TIME TRANSFER USING GEOSTATIONARY SATELLITES : IMPLEMENTATION OF A KALMAN FILTER

F. Meyer
Observatoire de Besançon,
41 bis Avenue de l'Observatoire, BP 1615
25010 Besançon Cedex, France

Abstract

Since 1988, various experiments [1, 2, 3, 4] have shown that the TV signals transmitted by direct TV satellites may easily be used to perform time transfers at the level of a few tens of nanoseconds, the main source of error being the uncertainty on the satellite position.

We first present the two methods used in our experiment to reduce the effects of the satellite residual motion : the first one consists in estimating the longitude variations of the satellite and then using this information to improve other measurements. This allows to reduce the uncertainty to values between 9 and 50 nanoseconds according to the position of the involved stations.

In the second method we determine the satellite position by using the data collected by three calibrated stations. Time transfer between each of these stations and a fourth one has been shown to be achievable at the precision level of ten nanoseconds.

A new approach based on the use of a Kalman filter is proposed in order to take into account the dynamics of the geostationary satellite.

The precisions on orbital elements and clock differences and rates determination given by the first simulated applications of the Kalman filter are presented and compared to those obtained by the other methods.

INTRODUCTION

The principle of this kind of time transfer consists in timing the arrival of a given pulse at two different locations and deducing the time difference of the two clocks from these data and from the co-ordinates of the receiving antennas and of the satellite [1].

But while the position of the stations are often known with a sufficient precision, it is not the case for the satellite position. The tolerance on the geostationary satellite longitude and latitude is generally ± 0.1 degree. This causes significant errors on time transfer as soon as the stations are distant from more than a few kilometers : between Paris and Besançon (324 km) the error can reach 3 microseconds.

Until now, different methods have been used to overcome that problem. Some of them imply external information on the satellite position such as laser ranging [4] or more directly position data from the satellite control center [3]. The method we carried out to determine the satellite position uses the time measurements made between three externally calibrated (GPS) stations to precise the satellite position and will be shortly outlined hereafter.

A second kind of method takes into account the known geometry of the involved links [2] and some properties of the satellite orbits [1] to remove the effects of unknown parameters. The performances of this last method will also be presented.

A third approach [3] simultaneously estimates the orbital elements of the satellite and the clock differences and rates from the time delays measured between three stations and from pseudo-ranging given by a two-way measurement performed at one of the station.

In section 3, we propose to do the same thing by using Kalman filtering, that is known as an excellent way of reconstructing satellite trajectory. The final aim of this method is to reach the limit of accuracy (a few ns) that can be obtained from passive use (one way) of the signals transmitted by geostationary satellites.

1. PREVIOUSLY USED METHODS

The results presented in this section are based on measurements provided by four receiving stations located in Besançon (OB), Paris (OP), Toulouse (CT), and in the Observatoire de la Côte d'Azur in Grasse referred to as OC. Those four stations track the TDF2 satellite, located at 18.9° W and transmitting a D2-MAC standard TV signal. The caesium clocks of these four stations are also linked by GPS allowing the calibration of our results.

1.1. Time transfer using longitude calibration

The main source of uncertainty on the time transfer is due to the residual motion of the satellite in its "parking box". For a given couple of stations it depends on two parameters : the difference between the two unit vectors station-satellite (see table 1, row 1) on the one hand, and the vector defined by the real position of the satellite and its mean position. Usually the satellite is maintained in a cell of ± 0.1° (i. e. ± 73.6 km at the geostationary radius) in longitude and latitude, what implies variations of ± 10 km in radius ; taking into

account the longitude of the satellite (about 18.9° W for TDF1-2) , we obtain values varying between 2.5 and 5.5 μs for the 6 different links of our experiment. The observed variations are well within these theoretical limits.

The satellite geostationary orbit is mainly composed of daily and half-daily periodic longitude, latitude and radius variations (due to residual eccentricity and inclination) and a non periodic longitudinal drift (due to irregularities of the gravitational potential). Consequently, it is possible to averaged out the influence of the periodic components, by averaging two measurement values separated by 12 hours. This reduces the errors on time transfer to values around 1 μs. Furthermore, if we assume that the link is calibrated by an external way (GPS), we can say that the observed variations of the residuals are the effect of the satellite longitudinal drift. The importance of this effect depends on the position on each of the two involved stations with respect to the satellite position and can be easily calculated for each couple of stations. So once the longitude variations have been determined by one particular link (figure 1), it is possible to correct the data concerning other links for the effects of these variations. According to the link corrected and the one used for calibration, the precision obtained varies from 9 ns to 50 ns in the worst case (Table 1). The Allan variance of the residuals for two configurations are shown on figure 2.

Calibration link / Corrected link	OP-CT	OB-OP	OP-OC	CT-OB	CT-OC	OB-OC
OP-CT	_	13	12	20	15	11
OB-OP	35	_	11	20	12	24
OP-OC	53	18	_	43	15	23
CT-OB	35	13	17	_	14	27
CT-OC	52	15	11	27	_	27
OB-OC	19	16	9	27	13	_

Table 1. Uncertainty for the corrected links (ns)
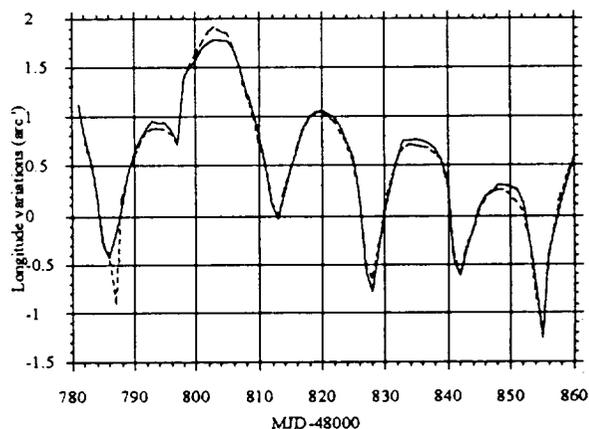
Figure 1 : Longitude variations determined by
OP-CT (dash line) and OP-OC (bold line)

1.2. Time transfer using explicit determination of the satellite position

This method uses three calibrated stations (two usable links) to reduce the uncertainty on the satellite position. It is as precise as the above method and far less sensitive to the position of the station to be linked. It consists in solving the geometric problem of finding the set of solutions of the system defined by the differential data obtained from the three calibrated stations. This gives the equation of a curve (intersection of two hyperboloïds) on which the

satellite must be found. The dispersion of the residuals is about 15 ns for the link between station OC and the triangle of stations OP-CT-OB. A better spatial resolution can be obtained by using the triangle OP-CT-OC that is a little bit more extended to reach a precision of about 11 ns for the link between the stations OB and OP.

Figure 2 (plots 2 and 3) shows that the accuracy reached by this method is of the order of a few 10^{-14} for the triangle OP-CT-OC as for OP-CT-OB.

1	——·——	Link CT-OB calibrated by OP-CT
2	———————	Link OB-OC using OP-CT-OB
3	— — — —	Link OB-OC using OP-CT-OC
4	··············	Link OB-OC calibrated by OP-OC
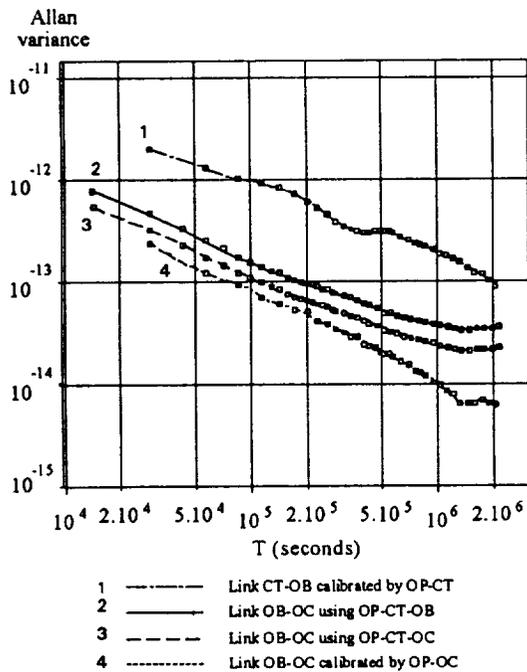
Figure 2 : Log-Log plots of Allan variance versus averaging time
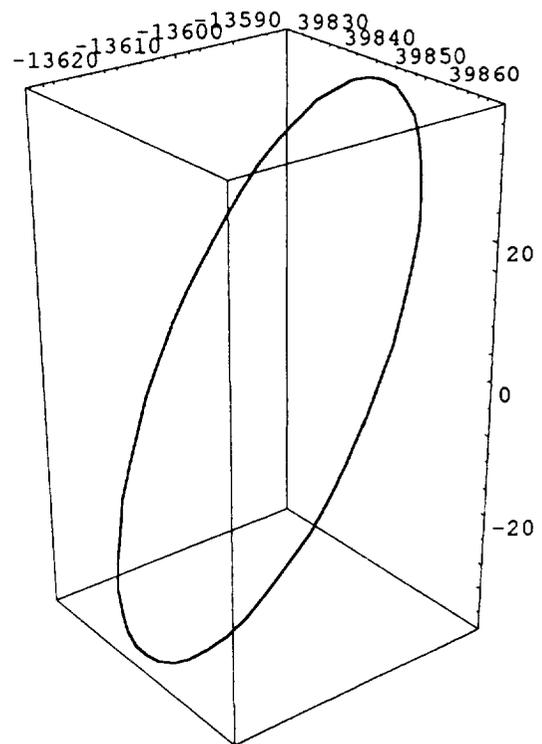
Figure 3 : Example of typical 24-hour residual orbit in a rotating reference frame (graduations in km).

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2. ORBIT MODEL

If we neglect at the present the different causes of perturbation of the satellite motion, this motion in a non-rotating geocentric reference frame is ruled by equation 1 :

$$\frac{d^2\vec{r}}{dt^2} = -\mu\frac{\vec{r}}{r^3}$$

where $\vec{r} = \begin{pmatrix} x_{sat} \\ y_{sat} \\ z_{sat} \end{pmatrix}$ and μ is a function of the semi-major axis a and period T. Associated to

initial conditions $(\vec{r}_0, \dot{\vec{r}}_0)$ this system defines a unique trajectory. If defined with classical Kepler's elements (semi-major axis a, eccentricity e, inclination i, longitude of the ascendant node ω, argument of the perigee Ω, mean anomaly M), this system is equivalent to :

$$\frac{d\vec{S}}{dt} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ n_0 \end{pmatrix} \quad \text{where} \quad \vec{S} = \begin{pmatrix} a \\ e \\ i \\ \Omega \\ \omega \\ M \end{pmatrix} \quad \text{and} \quad n_0 = \sqrt{\frac{\mu}{a_0^3}}$$

In the case of geostationary orbits, since e and i are very close to zero, it is usual to use the following state vector :

$$X = (a, e_x = e.\cos(\omega + \Omega), e_y = e.\sin(\omega + \Omega), h_x = i.\cos(\Omega), h_y = i.\sin(\Omega), L = \omega + \Omega + M)^T$$

We can then express $\vec{r}$ as a function of these elements :

$$\vec{r} = \begin{pmatrix} a\left[(\cos(L) - \frac{1}{2}(3e_x + e_x\cos(2L) + e_y\sin(2L))\right] \\ a\left[(\sin(L) - \frac{1}{2}(3e_y + e_y\cos(2L) + e_x\sin(2L))\right] \\ a\left[h_x\sin(L) - h_y\cos(L)\right] \end{pmatrix}$$

The time measurements ρ_{ij} made between two stations S_i $(\vec{r}_i)$ and S_j $(\vec{r}_j)$ at the instant t are a function of X and t that can be written as :

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$$\rho_{ij}(X,t) = \frac{\sqrt{(\vec{r} - \vec{r}_i)^2 - (\vec{r} - \vec{r}_j)^2}}{c}$$

where c is the speed of light. $\vec{r}_i$ and $\vec{r}_j$ are a function of the sidereal time (they would be constant in a rotating reference frame).

3. KALMAN FILTER SETUP

To apply the Kalman filter to a given system, we must be able to :

1. elaborate an evolution model of the system, i. e. estimate from a value at an instant k of a quantity X called state vector the value of X at the instant k+1. This simply reflects the dynamics of the system.

2. correct the preceding estimation with the help of measurements concerning a function of one or more of the components of X and of the prediction of what these measurements should be when taking into account the current state.

In our case, the system dynamics is simple in first approximation. We have :

$$\frac{dX}{dt} = (0,0,0,0,0,n_0)$$

Also, there is no problem to include in state vector X the clock difference and rate of two given stations. Then, if x_k stands for the estimated state vector at instant k, we can easily define the matrix $\varphi_{t_k, t_{k+1}}$ so that we have :

$$x_{k+1} = \varphi_{t_k, t_{k+1}} x_k$$

The observation equation however is not so simple since $\frac{d\rho_{ij}}{dt}(X,t)$ is not a linear function of the orbital elements. So the computation of the correction to the predicted measurement requires a linearisation around the current estimation x_k of these elements. This leads to the extend Kalman filter formulation [5] in which we have to determine the matrix :

$$H_k = \frac{\partial \rho_{ij}(X, t_k)}{\partial X}$$

	$\dfrac{\partial p_{ij}}{\partial L}\Delta L$ $\Delta L = 0.1°$	$\dfrac{\partial p_{ij}}{\partial a}\Delta a$ $\Delta a = 10^4 m$	$\dfrac{\partial p_{ij}}{\partial e_x}\Delta e_x$ $\Delta e_x = 7.10^{-5}$	$\dfrac{\partial p_{ij}}{\partial h_x}\Delta h_x$ $\Delta h_x = 10^{-3}$
OP-CT	0.4 μs	3 ns	70 ns	3 μs
OP-OB	2.0 μs	<1 ns	180 ns	1.2 μs
OP-OC	3.1 μs	15 ns	220 ns	2 μs
OB-OC	1.2 μs	16 ns	100 ns	1.6 μs
CT-OC	2.5 μs	15 ns	10 ns	2.4 μs
CT-OB	1.2 μs	30 ns	84 ns	1.6 μs

Table 2. Influence of the different orbital elements
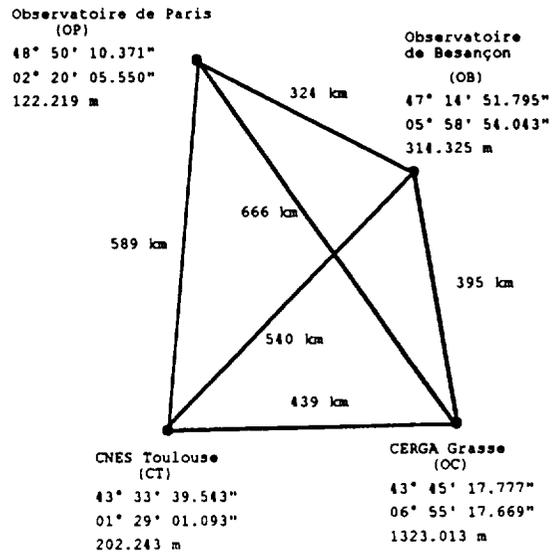
Figure 4 : Location of the stations

The influence of the variations of the different orbital elements on the measurements varies according to the relative geometry of the stations involved in the measurements. Table 2 shows the maximum error generated by the maximum authorised variations (in the sense of station keeping) of each orbital element for each of the 6 links of our experiment. The values obtained for e_x (resp. h_x) are of course the same for e_y (resp. h_y). We see that the main source of error are the mean longitude and the inclination vector. These values of $\dfrac{\partial p_{ij}}{\partial L}\Delta L$ confirm the ones obtained in a different way [1]. Table 2 also confirms the influence of the geometry of a given link on the partial derivatives (figure 4).

Because of a mistake on error estimation, the first simulation results are irrelevant and have to be recomputed.

Nevertheless, a separate study lead by the CNES demonstrates that when using the data from 4 calibrated stations, these elements can be determined with the imprecision given in table 3 [6], assuming an uncertainty on the measurements of a given link of 100 ns. In the near future, we expect to lessen this value to about 10 ns by performing a calibration of the 4 receiving sets.

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Errors	a (m)	e_x (10^{-6})	e_y (10^{-6})	h_x (10^{-3}deg)	h_y (10^{-3}deg)	L (10^{-3}deg)
4 days	1.4	8.1	8.0	1.2	1.1	2.5

Table 3 : Theoretic imprecision on the orbital elements.

CONCLUSION

We have presented the different methods we used to reduce the influence of the satellite motion on time delay determination. A data processing algorithm based on the application of an extended Kalman filter has been proposed and is currently being tested.

It could appear to be necessary to include in the model some known orbital perturbations, particularly those concerning the mean longitude L and to perform better calibration of the receiving sets [1], as this appears in [6] to be the most important source of uncertainty on the measurements, and then, in the determination of the satellite orbit.

A large amount of results have been obtained very recently by the CNES [6] and will be of great help in the continuation of the experiment.

ACKNOWLEDGEMENTS

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