.318	−.315	−.311	−.305	−.299
12	−.329	−.342	−.350	−.349	−.348	−.345	−.341	−.337	−.331	−.323	−.317
14	−.350	−.364	−.370	−.370	−.368	−.365	−.360	−.355	−.350	−.342	−.334
16	−.367	−.380	−.386	−.385	−.382	−.379	−.375	−.370	−.362	−.358	−.348
18	−.384	−.399	−.405	−.403	−.400	−.393	−.387	−.380	−.376	−.368	−.360
20	−.398	−.411	−.417	−.415	−.410	−.404	−.399	−.392	−.386	−.378	−.369
	SPANWISE STATION $\frac{y}{b/2}=0.2$										
2	−0.076	−0.080	−0.082	−0.085	−0.086	−0.086	−0.086	−0.085	−0.085	−0.084	−0.083
3	−.098	−.108	−.111	−.112	−.113	−.113	−.113	−.113	−.112	−.110	−.108
4	−.117	−.130	−.135	−.138	−.137	−.137	−.137	−.137	−.137	−.135	−.132
5	−.131	−.148	−.156	−.159	−.159	−.158	−.158	−.158	−.157	−.156	−.152
6	−.145	−.162	−.173	−.176	−.176	−.176	−.176	−.176	−.175	−.172	−.170
7	−.156	−.178	−.189	−.192	−.192	−.192	−.191	−.191	−.190	−.190	−.189
8	−.168	−.189	−.200	−.204	−.204	−.205	−.205	−.206	−.205	−.204	−.204
10	−.182	−.207	−.220	−.224	−.225	−.225	−.226	−.226	−.225	−.225	−.225
12	−.197	−.226	−.239	−.240	−.239	−.238	−.238	−.238	−.237	−.237	−.237
14	−.206	−.234	−.248	−.249	−.248	−.248	−.248	−.248	−.248	−.248	−.248
16	−.212	−.242	−.256	−.258	−.257	−.256	−.256	−.256	−.256	−.256	−.255
18	−.219	−.247	−.260	−.264	−.265	−.265	−.265	−.265	−.265	−.264	−.262
20	−.222	−.255	−.269	−.271	−.271	−.271	−.272	−.272	−.272	−.272	−.270
	SPANWISE STATION $\frac{y}{b/2}=0.4$										
2	−0.006	−0.011	−0.013	−0.015	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.015
3	−.002	−.010	−.012	−.015	−.016	−.016	−.016	−.016	−.017	−.018	−.018
4	0	−.006	−.011	−.012	−.016	−.016	−.018	−.019	−.020	−.020	−.021
5	.004	−.004	−.010	−.012	−.016	−.018	−.020	−.021	−.021	−.022	−.023
6	.009	−.002	−.008	−.012	−.016	−.018	−.020	−.021	−.022	−.024	−.026
7	.012	−.001	−.010	−.013	−.017	−.018	−.020	−.022	−.025	−.027	−.029
8	.014	0	−.008	−.012	−.017	−.019	−.021	−.025	−.029	−.030	−.030
10	.021	.007	−.002	−.010	−.017	−.020	−.022	−.027	−.030	−.032	−.032
12	.028	.009	−.001	−.010	−.017	−.021	−.025	−.029	−.032	−.036	−.038
14	.036	.013	0	−.010	−.017	−.021	−.028	−.031	−.035	−.040	−.042
16	.043	.019	.002	−.008	−.016	−.022	−.029	−.034	−.038	−.041	−.045
18	.049	.022	.004	−.008	−.015	−.022	−.031	−.038	−.041	−.043	−.046
20	.050	.023	.006	−.006	−.014	−.022	−.031	−.038	−.041	−.046	−.049

TABLE I.—BASIC SPAN LIFT-DISTRIBUTION DATA—Continued

VALUES OF L_b FOR TAPERED WINGS WITH ROUNDED TIPS $c_{l_b}=\frac{\epsilon a_0 S}{cb} L_b$

A \ c_t/c_s	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
						SPANWISE STATION $\frac{y}{b/2}=0.6$					
2	0.052	0.052	0.051	0.050	0.050	0.050	0.050	0.050	0.049	0.049	0.048
3	.070	.069	.068	.068	.068	.068	.068	.068	.068	.068	.068
4	.085	.082	.081	.080	.080	.080	.080	.080	.080	.080	.080
5	.099	.095	.092	.091	.091	.091	.091	.091	.090	.090	.090
6	.109	.107	.104	.102	.101	.101	.100	.100	.100	.100	.100
7	.119	.117	.114	.112	.111	.110	.110	.110	.110	.109	.108
8	.128	.122	.121	.120	.120	.119	.119	.118	.118	.117	.116
10	.139	.138	.135	.132	.131	.130	.130	.129	.128	.126	.124
12	.148	.145	.141	.140	.140	.139	.137	.135	.134	.132	.130
14	.155	.152	.150	.148	.145	.142	.141	.140	.139	.138	.135
16	.160	.158	.154	.151	.149	.146	.143	.141	.140	.139	.136
18	.165	.162	.160	.158	.152	.148	.145	.142	.140	.139	.138
20	.170	.169	.165	.159	.152	.148	.147	.143	.141	.140	.140
						SPANWISE STATION $\frac{y}{b/2}=0.8$					
2	0.072	0.079	0.080	0.082	0.083	0.085	0.085	0.086	0.086	0.084	0.081
3	.088	.098	.101	.102	.104	.108	.109	.110	.110	.108	.106
4	.100	.113	.120	.123	.125	.128	.128	.130	.130	.130	.129
5	.109	.125	.135	.138	.140	.143	.147	.148	.148	.148	.149
6	.115	.135	.148	.152	.156	.160	.160	.162	.163	.164	.165
7	.121	.142	.158	.163	.169	.172	.173	.173	.174	.174	.175
8	.126	.149	.164	.174	.180	.182	.182	.183	.183	.184	.184
10	.136	.160	.178	.188	.195	.200	.201	.202	.203	.201	.198
12	.145	.170	.188	.200	.208	.212	.214	.216	.216	.214	.210
14	.152	.182	.200	.210	.216	.221	.223	.227	.228	.225	.220
16	.159	.186	.205	.216	.222	.229	.232	.233	.236	.232	.229
18	.161	.197	.215	.224	.230	.235	.239	.242	.243	.242	.238
20	.166	.201	.220	.232	.237	.241	.245	.248	.248	.248	.247
						SPANWISE STATION $\frac{y}{b/2}=0.9$					
2	0.059	0.068	0.072	0.073	0.075	0.076	0.075	0.075	0.075	0.075	0.075
3	.068	.083	.092	.098	.099	.100	.100	.100	.100	.100	.100
4	.074	.098	.111	.118	.121	.122	.123	.123	.123	.123	.123
5	.081	.107	.122	.131	.138	.140	.141	.141	.142	.142	.142
6	.087	.117	.136	.148	.154	.159	.160	.160	.160	.160	.160
7	.090	.123	.146	.160	.167	.171	.171	.172	.172	.172	.172
8	.092	.131	.153	.170	.179	.182	.183	.184	.185	.186	.187
10	.098	.139	.166	.184	.197	.201	.203	.205	.207	.209	.210
12	.100	.147	.178	.198	.210	.218	.221	.225	.228	.229	.230
14	.102	.156	.188	.208	.220	.231	.238	.241	.243	.245	.246
16	.103	.161	.197	.219	.231	.241	.249	.253	.258	.259	.260
18	.105	.166	.202	.228	.243	.252	.260	.263	.269	.271	.275
20	.107	.172	.211	.233	.248	.260	.268	.273	.279	.282	.285
						SPANWISE STATION $\frac{y}{b/2}=0.95$					
2	0.038	0.051	0.058	0.059	0.060	0.060	0.060	0.060	0.059	0.059	0.058
3	.044	.063	.073	.078	.079	.080	.080	.080	.080	.079	.078
4	.050	.072	.076	.092	.095	.097	.099	.100	.100	.100	.099
5	.052	.083	.100	.107	.110	.112	.113	.114	.116	.117	.116
6	.054	.088	.109	.119	.122	.128	.130	.132	.132	.131	.130
7	.056	.093	.116	.130	.135	.140	.144	.148	.150	.149	.145
8	.057	.100	.125	.140	.146	.152	.158	.160	.161	.160	.159
10	.058	.107	.138	.152	.162	.171	.178	.182	.186	.187	.183
12	.059	.112	.143	.165	.179	.189	.198	.200	.202	.205	.204
14	.060	.116	.151	.174	.190	.202	.211	.215	.218	.221	.222
16	.061	.121	.159	.184	.203	.218	.222	.229	.233	.236	.238
18	.061	.126	.166	.194	.213	.229	.236	.241	.248	.251	.255
20	.061	.128	.173	.203	.225	.239	.245	.251	.259	.265	.271
						SPANWISE STATION $\frac{y}{b/2}=0.975$					
2	0.019	0.030	0.035	0.037	0.037	0.037	0.037	0.036	0.036	0.035	0.034
3	.022	.039	.045	.049	.050	.051	.052	.054	.053	.052	.051
4	.026	.043	.054	.060	.062	.064	.068	.069	.069	.068	.067
5	.029	.051	.065	.070	.071	.075	.078	.081	.082	.083	.083
6	.030	.055	.071	.079	.082	.088	.091	.094	.097	.097	.097
7	.030	.060	.078	.087	.091	.098	.101	.107	.110	.110	.110
8	.030	.062	.081	.091	.100	.107	.112	.120	.121	.121	.121
10	.031	.067	.090	.105	.115	.124	.132	.138	.141	.142	.143
12	.031	.069	.095	.115	.131	.141	.149	.153	.160	.161	.162
14	.031	.071	.102	.127	.143	.155	.163	.171	.175	.177	.178
16	.031	.077	.111	.138	.156	.169	.178	.182	.188	.190	.191
18	.032	.083	.121	.150	.169	.182	.191	.197	.200	.201	.202
20	.032	.086	.128	.158	.178	.193	.202	.208	.210	.212	.213

TABLE II.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA

VALUES OF L_a FOR TAPERED WINGS WITH ROUNDED TIPS, $c_{l_{a1}}=\frac{S}{cb}L_a$

A \ c_t/c_s	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	SPANWISE STATION $\frac{y}{b/2}=0$										
2	1.439	1.400	1.367	1.339	1.316	1.301	1.298	1.292	1.290	1.287	1.282
3	1.489	1.430	1.385	1.350	1.322	1.302	1.288	1.275	1.263	1.253	1.246
4	1.527	1.452	1.400	1.360	1.329	1.302	1.279	1.260	1.242	1.226	1.211
5	1.559	1.473	1.414	1.369	1.333	1.301	1.272	1.248	1.225	1.204	1.186
6	1.585	1.492	1.428	1.378	1.338	1.300	1.267	1.237	1.211	1.187	1.163
7	1.609	1.510	1.440	1.386	1.340	1.300	1.264	1.232	1.203	1.176	1.149
8	1.629	1.534	1.456	1.392	1.344	1.300	1.264	1.229	1.198	1.165	1.135
10	1.661	1.558	1.473	1.409	1.355	1.306	1.264	1.222	1.187	1.152	1.120
12	1.686	1.578	1.490	1.420	1.361	1.308	1.261	1.219	1.180	1.143	1.109
14	1.708	1.592	1.502	1.429	1.366	1.309	1.260	1.214	1.172	1.136	1.100
16	1.726	1.610	1.513	1.433	1.368	1.309	1.255	1.208	1.165	1.127	1.090
18	1.741	1.623	1.525	1.441	1.370	1.308	1.252	1.203	1.160	1.118	1.080
20	1.755	1.632	1.531	1.446	1.372	1.307	1.250	1.199	1.152	1.109	1.070
	SPANWISE STATION $\frac{y}{b/2}=0.2$										
2	1.369	1.329	1.300	1.279	1.267	1.260	1.258	1.256	1.253	1.250	1.248
3	1.405	1.346	1.308	1.279	1.260	1.248	1.241	1.234	1.228	1.221	1.214
4	1.434	1.363	1.318	1.284	1.260	1.243	1.232	1.220	1.209	1.198	1.186
5	1.459	1.377	1.324	1.288	1.260	1.240	1.223	1.208	1.194	1.181	1.168
6	1.477	1.388	1.329	1.290	1.259	1.236	1.218	1.200	1.184	1.169	1.151
7	1.491	1.393	1.332	1.291	1.259	1.236	1.214	1.193	1.174	1.157	1.138
8	1.502	1.401	1.338	1.294	1.261	1.236	1.212	1.189	1.168	1.148	1.129
10	1.513	1.411	1.347	1.299	1.265	1.236	1.209	1.182	1.158	1.137	1.114
12	1.520	1.417	1.349	1.302	1.265	1.233	1.202	1.172	1.148	1.126	1.102
14	1.527	1.423	1.354	1.307	1.268	1.232	1.201	1.170	1.144	1.119	1.094
16	1.532	1.428	1.358	1.308	1.269	1.232	1.199	1.164	1.135	1.110	1.087
18	1.539	1.429	1.359	1.309	1.270	1.231	1.195	1.160	1.130	1.103	1.078
20	1.547	1.431	1.360	1.311	1.271	1.230	1.190	1.155	1.123	1.098	1.069
	SPANWISE STATION $\frac{y}{b/2}=0.4$										
2	1.217	1.190	1.178	1.172	1.172	1.171	1.170	1.169	1.169	1.168	1.168
3	1.220	1.191	1.176	1.166	1.161	1.160	1.159	1.158	1.157	1.156	1.155
4	1.223	1.192	1.173	1.162	1.156	1.151	1.149	1.148	1.147	1.146	1.145
5	1.226	1.193	1.172	1.159	1.149	1.142	1.140	1.138	1.136	1.134	1.133
6	1.229	1.193	1.171	1.155	1.145	1.138	1.132	1.128	1.127	1.126	1.125
7	1.229	1.193	1.170	1.152	1.140	1.131	1.124	1.121	1.120	1.119	1.118
8	1.229	1.192	1.168	1.150	1.138	1.128	1.120	1.116	1.113	1.111	1.110
10	1.228	1.192	1.167	1.148	1.132	1.121	1.113	1.108	1.104	1.102	1.100
12	1.228	1.192	1.166	1.145	1.125	1.111	1.107	1.102	1.099	1.094	1.090
14	1.228	1.191	1.161	1.136	1.116	1.104	1.100	1.096	1.090	1.087	1.082
16	1.228	1.189	1.158	1.131	1.112	1.101	1.097	1.091	1.086	1.081	1.075
18	1.228	1.186	1.152	1.129	1.111	1.100	1.092	1.087	1.080	1.076	1.070
20	1.228	1.182	1.149	1.127	1.110	1.098	1.089	1.083	1.078	1.071	1.065
	SPANWISE STATION $\frac{y}{b/2}=0.6$										
2	0.970	0.976	0.984	0.992	1.003	1.010	1.012	1.014	1.016	1.018	1.019
3	.950	.962	.975	.985	.996	1.004	1.011	1.018	1.023	1.030	1.038
4	.932	.948	.962	.978	.992	1.002	1.008	1.014	1.023	1.035	1.050
5	.920	.938	.953	.971	.988	1.000	1.008	1.015	1.024	1.038	1.053
6	.909	.930	.949	.966	.981	.993	1.002	1.013	1.024	1.039	1.055
7	.900	.920	.940	.959	.975	.989	1.000	1.012	1.024	1.039	1.054
8	.891	.916	.938	.956	.972	.988	.999	1.011	1.024	1.039	1.053
10	.881	.907	.929	.947	.961	.976	.992	1.008	1.023	1.039	1.052
12	.872	.901	.923	.941	.958	.972	.989	1.006	1.022	1.038	1.051
14	.868	.895	.918	.937	.953	.969	.986	1.003	1.019	1.035	1.049
16	.861	.888	.912	.931	.948	.966	.983	1.000	1.017	1.033	1.048
18	.858	.883	.906	.925	.944	.963	.981	.998	1.015	1.032	1.047
20	.851	.876	.898	.920	.940	.959	.978	.995	1.012	1.028	1.046
	SPANWISE STATION $\frac{y}{b/2}=0.8$										
2	0.615	0.678	0.712	0.731	0.740	0.745	0.746	0.746	0.747	0.747	0.748
3	.589	.659	.700	.726	.743	.754	.764	.772	.782	.790	.799
4	.568	.644	.691	.723	.746	.764	.781	.795	.806	.816	.824
5	.548	.632	.685	.720	.748	.769	.790	.808	.822	.834	.845
6	.531	.619	.675	.717	.748	.775	.800	.820	.838	.851	.862
7	.517	.609	.670	.713	.748	.778	.802	.827	.845	.861	.875
8	.504	.600	.663	.710	.748	.779	.808	.834	.854	.872	.886
10	.486	.585	.653	.704	.748	.783	.815	.842	.868	.887	.905
12	.472	.576	.648	.702	.748	.788	.821	.850	.877	.899	.919
14	.462	.569	.641	.699	.748	.789	.825	.858	.887	.911	.933
16	.456	.564	.638	.698	.748	.791	.830	.862	.894	.921	.944
18	.450	.559	.636	.698	.750	.796	.835	.870	.901	.930	.953
20	.444	.545	.629	.698	.753	.801	.842	.878	.909	.937	.962

TABLE II.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA—Continued

VALUES OF L_a FOR TAPERED WINGS WITH ROUNDED TIPS, $c_{l_{a1}}=\frac{S}{cb}L_a$

A \ c_t/c_s	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	SPANWISE STATION $\frac{y}{b/2}=0.9$										
2	0.378	0.465	0.508	0.525	0.531	0.534	0.535	0.536	0.537	0.538	0.539
3	.352	.447	.500	.528	.543	.552	.559	.564	.568	.571	.575
4	.331	.435	.495	.532	.554	.569	.581	.590	.598	.603	.609
5	.314	.424	.490	.531	.560	.583	.600	.613	.622	.630	.636
6	.300	.416	.487	.531	.565	.595	.615	.631	.643	.652	.659
7	.290	.410	.484	.535	.572	.603	.628	.646	.660	.671	.678
8	.282	.403	.481	.536	.579	.612	.638	.658	.673	.686	.696
10	.266	.383	.472	.541	.590	.628	.656	.679	.698	.712	.723
12	.253	.376	.469	.542	.597	.639	.669	.698	.718	.736	.751
14	.245	.370	.468	.545	.602	.648	.684	.715	.739	.759	.776
16	.239	.366	.468	.547	.609	.659	.698	.729	.756	.780	.801
18	.234	.367	.470	.552	.618	.669	.710	.743	.773	.800	.822
20	.231	.368	.473	.560	.625	.679	.722	.759	.791	.819	.846
	SPANWISE STATION $\frac{y}{b/2}=0.95$										
2	0.231	0.296	0.334	0.358	0.370	0.379	0.381	0.383	0.386	0.388	0.390
3	.209	.290	.339	.369	.389	.401	.407	.412	.416	.418	.420
4	.191	.286	.342	.378	.402	.420	.428	.434	.440	.444	.446
5	.176	.281	.344	.384	.415	.436	.449	.458	.463	.469	.471
6	.166	.278	.346	.392	.428	.451	.466	.475	.482	.490	.496
7	.155	.272	.346	.398	.438	.464	.481	.494	.502	.510	.515
8	.148	.261	.346	.403	.446	.475	.495	.510	.521	.529	.534
10	.138	.255	.346	.410	.460	.495	.520	.538	.553	.566	.575
12	.132	.254	.348	.419	.473	.511	.542	.566	.583	.598	.608
14	.129	.252	.349	.423	.482	.529	.562	.588	.609	.628	.640
16	.126	.252	.351	.432	.495	.546	.581	.610	.635	.655	.671
18	.122	.254	.357	.439	.503	.558	.598	.629	.658	.682	.702
20	.121	.258	.364	.449	.516	.569	.613	.648	.680	.707	.730
	SPANWISE STATION $\frac{y}{b/2}=0.975$										
2	0.132	0.172	0.207	0.239	0.263	0.272	0.274	0.277	0.279	0.281	0.282
3	.119	.166	.210	.250	.278	.289	.291	.294	.298	.300	.301
4	.107	.163	.214	.258	.288	.304	.308	.311	.315	.319	.322
5	.098	.158	.217	.269	.304	.320	.322	.328	.333	.338	.342
6	.089	.158	.219	.272	.314	.332	.340	.344	.350	.357	.361
7	.081	.158	.222	.278	.320	.342	.351	.359	.366	.373	.381
8	.077	.158	.228	.283	.328	.352	.363	.374	.383	.391	.400
10	.069	.158	.233	.295	.343	.373	.390	.403	.415	.428	.438
12	.068	.161	.242	.308	.360	.395	.413	.430	.448	.461	.473
14	.066	.163	.248	.320	.376	.413	.438	.458	.478	.495	.510
16	.064	.166	.255	.331	.394	.435	.463	.488	.510	.529	.546
18	.063	.169	.263	.346	.412	.461	.492	.518	.539	.560	.580
20	.062	.171	.271	.363	.435	.483	.515	.544	.570	.593	.615

TABLE III.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA FOR THE ELLIPTICAL WING, $c_{l_{a1}}=\frac{S}{cb}L_a$

$\frac{y}{b/2}$	L_a
0	1.273
.2	1.248
.4	1.167
.6	1.019
.8	.764
.9	.555
.95	.398
.975	.283

TABLE IV.—CALCULATION OF LIFT DISTRIBUTION FOR ILLUSTRATIVE EXAMPLE

$\frac{y}{b/2}$	c	a_0	L_b	L_a	c_{l_b} [1]	$c_{l_{a1}}$ [2]	$C_L \times c_{l_{a1}}$	c_l	l_b	l_a	l	$c_{m_{a.c.}}$	$c_{m_{a.c.}} \times c^2$
0	9.13	0.097	−0.252	1.300	0.127	0.950	1.140	1.267	11.59	104.0	115.6	−0.083	−6.92
.2	8.22	.097	−.176	1.236	.098	1.003	1.205	1.303	8.05	99.0	107.2	−.075	−5.06
.4	7.30	.098	−.018	1.138	.012	1.039	1.248	1.260	.88	91.0	92.0	−.067	−3.57
.6	6.39	.098	.101	.993	−.073	1.036	1.242	1.169	−4.66	79.6	74.7	−.060	−2.45
.8	5.42	.099	.160	.775	−.138	.954	1.145	1.007	−7.48	62.0	54.6	−.052	−1.53
.9	4.49	.099	.159	.595	−.165	.884	1.061	.896	−7.41	47.7	40.3	−.048	−.97
.95	3.43	.099	.128	.451	−.175	.877	1.053	.878	−6.01	36.2	30.1	−.046	−.54
.975	2.47	.099	.088	.332	−.167	.896	1.076	.909	−4.13	26.6	22.4	−.045	−.27
1.0	0	(.099)	0	0	0	0	0	0	0	0	0	(−.044)	0

[1] $c_{l_b}=\frac{\epsilon a_0 S}{cb}L_b=-47.3\frac{a_0}{c}L_b$

[2] $c_{l_{a1}}=\frac{S}{cb}L_a=\frac{6.67}{c}L_a$

TABLE V.—SUMMARY OF TEST RESULTS

[Effective Reynolds number, approximately 8,000,000]

Wing[1]	$C_{L_{max}}$	$C_{D_{0_{min}}}$	$C_{L_{max}}/C_{D_{0_{min}}}$	[2] $\frac{p}{S/b}$	[2] $\frac{h}{S/b}$	C_{m_0}
00-0-0	1.53	0.0076	201	0.320	0.047	0
24-0-0	1.68	.0077	218	.312	.051	−.040
24-15-0	1.63	.0076	215	.685	.051	−.043
24-30-0	1.43	.0076	188	1.108	.084	−.042
24-30-8.50	1.51	.0084	180	1.119	.040	.002
$2R_1$-15-8.50	1.59	.0092	173	.681	.055	.003
$2R_2$-15-0	1.50	.0078	192	.684	.084	.004
00-15-3.45	1.48	.0081	183	.679	.068	.007
00-15-3.45(4-1)	1.32	.0082	161	.667	.059	.005

[1] The first group of numbers designates the mean line of the airfoil sections; the next group gives the angle of sweepback in degrees; the last group gives the angle of washout in degrees.

[2] Coordinates of the aerodynamic center: p is the distance from the leading edge of the root chord; and h is the distance above the root chord.

TABLE VI.—COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES

Wing	C_{m_0}		$\frac{x_{a.c.}}{S/b}$		$\alpha_{s_{(L=0)}}$		a	
	Calculated	Experimental	Calculated	Experimental	Calculated	Experimental	Calculated	Experimental
00-00	0	0	0	−0.014	0	0	0.074	0.075
24-0-0	−.043	−.040	0	−.022	−1.7	−1.7	.074	.074
24-15-0	−.043	−.043	.345	.352	−1.7	−1.9	.074	.075
24-30-0	−.043	−.042	.744	.775	−1.7	−1.9	.074	.072
24-30-8.50	.010	.002	.744	.786	.9	.7	.074	.076
$2R_1$-15-8.50	.006	.003	.345	.348	1.1	1.2	.074	.076
$2R_2$-15-0	.004	.004	.345	.351	−.6	−.7	.074	.078
00-15-3.45	.010	.007	.345	.346	1.1	1.0	.074	.076
00-15-3.45(4-1)	-------	.005	.345	.334	-------	.7	.075	.076