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NOISE AND DRIFT ANALYSIS OF NON–EQUALLY SPACED TIMING DATA

F. Vernotte, G. Zalamansky
Observatoire de Besançon
41 bis Avenue de l'Observatoire
BP 1615, 25010 Besanqn
Cedex — France

E.Lantz
Laboratoire d'Optique P. M. Duffieux
U.A. CNRS 214
Facult des Sciences et des Techniques
25030 Besanqn
Cedex — France

Abstract

Generally, it is possible to obtain equally spaced timing data from oscillators. The measurement of the drifts and noises affecting oscillators, is then performed by using a variance (Allan variance, modified Allan variance, Time variance) or a system of several variances (multi-variance method[1, 2]*).*

However, in some cases, several samples, or even several set of samples, are missing. In the case of millisecond pulsar timing data, for instance, observations are quite irregularly spaced in time. Nevertheless, since some observations are very close together (1 minute) and since the timing data sequence is very long (more than 10 years), information on both short–term and long–term stability is available. Unfortunately, a direct variance analysis is not possible without interpolating missing data.

We used different interpolation algorithms (linear interpolation, cubic spline) to calculate variances in order to verify that they do neither lose information nor add erroneous information. A comparison of the results of the different algorithms will be given in the paper.

Finally, we adapted the multi–variance method to the measurement sequence of the millisecond pulsar timing data: we calculated the responses of each variance of the system for each type of noise and drift, with the same missing samples as in the pulsar timing sequence. An estimation of precision, dynamics and separability[1] *of this method will be given in the paper.*

INTRODUCTION

The time stability measurement of oscillators is well known in the case of signals composed of equally time–spaced data[1, 2, 3, 4]. However, in some cases, e. g. the millisecond pulsar timing data, the sequence of time error measurement is not regularly spaced. It is then necessary to reconstruct a sequence of equally time–spaced data from the original sequence. The aim of this paper is the study of different ways of sequence reconstruction.

1. DIFFERENT METHODS OF SEQUENCE RECONSTRUCTION

The goal of the reconstruction is to get N equally time–spaced data from M irregularly spaced data without losing information or adding information.

From M data we measure τ_0, the smallest interval between 2 consecutive timing data. This smallest interval τ_0 is the basic interval between all consecutive data of the reconstructed sequence. N, the new number of data, is then equal to the duration of the sequence divided by τ_0. Actually, in order to easily perform Fast Fourier Transform over the reconstructed data, we choose as N the first power of two greater than the duration of the sequence divided by τ_0. Let us define τ_0 the date of the first sample of the sequence and τ_{M-1} the date of the last sample of the sequence, N is given by the relationship :

$$N = 2^{\mathrm{int}\left(\log_2 \frac{(t-M-1-t_0)}{\tau_0}\right)+1} \quad (1)$$

Generally, the available data are time error $x(t)$ measurements between the oscillators and a reference oscillator. However, the time stability is mainly studied from the instantaneous normalised frequency deviation samples $\bar{y}_k$, obtained from the $x(t)$ data by the relationship:

$$\bar{y}_k = \frac{x(t_k + \tau) - x(t_k)}{\tau} \quad (2)$$

Reconstructing equally spaced data from the $x(t)$ data or from the $\bar{y}_k$ samples yields different ways of reconstruction.

1.1. Reconstruction by linear interpolation of the $x(t)$ data

This first method (see Fig. 1, left) keeps the same $\bar{y}_k$ samples as in the original irregularly spaced sequence. The only difference this method yields, is the division of each initial $\bar{y}_k$ sample into several τ_0–long samples with the same value. Thus, the added information is the constancy of the frequency deviation during the initial samples.

1.2. Reconstruction of the $x(t)$ data by cubic spline functions

Obviously, the real frequency deviation $y(t)$ is not constant over the time interval of each initial $\bar{y}_k$ samples. In order to avoid this hypothesis of constant samples within each initial sample, it is possible to fit the $x(t)$ data with cubic spline functions (see Fig. 2, right). The new τ_0–long samples vary smoothly while preserving the same average over the initial samples. The added information is then an hypothesis of continuity (and derivability) of the $\bar{y}_k$ samples, due to the continuous variation (derivability of second order) of $x(t)$.

Although the $x(t)$ samples are strongly correlated for the low frequency noises, the hypothesis of continuous variation of the $x(t)$ samples is completely wrong in the case of a white noise! Since the types of frequency noises can vary from f^{-3} (only in the case of millisecond pulsars[5, 6]) to f^{+2}, i. e. from f^{-5} to f^{0} phase noises, this method may be justified only for correlated $x(t)$ data, but not in the case of a white phase noise (f^{+2} frequency noise).

1.3. Reconstruction by linear interpolation of the $\bar{y}_k$ samples

On the other hand, it is possible to reconstruct directly the $\bar{y}_k$ samples by linear interpolation. The new yk sequence is then continuous but not derivable. The $x(t)$ sequence is obtained by the relationship :

$$x(t_k + \tau_0) = x(t_k) + \tau_0 \bar{y}_k \tag{3}$$

In this case, the $x(t)$ function is only derivable once. However, the hypothesis of continuity of the $\bar{y}_k$ samples is wrong in the case of a white frequency noise (f^{-2} phase noise) or higher frequency noise. This method may only be applied to low frequency noises.

1.4. Reconstruction of the $\bar{y}_k$ samples by cubic spline functions.

Theoretically, this method could only be justified for very low frequency noises (f^{-3} frequency noise) ; nevertheless we decided to observe the behaviour of such a method for all the types of noises in order to confirm our theoretical considerations.

2. Analysis method

2.1. Use of the multivariance method

The multivariance method uses a system of several variances, calculated for several integration values τ, over the same signal [1, 2]. The results are the most probable (in the sense of the least squares) set of h_α noise coefficients and drift coefficients. Moreover, this method yields an estimation of the confidence interval of each coefficient.

In order to study the influence of the reconstruction way by the multivariance method, we generated several sequences of 8192 simulated $x(t)$ data. Each of these sequences was composed of one only pure noise (one sequence of f^{-3} frequency noise, ..., one sequence of f^{+2} frequency noise). Then, we removed a lot of data according to a real pulsar timing sequence : we kept only 167 irregularly spaced $x(t)$ data from the 8192 ones (see Fig. 2).

2.2. Responses of variances for the different reconstruction method

Figure 3 shows the responses of the modified Allan variance[7] for the different types of noises and for a linear frequency drift. On each graph, the response of this variance for one type of noises with equally time-spaced data (continuous line) is compared with the responses obtained with the different reconstruction methods. Actually, each curve is the average of the results for 100 different realizations of these noises.

For f^{-3} frequency noise and linear frequency drift, the reconstruction from the $\bar{y}_k$ yields curves closer to the reference curve (corresponding to equally spaced data) than the curves due to the reconstruction from the $x(t)$ data. In these cases, the $\bar{y}_k$ samples are strongly correlated with their neighbours and the smoothest reconstruction methods provide the best results. Moreover, for τ values greater than $50\tau_0$, which is about the ratio of 8192 over 167, the different curves converge to the reference one.

However, for the higher frequency noises, the curves corresponding to the reconstruction by linear interpolation of the $x(t)$ data, remains the closest to the reference curves. The only important difference is visible in the case of f^{+2} frequency noise : although the slope is the same as the one of the reference curve, the variance measurement are about 100 times greater than the reference variance measurements. We may also notice that the curves corresponding to the reconstruction by cubic spline functions of the $x(t)$ data are not very far from the curves corresponding to the reconstruction by linear interpolation of the $x(t)$ data.

On the other hand, the results given by the reconstruction of the $\bar{y}_k$ samples for f^0, f^{+1} and f^{+2} frequency noises always yield the same behaviour. Therefore, these 2 reconstruction methods should not be able to separate these 3 types of noises. Of course, interpolating high frequency noises by linear interpolation or, *a fortiori*, by cubic spline functions completely modifies the information about the initial data.

2.3. Generating a model of variance responses

In order to increase the sensitivity of the multivariance method, we used the results shown in Figure 3 as the theoretical responses of the different variances for the different types of noises and for the different reconstruction method. Thus, the determination of the noise and drift coefficients of a signal, will be obtained by minimizing the differences between the variance results for this signal and the new theoretical responses of variances.

Therefore, if we choose for instance the reconstruction by linear interpolation of the $x(t)$ for analysing a signal mapped as in Figure 2 (with 167 data obtained for the same date as in Figure 2), we will compare the variance results with the new model corresponding to this type of interpolation and not with the classical theoretical variance responses. Consequently, this method requires the calculation of the corresponding model for each irregularly spaced sequence.

3. Results and discussion

3.1. Results for pure noises

Figure 4 shows histograms of values obtained for 100 realizations of the same pure noises (only f^{-3} noise, ... , only f^{+2} noise) and for the different reconstruction methods. For the low frequency noises (f^{-3} and f^{-2} frequency noises), the different methods yield histograms similar to the reference one (in front), i. e. the histogram obtained with 8192 equally spaced data. The histogram corresponding to the reconstruction of the $x(t)$ data by cubic spline functions (3rd in order of depth) seems to be slightly better than the ones of the other methods.

For a f^{-1} frequency noise, the histogram corresponding to the reconstruction of the $\bar{y}_k$ by cubic spline functions (last in order of depth) is already larger as the other ones. These other histograms remains similar to the reference one.

Finally, for a f^{+2} frequency noise, the dispersion is very important for the different methods. Only the histogram of the reconstruction of the $x(t)$ by linear interpolation (2nd in order of depth) seems to be interesting.

3.2. Results for a signal composed of all types of noises

Figure 5 shows histograms obtained for 100 realizations of a signal composed of 6 types of noises:

$$S_y(f) = h_{-3}f^{-3} + h_{-2}f^{-2} + h_{-1}f^{-1} + h_0 f^0 + h_{+1}f^{+1} + h_{+2}f^{+2} \tag{4}$$

with

$$h_{-3} = 1.2 \times 10^{-7};\ h_{-2} = 3.1 \times 10^{-7};\ h_{-1} = 0.002;\ h_0 = 0.031;\ h_{+1} = 0.25;\ h_{+2} = 1$$

With these values, each type of noise prevails over the other within an interval of the studied range of frequencies.

Although the f^{-3} noise is detected by all different methods, the measurement of the h_{-2} coefficient is difficult, even in the case of 8192 equally spaced data. The best method seems to be the interpolation of $x(t)$ by cubic spline functions (3rd in order of depth), because the number of non–null measurement is about 60%, and the maximum of the histogram is about the entered value. However, for f^{-1} and, *a fortiori*, for f^{+2} frequency noises, the measurement is almost impossible (from 70 to 90% of null measurement).

Conclusion

The results obtained for pure noises shows that the 4 methods are able to measure a signal over which a low frequency (from f^{-3} to f^{-1} frequency noises) prevails. For higher frequency noises (f^0 to f^{+2} frequency noises), only the methods of reconstruction of the $x(t)$ seem to be reliable.

On the other hand, in the case of a signal composed of 6 different noises with noise coefficients of equivalent levels, only the f^{-3} frequency noise can be determined by the 4 methods and sometimes the f^{-2} frequency noise by the spline reconstruction of the $x(t)$. It may appear that these results are poor for a classical oscillator measurement.

However, in the case of the millisecond pulsars, we are only interested in the very low frequency noises (f^{-3} and f^{-2} frequency noises). The interest of the millisecond pulsars is their great long term stability. Especially, the question is: does the stability curve of the millisecond pulsars will continue to go down versus time, under the estimated threshold of the International Atomic Time stability, or will it change of slope and go up because of f^{-2} or f^{-3} frequency noises? Perhaps is it already possible to answer!

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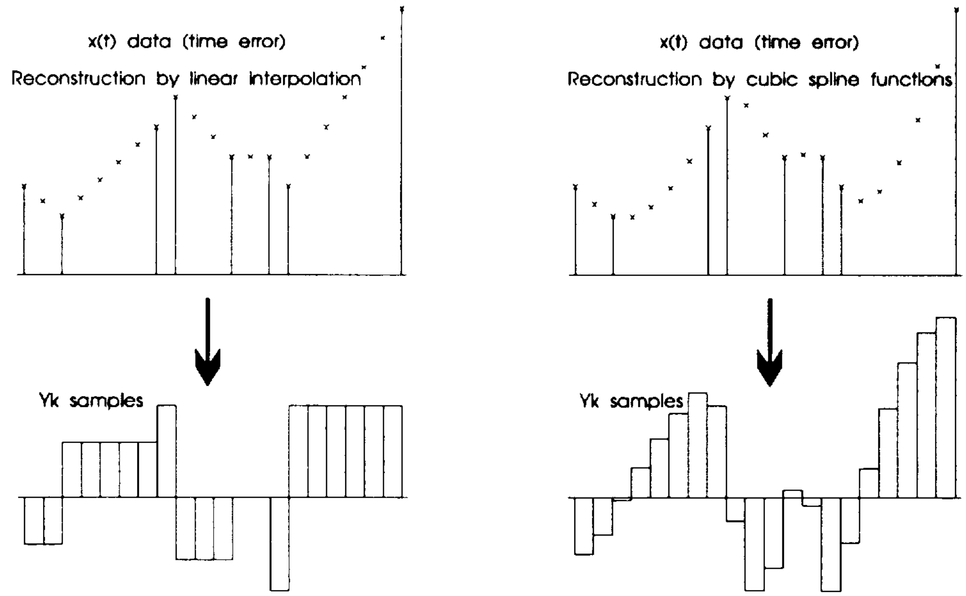

Figure 1 : Reconstruction by linear interpolation of the time error data (left) and by cubic spline functions of the time error data (right).

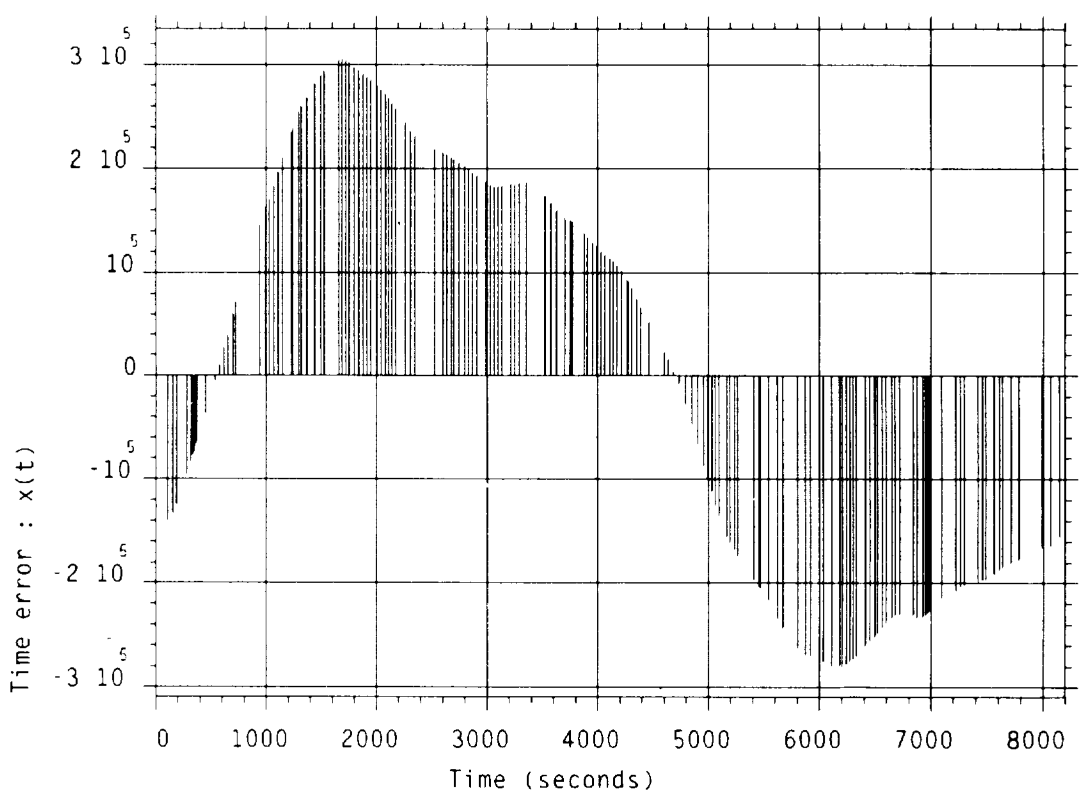

Figure 2 : Sequence of irregularly time-spaced data (167 data).

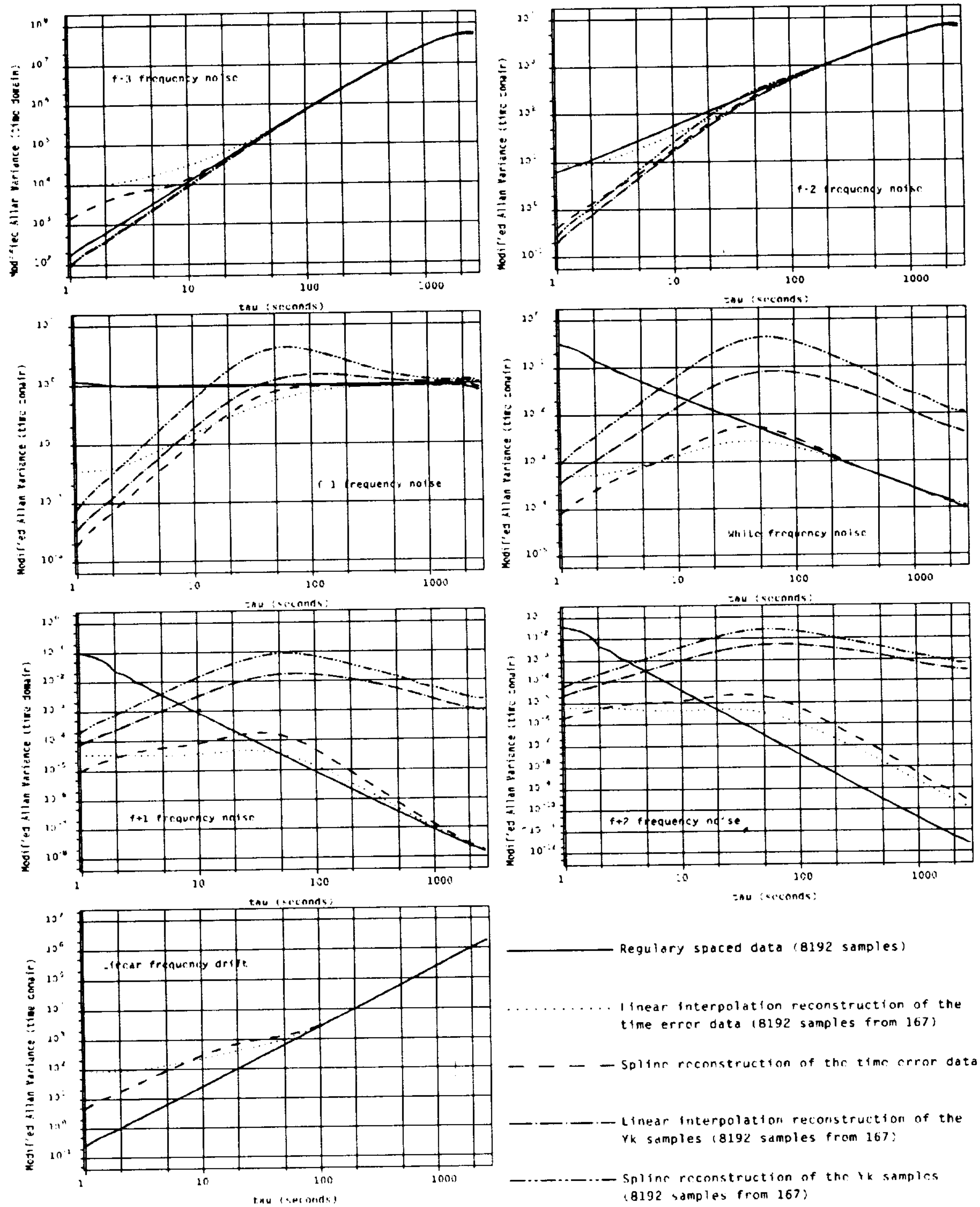

Figure 3 : Responses of the Modified Allan Variance for the different types of noises and for the different ways of réconstruction.

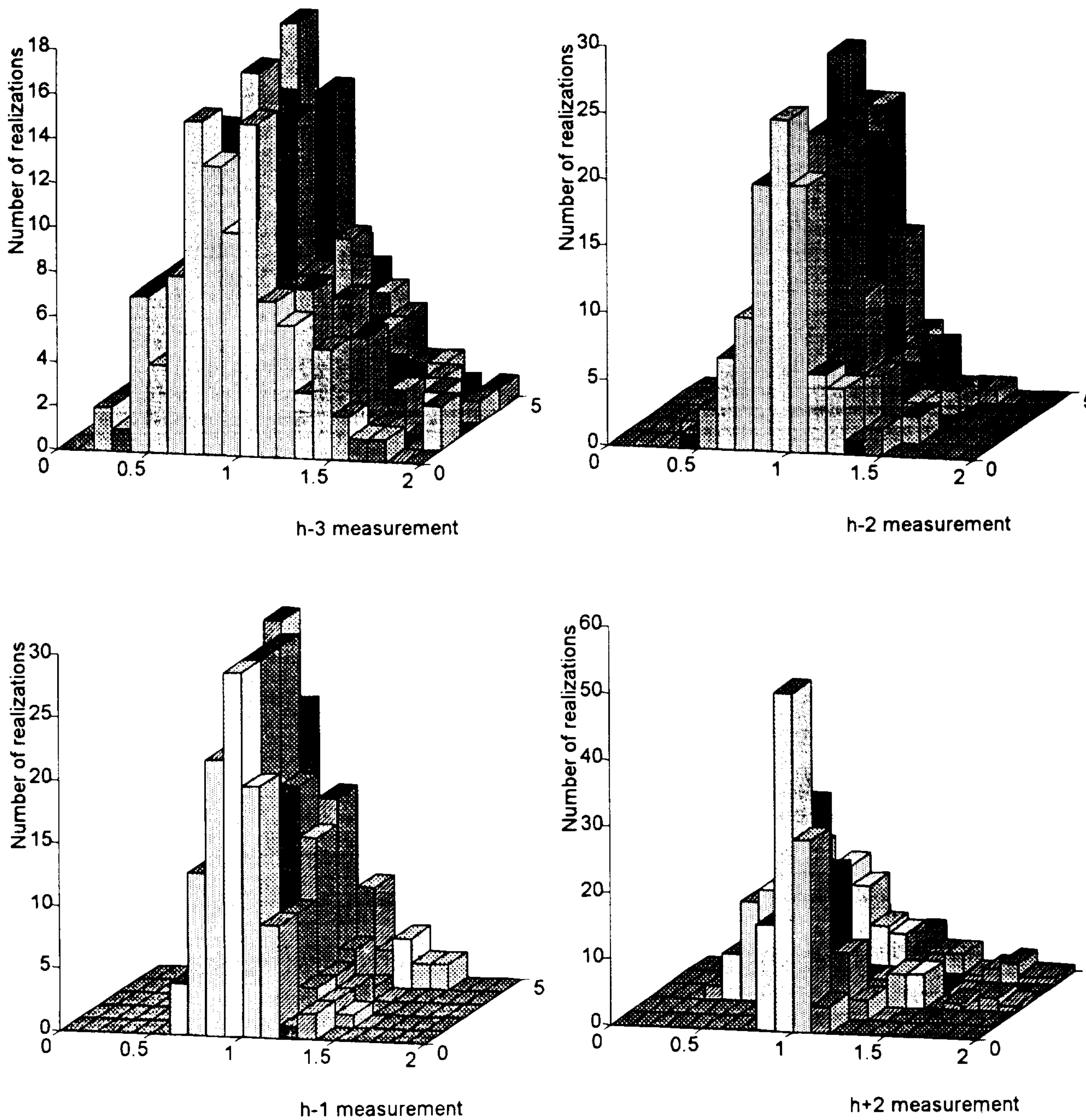

Figure 4 : Histograms of measurements of 100 realizations of pure noises. The first ones (in front) correspond to equally spaced data ; the second ones : reconstruction by linear interpolation of the x(t) ; the third ones : reconstruction of the x(t) by spline ; the fourth : reconstruction by linear interpolation of the $\overline{y_k}$ *; the last (in the back) : reconstruction of the* $\overline{y_k}$ *by spline.*

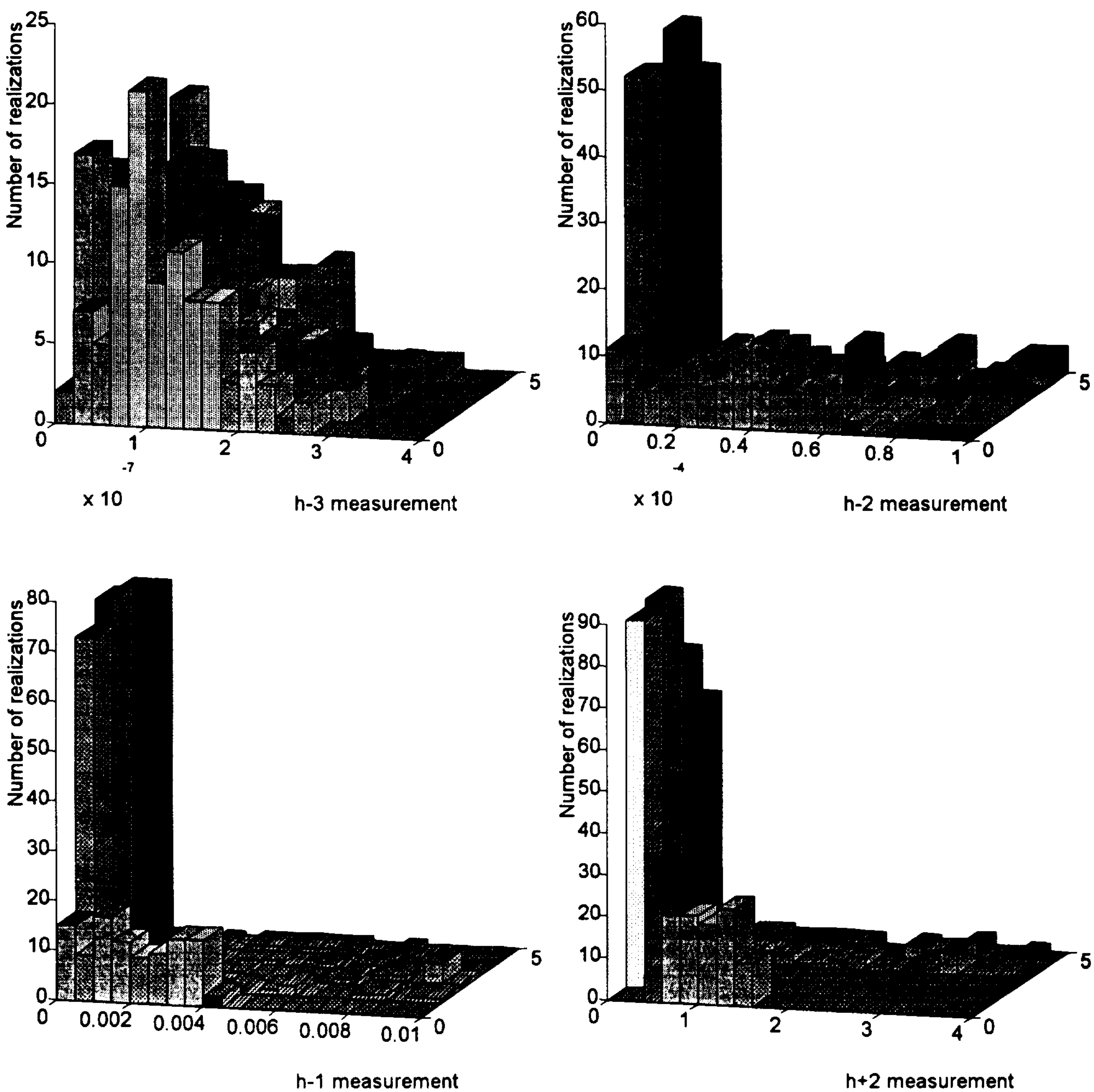

Figure 5 : Histograms of measurements of 100 realizations of a signal composed of 6 types of noises. The first ones (in front) correspond to equally spaced data ; the second ones : reconstruction by linear interpolation of the x(t) ; the third ones : reconstruction of the x(t) by spline ; the fourth : reconstruction by linear interpolation of the $\overline{y_k}$ *; the last (in the back) : reconstruction of the* $\overline{y_k}$ *by spline.*