

## Diffraction Optics in Adverse Environments

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Gregory P. Behrmann  
 Army Research Laboratory  
 AMSRL-SS-SF  
 2800 Powder Mill Road  
 Adelphi, MD 20783  
 (301) 394-3800

### Abstract

An investigation at the Army Research Laboratory is in progress to characterize DOE performance in mil-spec environments. One of the most significant environmental influences is temperature. An analysis of a diffractive lens is presented in which optical performance is described as a function of temperature. In particular, we review the thermal dependence of focal length and diffraction efficiency. It is shown that the change in these parameters is independent of lens shape and relates only to material properties. Athermalized hybrid refractive/diffractive designs are discussed.

### 1.0 Introduction

For military and aerospace applications, temperature is one of the more important environmental influences. Typical military specifications can require a system to operate over a temperature range of  $-30^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . In fact, it is not uncommon for commercial products to operate over a significant temperature range. Sometimes, the combination of changes in dimension and index can affect the optical performance of a system.<sup>1</sup>

To date, the effects of temperature on DOEs have received little attention.<sup>2,3</sup> Recently, several researchers<sup>4,5,6,7</sup> have independently examined the relationship among temperature, material properties, and optical performance.

In this paper, we will describe focal length as a function of temperature and describe the temperature dependence of diffraction efficiency. We will show how an athermalized lens can be designed by combining refractive and diffractive surfaces and discuss its limitations. In addition, we will present a new design that is both athermal and achromatic. Finally, we will end with some concluding remarks.

<sup>1</sup>D. S. Grey, "Athermalization of Optical Systems," *JOSA* 38 (1948), p. 542.

<sup>2</sup>A. McKay and J. White, "Effects of Simulated Space Environments on Dichromated Gelatin Holograms," *SPIE Proc.* 1044 (1989), pp. 269-275.

<sup>3</sup>M. Tanigami, S. Ogata, S. Aoyama, T. Yamashita, and K. Imanaka, "Low-Wavefront Aberration and High Temperature Stability Molded Micro Fresnel Lens," *IEEE Photonics Tech. Lett.* 1 (1989), pp. 384-385.

<sup>4</sup>J. Jahns, Y. H. Lee, C. A. Burrus Jr., and J. Jewell, "Optical Interconnects Using Top-Surface-Emitting Microlasers and Planar Optics," *Appl. Opt.* 31, (1992), pp. 592-597.

<sup>5</sup>C. Londono, W. T. Plummer, and P. P. Clark, "Athermalization with Diffractive Optics," *Diffractive Optics: Design, Fabrication, and Applications 1992*, Technical Digest Series 9 (OSA, Washington D.C., 1992), p. 7.

<sup>6</sup>G. P. Behrmann and J. P. Bowen, "Thermal Effects in Diffractive Lenses," *Diffractive Optics: Design, Fabrication, and Applications 1992*, Technical Digest Series 9 (OSA, Washington D.C., 1992), p. 8-10.

<sup>7</sup>G. P. Behrmann and J. P. Bowen, "The Influence of Temperature on Diffractive Lens Performance," accepted for publication in *Applied Optics*.

## 2.0 Opto-thermal Expansion Coefficient

The opto-thermal expansion coefficient for a refractive lens,  $x_{f,r}$ , as defined by Jamieson,<sup>8</sup> relates the change in focal length to a change in temperature. This coefficient is a measure of a system's sensitivity to temperature. In this section we examine the opto-thermal expansion coefficient for both the refractive and diffractive lens.

The opto-thermal expansion coefficient for a thin refractive lens,  $x_{f,r}$  is given by

$$x_{f,r} = \frac{1}{f} \frac{df}{dT} = \alpha_g - \frac{1}{n - n_o} \left( \frac{dn}{dT} - n \frac{dn_o}{dT} \right), \quad (1)$$

where  $\alpha_g$  is the coefficient of thermal expansion of the lens,  $n$  is the refractive index of the lens, and  $n_o$  is the refractive index of the image space. Note that the opto-thermal expansion coefficient is independent of the shape of the lens and is dependent only on material properties. Since  $x_{f,r}$  is normalized by  $f$  it can be used to calculate the change in focal length by

$$\Delta f = f x_{f,r} \Delta T. \quad (2)$$

For a more complex system, the opto-thermal expansion coefficient is a function of the thermal properties of all lenses and spaces. To evaluate the effects of temperature, it is necessary to determine the opto-thermal expansion coefficient for the entire system.

In analyzing systems that include diffractive lenses it is useful to have a similar expression. With an opto-thermal expansion coefficient for a diffractive lens,  $x_{f,d}$ , the techniques that have been developed to evaluate and athermalize refractive systems can be applied.

In the paraxial region, the focal length of a diffractive lens can be expressed as a function of the zone radius:

$$f = \frac{n_o r_m^2}{2m\lambda_o}, \quad m = 1, 2, 3, \dots \quad (3)$$

As the temperature changes, the zone spacing will expand and contract. The zone radius,  $r_m$ , can, to the first order, be expressed as

$$r_m(T) = r_m(1 + \alpha_g \Delta T). \quad (4)$$

Additionally, the index of the image space will change with temperature by

$$n_o(T) = n_o + \frac{dn_o}{dT} \Delta T. \quad (5)$$

By incorporating Eqs. 4-5, the opto-thermal expansion coefficient for a diffractive lens can be expressed as

$$x_{f,d} = 2\alpha_g + \frac{1}{n_o} \frac{dn_o}{dT}. \quad (6)$$

See References 4 or 7 for a more complete derivation.

Note that the change in focal length of a diffractive lens is only a function of  $\alpha_g$  and index changes of the image space. It is not a function of temperature-induced changes in the refractive index of the lens material. This is a fundamental difference between the thermal behavior of

<sup>8</sup>T. H. Jamieson, "Thermal Effects in Optical Systems," Opt. Engr.20 (1981), pp. 156-160.

refractive and diffractive lenses. Table 1 shows the relevant parameters and the opto-thermal expansion coefficients for several optical materials.

Opto-thermal expansion coefficients can be used by a designer to athermalize a system. One way in which this is achieved is by matching the opto-thermal expansion coefficient of the lens system to the coefficient of thermal expansion of the mounting material so that the change in image position corresponds to the change in position of the focal plane. See Fig. 1.

Because  $x_{f,r}$  for a refractive singlet and  $x_{f,d}$  for a diffractive singlet made of the same material can be different, it is theoretically possible to design an athermalized hybrid refractive/diffractive element. Such an element has a refractive power due to the curvature of the surfaces and a diffractive power due to surface-relief structure. This is pictorially represented in Fig. 2. The practical limitations on such designs are discussed in the Sec. 4.0.

### 3.0 Diffraction Efficiency

Diffraction efficiency is also an important consideration for systems. In this section we examine the temperature-dependent parameters that are associated with diffraction efficiency and present an analysis that determines the effect of temperature.

To attain a diffraction efficiency of 100 percent, a continuous-phase profile must have a blaze height given by

$$h = \frac{\lambda_o}{n - n_o}, \quad (7)$$

where  $\lambda_o$  is the design wavelength,  $n$  is the index of refraction of the lens material and  $n_o$  is the index of the image space. A temperature change will cause the blaze height to change. An incorrect blaze height will reduce the diffraction efficiency. Additionally, the index of both the lens material and the image space will change with temperature, influencing the diffraction efficiency.

For a relative error,  $\epsilon$ , in the blaze height, Swanson<sup>9</sup> has shown from scalar diffraction theory that the first order diffraction efficiency,  $\eta_1$ , is

$$\eta_1 = \left[ \frac{\sin(\pi\epsilon)}{\pi\epsilon} \right]^2. \quad (8)$$

By examining the index and physical expansion effects separately, a desired height,  $h_d$ , and an actual height,  $h_a$ , can be defined. Considering only the changes in index,  $h_d$  can be thought of as a desired height that would be required to maintain maximum diffraction efficiency:

$$h_d = \frac{\lambda_o}{\left[ n + \left( \frac{dn}{dT} \right) \Delta T \right] - \left[ n_o + \frac{dn_o}{dT} \Delta T \right]}. \quad (9)$$

The actual height,  $h_a$ , is associated with the physical expansion of the lens material with a temperature change. It can be expressed as

$$h_a = h(1 + \alpha_g \Delta T). \quad (10)$$

Therefore, the true error,  $\delta$ , after a temperature change is

$$\delta = h_a - h_d. \quad (11)$$

<sup>9</sup>G. J. Swanson, "Binary Optics Technology: The Theory and Design of Mult-Level Diffractive Optical Elements," MIT Lincoln Laboratory Tech. Rep. 854 (1989).

The relative error,  $\epsilon$ , in Eq. (10) is

$$\epsilon = \frac{|\delta|}{h_d} \quad (12)$$

If we consider optical systems that operate in air, we have found that, for most materials, the change in diffraction efficiency due to temperature effects is negligible. This is true even for plastics which have both a high  $\alpha_g$  and  $dn/dT$ .

For example, acrylic has a coefficient of thermal expansion of  $65.5 \times 10^{-6} C^{-1}$  and a  $dn/dT$  of  $-125 \times 10^{-6} C^{-1}$ . If the temperature of a diffractive lens made of acrylic is increased by  $30^\circ C$ ,  $\epsilon$  would be 0.0056 and the change in diffraction efficiency would be less than 0.1 percent.

#### 4.0 Design Examples

In the following section, we show how opto-thermal expansion data can be used to design hybrid athermalized lens systems. Here we restrict our analysis to the case of an athermalized doublet and an athermalized/achromatized triplet. For systems containing more elements or airspaces, the same methodology applies.

The total power of a lens,  $\phi_t$ , immersed in air, can be expressed as

$$\phi_t = \frac{1}{f_t} \quad (13)$$

where  $f_t$  is the focal length of the lens. It follows that for a  $k$ -element lens,  $\phi_t$  can be determined by

$$\phi_t = \phi_1 + \phi_2 + \dots + \phi_k \quad (14)$$

In Eq. 14,  $\phi_1$  is the power of the first lens, and  $\phi_2$  is the power of the second lens and so on.

An opto-thermal expansion coefficient for the lens,  $x_{f,t}$ , is given by

$$x_{f,t} = f_t \phi_1 x_{f1} + f_t \phi_2 x_{f2} + \dots + f_t \phi_k x_{fk} \quad (15)$$

In Eq. 15,  $x_{f1}$ ,  $x_{f2}$ , and  $x_{fk}$  are the corresponding opto-thermal expansion coefficients of the individual lens elements.

Consider lenses made from polycarbonate. This material offers several advantages: it is inexpensive, lightweight, and easy to shape. However, for the refractive case, polycarbonate has an extremely high opto-thermal expansion coefficient ( $245 \times 10^{-6} C^{-1}$ ). This can be attributed to its large coefficient of thermal expansion and its large  $dn/dT$ . For the diffractive case, the opto-thermal expansion coefficient is also quite large ( $130 \times 10^{-6} C^{-1}$ ). Therefore, polycarbonate lenses are difficult to incorporate in optical systems that must operate over wide temperature ranges.

If it were necessary to athermalize a polycarbonate lens, a hybrid doublet could be designed to meet this requirement. Consider an  $f/5$  doublet with a focal length of 100 mm operating at  $\lambda_0$  equal to 632.8 nm. By using Eqs. 14 and 15, we can select powers in the refractive surface and diffractive surface such that the net opto-thermal expansion coefficient matches the mounting material. For an aluminum mount, Table 2 shows the coefficients and calculated powers required for athermalization.

As a basis of comparison, it is appropriate to compare the performance of the athermalized hybrid to an all refractive singlet made from polycarbonate mounted in aluminum.

The thermal performance of a refractive singlet is shown in Fig. 3. It is assumed that the system is perfectly aligned at  $20^\circ C$ . If focal length and detector position are plotted as a function

of temperature, it is seen that when the system is heated to  $50^{\circ}\text{C}$ , the distance between detector and focal length is 0.67 mm. This separation is more than 20 times the tolerable depth of focus for diffraction limited performance.

On the other hand, Fig. 4 is a plot of focal length and detector position for the athermalized hybrid. For a first-order approximation of material properties, the distance between the two remains the same.

This athermalization technique is similar to one used to achromatize refractive and diffractive surfaces.<sup>10</sup> In achromatization, power is distributed in the refractive and diffractive surfaces based on the relative dispersions of the lens materials. Typically, the dispersion of a lens material is measured by its Abbe number,  $V$ . Therefore, an achromatized lens with a real object should meet the following condition:

$$0 = \frac{\phi_1}{V_1} + \frac{\phi_2}{V_2} + \dots + \frac{\phi_k}{V_k}. \quad (16)$$

For the refractive surface, the Abbe number of polycarbonate is 30. The diffractive surface has an Abbe number of -3.45. Note that in the athermalized hybrid, the refractive power is negative and the diffractive power is positive. Therefore, the conditions of Eq. 16 are not satisfied and chromatic aberrations must be addressed.

For diffraction limited performance, the refractive singlet has a bandwidth of  $\pm 2.94$  nm, and the athermalized doublet has an equivalent bandwidth of  $\pm 0.1$  nm. Thus, the chromatic aberrations are more severe in the athermalized hybrid element. For polycarbonate, this athermalization technique may only be useful for monochromatic applications.

Estelle<sup>11</sup> has examined the requirements for simultaneously athermalizing and achromatizing refractive lens systems. For the most part, a three-material triplet is required. With three materials, one can use Eqs. 14-16 to set up a system of 3 equations and 3 unknowns in which to solve for the necessary power in each element. Other aberrations, for example spherical aberration, can be reduced by solving for the optimal bending of the lens elements. Estelle studied a family of triplet solutions consisting of two plastics and one glass element. In the following, due to the unique dispersive and thermal behavior of the diffractive lens, we examine the possibility of replacing one of the three materials with a diffractive surface. See Fig. 5.

Assume now that it were necessary to design an  $f/5$ , 100 mm focal length lens mounted in aluminum. In this case, the lens must be achromatized over the visible range. If we use BK-7 and polycarbonate, both inexpensive materials, an element can be designed that has the mass and volume of a doublet but the performance of triplets discussed by Estelle. Table 3 shows the calculated powers for a solution with the glass element in front and the diffractive surface on the back. Note that the diffractive surface is relatively weak and would therefore be easier to fabricate. Additionally, the powers in the refractive components are manufacturable. In fact, similar levels of performance can be attained in IR lenses.<sup>12</sup> Infrared materials exhibit large changes in index with temperature. Traditionally it has been difficult to athermalize such systems.

<sup>10</sup>T. Stone and N. George, "Hybrid diffractive-refractive lenses and achromats," *Applied Optics*, 27, (1988) pp. 2960-2971.

<sup>11</sup>L. R. Estelle, "Third-Order Theory of Thermally Controlled Plastic and Glass Triplets," *SPIE Proceedings*, 237, (1980) pp. 392-401.

<sup>12</sup>G. Behrmann, "Color Correction in Athermalized Hybrid Lenses," to be presented at OSA Topical Meeting, Design for Photonics, March 22, 1993.

## 5.0 Conclusions

We have performed an analysis of thermal effects and their influence on the performance of diffractive lenses. Formulas describing the change in first-order properties have been presented. These expressions are useful for designing systems that must operate over wide temperature ranges.

The opto-thermal expansion coefficient for a diffractive lens has been derived. It is different from its refractive counterpart in that the focal length of a diffractive lens is not dependent on temperature-induced changes in refractive index of the lens material. An analysis of the change in diffraction efficiency due to temperature was performed. We have found for systems operating in air that the changes are insignificant.

A design technique was presented that shows how refractive and diffractive surfaces can be used together to produce hybrids with desirable thermal behavior. For a single hybrid, the amount of chromatic aberration will usually be increased if the athermal conditions are imposed. It was also shown that a diffractive surface can be used to significantly reduce the mass and volume of all-refractive triplet solutions that are both athermal and achromatic.

Material	n	$\alpha$	dn/dT	$X_{f,r}$	$X_{f,d}$
$\lambda = 546.1 \text{ nm}$					
FK52	1.487	14.4	-7.8	27.64	27.89
PSK2	1.571	6.4	1.5	1.28	11.89
BK7	1.519	7.1	1.7	0.98	13.89
K10	1.503	6.5	3.0	-2.17	12.09
BaF9	1.646	6.5	4.0	-2.00	12.09
F5	1.607	8.0	3.5	-0.17	15.09
LaFN2	1.748	8.2	0.7	5.14	15.49
SF11	1.792	6.1	11.4	-10.35	11.29
Acrylic	1.495	67.9	-105	227.40	134.89
Polycarb.	1.585	65.5	-107	245.80	130.04
$\lambda = 10 \mu\text{m}$					
ZnS	2.199	6.9	50	-36.45	12.91
ZnSe	2.401	7.7	48	-28.09	14.51
Ge	4.003	6.1	390	-124.95	11.31
KRS-5	2.370	58	-234	290.0	115.11

Table 1: Opto-Thermal and Thermal Properties of Various Lens Materials  
 $\alpha$ , dn/dT,  $X_{f,r}$  and  $X_{f,d}$  in ppm/°C

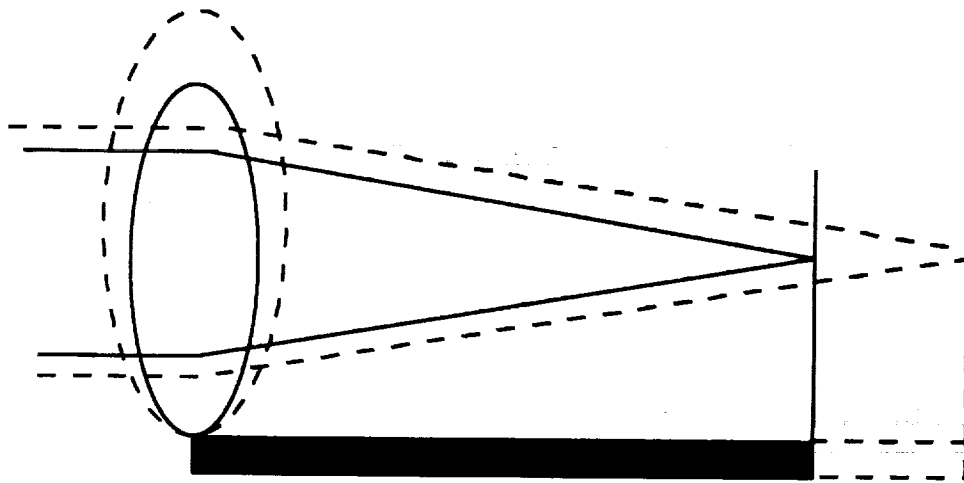


Fig. 1 Athermalized Lens System

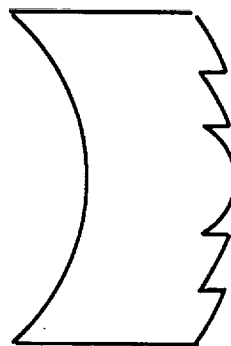


Fig. 2 Athermalized Hybrid Lens

$\alpha_{\text{mount}}$	$X_{f,r}$	$\phi_r$	$X_{f,d}$	$\phi_d$
23 ppm/°C	246 ppm/°C	-0.0092	130 ppm/°C	0.0192

Table 2 Coefficients, Refractive and Diffractive Power for 100 mm focal length hybrid polycarbonate lens, athermalized to aluminum.



# Polycarbonate Singlet

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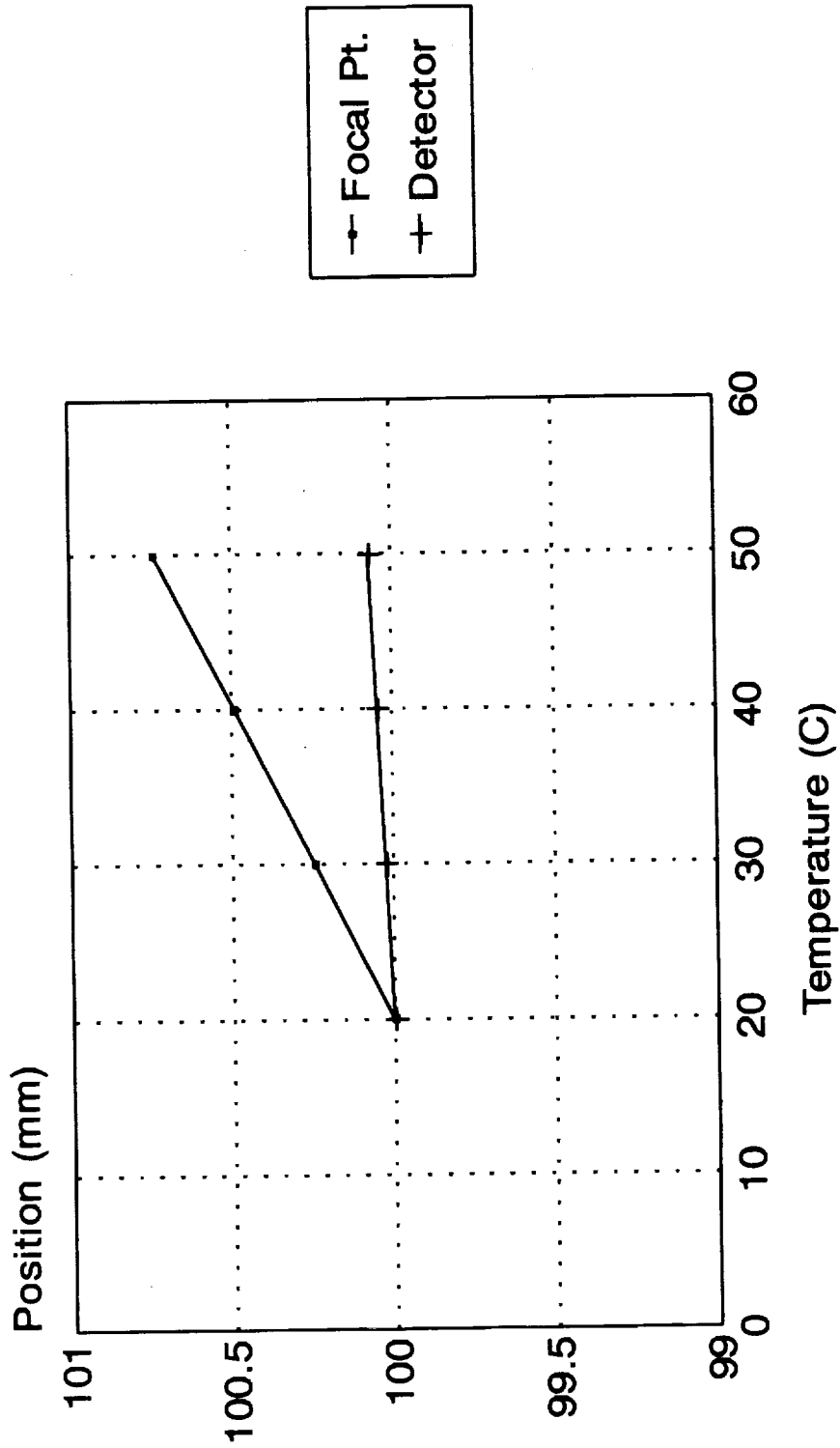


Fig. 3 Focal Point and Detector Position for 100 mm focal length, all-refractive, polycarbonate lens, aluminum mount.

# Athermalized Hybrid Polycarbonate Lens

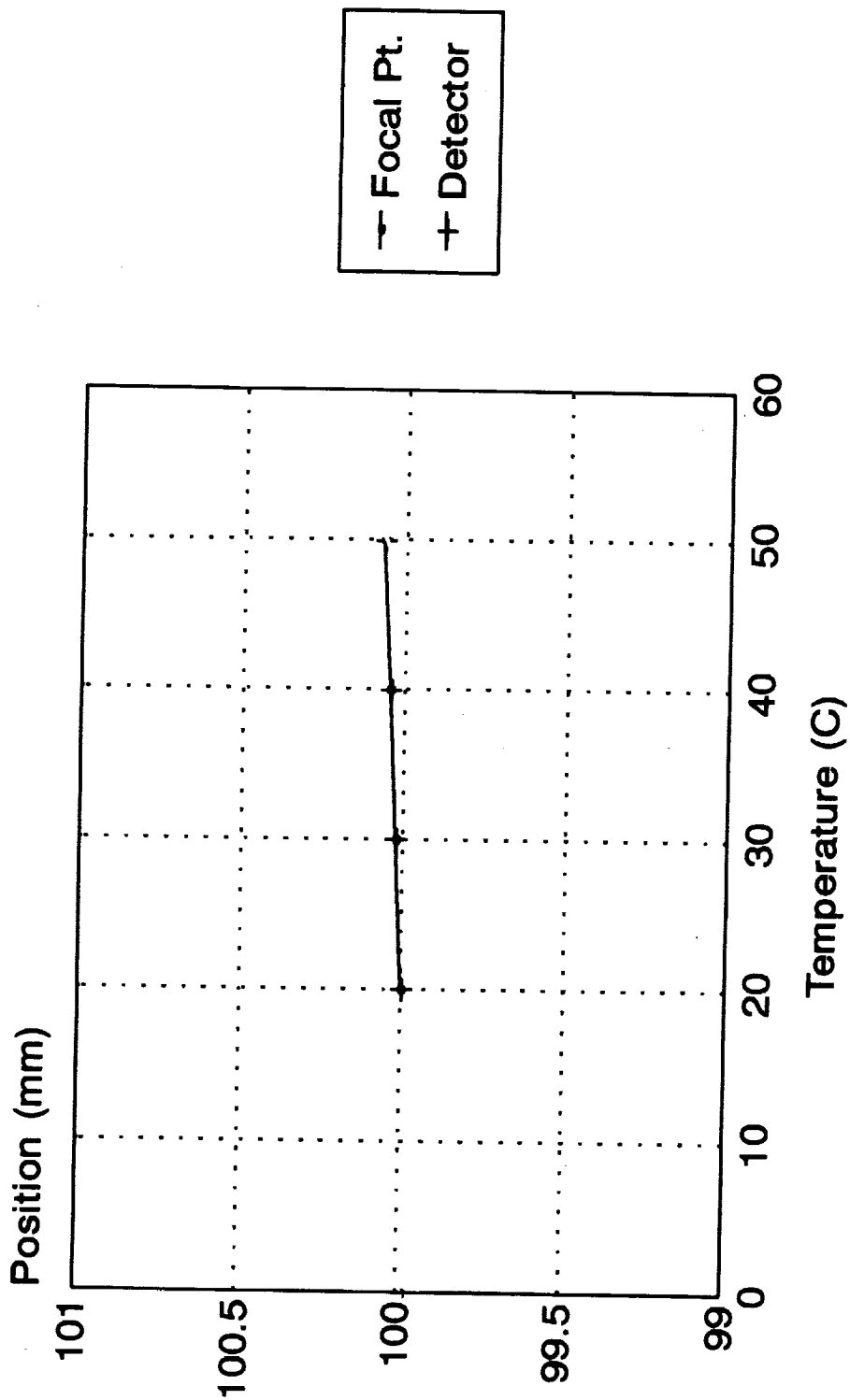
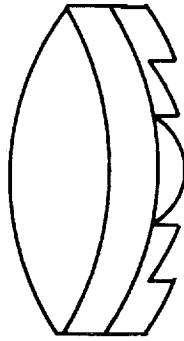


Fig. 4 Focal Point and Detector Position for 100 mm focal length, athermalized hybrid, polycarbonate lens, aluminum mount (the two lines are superimposed).



**Fig. 5 Athermal/Achromatic Hybrid Lens**

<b>Mat'l</b>	<b><math>X_f</math></b>	<b>V</b>	<b><math>\phi</math></b>
<b>BK-7</b>	<b>0.98 ppm/°C</b>	<b>64.2</b>	<b>0.0088</b>
<b>Polycarb.</b>	<b>246 ppm/°C</b>	<b>30.0</b>	<b>0.0006</b>
<b>DOE</b>	<b>130 ppm/°C</b>	<b>-3.45</b>	<b>0.0006</b>

**Table 3 Athermal/Achromatic Hybrid Lens, 100 mm focal length, athermalized to aluminum.**

