N94- 35608

# High Earth Orbit Design for Lunar Assisted Small Explorer Class Missions*

M. Mathews and M. Hametz
Computer Sciences Corporation (CSC)
Lanham-Seabrook, Maryland, USA

J. Cooley and D. Skillman
Goddard Space Flight Center (GSFC)
Greenbelt, Maryland, USA

## Abstract

Small Expendable launch vehicles are capable of injecting modest payloads into high Earth orbits having apogee near the lunar distance. However, lunar and solar perturbations can quickly lower perigee and cause premature reentry. Costly perigee raising maneuvers by the spacecraft are required to maintain the orbit. In addition, the range of inclinations achievable is limited to those of launch sites unless costly spacecraft maneuvers are performed. This study investigates the use of a lunar swingby in a near-Hohmann transfer trajectory to raise perigee into the 8 to 25 $R_e$ range and reach a wide variety of inclinations without spacecraft maneuvers. It is found that extremely stable orbits can be obtained if the postencounter spacecraft orbital period is one-half of a lunar sidereal revolution and the Earth-vehicle-Moon geometry is within a specified range.

Criteria for achieving stable orbits with various perigee heights and ecliptic inclinations are developed, and the sensitivity of the resulting mission orbits to transfer trajectory injection (TTI) errors is examined. It is shown that carefully designed orbits yield lifetimes of several years, with excellent ground station coverage characteristics and minimal eclipses. A phasing loop error correction strategy is considered with the spacecraft propulsion system $\Delta V$ demand for TTI error correction and a postlunar encounter apogee trim maneuver typically in the 30- to 120-meters-per-second range.

## I. Introduction

The low Earth orbit environment presents significant problems for spacecraft that observe astronomical objects. The Earth is very bright and can scatter light into the telescope, so only objects toward the local zenith are unaffected. The apparent motion of the Earth soon causes the target to approach the bright Earth limb and target occultation soon follows. While in low Earth orbit the spacecraft experiences significant gravity gradient torques, atmospheric torques, and perhaps is exposed to radiation during passage through the South Atlantic Anomaly.

The target observability constraints severely limit the overall efficiency of the observatory. The environmental torques disturb the body orientation, which can blur the images. Radiation passages can cause a cessation of observing until the background count rate subsides.

Low Earth orbit is inevitable for observatories that optimize toward large optics or spacecraft complexity [as for the Hubble Space Telescope (HST)]. Smaller missions that concentrate on single instruments and modest apertures can take advantage of even small launchers to take them to higher orbits.

A higher orbit is optimal only if it can avoid Van Allen belt radiation, gravity gradient effects, and atmospheric torques. This implies that useful orbits should have perigee at or above the synchronous satellite altitudes (~6 $R_e$). At these altitudes, there is only rare exposure to the radiation belts (during solar storms), small gravity gradient effects, and no atmospheric torques. At high altitudes the trapped radiation effects are largely absent, as are the gravity gradient and atmospheric effects.

* This work was supported by the National Aeronautics and Space Administration (NASA)/Goddard Space Flight Center (GSFC), Greenbelt, Maryland, under Contract NAS 5-31500

Some missions (Far Ultraviolet Spectroscopic Explorer) have taken the middle path via highly elliptical orbits that spend a large fraction of time above 6 $R_e$ , but which do suffer repeated loss of observing time while passing through the radiation belts twice each orbit.

Our goal was to find a high Earth orbit (HEO) that optimizes for the following:

a) minimal atmospheric torques
b) minimal exposure to trapped radiation
c) minimal gravity gradient effects
d) minimal line-of-sight interference by the Earth
e) maximal observing time-on-target
f) minimal eclipse time in Earth or Moon shadow
g) minimal launch energy
h) mission lifetime greater than 1 year
i) data rate potential of at least 1 Mbs with modest RF system
j) maximal inclination options
k) minimal stationkeeping
l) single station support.

Goals a and c impact the fine pointing ability of the spacecraft. Goal k is not really necessary for HEOs considered here since the orbits can be maintained without stationkeeping. However, some stationkeeping capability is assumed necessary in case the ability to avoid eclipses or perform other contingency maneuvers becomes necessary.

High Earth orbits, here defined to be an orbit with perigee radius greater than 6 $R_e$, thus have the advantage of remaining above the radiation belts and generally providing excellent station coverage and communication links. Additionally, they can be advantageous for missions with constraints on the amount of time the spacecraft can spend in shadow. The relative freedom of the HEO from environmental disturbances (e.g., aerodynamic and gravity gradient torques) results in a stable observing platform with excellent pointing stability.

The principal disadvantages of traditional HEO concepts are that, typically, a $\Delta V$ on the order of 1 km/s or greater is needed to raise the perigee height of the transfer orbit to the final mission specific value and strong luni-solar perturbations can quickly drive perigee down and cause premature reentry unless the launch epoch is carefully chosen.

## II. Concept Proposed in This Paper

The work described here expands the concept of perigee raising through a near lunar encounter developed in Reference 1 and explores the use of modest capability launch vehicles ($C_3 \sim -2$ km$^2$/s$^2$) to place a spacecraft in a transfer orbit resulting in a low relative velocity lunar swingby targeted to meet other mission requirements than perigee height. Typical values of $C_3$ for the missions examined actually ranged from –2.2 to –1.8 km$^2$/s$^2$. An example of such a mission would be the proposed Prometheus mission with a Pegasus-XL launch vehicle and a spacecraft of approximately 100 kg. The Prometheus mission, which will conduct a wide field ultraviolet sky survey, uses a lightweight, three-axis, sub-arcsecond stabilized spacecraft in a 19 $R_e$ perigee, 57 $R_e$ apogee, 15-day period orbit. The high angular resolution of the Prometheus instrument makes pointing stability an important concern, and weight limitations necessitate minimizing spacecraft propulsion system demands. It will be shown that using a lunar encounter permits launch of useful payloads with small expendable launch vehicles while achieving the desired perigee height without placing the demand for a perigee-raising maneuver on the spacecraft propulsion system. In fact, once transfer trajectory injection (TTI) errors have been corrected, no further spacecraft $\Delta V$ is required until one of the first few postencounter perigee passages. This scenario permits the control of orbit parameters to meet mission objectives and proper phasing with the Moon. Additionally, apogee can be moved out of the ecliptic plane to minimize the impact of shadows. Relatively large inclination changes can be produced as well. In fact, a swingby can be tailored to place the spacecraft in a polar orbit about the Earth, a mission with an, as yet, unexplored potential to the Earth observing community.

In general, after the lunar encounter the spacecraft will be in an elliptical Earth orbit with apogee distance somewhat greater than the mean lunar distance. Without further adjustment this orbit would, at times, produce long lived but chaotic orbits that are subject to strong lunar perturbations or even close encounters with the Moon that can result in reentry or even expulsion of spacecraft from the Earth-Moon system.

If, however, the spacecraft and Moon are approximately aligned along the line of sight from the Earth to the Moon when the spacecraft is at perigee and a maneuver is performed to lower apogee so that the resulting orbit has a period equal to half the lunar sidereal period of revolution, then at each subsequent apogee, the Moon will alternately lead or lag the spacecraft by 90 deg. Since the strongest lunar perturbations occur when the spacecraft is near apogee, this phasing will keep the spacecraft far from the Moon at apogee and the perturbative effects at one apogee will tend to be cancelled at the next.

With the proper phasing, a half-lunar period orbit is extremely stable for intervals up to several years, requiring essentially no stationkeeping for orbit maintenance. The secular variation of the Keplerian elements is small so that rotation of the line of apsides to bring apogee into the southern hemisphere with consequent impact on station coverage (the spacecraft spends much time near apogee and station coverage is better from the northern hemisphere) can be avoided.

Figure 1a shows, schematically, as viewed from the north ecliptic pole, the sequence of events involved in establishing a HEO mission. Starting from TTI at 1, the spacecraft encounters the Moon at 2, followed by an apogee adjusting maneuver at 3 to establish a one-half lunar period mission orbit. An important parameter for stability considerations is the angle between the spacecraft to Moon and the spacecraft to Earth vectors at the postencounter perigee. This Earth-vehicle-Moon angle is 180 deg if the Earth, vehicle, and Moon lie along a straight line, but a 180-deg Earth-vehicle-Moon angle can only be achieved if the spacecraft is in the Earth-Moon orbit plane at P1. Figure 1b shows the same sequence as viewed in the Moon's orbit plane. For this particular example, the mission was designed to have the final orbit in the Moon's orbit plane in order to achieve an Earth-vehicle-Moon angle of as close to 180 deg as possible.

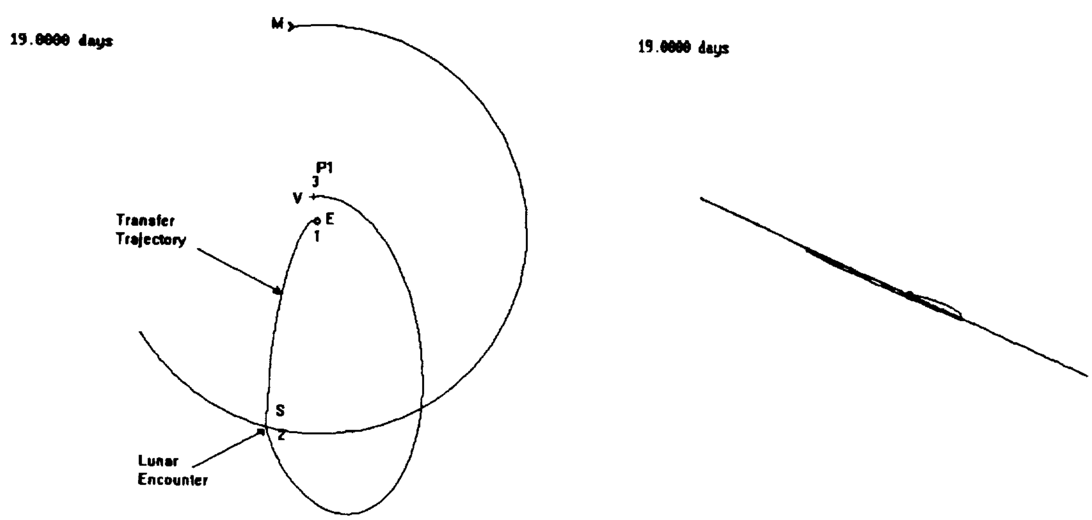


**Figure 1a: View From Ecliptic Pole** **Figure 1b: View in Plane of Moon's Orbit**

**Figure 1: HEO Transfer Trajectories to Mission Orbit**

The work summarized here assumes insertion from a circular parking orbit into a 5-day lunar transfer trajectory. Such slow transfers require the minimum capability from the launch vehicle and result in an encounter with low spacecraft inertial velocity. The low velocity encounter permits large changes in perigee height in the postencounter orbit while keeping apogee near the lunar distance. Avoiding a large postencounter semimajor axis (SMA) is desirable to minimize the subsequent $\Delta V$ needed to establish the one-half lunar period orbit.

Actually, the total mission $\Delta V$, including that provided by the launch vehicle is only less with the lunar assist if the final mission perigee radius is greater than a certain value. Below that limit, the lunar assist is more costly but, for all HEO, the lunar assist is the most economical means of establishing the orbit. Figure 2 compares the $\Delta V$ costs for our standard mission* using both a direct transfer to HEO (i.e., the spacecraft provides the $\Delta V$ for the perigee-raising maneuver ) and a lunar assist ( the Moon provides the $\Delta V$ for the perigee-raising maneuver ) targeted to different postencounter perigee radii. No attempt was made to target to a particular value of inclination and in both instances the initial state used was a 300-km circular parking orbit. For the particular example selected here, the lunar assisted transfer is less costly whenever the mission perigee radius is greater than approximately 10,000 km. For comparison, the $\Delta V$ needed to establish a geostationary orbit via a Hohmann transfer from our parking orbit is included. Even though the geostationary radius is only 6.67 $R_e$, its $\Delta V$ is significantly higher than that required to establish the HEOs because the geostationary orbit is circular.

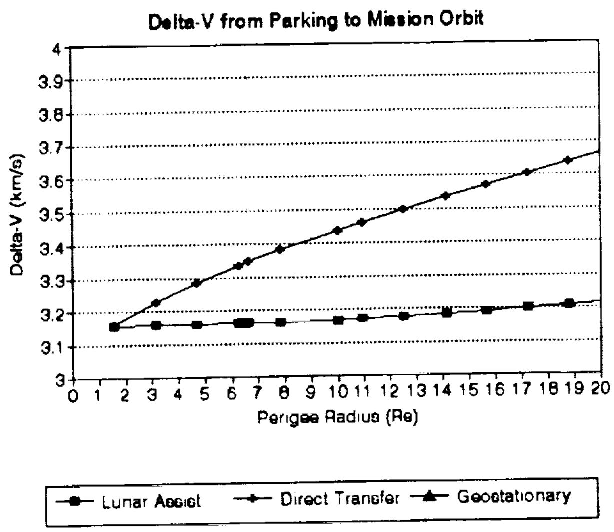


**Figure 2: $\Delta V$ To Achieve HEO From Parking Orbit**

Trajectories were modeled with the Mission Analysis and Design Tool, Swingby, using an eighth-order Runge-Kutta-Nystrom propagator with adaptive step size control through sixth order. Targeting to the initial lunar encounter conditions and then for various mission parameters (e.g., perigee radius and inclination) in the postencounter orbit was performed using the differential corrections targeting scheme in Swingby. For all calculations, point mass gravities were assumed for the Sun and Moon, and a 9x9 geopotential model was used for the Earth's gravity field. Solar radiation pressure and atmospheric drag were not modeled since their effects were found to be insignificant. Swingby was used to generate trajectory and ephemeris files which were then used as input to the Acquisition Data Program (ACQSCAN) to study shadows and station coverage.

## III. Dynamics of the Lunar Encounter

A typical transfer orbit to establish a HEO type mission is shown in Figure 3. The transfer orbit is a Hohmann transfer in the sense that it has just sufficient energy for the apogee to be at approximately the lunar distance. The transfer requires approximately 4.7 days flight time from perigee of the initial state to lunar encounter. It is important to realize that the HEO transfer requires no $\Delta V$ capability of the spacecraft at the lunar encounter and

* Most of the figures in this document were created with data generated from trajectories derived from an initial state described by the following osculating elements: Epoch UTC 1993/12/16 12:00:00.0, $a$=217831.93226 km, $e$=0.96934571, $i$=5.0 deg, $\Omega$=359.503048 deg, $\omega$=170.1790919 deg, $TA$=0 deg, Mean-of-J2000 Earth equatorial coordinates. The encounter parameters were changed slightly to vary mission parameters for the particular situation to be illustrated. This initial state is referred to as our "standard mission."

the mission orbit is not circular. The $\Delta V$ needed to raise the perigee to the final mission value is provided by a momentum exchange with the Moon.

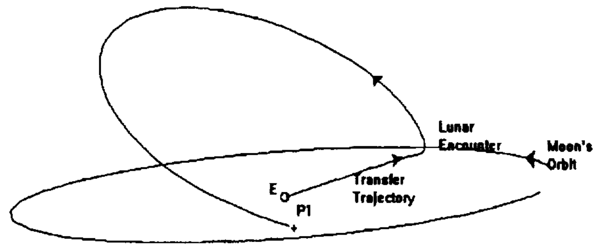


**Figure 3. HEO Orbit Transfer to First Perigee**

Although the results are not sufficiently accurate for detailed mission design, insight into the dynamics of the encounter can be gained by treating it as a classical two-body, zero sphere of influence collision of point masses. This approach is examined in some detail in Reference 7. Figure 4a shows the velocities of the spacecraft relative to an Earth-centered inertial reference frame just prior to the lunar encounter. In the Earth-centered inertial frame, the pre- and postencounter spacecraft trajectories are ellipses. In the selenocentric frame, however, the trajectory of the spacecraft is hyperbolic with an asymptotic approach velocity relative to the Moon given by

$$\boldsymbol{V}_{br} = \boldsymbol{V}_b - \boldsymbol{V}_m$$

where $\boldsymbol{V}_b$ is the velocity of the spacecraft in the Earth-centered inertial frame before the encounter and $\boldsymbol{V}_m$ is the corresponding Earth-centered inertial velocity of the Moon. $\boldsymbol{V}_{br}$ and $\boldsymbol{V}_{ar}$ are the spacecraft velocities just before and just after the lunar encounter relative to the selenocentric frame. Since the spacecraft energy is conserved in the selenocentric frame during the encounter, the asymptotic departure velocity is equal in magnitude to the approach velocity, but from Figure 4b, it is apparent that the relative velocity vector has been rotated by the encounter through the angle $\Theta$ in the plane of relative motion even though its magnitude is unchanged.

Since the spacecraft velocity in the Earth-centered inertial frame after the encounter is

$$\boldsymbol{V}_a = \boldsymbol{V}_m + \boldsymbol{V}_{ar}$$

it is obvious from Figure 4c that the encounter changes the spacecraft Earth-centered inertial velocity by $\Delta \boldsymbol{V}$ where

$$\Delta V = 2 * V_{br} * \sin(\Theta/2)$$

where

$$\Theta/2 = \arcsin(1/e) \ and \ e = 1 + R_p * V_{br}^2/\mu$$

For the HEO considered in this analysis, with encounter distances ~ 15000 km and relative encounter velocities ~ 1 km/s, typically $\Delta V$ of approximately 500-1000 m/s are obtained from the encounter. This is sufficient to raise perigee to 10–25 $R_e$.

It is instructive to consider two special cases. If the motion is such that the result is a pure rotation of the relative velocity vector about the Moon's velocity vector, then the magnitude of spacecraft inertial velocity is unchanged. Such an encounter would change the trajectory plane (inclination) but not the energy (SMA). If the encounter were to take place so that the relative velocity vector is rotated entirely in the initial trajectory plane, then the inclination is unaffected by the encounter. Usually, an encounter changes both the inclination and SMA and the change in each parameter can be controlled by appropriate B-plane targeting* of the incoming velocity asymptote.

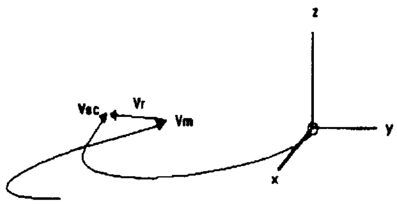


**Figure 4a**

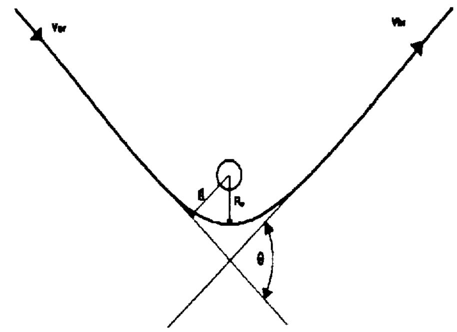


**Figure 4b**

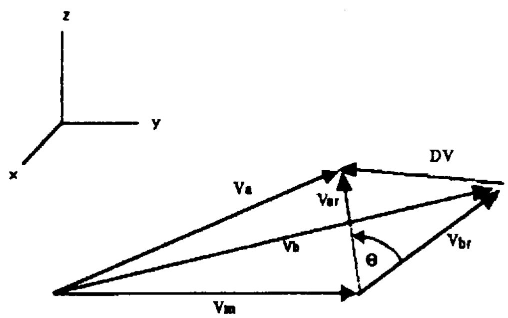


**Figure 4c**

**Figure 4. Relative and Inertial Velocities During Encounter**

Although the interrelationships of the orbital elements during the encounter are complicated, it is easy to show that the trajectory can be targeted to achieve the desired inclination to the Earth-Moon orbit plane and perigee radius by passing the Moon at the appropriate distance and height above or below the lunar orbit plane. The lunar swingby can be tailored to change the angular momentum and energy of the spacecraft to achieve the trajectory goals. Energy in the Earth-centered inertial reference frame is conserved but energy exchange between the Moon and spacecraft can produce large changes in the spacecraft orbit. A trailing edge swingby will add energy to the orbit, thus increasing the SMA and perigee height. The magnitude of the energy increase is controlled by appropriate choice of distance of closest approach. The closer the approach to the Moon, the greater the energy transfer and increase in SMA. The inclination of the postencounter trajectory is essentially controlled by varying the distance of the spacecraft above or below the Earth-Moon orbit plane while still achieving the targeted closest approach distance. Figures 5a and 5b show the behavior of the postencounter orbit inclination and perigee radius for the standard HEO as the targeted B-plane parameters are varied. To generate the data for Figure 5a, the lunar encounter was in the Moon's orbit plane, and only the miss distance was varied. In this case, the postencounter inclination changes only slightly while the perigee radius varies by more than a factor of 2. Conversely, in Figure 5b, the encounter distance projected in the lunar orbit plane was kept constant while the distance above or below

---

* A thorough discussion of B-Plane targeting parameters is given in Reference 8. The $\boldsymbol{B}$ vector is directed from the center of the targeted body to the point of closest approach the spacecraft would have if the target had no gravity. For targeting parameters, we frequently used the $R$ and $T$ components of $\boldsymbol{B}$ where $\hat{\boldsymbol{R}}$ is along the negative orbit normal and $\hat{\boldsymbol{T}}$ is perpendicular to $\hat{\boldsymbol{R}}$ and coplanar with $\boldsymbol{B}$ and $\hat{\boldsymbol{R}}$. In our cases, $\hat{\boldsymbol{T}}$ lies essentially in the Moon's equatorial plane and is positive toward the trailing edge.

the plane at encounter was varied. Clearly this primarily affects the inclination while leaving the postencounter perigee radius unchanged.

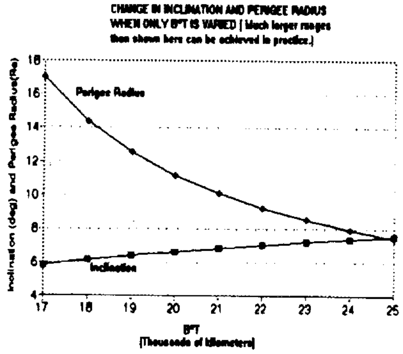


Figure 5a

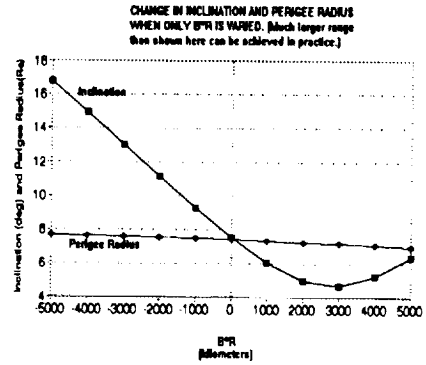


Figure 5b

**Figure 5. Variation of Inclination and Perigee Radius With B-plane Parameters**

The greater the number of elements that are targeted to specific values, the greater the difficulty in obtaining a suitable mission orbit. In general, it is not possible to target to an arbitrarily selected set of postencounter orbital elements and phase with respect to the Moon. This is a consequence of the constancy of the Jacobian integral in the restricted three-body problem.

This can be shown by considering the formulation of the Jacobian integral in the three-body problem commonly referred to as "Tisserand's Criterion for the Identification of Comets" (Reference 5). This can be expressed as

$$1/a+2 * \sqrt{a * (1-e^2)/p^3} * \cos(i) = \text{const}$$

where $p$ is the mean Earth-Moon distance, $a$ is the SMA, $e$ is the eccentricity, and $i$ is the inclination with respect to the lunar orbit plane. The constant retains approximately the same value in both the transfer orbit and the postencounter orbit. The expression can be written in terms of $i$, $a$, and initial perigee radius, $R_p$, by eliminating $e$ with the aid of $R_p = a(1-e)$. The end result is a constant of the motion involving only the SMA, inclination, and initial perigee radius. That this result constrains the achievable inclinations follows directly from the above equation. For a fixed postencounter perigee radius, the $1/a$ term contributes most for different choices of SMA. To achieve a high postencounter inclination, $\cos(i)$ should be small, which is most easily accomplished by decreasing the value of $a$. This was observed in the detailed mission design. For sequences of trajectories targeted to a fixed postencounter perigee radius, but with a range of inclinations, the postencounter SMA decreased with increasing inclination.

If particular values of perigee radius and inclination are selected from mission requirement considerations, then the SMA of the postlunar encounter orbit would be determined by the particular values selected. The resulting $\Delta V$ required to establish the final mission orbit of 241,000-km SMA after the apogee-adjusting maneuver would depend directly on the difference of the SMAs in the postencounter orbit and the final mission orbit.

In general, it will be found that if two of the mission parameters are fixed, then the choice of others is greatly restricted. For example, if perigee radius and Earth-vehicle-Moon angle are specified, then only a restricted range of inclinations may be achievable (fixing the Earth-vehicle-Moon angle is equivalent to choosing the postencounter SMA that determines the time of flight from encounter to $P1$). Conversely, precisely specifying perigee radius and inclination may make a viable Earth-vehicle-Moon angle unobtainable. Nevertheless, it is generally possible to find a wide range of orbits that meet mission objectives of lifetime and stability while, at the same time, having apogee rotated far enough out of the ecliptic plane and inclinations sufficient to minimize eclipses and maximize station coverage.

Typically we targeted pairs of elements. For example, perigee radius and inclination, or perigee radius and time of flight (phasing) to the first postencounter perigee were frequent choices for targeting parameters. Although most of the results presented in this paper are derived from a single example, many different trajectories and sets of mission elements were considered in Reference 4. In this study (Reference 4), the relationship between targeting various mission parameters and the required spacecraft capability are examined in detail for ecliptic inclinations ranging from 0 to 75 deg and postencounter perigee radii ranging from 8 to 20 $R_e$. By choosing the launch geometry and epoch appropriately, we have found it possible to achieve inclinations relative to the Earth equatorial reference frame of 98 deg. Higher inclinations may be possible. Orbits with equatorial inclinations this high are tantalizing candidates for Earth observing missions that may want to study the polar regions of the Earth synoptically. To date, the emphasis of Earth observing missions has been on low Earth orbits.

## IV. Error Correction Schemes

Detailed mission planning must consider and budget for maneuvers to correct TTI errors. Detailed analysis of the sensitivity of mission parameters such as perigee radius and Earth-vehicle-Moon angle to TTI errors is presented in Reference 4. It was found that the targeting of a typical HEO was quite sensitive to hot and cold burn errors along the velocity vector, but that a potentially useful HEO could still be reached without correction for pointing errors up to several degrees. Hot and cold burn errors of more than a few meters per second would probably need correction.

Two schemes for correcting TTI errors (hot and cold burns) are considered here. A transfer proceeding directly from the parking orbit in a transfer ellipse with apogee near the lunar distance and an encounter approximately 5 days after TTI is referred to as a "direct transfer." A transfer involving insertion into an orbit with apogee near the lunar distance in which the spacecraft completes one or more revolutions or "phasing loops" prior to the lunar encounter is referred to as a "phasing loop transfer."

*Direct Transfer Post-TTI Error Correction*

Figure 6 shows the $\Delta V$ needed to correct to the original targeted trajectory as a function of time from TTI for a representative mission assuming a 15 m/s under- or over-burn. Considering the previously demonstrated sensitivity to this kind of error, it is quite possible that correction may be necessary and, to avoid a sizable $\Delta V$ penalty (> 100 m/s after 20 hrs), the correction should be done soon after TTI; otherwise, a phasing loop mission scenario should be considered. It should be noted that corrections may not be as large or as critical as Figure 6 might seem to imply, because it may only be necessary to correct to any acceptable mission orbit and not necessarily to a precisely specified state. This concept is briefly considered in a subsequent section. There is considerable latitude if this is the case. An in-depth analysis of the various error correction scenarios should be explored in future studies.

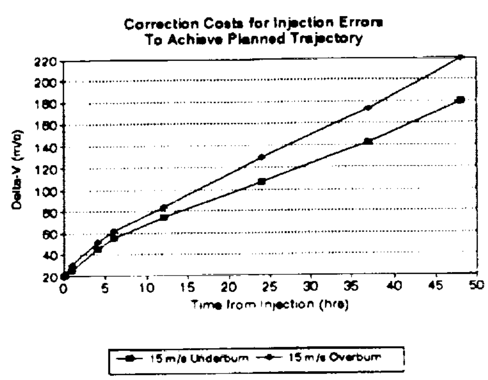


**Figure 6. Direct Transfer Error Correction Costs**

*Phasing Loop Transfer Error Correction*

Figure 6 indicates rapid growth with time from launch of the $\Delta V$ required to correct for hot or cold burns at TTI. Unless there are mission constraints to the contrary, correcting for TTI errors over two or more phasing loops could be a good strategy to avoid a potentially large fuel expenditure for error correction.

Essentially, a hot or cold burn results in a need for two corrections: phasing with respect to the Moon because the error results in early or late arrival at the lunar distance; and correcting energy so that the spacecraft has the proper velocity with respect to the Moon at encounter. A correction scheme using phasing loops would use burns at two perigee passes to correct the burn error. At the first perigee pass, a burn would be performed to adjust the period of the spacecraft so that the proper phasing can be regained by the time the spacecraft reaches the next perigee. At the second perigee pass, the spacecraft would then be given the proper energy for the encounter. Both the period change and the energy change can be accomplished by changing the size of the SMA.

To obtain a rough estimate of the total $\Delta V$ needed to implement this 2.5-loop phasing trajectory strategy, ± 15-m/s impulsive $\Delta V$ perigee burns were applied at the initial epoch of a 15 $R_e$ perigee, 50-deg initial inclination case to simulate hot/cold burn errors. The trajectories were then modeled with Swingby through the phasing loops with a period-changing maneuver performed at the first perigee to correct the arrival at the second perigee to the nominal epoch, and a final energy adjusting maneuver performed at the second perigee to correct to the proper energy for the lunar encounter. For the particular case and error correction strategy chosen, the total $\Delta V$ needed to correct for a 15-m/s overburn was found to be 65.8 m/s, while the amount needed to correct for the underburn was 36.0 m/s.

Although this was a simplified specific case, the results are consistent with results obtained for other missions such as the Geomagnetic Tail Laboratory (GEOTAIL) (Reference 6). It might be possible to reduce the $\Delta V$ by using other strategies, such as a 4.5-loop phasing trajectory strategy. For the purposes of this study, however, it seems reasonable to conclude that a phasing loop error correction strategy can be expected to require $\Delta V$ in the range of 20-80 m/s to correct TTI errors. In order to keep the direct transfer TTI error correction costs to this level, the corrections must be made within the first 10 hours after TTI. The disadvantage of the phasing loop strategy is that a time interval of approximately 35 days must elapse before the lunar encounter thus delaying the start of the science mission, as opposed to only 5 days if the errors are corrected during a direct transfer. This also results in some additional time being spent by the spacecraft in the radiation belts, but, because the spacecraft would be near perigee when this occurred, that time might not be excessive.

## V. Shadows

ACQSCAN was used to determine periods when the spacecraft is in either Earth or Moon shadow. The cumulative eclipse summaries presented in this section are for a sequence of missions with 8 $R_e$ perigee radius and ecliptic inclinations ranging from 0 to 70 deg that were analyzed in the course of the work done in preparing Reference 4. Figure 7 shows the total amount of time the spacecraft was in shadow during the 500-day interval covered by the ephemerides at each inclination. This was obtained by summing the individual eclipse durations for each 500-day mission at each inclination. No attempt was made to subdivide the totals into time spent in the umbral and penumbral regions. It is obvious from Figure 8 that eclipses become very infrequent when the orbit plane is tipped out of the ecliptic plane by only a small amount. Rotating apogee out of the ecliptic plane further reduces the likelihood of eclipses. The low-inclination orbits showed eclipses with durations of up to 10 hrs, but for inclinations above 20 deg, the longest eclipses observed were approximately 2 hrs, similar to the duration of total lunar eclipses.

It was initially thought that periods spent in the Moon's shadow might be significant. For the 0-, 10-, and 20-deg inclination cases, shadowing by the Moon for an interval of 36–42 min was observed approximately 5 days after launch during the lunar encounter. Only the 8 $R_e$, 10-deg initial inclination case showed any shadowing due to the Moon at other times, and this was a single 33-min interval approximately 4.5 months into the mission lifetime. No shadowing by the Moon at all was observed for the 8 $R_e$ perigee, inclinations greater than 20-deg cases.

One goal of the study described in Reference 4 was to determine whether it was possible to achieve a mission orbit with no eclipses during the first year. This is possible and is most easily achieved by choosing a mission orbit with an ecliptic inclination greater than 20 deg and apogee rotated out of the ecliptic plane. We targeted our standard orbit to an ecliptic inclination of 45 deg and a perigee radius of 25 $R_e$ and then propagated the mission for 365 days. Figure 8 shows this trajectory propagated from the initial epoch to a stop on the first apogee after $P1$. The inclination of the orbit and the elevation of apogee above the ecliptic plane are clearly shown. No Earth shadows were encountered during the propagation, although the spacecraft did spend one 92-min interval in the Moon's penumbra.

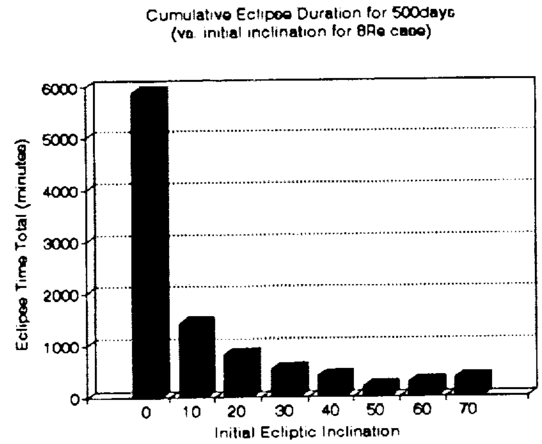


**Figure 7. Cumulative Eclipse Duration for a 500-Day Mission**

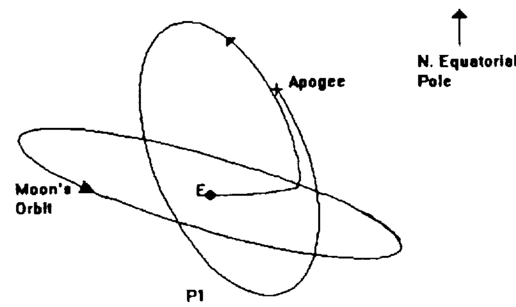


**Figure 8. HEO for Eclipse and Station Coverage Analysis**

## VI. Station Coverage

ACQSCAN was used in conjunction with ephemerides generated with Swingby in order to obtain contact times for the Wallops Island and Canberra satellite tracking stations. The ephemerides used were obtained by starting from the initial state of the standard mission and retargeting to generate a mission orbit with a 28 $R_e$ perigee radius and an Earth equatorial inclination of 70 deg. A plotting utility was used to graphically display the data for selected intervals during the mission lifetime. Figure 9 shows that the two stations selected complement each other quite well and generally provide from 20 to 24 hrs of combined coverage per day. The coverage provided by each station depends on the value of the argument of perigee (which determines whether apogee is located in the northern or southern hemisphere), the inclination, and the perigee radius (which determines the apparent angular rate of the spacecraft).

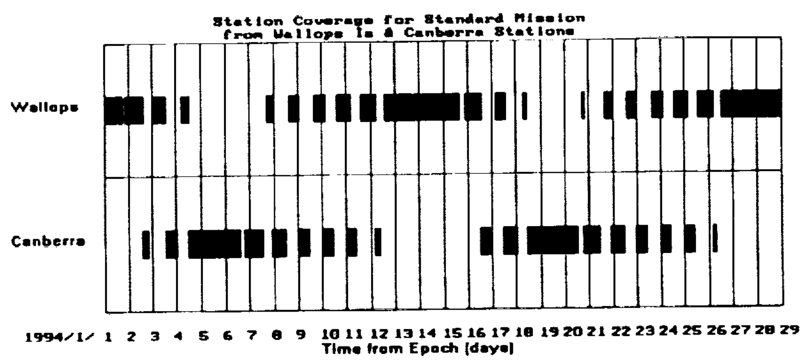


**Figure 9. Station Coverage for the Standard HEO Mission**

Because of its high altitude, even at perigee the spacecraft would be visible from approximately 92 percent of the hemisphere of the Earth facing it. Additionally, the apparent motion of the spacecraft is not great since, at perigee, its angular rate relative to the Earth-centered inertial coordinate system is only a few degrees per hour so that its diurnal motion is somewhat Moon-like.

Figure 10 shows the declination of the spacecraft over a complete orbit. Since Wallops Island is at a latitude of 38 deg, the spacecraft will be circumpolar when its declination is greater than approximately 52 deg. Thus for several days, the spacecraft will be continuously visible. From Canberra, at latitude 35 deg south, this situation will occur whenever the declination is south of approximately –55 deg. The longer contact times for Wallops are because, for this case, apogee is in the northern hemisphere and the spacecraft remains near apogee for a considerably longer period than near perigee.

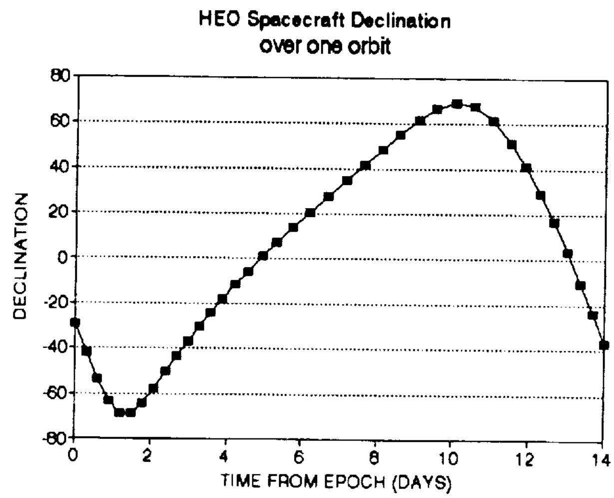


**Figure 10. HEO Spacecraft Declination vs. Time From Jan. 4.5, 1994**

The gaps in station coverage occur when the spacecraft is too far south from Wallops or north from Canberra to be seen. For two-station coverage, the choice of the Wallops Island and Canberra tracking stations is almost ideal since they complement each other extremely well. If tracking is to be done from a single station, an inclination could be chosen that avoids the coverage gaps and maximizes daily coverage for that particular station. In order to guarantee at least some coverage each and every day from Wallops Island, the equatorial inclination of the mission orbit would have to be less than 50 deg. Additionally, a lower perigee radius might be chosen to increase the time the spacecraft spends near apogee. This would shorten the periods of worst coverage.

## VII. Orbit Stability

Were it not for the lunisolar perturbations, the Keplerian elements of a typical HEO would be quite stable since secular perturbations due to the geopotential are inversely proportional to the SMA raised to the 7-halves power (Reference 5, pg. 290), and the HEO SMA is large. Since the HEO examined here are designed to minimize such perturbations, they should show generally small secular changes in the elements. Figures 11a through 11e show the time variation of the elements for a 500-day propagation from the apogee-adjusting maneuver to establish the one-half period orbit. It is apparent that the changes are small compared to those customarily found for low Earth orbits.

Note the small secular component of the variation of the elements because of the large SMA. The phasing of the HEO orbit essentially cancels the strong lunar perturbations that would otherwise cause large variations.

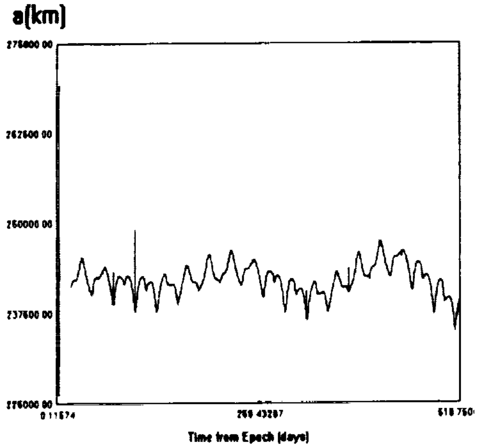


**Figure 11a**

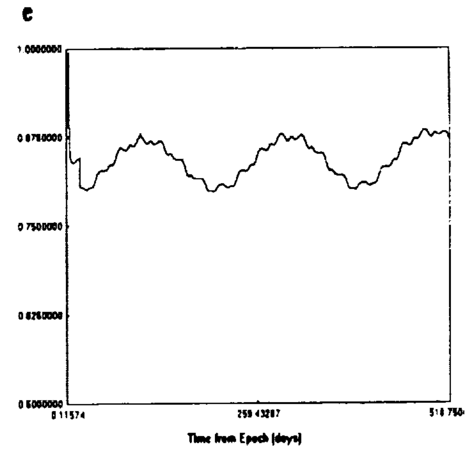


**Figure 11b**

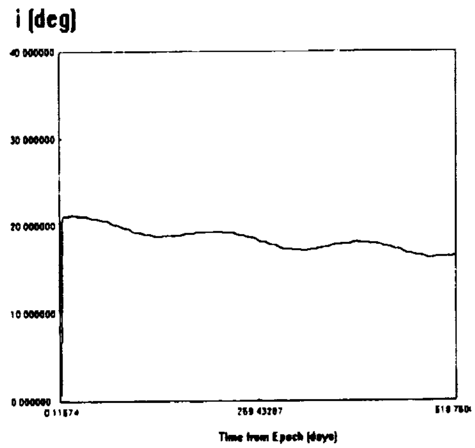


**Figure 11c**

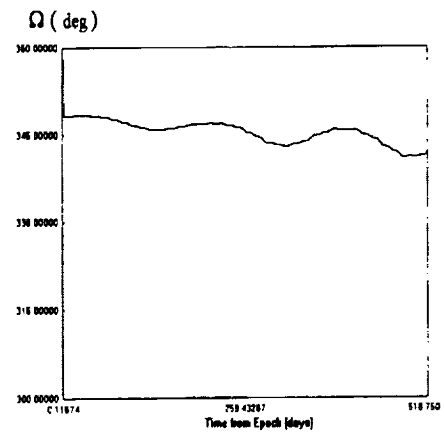


**Figure 11d**

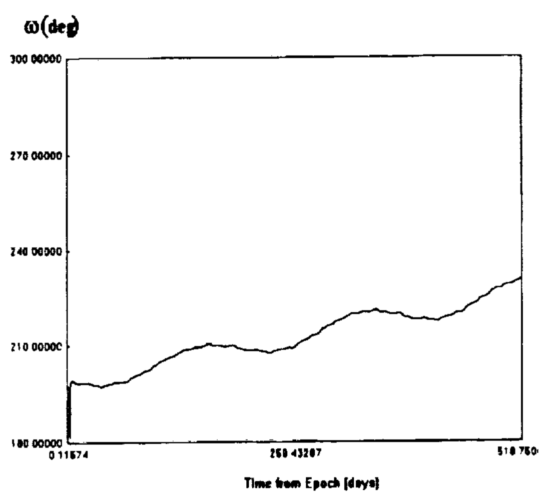


**Figure 11e**

**Figure 11. Time Variation of HEO Orbital Elements**

The long-term stability of a HEO is easily demonstrated by simply producing a trajectory plot of a propagation for the duration of the mission lifetime. Figures 12a and 12b show the trajectory propagated from the transfer trajectory through the apogee adjusting maneuver at the first post-encounter perigee and then for an additional 365 days. The time from TTI to *P*1 is very nearly 19 days in all cases where a maximum Earth-vehicle-Moon angle was achieved. Figure 12a is a view from the north ecliptic pole while Figure 13b shows the trajectory as seen in the plane of the Moon's orbit. This particular case was purposely targeted to achieve a final mission orbit coplanar with the Moon's orbit in order to obtain an Earth-vehicle-Moon angle as near 180 deg as possible. The greatest Earth-vehicle-Moon angle obtained for this case was 179.5 deg.

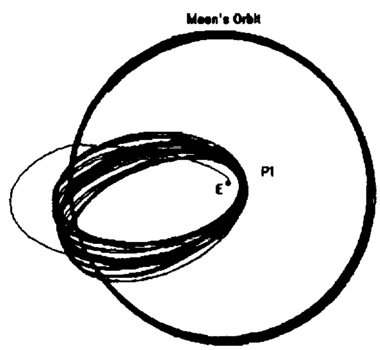


**Figure 12a**

**Figure 12b**

**Figure 12. 365-Day Propagation of the Standard HEO**

An interesting way of showing the stability of a HEO is to view the trajectory when propagated in an Earth-Moon rotating reference frame. Figure 13 is such a plot for the standard mission. Note that the spacecraft, when at apogee, clearly remains approximately 90 deg different in phase from the Moon for the entire mission duration.

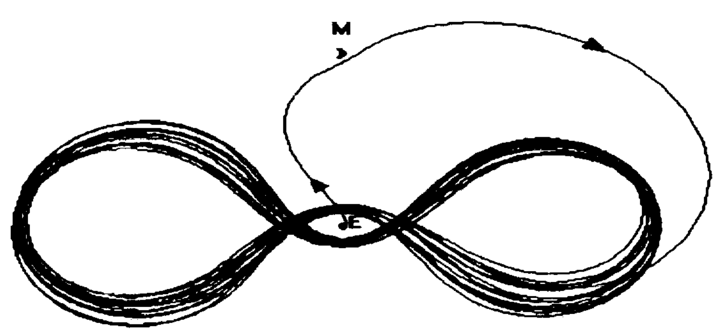


**Figure 13. HEO Propagation for 365-Day Mission in Earth-Moon Rotating Coordinates**

## VIII. Other Possible Harmonic Orbits

It is certainly possible to establish orbits with fractional lunar periods that are ratios other than 1:2. The work described in Reference 1 examined orbits that had the lunar encounter at less than the lunar distance and were, thus, strictly speaking not swingbys, and it was not fully realized at that time that a one-half lunar period orbit gave the most stable mission configuration. The orbits of Reference 1 were not required to remain above 6 $R_e$ perigee radius, so the lifetimes reported there were to impact and not really comparable to the ones studied here. Nevertheless, many of the desired HEO traits were obtained.

We looked briefly at orbital period ratios of 1:3 and 1:4 the lunar sidereal period. The 1:3 ratio orbit was not stable and its elements changed rapidly under the influence of lunar perturbations. The 1:4 ratio orbit was quite stable and could be a potential candidate for the appropriate mission. Figures 14a and 14b show the evolution of the 1:3 and the 1:4 ratio orbits in a co-rotating Earth-Moon reference frame for a 365-day propagation. The differences in stability are evident.

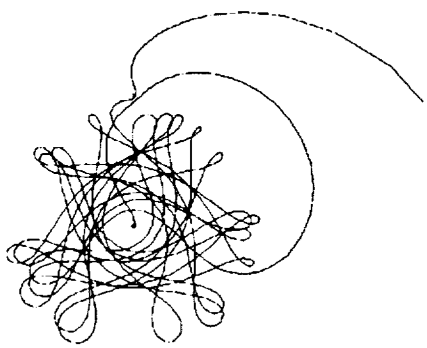

**Figure 14a. 1/3 Lunar Period HEO Orbit**

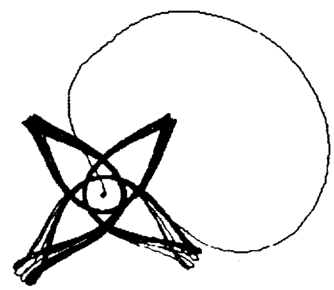

**Figure 14b. 1/4 Lunar Period HEO Orbit**

**Figure 14. 365-Day Propagations of Non-Half-Period HEO Orbits**

The $\Delta V$ required at $P1$ to adjust apogee to the appropriate mission orbit value are 43.5 m/s, 109.5 m/s, and 169.8 m/s for the 1:2, 1:3, and 1:4 orbits respectively.

## IX. Conclusion and Summary

We feel that the potential utility of high Earth orbits for future missions within the Earth-Moon system above the radiation belts has been convincingly demonstrated. Trajectories can be designed that show great stability for up to several years with no $\Delta V$ required either at the lunar encounter or for orbit maintenance maneuvers. The lunisolar perturbations that can cause dramatic changes in perigee height and other orbit parameters are effectively neutralized in these orbits. An additional benefit is that the long-term stability of the orbital elements for a properly designed trajectory can be used to enhance the already excellent ground station coverage in these orbits by maintaining apogee within a chosen hemisphere.

It would be presumptuous to say we had found orbits that optimize for all the parameters listed in Section I, but it seems clear that, with perigee radii near 100,000 km, the effect of environmental torques and the radiation belts should be minimal. With no $\Delta V$ required at the lunar swingby or for stationkeeping, and with $C_3$ near $-2$ km$^2$/s$^2$, there are no great demands on either launch vehicle or spacecraft propulsion system. Adequate coverage from a single groundstation can be obtained by a judicious choice of inclination, but two stations can provide nearly continuous coverage. An orbit was easily found that did not result in eclipses for a nominal mission lifetime of 1 year, but that does not imply that the orbit was stable for only 1 year. The same trajectory was propagated for 10 years and, while the node and perigee cycled slowly, the SMA and Earth-vehicle-Moon angle at perigee changed by only 0.6 and 6.0 percent respectively, demonstrating great stability of the mission orbit.

Although this and other work have established the feasibility of high-Earth type orbits and many of their general properties, much analysis remains to be done. Further investigations into optimizing high-Earth orbit parameters through choice of launch date and various Earth-Moon-Sun configurations are needed, because only a handful of launch epochs have been considered. In-depth error and phasing loop analyses need to be performed to refine estimates of required mission $\Delta V$ budgets. Required specific launch vehicle energy ($C_3$) should be investigated for a variety of initial Earth-Moon configurations to identify favorable launch geometries. The work documented in Reference 4 revealed an apparent correlation between perigee radius, inclination, and large Earth-vehicle-Moon angles. For larger perigee radii, the greatest angles occur for inclinations near 50 deg but lower inclinations tend to be favored for smaller perigee radii. Because the most stable orbits result from Earth-vehicle-Moon angles near 180 or 0 deg, this correlation should be investigated in greater detail and any dependence on launch date determined so that the most favorable launch dates may be identified.

## Acknowledgments

The authors wish to gratefully acknowledge the preliminary efforts and insights of Peter Sharer, Bob Sperling, Steve Stalos, and Jeff Zsoldos.

## References

1. Computer Sciences Corporation, CSC 53704 14, "HEO Orbit Analysis" (memorandum), S. Stalos, January 6, 1993

2. Computer Sciences Corporation, CSC 53704 49, "HEO Orbit Feasibility and Strategy Analysis" (memorandum), J. Zsoldos, and R. Sperling, June 17, 1993

3. Computer Sciences Corporation, CSC 53704 50, "Potential HEO Orbit Geometries" (memorandum), M. Hametz, J. Zsoldos, and R. Sperling, July 7, 1993

4. Computer Sciences Corporation, CSC 54704-08, "High Earth Orbit Study" (memorandum), M. Mathews and R. Sperling, December 30, 1993

5. A. E. Roy, *Orbital Motion*, p. 130, New York: Adam Hilger, 1988

6. D. W. Dunham and Shao-Chiang Jen, "A Launch Window Study for Geotail's Double Lunar Swingby Trajectory" International Astronautical Federation, 41st Congress Paper IAF-90-309, October 6-12, 1990

7. C. Uphoff, P. H. Roberts, and L. Friedman, "Orbit Design Concepts for Jupiter Orbiter Missions," *Journal of Spacecraft and Rockets*, Vol. 13, pp 348-355, June 1976

8. Goddard Space Flight Center, Flight Dynamics Division, 553-FDD-92/058R0UD0, *Mission Analysis and Design Tool (Swingby) Mathematical Principles, Release 1*, J. Carrico (CSC) and C. Schiff (CSC), prepared by Computer Sciences Corporation, July 1992