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A PC PROGRAM TO OPTIMIZE SYSTEM CONFIGURATION FOR DESIRED RELIABILITY AT MINIMUM COST

P. 8

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ABSTRACT

High reliability is desired in all engineered systems. One way to improve system reliability is to use redundant components. When redundant components are used, the problem becomes one of allocating them to achieve the best reliability without exceeding other design constraints such as cost, weight, or volume. Systems with few components can be optimized by simply examining every possible combination but the number of combinations for most systems is prohibitive. A computerized iteration of the process is possible but anything short of a super computer requires too much time to be practical. Many researchers have derived mathematical formulations for calculating the optimum configuration directly. However, most of the derivations are based on continuous functions whereas the real system is composed of discrete entities. Therefore, these techniques are approximations of the true optimum solution. This paper describes a computer program that will determine the optimum configuration of a system for multiple redundancy of both standard and optional components. The algorithm is a pair-wise comparative progression technique which can derive the true optimum by calculating only a small fraction of the total number of combinations. A designer can quickly analyze a system with this program on a personal computer.

INTRODUCTION

Historically, systems have been designed and prototypes tested before the system reliability was analyzed. Some organizations use reliability engineers armed with component reliability data to analyze systems before prototyping, but the design engineer still does not have a tool to help in the preliminary design process. Customers of most modern systems expect a quantified reliability number and many engineers would like to design to specified reliability. In order for a designer to proceed toward a predetermined desired reliability, a tool for incorporating reliability analysis must be used early in the design process. The SYstem Reliability OPtimizer (SYROP) is such a tool.

There are several methods commonly used to analyze system reliability. The one chosen for this application is the reliability block diagram (RBD). The RBD shows the success paths for the system which are generated from the functional block diagram. The connectivity of the blocks is often the same as the functional block diagram. Development of the functional block diagram is one of the first steps in a system design. Therefore, the RBD can also be produced very early in the design. The reliability of a component is defined as the probability of it performing the required function under stated conditions for a stated period of time. For components with constant failure rates this can be expressed as $R=e^{-\lambda t}$, where λ is the statistical failure rate and t is the time. With this type of data and the RBD, SYROP can be used very early in the design process to analyze the system reliability and optimize the configuration of the system. The SYROP model is quickly executed on a personal computer (PC) and is easily changed to accommodate the progress of the design or to perform parametric studies.

System reliability can be improved in several ways. Some procedures are implemented after the system is designed and fabricated. These include scheduled inspection, testing, and preventative maintenance. Two ways to improve reliability in the design phase are to use more reliable components and to use redundant components. When a designer uses one or both of these methods, the problem becomes one of optimizing the allocation of redundant or improved components to achieve the necessary reliability without exceeding other design constraints such as cost, weight, or volume. For systems with few components and few options, this optimization can be simply performed by examining every possible combination. However, for most systems, the number of combinations is astronomical. For example, a system with 20 different components and 6 possible configurations for each component has 6^{20}, or 3.6×10^{15} possible combinations. That is why many researchers have derived mathematical formulations for calculating the optimum configuration directly. Some of the derivations have been based on continuous functions and others have attempted to account for the fact that component redundancy affects the system reliability in discrete increments. These approaches range from ones that simply rank the components in order of influence each one has on the overall system reliability to ones that attempt to optimize the redundancy allocation. Mohamed, Leemis, and Ravindran [1] provide a review of 62 papers and Tillman, Hwang, and Kuo [2] reviewed 144 papers describing techniques such as integer programming, mixed integer programming, dynamic programming, maximum principle, linear programming, geometric programming, sequential unconstrained minimization technique, modified sequential simplex pattern search, Lagrange multipliers, Kuhn-Tucker conditions, generalized Lagrangian function, generalized reduced gradient, heuristic approaches, parametric approaches, pseudo-Boolean programming, Hooke and Jeeves pattern search, and combinations of the above.

Designing for a specified reliability has also been discussed in the literature. Aggarwal [3] developed a method that would use the cost-reliability curve in the form of a mathematical function. Aggarwal and Sharma [4] proposed that the PC could be used with an incremental technique. Rao and Dhingra [5] developed two optimization techniques coupled with heuristic approaches to solve the mixed integer nonlinear programming problem of multistage systems. Numerous other papers can be found on the subject. The problem is that none of these approximations has produced a practical tool for the designer working with actual components that have distinct physical properties such as cost, weight, volume, and reliability.

A computer program is the obvious answer to analyze all the combinations and determine the true optimum solution. However, calculation of the system reliability and cost for the many (e.g. 3.6×10^{15}) combinations would take a PC several hours and a super computer is not readily available to most engineers. Therefore, an effort was made to develop an algorithm or procedure that would determine the true optimum redundancy allocation while minimizing at least one system constraint without having to calculate every combination. This program must be based on using actual component reliability and physical parameters available from vendor data or other engineering analyses.

PAIR-WISE COMPARATIVE, PROGRESSIVE DOMINANCE ALGORITHM

For this analysis, all component failures are assumed to be independent and redundant components are assumed to be in parallel, not standby mode. The reliability of the block (in the RBD) to which a redundant component is added will be increased by the second component functioning in parallel because failure of that block would require both components to fail. If the reliability of a component is R, the reliability of two identical components in parallel is $R' = 1 - (1 - R)^2$ and the reliability of three identical components in parallel is $R'' = 1 - (1 - R)^3$.

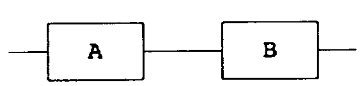


Figure 1. A Two-Component System.

Consider a system of two serial components whose RBD is shown in Figure 1. System failure would occur if either one failed because the success path through the RBD must include both A and B. The reliability of the serial system is the product of the reliability of each component; $R_S = R_A R_B$. In other words, the reliability of the two components in series is less than the reliability of either one alone. In order to increase the system reliability redundant components may be added to either A or B, components of higher reliability may be substituted for either A or B, or a combination of redundant and optional components may be used. SYROP is arbitrarily limited to adding no more than two redundant components to each block. Two redundant components implies that there are three identical components in parallel. This limit was imposed because more than three parallel components will not add significant reliability to a system unless the component reliability is very low. The number of optional components is limited to one. Therefore, with both standard and optional components available and with redundancies of 0, 1, or 2 possible for each, there are 6 potential configurations for each block of the RBD. Six configurations for each block results in 36 possible configurations for the two-component system.

Next, examine the system in Figure 1 with some assumed values of component reliability and cost. Assume that there are standard and optional components available for both A and B with values as listed at the top of Figure 2. The 36 possible combinations for this system are also shown in Figure 2 and the system reliability versus cost is plotted in Figure 3 with each point labeled by its configuration number. Investigation of this graph reveals that the sequence of 10 configurations connected by the dashed line "dominates" all other configurations. One configuration dominates another if it has higher reliability at no more cost or no less reliability at less cost. For instance, configurations 4, 10, and 19 have equal cost but 4 has higher reliability and is, therefore, the dominant one. Also, configurations 3, 16, and 22 have equal cost but configuration 5, which has lower cost, dominates because it has higher reliability. Kettelle [6] showed that, for a system of serial components, the complete set of dominant configurations is composed of subsets of the dominant sequences of subsystems of the whole system. It can also be shown that this holds true for all subsystems, whether they are in series or parallel, as long as each subsystem is either all parallel or all serial [7].

The pair-wise comparative, progressive dominance algorithm used in SYROP calculates reliability and cost for only two components or nodes at a time (hence, the term "pair-wise comparative") and progresses from the lowest level of the model to the top level of the model, selecting the dominant sequences at each node (hence, the term "progressive dominance"). Only the dominant sequence of configurations is used to calculate the reliability and cost of the next higher level. This allows the dominant sequence of the top level to be determined from a much smaller number of calculations than if all configurations were calculated. As systems increase in size, the required number of calculations may increase but the ratio of required number to total possible will decrease. With this algorithm a PC can analyze a system in seconds that would take hours if all combinations were calculated.

COMPUTER PROGRAM

The nodal model is developed by first dividing the system into serial "stages" where each stage is the smallest group of components that can be kept in series with other stages. Each stage is then broken into parallel "legs" of components and each leg is broken into serial "subsystems" and each subsystem is separated into parallel components. This will be illustrated by the following hypothetical example. Suppose that a system consists of twelve components and its RBD is drawn in Figure 4. The designer has made an initial selection of standard components with reliability and cost data as listed in Table 1. Seven of the components also have optional values to consider. This RBD has three serial stages. The first stage is only component 1, the second stage contains components 2 through 6, and the third stage is components 7 through 12. Each of these stages is the minimum set of components that can be grouped in series with each other. The first stage has no subsets, but the second stage has two parallel legs of subsystems and the third stage has three parallel legs of subsystems. The subsystems of all legs are single components except for the bottom leg of stage 2. The second subsystem in this leg is composed of two components in parallel.

It takes just a few minutes to input this data to SYROP and then it will determine the sequence of configurations that dominate all possible combinations. The combinations include six different configurations for each of the first seven components but only three configurations for components 8 through 12 because there are no optional devices available. Therefore, the total number of combinations is $6^7 \cdot 3^5$=68,024,448. Of course, the pair-wise comparative, progressive dominance algorithm does not make that many calculations and it takes an 80386 PC about four seconds to determine the dominant sequence and write the data files for this example. The first output data is a plot of the 82 dominant configurations as shown in Figure 5. The dominant sequence ranges from the initial configuration with no redundant or optional components (R_S=.8068, C_S=13) to the one with

		Reliability	Cost
Component A	Standard	0.80	100
	Optional	0.85	200
Component B	Standard	0.75	150
	Optional	0.85	250

36 Possible Configurations: S denotes a standard component and O denotes an optional component.

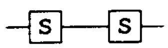
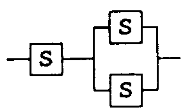
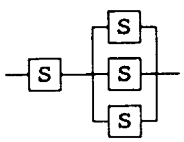
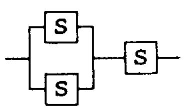
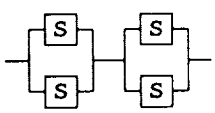
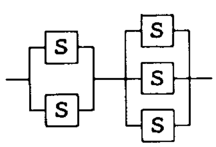
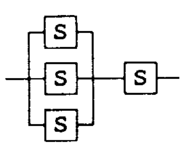
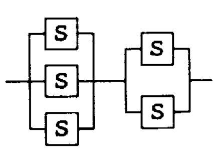
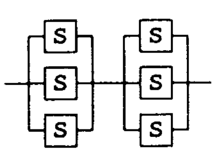
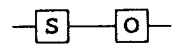
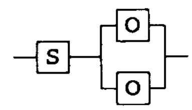
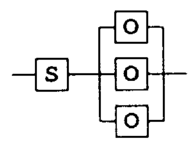
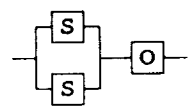
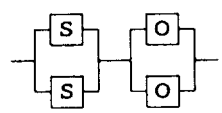
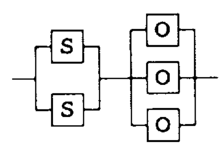
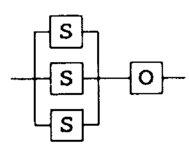
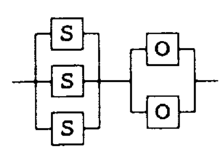
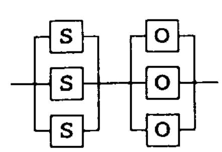
	Configuration	Reliability	Cost
(1)		.600	250
(2)		.750	400
(3)		.787	550
(4)		.720	350
(5)		.900	500
(6)		.945	650
(7)		.744	450
(8)		.930	600
(9)		.977	750
(10)		.680	350
(11)		.782	600
(12)		.797	850
(13)		.816	450
(14)		.938	700
(15)		.957	950
(16)		.843	550
(17)		.970	800
(18)		.989	1050

Figure 2. Possible System Configurations for the Two-Component Example

		Reliability	Cost
Component A	Standard	0.80	100
	Optional	0.85	200
Component B	Standard	0.75	150
	Optional	0.85	250

36 Possible Configurations: S denotes a standard component and O denotes an optional component.

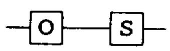
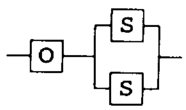
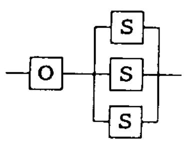
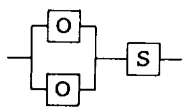
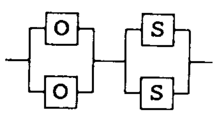
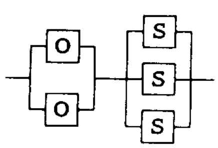
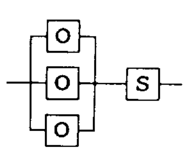
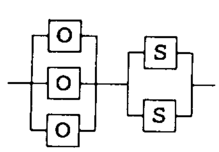
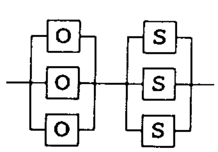
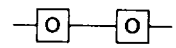
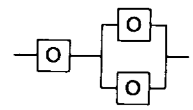
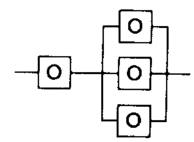
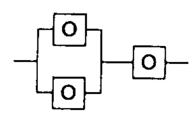
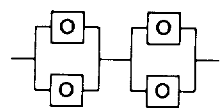
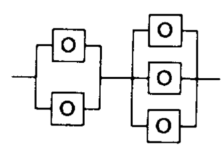
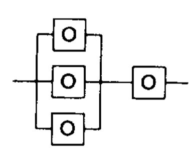
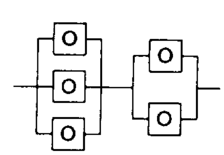
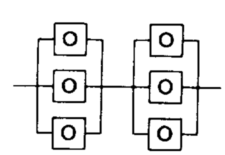
	Configuration	Reliability	Cost
(19)		.638	350
(20)		.797	500
(21)		.837	650
(22)		.733	550
(23)		.916	700
(24)		.962	850
(25)		.747	750
(26)		.934	900
(27)		.981	1050
(28)		.723	450
(29)		.831	700
(30)		.847	950
(31)		.831	650
(32)		.956	900
(33)		.974	1150
(34)		.847	850
(35)		.974	1100
(36)		.989	1350

Figure 2. (continued) Possible System Configurations for the Two-Component Example

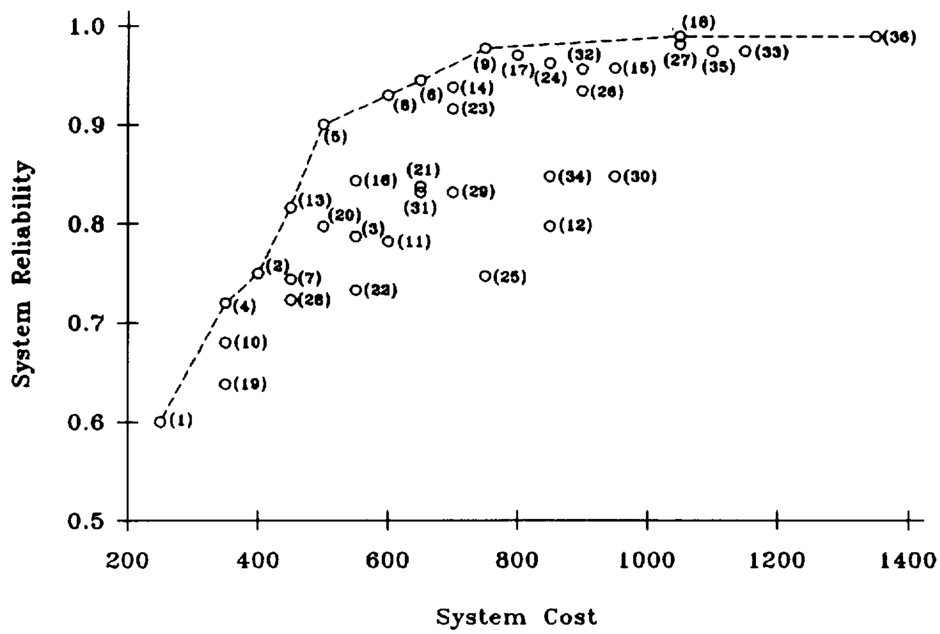


Figure 3. Reliability vs. Cost for the Two-Component Example.

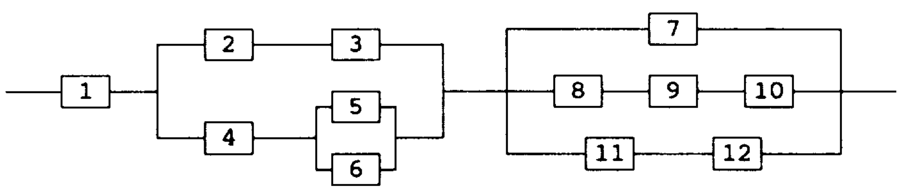


Figure 4. Reliability Block Diagram for the Hypothetical Example.

Component Number	Standard Values Reliability	Standard Values Cost	Optional Values Reliability	Optional Values Cost
1	.90	1.0	.95	2.0
2	.80	1.0	.90	2.5
3	.80	1.0	.95	3.0
4	.80	1.0	.85	1.5
5	.75	1.5	.80	2.0
6	.75	1.5	.80	2.0
7	.85	1.0	.95	2.0
8	.85	1.0	-	-
9	.90	1.0	-	-
10	.90	1.0	-	-
11	.80	1.0	-	-
12	.85	1.0	-	-

Table 1. Reliability and Cost Data for the Hypothetical Example

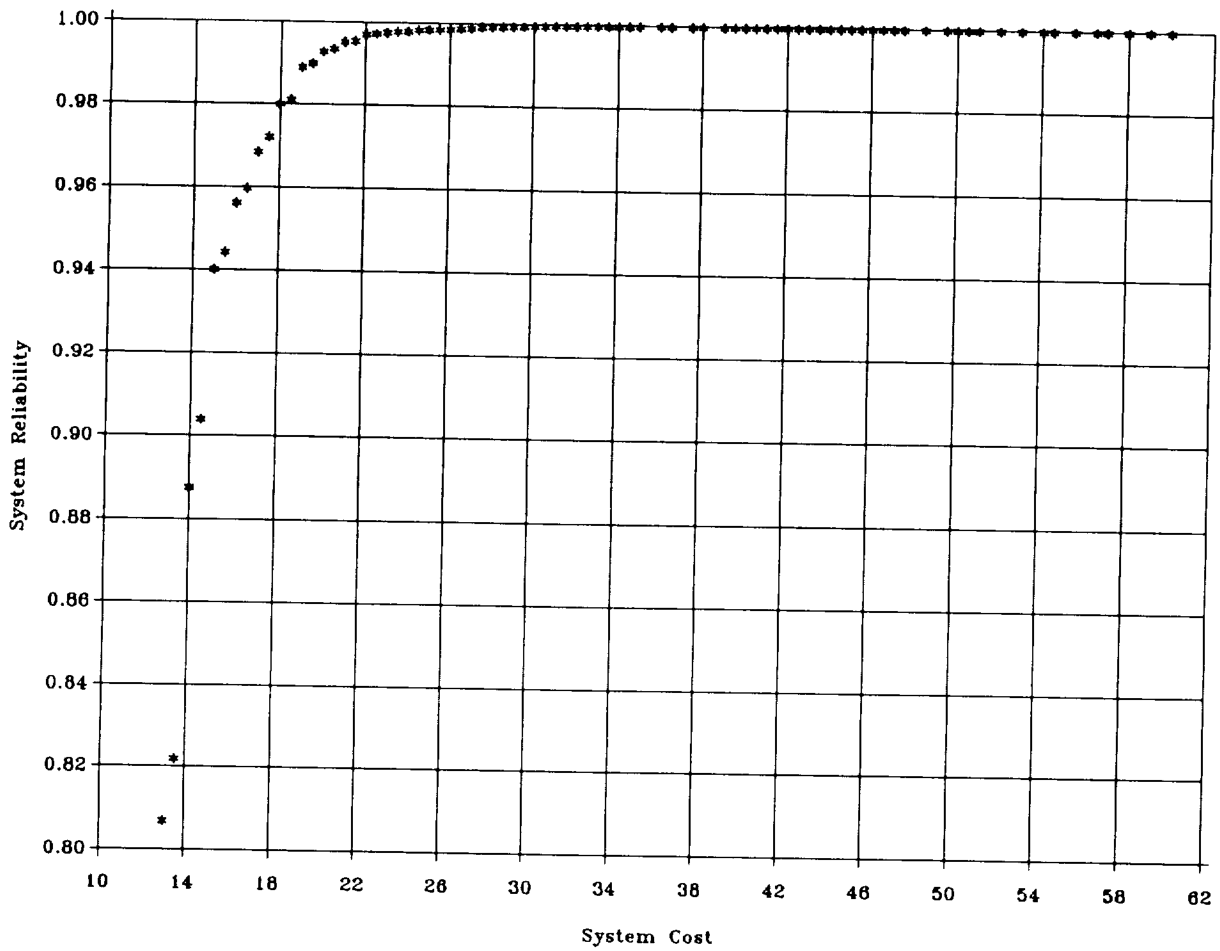


Figure 5. Reliability vs. Cost for the Dominant Configurations of the Hypothetical Example.

double redundancy for all components and optional ones where available (R_S=.9999, C_S=60). This picture very quickly illustrates the reliability/cost options for the system. SYROP then displays the system configuration details when the user inputs the desired system reliability. For example, if a desired reliability of 0.97 is selected, SYROP displays the configuration as shown in Figure 6. The calculated reliability of the system for this configuration is 0.972 and the cost is 17.5. This configuration includes redundant standard components for components 1, 2, 3, and 7 and a single optional component in place of number 4. The other components are unchanged. Additional values of desired reliability can be quickly investigated once the dominant sequence is determined. The user can also print a list of the dominant sequence from lowest reliability to highest reliability with the configuration of each component given. This allows the system designer to see any trends that may be in the system. The data in SYROP can be easily changed to make a parametric study of the system components.

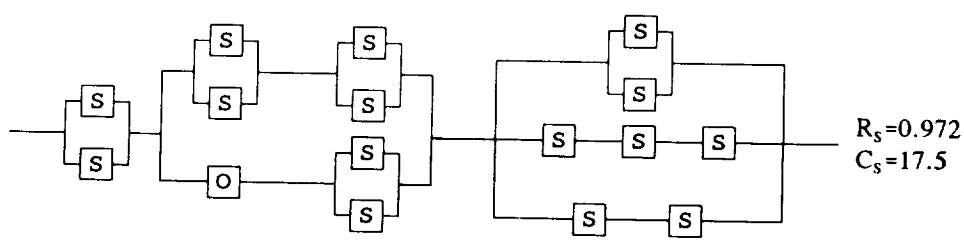


Figure 6. System Configuration of Hypothetical Example for Desired Reliability of 0.97.

FURTHER WORK

There are several things planned to improve SYROP. The most basic is the user interface. The data input graphics, file management, and information output are currently being enhanced. Other functional improvements that are under consideration include:

Limit redundancy for specific components. There may be physical constraints on some components of a system that do not allow redundant components or may allow only one redundant component. SYROP could have the option to set the redundancy limit for specific components.

Link two blocks of the RBD together logically. The functional block diagram of a system may dictate that the same block appears twice in the RBD. If this is the case, the configuration of both of these blocks would have to be the same in order to be physically correct.

Consider mixed redundancy. For some systems there may be an advantage to have a redundant component that is different than the initial component rather than forcing all redundant components to be the same, as SYROP currently does. This could be especially true for standby components.

Analyze standby systems. All calculations in SYROP currently assume pure parallel redundancy. An analysis technique for standby systems would be useful.

Analyze dual mode failure. Some components may have two modes of failure. For example, hydraulic valves or electrical switches may fail to close on demand or fail to open on demand and the time dependent or demand dependent failure rate is often different for each mode. A technique has been developed [7] to analyze dual mode failure and it may be possible to incorporate this into SYROP.

In addition to the ideas mentioned above, SYROP needs to be evaluated for several actual systems to determine its strengths, weaknesses, and applicability. Development will continue in order to establish this as a useful tool for design engineers.

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