N94- 35620

Improved Solution Accuracy for TDRSS-Based TOPEX/Poseidon Orbit Determination*

C. E. Doll, G. D. Mistretta, and R. C. Hart
Goddard Space Flight Center (GSFC)
Greenbelt, Maryland, USA

D. H. Oza, D. T. Bolvin, C. M. Cox, M. Nemesure,
D. J. Niklewski, and M. V. Samii
Computer Sciences Corporation (CSC)
Lanham-Seabrook, Maryland, USA

Abstract

Orbit determination results are obtained by the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD) using a batch-least-squares estimator available in the Goddard Trajectory Determination System (GTDS) and an extended Kalman filter estimation system to process Tracking and Data Relay Satellite (TDRS) System (TDRSS) measurements. GTDS is the operational orbit determination system used by the FDD in support of the Ocean Topography Experiment (TOPEX)/Poseidon spacecraft navigation and health and safety operations. The extended Kalman filter was implemented in an orbit determination analysis prototype system, closely related to the Real-Time Orbit Determination System/Enhanced (RTOD/E)** system. In addition, the Precision Orbit Determination (POD) team within the GSFC Space Geodesy Branch generated an independent set of high-accuracy trajectories to support the TOPEX/Poseidon scientific data. These latter solutions use the Geodynamics (GEODYN) orbit determination system with laser ranging and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) tracking measurements.

The TOPEX/Poseidon trajectories were estimated for November 7 through November 11, 1992, the timeframe under study. Independent assessments were made of the consistencies of solutions produced by the batch and sequential methods. The batch-least-squares solutions were assessed based on the solution residuals, while the sequential solutions were assessed based on primarily the estimated covariances. The batch-least-squares and sequential orbit solutions were compared with the definitive POD orbit solutions. The solution differences were generally less than 2 meters for the batch-least-squares and less than 13 meters for the sequential estimation solutions. After the sequential estimation solutions were processed with a smoother algorithm, position differences with POD orbit solutions of less than 7 meters were obtained. The differences among the POD, GTDS, and filter/smoother solutions can be traced to differences in modeling and tracking data types, which are being analyzed in detail.

1.0 Introduction

This paper assesses the Ocean Topography Experiment (TOPEX)/Poseidon orbit determination accuracy of the Tracking and Data Relay Satellite (TDRS) System (TDRSS)-based orbit solutions using an operational batch-least-squares system and a prototype sequential orbit determination system within the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD). The TDRSS-based orbit solutions are compared with the high-precision orbit solutions obtained by the GSFC Space Geodesy branch using laser and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) tracking measurements.

TDRSS is a geosynchronous relay satellite network, which currently consists of five geosynchronous spacecraft and the White Sands Ground Terminal (WSGT) at White Sands, New Mexico. Of the five TDRSs, three (TDRS-East, TDRS-West, and TDRS-Spare, located at 41 degrees, 174 degrees, and 62 degress west longitude, respectively) actively support tracking of TDRSS-user spacecraft. Of the two remaining TDRSs, one TDRS (located at 275 degrees west longitude) is used only for satellite communications, while the other TDRS (located at 46 degrees west longitude) is being reserved for future use. TDRSS can provide 85-percent to 100-percent coverage, depending on spacecraft altitude.

The Bilateration Ranging Transponder System (BRTS) provides range and Doppler measurements for determining each TDRS orbit. The ground-based BRTS transponders are tracked as if they were TDRSS-user spacecraft. Since the positions of the BRTS transponders are known, their ranging data can be used to precisely determine the trajectory of the TDRSs.

* This work was supported by the National Aeronautics and Space Administration (NASA)/Goddard Space Flight Center (GSFC), Greenbelt, Maryland, under Contract NAS 5-31500.

** RTOD-E is a copyrighted product of Applied Technologies Associates, Incorporated (ATA).

The accuracy requirements on the Space Geodesy Branch Geodynamics (GEODYN) (Reference 1) orbit determination solutions, used to analyze the sea surface height measurements obtained by the TOPEX/Poseidon radar altimeter, are extremely stringent. The definitive orbit determination requirements for the TOPEX/Poseidon mission science data include a maximum 13-centimeter (1σ) radial position error. The accuracy of the precision orbit ephemerides (POEs) is being verified through the use of the TOPEX/Poseidon science data. Radar altimeter measurements over known overflight verification sites and the ocean surface are taken and then compared with coincident definitive TOPEX ephemerides generated using the ground-based laser and DORIS tracking. The resulting high-accuracy ephemerides are used to assess the accuracy of FDD-generated orbit determination solutions. The availability of the independent orbit determination solutions generated by the Space Geodesy Branch provides a unique opportunity to evaluate the accuracy of the orbit determination systems used by the FDD for operational navigation and analysis support.

This paper presents recent results of the TDRSS-based orbit determination accuracy analysis using the batch-least-squares method that is used for operational orbit determination support in the GSFC Flight Dynamics Facility (FDF). The batch-weighted-least-squares algorithm implemented in the Goddard Trajectory Determination System (GTDS) (Reference 2) estimates sets of orbital elements, force modeling parameters, and measurement-related parameters.

The sequential estimation algorithm is implemented in a prototype system, referred to as the Prototype Filter Smoother (PFS) filter. The PFS filter, which is closely related to the Real-Time Orbit Determination/Enhanced (RTOD/E) system (Reference 3), simultaneously estimates the TDRSS user and relay spacecraft orbital elements and other parameters in the force and measurements models at each tracking measurement time (Reference 4). It performs forward filtering of tracking measurements using the extended Kalman filter with a process noise model to account for serially correlated, geopotentially induced errors (Reference 4), as well as Gauss-Markov processes for drag, solar radiation pressure, and measurement biases. The PFS filter incorporates the same essential estimation algorithm as RTOD/E. It differs from RTOD/E in four significant ways: (1) the PFS filter executes on a mainframe computer whereas RTOD/E executes on a personal computer (PC); (2) the PFS filter lacks a maneuver model; (3) the PFS filter does not process one-way return Doppler TDRSS measurements; and (4) PFS includes a smoother and does not have a spacecraft antenna offset. The main features of RTOD/E can be found in Reference 5. To gain further insight into the comparison results, auxiliary sequential estimation solutions were generated with a smoother algorithm implemented in a system referred to as the PFS smoother. These solutions were compared with the POD solutions as well.

The estimated TOPEX/Poseidon ephemerides were obtained for the period November 7 through November 11, 1992. This timeframe was chosen because this period was relatively free of TOPEX attitude events and was well characterized through previous analyses (Reference 6). Independent assessments were made to examine the internal consistencies of results obtained by the batch and sequential methods.

This paper describes the POD solutions (Reference 7), describes the batch-least-squares and sequential orbit determination and evaluation procedures used in this study, provides an accuracy assessment of the POD solutions, describes the results obtained by the batch-least-squares and sequential estimation methods, provides the resulting consistency and comparisons with the POD solutions, and presents the conclusions of this study.

2.0 Analysis Procedures

This section describes the analysis procedures used in this study and provides a description of the tracking measurements and orbit determination and modeling methods.

2.1 Tracking Measurements

The TOPEX/Poseidon spacecraft was launched on an Ariane 42P expendable launch vehicle in August 1992. In October 1992, maneuvers were completed that moved the spacecraft into its operational orbit, which is circular with an inclination of 66 degrees, an altitude of 1336 kilometers, a period of 112 minutes, and a 10-day ground track repeat period. The time period chosen for this study was from 00:00 hours coordinated universal time (UTC) on November 7, 1992, through 21:33 hours UTC on November 11, 1992, which corresponds to the latter portion of the fifth 10-day ground track repeat cycle, hereafter referred to as Cycle 5.

Tracking measurements from TDRSS, used for TOPEX/Poseidon operational orbit navigation support by the FDF, were used to estimate the GTDS and filter definitive ephemerides. The GTDS orbit solutions were obtained using two-way range and one-way return and two-way Doppler data from TDRSS in addition to two-way range data from BRTS for estimation of the TDRS locations. The sequential estimation solutions were generated using two-way range and two-way Doppler data from TDRSS and BRTS, but no one-way return Doppler data were used. This restriction was necessary because the PFS filter/smoother combination, currently the only means available for studying smoothing processes, does not accommodate one-way return Doppler tracking measurements. The inability of the PFS filter/smoother to accommodate TDRS maneuvers imposed an additional restriction on the time period processed.

The tracking consisted of an average of 10 passes of one-way return Doppler measurements and 11 passes of two-way range and Doppler measurements per day, with the average pass lasting 40 minutes. During selected tracking passes, TOPEX/Poseidon science data are downlinked. A representative daily TDRSS tracking data distribution is shown in Figure 1. Passes labeled "2" consist of two-way range and Doppler measurements, while passes labeled "1" consist of one-way return Doppler measurements. BRTS tracking coverage of each TDRS spacecraft typically consists of twelve to fifteen 5-minute passes per day.

The POD team uses ground-based laser ranging and one-way forward Doppler measurements from the DORIS system to generate the POEs. The laser tracking data network consists of approximately 50 ground stations located around the world. Fifteen of these stations are specifically designated to support TOPEX/Poseidon tracking. Most of the stations are located in the United States, Europe, and Australia. For Cycle 5, 171 tracking data passes were taken from 25 laser tracking stations. A typical pass of laser ranging data lasts from 10 to 15 minutes.

The DORIS tracking system, developed by the Centre Nationale d'Etudes Spatiales (CNES), consists of a global network of approximately 50 ground-based transmitter beacons that provide one-way ground-to-spacecraft Doppler tracking measurements. During a typical 10-day cycle, tracking measurements are obtained using approximately 40 of these ground beacons, which generate a total of about 1300 tracking passes per cycle. For Cycle 5, 1071 tracking data passes were taken using 42 DORIS tracking stations. Each pass is approximately 10 minutes in duration.

2.2 Orbit Determination Methods and Modeling

This section describes the orbit determination methods and the modeling used to generate the POEs and the GTDS batch-least-squares, and sequential estimation TOPEX/Poseidon solutions and ephemerides.

2.2.1 Precision Orbit Ephemerides

The POEs are generated by the Space Geodesy Branch POD team using the GEODYN program. Each POE spans a 10-day period coincident with a project-defined beginning and end of a repeatable ground track cycle. GEODYN, like GTDS, uses a batch-least-squares estimation process to fit the tracking measurements and estimate a solution. The POE used in this analysis covers the period from 17:32 hours UTC on November 1, 1992, through 21:33 hours on November 11, 1992. This timespan corresponds to the fifth 10-day ground track repeat cycle. The POEs for Cycles 4 and 6 were also used for additional comparisons with the filter/smoother solutions.

The POEs used in this study represent the most refined POD solutions used to support the TOPEX/Poseidon science data. The quality of these POEs is discussed later in the paper.

The important force models and parameters used in the POE are given in Table 1. The TOPEX/Poseidon dynamic solve-for parameters consist of the TOPEX/Poseidon spacecraft state vector, one once-per-revolution along-track acceleration per day, one once- per-revolution cross-track acceleration per day, and one constant along-track acceleration per day. These

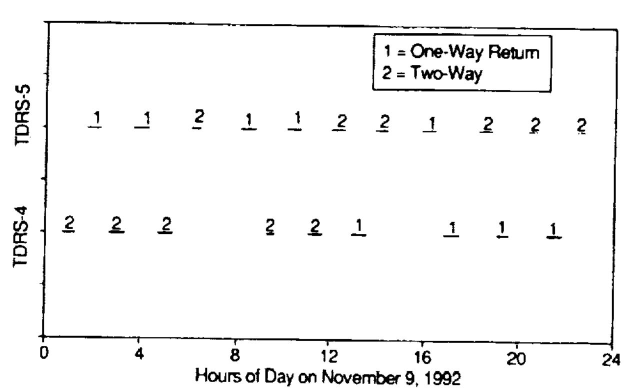

Figure 1. Typical TDRSS Tracking Scenario for TOPEX/Poseidon

Table 1. Force Modeling and Parameters Used in the POEs

Orbit Determination Parameter or Option	POE Values
Estimated parameters	Orbital state, along-track accelerations, cross-track acceleration
Integration type	11th-order fixed-step Cowell
Coordinate system of integration	True-of-reference
Integration step size	30.0 seconds
Tracking data	Ground-based laser ranging and DORIS data
Data rate	1 per 30 seconds
Differential correction convergence parameter	2 percent between iterations
Editing criterion	3.5 σ
Satellite area model	Box/wing model
Geopotential model	70 × 70 Joint Gravity Model-2 (JGM-2)
Atmospheric density model	Drag temperature model (DTM)
Coefficient of atmospheric drag	2.3
Coefficient of solar radiation pressure	1.0
Solar and lunar ephemerides	JPL Developmental Ephemeris-200 (DE-200)
Tropospheric refraction correction	Yes
Polar motion correction	Yes
Solid Earth tides	Yes
Ocean tides	Yes
Plate motion	Yes
Earth radiation pressure	Yes

once-per-revolution along-track and cross-track accelerations were introduced to better model an anomalous spacecraft body-fixed acceleration discovered shortly after launch. Atmospheric drag and solar radiation forces are applied but are not solved for. The constant along-track acceleration was introduced as an adjustment for atmospheric drag.

2.2.2 Batch-Least-Squares Estimation

The batch-least-squares estimation algorithm used by GTDS for this analysis is the same as that used for operational navigation support of the TOPEX/Poseidon mission by the GSFC FDF. The procedure used for operational support includes solving for the TOPEX/Poseidon spacecraft state, onboard ultrastable oscillator (USO) frequency bias and drift parameters, and an along-track thrust estimation parameter using two-way and one-way return Doppler measurements. TOPEX/Poseidon range measurements are excluded from the solutions because covariance analysis shows no improvement in accuracy and to avoid operational limitations in solving for uncorrected biases, which have been found to reduce the orbit solution quality. TDRS spacecraft trajectories are determined separately using the BRTS ranging and Doppler measurements.

The modeling and state estimation parameters used for this analysis have been modified and enhanced to provide more accurate results and to take advantage of modeling and techniques not currently in operational use. Specifically, the TOPEX/Poseidon state space was expanded to include estimation of the coefficient of solar radiation pressure in addition to multiple along-track thrust parameters that were intended to compensate for the anomalous acceleration acting on the spacecraft. Analysis of the operational TOPEX/Poseidon orbit solutions has indicated the presence of an unmodeled spacecraft body-fixed force with a day-to-day variability. Analysis performed by the Jet Propulsion Laboratory (JPL) has indicated that the unmodeled force is dependent on the angle between the orbit plane and the Sun (Reference 8). Consequently, in addition to an applied drag force, a series of thrust scale factors (referenced to a 1-micronewton continuous along-track thrust) was estimated.

TDRS Orbit Determination

TDRS spacecraft trajectories were estimated simultaneously with TOPEX/Poseidon using both BRTS range and TOPEX/Poseidon two-way range and two-way and one-way return Doppler data to determine the best possible TDRS trajectories for use in the TOPEX/Poseidon-only batch estimation. The modeling, data types, and other orbit determination options used for the TDRSs and TOPEX/Poseidon in the simultaneous solution are presented in Table 2. The data span chosen was 5 days, with one thrust correction factor per day. The simultaneous TDRS/TOPEX solution arcs were selected to avoid all maneuvers and angular momentum unloads, where possible, while maintaining the longest possible data spans. In addition, central angle editing was used to mitigate the effects of ionospheric refraction on the TDRS-to-TOPEX/Poseidon tracking link. The central angle chosen was designed to eliminate all data below the TOPEX/Poseidon local horizon.

Numerous transponder delay corrections were necessary to resolve biases between the BRTS and TOPEX/Poseidon range measurement types in the simultaneous solutions. These transponder delays included the individual transponder delays for each BRTS ground transponder and a transponder delay on each TDRS. In addition, a TOPEX/Poseidon spacecraft transponder delay correction value was applied to reduce the effects of ranging calibration errors on the TDRS and TOPEX/Poseidon orbit solutions.

Table 2. Parameters and Options Used in the GTDS Solutions

Orbit Determination Parameter or Option	GTDS Values*	
	TOPEX	TDRS-East, TDRS-West
Estimated parameters	Orbital state, thrust coefficients, coefficient of solar radiation pressure (C_R), USO bias and drift	Orbital state, coefficient of solar radiation pressure (C_R), spacecraft transponder delay, BRTS transponder delays
Integration type	Cowell 12th order	Cowell 12th order
Coordinate system of integration	Mean-of-J2000.0	Mean-of-J2000.0
Integration step size (seconds)	60 seconds	600 seconds
Tracking measurements	TDRSS two-way Doppler TDRSS one-way return Doppler TDRSS two-way range	BRTS two-way range
Data span	See text	See Section 3.2 of text
Data rate	1 per minute	1 per 20 seconds
DC convergence parameter	0.00005	0.00005
Editing criterion	3σ	3σ
Ionospheric editing criterion	Central angle greater than 79.48 degrees	–
Measurement weight sigmas	Doppler: 10 millihertz Range: 1.5 meters	2 meters
Satellite area model	Variable mean area model	Constant, 40 meters^2
Satellite mass	2417.2 kilograms	TDRS-5: 1973.1 kilograms TDRS-4: 1853.6 kilograms
Geopotential model	50 × 50 JGM-2	20 × 20 JGM-2
Atmospheric density model	Jacchia-Roberts	N/A
Solar and lunar ephemerides	DE-200	DE-200
Coefficient of drag (C_D)	2.3 applied	N/A
Ionospheric refraction correction Ground-to-spacecraft Spacecraft-to-spacecraft	Yes No (central angle edit instead)	Yes N/A
User-spacecraft antenna offset correction	Constant radial, along-track, cross-track	No
Tropospheric refraction correction	Yes	Yes
Polar motion correction	Yes	Yes
Solid Earth tides	Yes	Yes
Ocean tides	No	Yes
Plate motion	No	No
Earth radiation pressure	No	No

*JGM = Joint Gravity Model; N/A = not applicable

Application of at least a single BRTS transponder delay is necessary to prevent the orbit solutions from being ill-determined. Residuals analysis, supported by comparison with the precision ephemerides, indicated that the default WSGT BRTS transponder delays provided optimal TOPEX/Poseidon estimation. Estimation of the Alice Springs, Australia, BRTS site transponder delay was found to have little impact on the TOPEX/Poseidon estimation accuracy. The applied TOPEX/Poseidon transponder delay correction was modeled as a range bias and was determined based on an auxiliary solution where BRTS and TOPEX/Poseidon range measurement biases were estimated instead of the BRTS and TDRS transponder delays.

Topex/Poseidon Orbit Determination

After the TDRS trajectories were estimated in the simultaneous solution, they were applied in a TOPEX/Poseidon-only solution that used the one-way and two-way Doppler data only. This was done to minimize the effect of TOPEX/Poseidon range data bias modeling errors on the TOPEX/Poseidon trajectory. The span of this solution was only 4 days, and it was selected to reduce the dynamical modeling errors and to simplify the thrust estimation parameter selection. Force modeling for the TOPEX/Poseidon-only solution is the same as that used for the simultaneous solutions with TDRS (see Table 2), with the exception that only two thrust correction factors were estimated for the 4-day data span.

Solution Evaluation

Since adjoining and overlapping solutions were not calculated for this analysis, evaluation of the solution quality was performed based on comparison with the POEs and the final solution observation residuals. This was performed using average and standard deviation summary statistics and was shown graphically using plots of the individual data point residuals. Only data points that were not edited from the solution due to data validity flagging, 3σ editing, user editing, or central-angle editing were used in the evaluation of the residuals. However, the graphical evaluation did consider the possible need to use data eliminated by the 3σ editing, to preclude the elimination of potentially useful data that may have been edited as the result of a mismodeled bias.

2.2.3 Sequential Estimation

The improvement in the POE-filter comparison results brought about by application of a smoother was studied using Cycle 4, 5, and 6 POEs and the PFS filter/smoother. The general approach was to generate several PFS filter and smoother solutions for portions of Cycles 4, 5, and 6 and to compare these solutions with the respective POEs. The PFS filter was run for a period several days long, and a series of PFS smoother runs was made for progressively longer spans, each ending at the same epoch. Previously, sequential TOPEX orbit solutions for the same time period were generated using RTOD/E (Reference 6). A more realistic operating mode was achieved for these earlier solutions, for example, by processing for extended periods (more than 1 month) and by suspending RTOD/E execution at various points to accommodate maneuvers and adjust tuning parameters and, when necessary, for complete reinitialization. The PFS filter/smoother system is currently the only means to study the smoothing process, a paramount objective of the current study. Although limitations of the PFS filter/smoother (i.e., inability to process one-way TDRSS measurements and lack of a maneuver model) precluded replication of the RTOD/E solutions used in the earlier study, ephemeris consistency tests showed that solutions generated with the PFS filter were essentially reproducible with RTOD/E when common tracking data sets were used. Valid conclusions about the potential for improvement in RTOD/E solutions (for example, those discussed in Reference 6) could thus be drawn.

The filter was initialized for TOPEX, TDRS-Spare, and TDRS-West for October 22, 1992, 19:00:00 UTC, and run to October 27, 1992, 00:00:00 UTC (no two-way TOPEX measurement data were encountered until October 24, 1992, 03:32:00 UTC). The smoother was run from October 27, 1992, 00:00:00 UTC back to October 24, 1992, 00:00:00 UTC. This period is contained within Cycle 4. Although tracking measurements from both TDRS-Spare and TDRS-West were included, TDRSS and BRTS tracking measurements for TDRS-West that occurred after a TDRS-West maneuver at about 14:00:00 UTC on October 26, 1992, were rejected by the editing process due to the inability of the software to model maneuvers. A second filter run was initiated for TOPEX, TDRS-East, and TDRS-West for November 5, 1992, 00:00:00 UTC, and run for 14 days. This second processing period overlaps Cycles 5 and 6. For either run, two generic initial orbit RIC [radial, in-track (along-track), and cross-track] covariance matrices, one for TOPEX and one for the TDRSs, were used for each initialization.

Tables 3 and 4 provide detailed information on the models and options used for the filter smoother solutions. The solution state included orbital elements for TOPEX and each of two TDRSs. Other estimated quantities included a coefficient of atmospheric drag for TOPEX and a coefficient of solar radiation pressure for each of the three satellites.

Table 3. Parameters and Options for PFS Solutions

Orbit Determination Parameter or Option	PFS Filter/Smoother Values	
	TOPEX	TDRS-East/TDRS-West/TDRS-Spare
Estimated parameters	Orbital state, coefficients of drag and solar radiation pressure, TDRSS range and Doppler tracking measurement biases	Orbital state, coefficient of solar radiation pressure, BRTS range and Doppler tracking measurement biases
Integration type	Variation of Parameters (VOP)	VOP
Coordinate system of integration	Mean of 1950.0	Mean of 1950.0
Integration step size	60.0 seconds	60.0 seconds
Tracking data	TDRSS two-way range and Doppler	BRTS range and Doppler
Data rate	1 per minute	1 per minute
Editing criterion	3σ	3σ
Gravity error autocorrelation values	R: 2.828 minutes I: 0.001 minute C: 5.611 minutes Errors of omission and commission	Not applicable (N/A)
Measurement sigmas: Range Doppler	 0.50 meter 0.010 hertz	 0.25 meter 0.002 hertz
Gauss-Markov parameters: Drag half-life Drag sigma C_R half-life C_R sigma Range bias half-life Range bias sigma Doppler bias half-life Doppler bias sigma	 840.0 minutes 0.400 1440.0 minutes 0.200 60.0 minutes 6.0 meters 8 minutes 0.034 hertz	 N/A N/A 11520.0 minutes 0.200 60.0 minutes 7.0 meters 60 minutes 0.030 hertz
Standard deviation of the Earth's gravitational constant	0.005 kilometers^3/second^2	0.005 kilometers^3/second^2

Table 4. PFS Force and Measurement Model Specifications

Model or Options	PFS Values	
	TOPEX	TDRS-East/West
Geopotential model	GEM-T3 (50 × 50)	GEM-T3 (8 × 8) (truncated)
Atmospheric density model	CIRA 72*	N/A
Solar and lunar ephemerides	Analytic	Analytic
Coefficient of drag	Estimated with a priori value of 2.3	N/A
Coefficient of reflectivity	Estimated with a priori value of 1.25	Estimated with a priori value of 1.4
Ionospheric refraction correction	No	No
Tropospheric refraction correction	Yes	Yes
Antenna mount correction	No	No
Polar motion correction	Yes	Yes
Earth tides	No	No

*CIRA = Committee on Space Research (COSPAR) International Reference Atmosphere; GEM = Goddard Earth Model

A comparison between the filter, smoother, and the POEs, resolved in orbit-plane principal directions, provided the primary means of gauging the sequential orbit determination accuracy. The comparisons were performed in the J2000.0 true-of-date (TOD) coordinate frame. Other indicators of solution quality were provided by the diagonal elements of the state error covariance matrix (Reference 9), the integrity of the drag coefficient estimates, and an examination of the residual statistics.

3.0 Results and Discussion

This section presents the TOPEX/Poseidon accuracy assessment analysis results, an assessment of the consistency of the TOPEX/Poseidon ephemerides, and the ephemeris comparison results.

3.1 Accuracy Assessment of the POEs

To support the science objectives of the TOPEX/Poseidon mission, the POD team is required to produce POEs that are accurate to 13 centimeters (1σ) in the radial component. Comparisons of the POEs with actual TOPEX/Poseidon radar altimeter data show agreement to within 12 centimeters. These comparisons, in conjunction with a battery of other verification tests, provide strong evidence that the POEs are sufficiently accurate to meet the 13-centimeter (1σ) requirement. The tests also indicate that the along- track component is three to four times less accurate than the radial component, while the cross-track component is one to three times less accurate than the radial component (Reference 7).

One aspect of the POE verification involves performing overlap comparisons to assess solution consistency between the POEs and specially generated overlap solutions. A special 10-day overlap solution, which overlaps the first 5 days of the Cycle 5 POE, was generated and compared with the Cycle 5 POE in the common interval. The results show an average root-mean-square (RMS) overlap radial position consistency of 0.6 centimeter, which is substantially less than the 13-centimeter (1σ) accuracy requirement. In addition, the average RMS overlap along-track and cross-track position consistencies are 3.3 and 3.8 centimeters, respectively (Reference 7).

3.2 Summary of the Batch-Least-Squares Estimation Results

The simultaneous TDRS/TOPEX solution spanned November 7 through November 12, 1992. This period was chosen since it provided TDRS data spans with few momentum unloads, which normally occur every 1.5 to 2.5 days. The separate TOPEX/Poseidon-only solution spanned November 7 through November 11, 1992, maximizing the measurement span for which there were two valid TDRS trajectories available from the simultaneous solution. Shortening the solution span to 4 days also reduces the effect of known dynamical modeling errors by approximately 20 percent compared with a 5-day span.

Solution residuals are presented for the simultaneous TDRS/TOPEX orbit solution, used primarily for the TDRS trajectory estimation, and for the separate TOPEX/Poseidon orbit determination solution. Both solutions correspond to the latter half of the TOPEX/Poseidon ground track Cycle 5. There were three TDRS momentum unloads during this period, each having a different impact on the orbit determination performance. A TDRS-West momentum unload on November 10 at 17:00 UTC had a significant impact on the solution residuals; therefore, all TDRS-West tracking data after that time were edited from the simultaneous solution, but the residuals were calculated to illustrate the effect of the momentum unload. TDRS-East momentum unloads on November 7 at 06:35 UTC and November 9 at 19:45 UTC had little impact on the residuals. Exclusion of the TDRS-East momentum unloads was found to reduce the solution quality because of the shortened TDRS data spans and the lack of significant two-TDRS tracking of TOPEX/Poseidon.

Figure 2 illustrates each TOPEX/Poseidon two-way range residual from the simultaneous solution. Tracking data from both TDRS-West and TDRS-East are included. The edited data after the TDRS-West momentum unload was the result of the manual exclusion of the TDRS-West data due to the momentum unload. The mean of the accepted data residuals is approximately 0 meters, and the residuals are generally within ± 4 meters. The residual statistics reported from the solution were 0.022 ± 2.219 meters (in the form of the mean $\pm$ the standard deviation). There appears to be a 24-hour periodicity to the residuals until the TDRS-East momentum unload on November 9 at 19:45 UTC. Given the 24-hour periodicity, the most likely cause is a modeling error common to both TDRS spacecraft trajectories. Further analysis is needed to positively identify the cause. The TDRS-West momentum unload on November 10 at 17:00 UTC resulted in either an increase in the amplitude of the residuals from ± 5 meters to ± 10 meters, assuming that the pass with a residual of 10 meters near noon on November 11 was good, or an introduction of a secular rate of approximately –5 meters per day in the residuals, assuming the pass was randomly biased. Either way, inclusion of the TDRS-West data after the momentum unload degraded the solution. The TDRS-East momentum unloads on November 7 at 06:35 UTC and on November 9 at 19:45 UTC had little effect on the magnitude of the residuals.

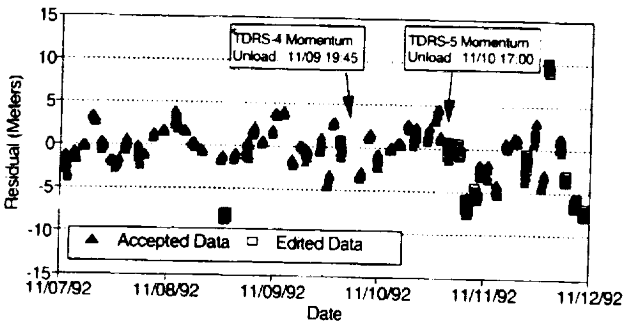

Figure 2. TOPEX/Poseidon Two-Way Range Residuals (Simultaneous Solution)

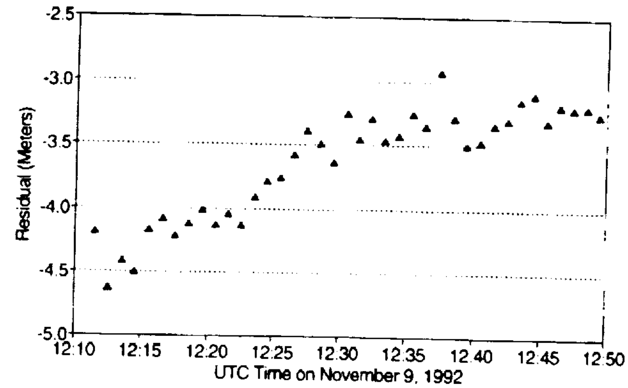

Figure 3. TOPEX/Poseidon Two-Way Range Residuals From TDRS-East (Simultaneous Solution)

A representative pass of TOPEX/Poseidon two-way range data residuals from TDRS-East is plotted in Figure 3. The data used in the solution were sampled at the rate of one point every minute out of data that were available at 10-second intervals. No smoothing was performed in the sampling of the 10-second data. As can be seen, there is still significant structure to the residuals with evidence of very little noise.

Two-way BRTS range residuals for TDRS-East are given in Figure 4. As with the TOPEX/Poseidon range data, the residuals generally do not exceed 5 meters. There is a significant amount of structure left in the residuals, which exhibit a 24-hour periodicity. Unlike the TDRS-West momentum unload on November 10 at 17:00 UTC, which resulted in an increase in the amplitude of the residuals, the TDRS-East momentum unloads did not. Each vertical block of data points typically represents two 5-minute adjacent passes. Assuming that the geostationary TDRS spacecraft are not moving significantly with respect to the ground during the passes, the vertical scatter in the data points is the result of noise in the data. Inspection of the passes on a pass-by-pass basis confirms that the 3σ noise is approximately 1 meter. The BRTS range measurement weight sigma was 2 meters. Some discontinuities are evident in the data, implying that there are biases in the data that are not entirely constant over the solution data span. The cause of the bias changes needs to be investigated further. The combined TDRS-East and TDRS-West BRTS range residual average is -0.016 ± 3.008 meters, slightly larger than the TOPEX/Poseidon range values.

The S-Band (2287-megahertz) return-link two-way TOPEX/Poseidon Doppler tracking residuals for the simultaneous solutions were generally bounded by 30 millihertz, with the average being 0.0 ± 9.7 millihertz. One-way return residuals averaged 0.0 ± 12.8 millihertz. These values correspond to range-rate values of 0.0 ± 1.27 millimeters per second for the two-way Doppler and 0.0 ± 1.68 millimeters per second for the one-way return Doppler. Most of the residuals have structure, implying that mismodeling, rather than noise, is the dominant source of error. Overall, these figures are approximately 40 percent of the values from previously reported results (Reference 6).

Two-way Doppler residuals for the 4-day TOPEX/Poseidon-only orbit solution average to 0.0 ± 8.5 millihertz, a little more than 10 percent better than the simultaneous solution. The one-way Doppler residuals average was 0.0 ± 12.7 millihertz. A representative pass of two-way Doppler data is given in Figure 5, illustrating the structure left in the residuals. Noise in the Doppler data appears to be limited to 1 to 2 millihertz, making noise only 10 percent of the observed residuals. Since the observed residuals appear to be highly structured, it should be possible to improve the modeling to minimize solution errors.

Overall, the solution range residuals show an approximately 2.0-meter 1σ error for the TOPEX/Poseidon range data, while the TDRS BRTS range data had an approximately 3.0-meter 1σ error. The cause of the higher error level for the BRTS range data appears to be the result of noise; otherwise, it is comparable to the TOPEX/Poseidon range data in quality. Based on the presence of a 24-hour periodicity in the residuals, most of the user range residual structure appears to be caused by TDRS trajectory error. The TOPEX/Poseidon Doppler residuals were of the order of 10 millihertz (1σ), with most of the residuals having significant structure and little noise. Some improvement in the Doppler residuals was observed when the TDRS estimation and range data were eliminated from the solution, using the previously estimated TDRS trajectories.

3.3 Summary of Sequential Estimation Results

Several indicators were available to assess the quality of the filter solutions independent of other orbit determination systems. Among such performance criteria are the diagonal components of the state error covariance matrix, more specifically, the square root of these values (standard deviation) (Reference 9). Figure 6 shows the time-evolution of the 1σ root-sum-square

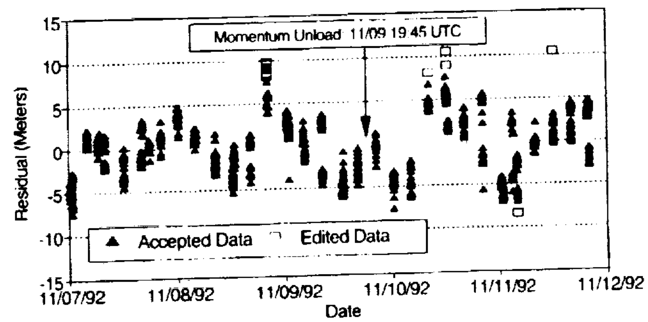

Figure 4. TDRS-East Two-Way Range Residuals From BRTS (Simultaneous Solution)

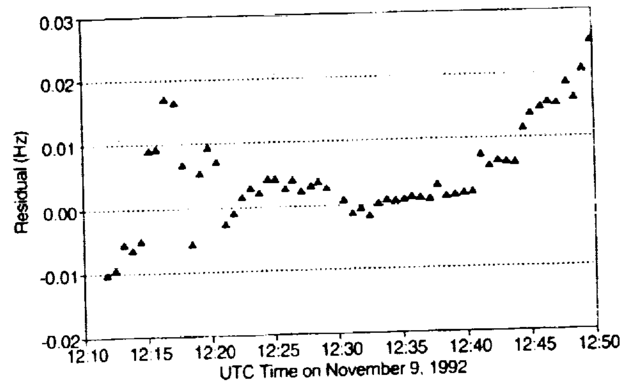

Figure 5. TOPEX/Poseidon Two-Way Doppler Residuals via TDRS-East

(RSS) position error estimate for TOPEX during the 4-day period beginning at 00:00:00 hours UTC on November 7, as computed by the PFS filter and the PFS smoother. Filter settling is apparent throughout the 4 days shown in Figure 6. Correspondingly, improved solution quality is expected as the smoother interval is increased. The root-variance estimates shown in Figure 6 indicate optimum accuracy near the middle of the smoother span, a characteristic predicted by theory. Averages of the standard deviations over the orbital periods (112 minutes) were computed to produce this plot. Perfect agreement in smoother and filter covariance at the start of a filter run, as evident in the figure, is a direct reflection of the smoothing algorithm.

Additional evidence of solution quality was sought by examining the residual statistics. Smoother postfit residual standard deviations for the period from November 7, 00:00:00 UTC, to November 8, 00:00:00 UTC, are shown in Table 5. These are typical for the timespan studied. A high degree of similarity can be seen among the values for the various smoother runs.

Excessive variations in the estimates for the coefficient of the solar radiation pressure and the coefficient of atmospheric drag for TOPEX can indicate problems with solution quality. When properly tuned, the estimated values of the drag and solar radiation coefficients should accommodate mismodeling of the atmospheric density and uncompensated variations in the solar radiation force model, respectively. In addition to atmospheric modeling and solar flux level uncertainties, changes in the spacecraft attitude can be expected to induce variation in the coefficient estimates (the PFS filter uses a constant-area cross-section for both the drag and solar radiation pressure computations). Given these factors, the observed variation in the coefficient estimates was judged to be reasonable, although a nonoptimum process noise tuning parameter C_D is indicated.

3.4 Results of POE and GTDS Solution Comparisons

Two GTDS ephemerides, spanning the latter portion of Cycle 5, were compared with the Cycle 5 POE. The ephemerides were compared at 10-minute intervals in orbit plane coordinates over their common definitive spans.

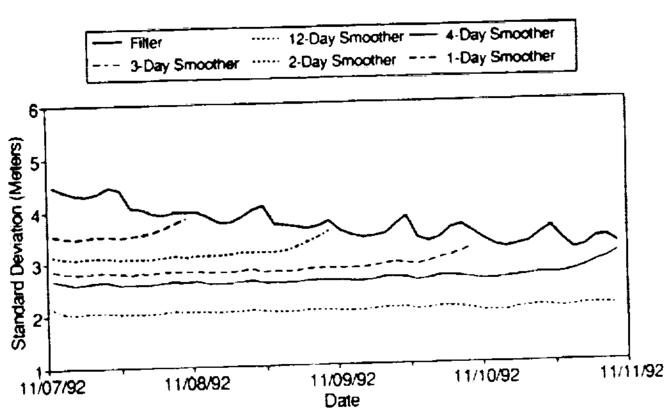

Figure 6. TOPEX/Poseidon Filter/Smoother RSS Position Error Estimates

Table 5. Typical PFS Filter/Smoother Residual Statistics

Estimation Type	Residual Statistics			
	TDRSS Range σ (meters)	TDRSS Doppler σ (hertz)	BRTS Range σ (meters)	BRTS Doppler σ (hertz)
Filter (prefit)	0.38680	0.011474	0.79318	0.014827
Filter (postfit)	0.08234	0.002383	0.12289	0.002569
Smoother (12-day)	0.10093	0.002406	0.17754	0.002893
Smoother (4-day)	0.10052	0.002352	0.17690	0.002866
Smoother (2-day)	0.10051	0.002223	0.17570	0.002903
Smoother (1-day)	0.10172	0.002463	0.17471	0.002986

NOTE: The values in the table above are for all residuals between 11/7/92 00:00:00 and 11/8/92 00:00:00 UTC, except for the first ones in passes. Thus, all are based on the same set of measurements over a 24-hour period. There were 450 TDRSS measurement pairs in this sample and 90 BRTS measurement pairs.

The first GTDS ephemeris, which corresponds to the TOPEX/TDRS simultaneous solution used to obtain the optimal TDRS orbits, is approximately 5 days long and spans the period 00:00 hours UTC on November 7, 1992, through 21:33 hours UTC on November 11, 1992. The RSS position differences between this GTDS ephemeris and the Cycle 5 POE are shown in Figure 7. The average RSS position difference is 1.1 meters, with a maximum difference of 2.9 meters.

The second GTDS ephemeris, which corresponds to the separate TOPEX solution and represents the best currently available TOPEX orbit, is 4 days long and spans the period 00:00 hours UTC on November 7, 1992, through 00:00 hours UTC on November 11, 1992. The RSS position differences between the second GTDS ephemeris and the Cycle 5 POE are shown in Figure 8. The average RSS position difference is 1.0 meter, with a maximum difference of 2.0 meters.

Note that the GTDS/POE differences shown in Figures 7 and 8 are similar except near the ends of the solution arc, where the differences for the separate GTDS solution/POE ephemeris comparison are somewhat smaller. This can be attributed to the reduced number of solved-for thrust coefficients in the separate TOPEX solution, which allows for the increased observability of, and a better estimate for, the unmodeled along-track accelerations acting on the spacecraft in addition to uncertainties related to the TDRS estimation.

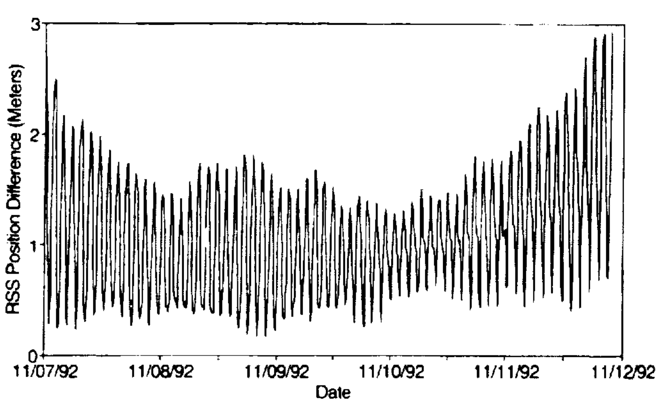

Figure 7. Position Differences Between POE and GTDS TOPEX Ephemerides (Simultaneous Solution)

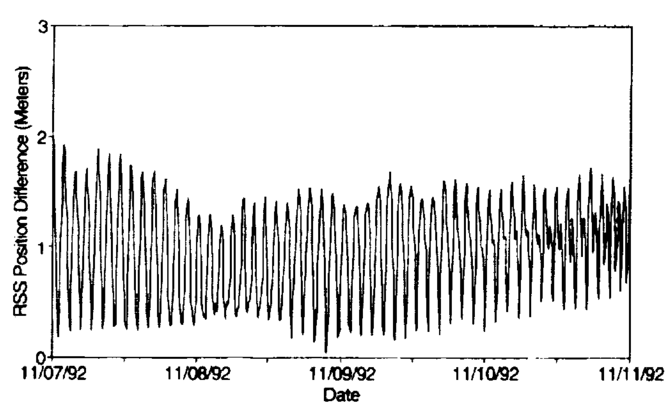

Figure 8. Position Differences Between POE and GTDS TOPEX Ephemerides (Separate Solution)

Figure 9 shows the representative differences in the radial, cross-track, and along-track directions for the separate TOPEX solution, on November 9, 1992. The maximum radial difference is 0.5 meter, while the maximum cross-track difference is 1.6 meters. The maximum along-track difference, which is the largest of the three components, is about 1.9 meters. The differences in the along-track and cross-track components have an average value of –0.5 meters and 0.3 meter, respectively, while the average difference in the radial component is nearly zero.

Some of the difference in the along-track component is likely due to differences in the modeling of along-track accelerations. The POEs estimate a daily once-per-revolution along-track acceleration, consisting of two solved-for parameters per day, and a daily constant along-track acceleration to accurately model the effects of the anomalous spacecraft forces as well as atmospheric drag perturbations. This represents a total of 30 solve-for parameters to characterize the along-track accelerations. The separate GTDS TOPEX solution, however, estimates only two thrust scale factors to characterize the along-track forces. Similarly, the POEs estimate a daily once-per-revolution cross-track acceleration, consisting of two solved-for parameters per day, to characterize the cross-track accelerations. The separate GTDS TOPEX solution, however, estimates no cross-track accelerations. Along-track and cross-track component differences can, in part, also be attributed to the differences in the modeling of the attitude changes resulting from the yaw-steering feature. These would affect both the measurement modeling and the atmospheric drag and solar radiation pressure force modeling. The POEs model the instantaneous changes in the spacecraft cross-sectional areas for drag and solar radiation pressure evaluation resulting from the yaw steering. The separate GTDS TOPEX solution uses the variable mean area model, which provides mean orbital values of the drag and solar radiation pressure cross-sectional areas.

3.5 Comparison Between POEs and Sequential Ephemerides

Ephemeris comparison results for the Cycle 5/6 period are illustrated in Figures 10 and 11. Figure 10 shows the TOPEX position difference between the POEs and the filter ephemeris and between the POEs and the smoother ephemerides for a 12-day span. The position differences are represented with an orbital average of the RSS position difference computed over 110-minute periods (the TOPEX orbital period is approximately 112 minutes). The figure also displays results for smoother runs of 1 through 12 days, each ending on November 7, 00:00:00 UTC. Figure 11 shows the radial, cross-track, and along-track components of the position difference between the Cycle 5 POE and the 4-day smoother ephemeris during a representative day (November 9, 1992).

The average 1-day RSS position difference was under 7 meters for all but the 1-day smoother run. In Figure 10, a reduction in the position difference is evident. The maximum difference for the filter was approximately 15 meters, while for the smoother it was approximately 8 meters (after the smoother settled). Thus, a categorical improvement in agreement with the POEs resulted from the application of the smoother to the sequential estimation solutions.

As seen previously for RTOD/E results, a significant cross-track position difference is observed for the PFS filter results. While at a reduced level, the cross-track component is proportionately similar for the smoother results.

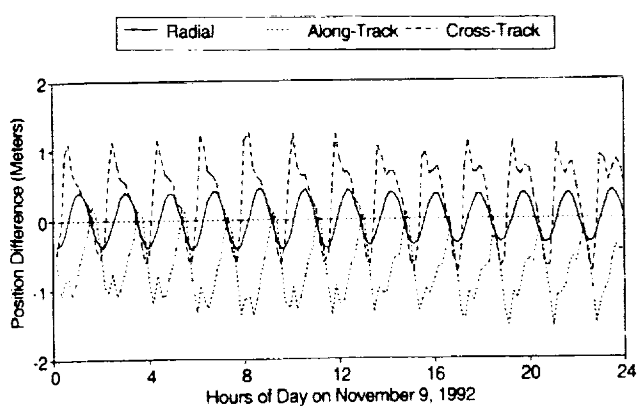

Figure 9. Position Differences by Component Between POE and GTDS Ephemerides for November 9, 1992

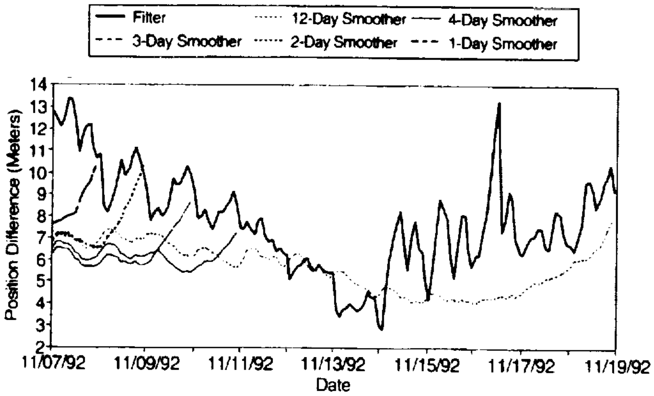

Figure 10. Position Differences Between POE and PFS Filter/Smoother Ephemerides

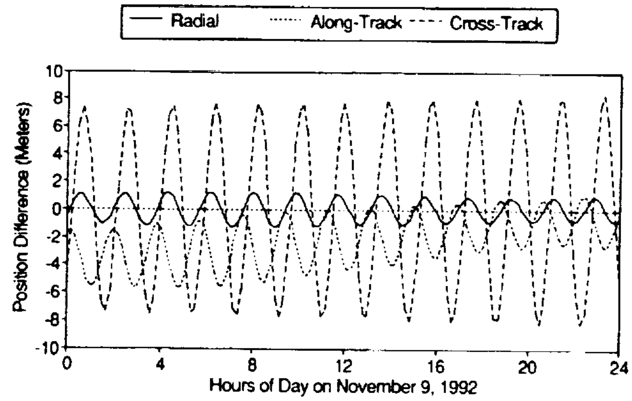

Figure 11. Position Differences by Component Between POE and PFS Filter/Smoother Ephemerides for November 9, 1992

3.6 Remarks on Supporting Analysis

Batch-least-squares covariance analysis was performed to analyze the GTDS solutions. The modeling for the covariance analysis was made as close as possible to the GTDS modeling. The 3σ RSS position uncertainty was found to vary between 7 and 15 meters. By components, the maximum 3σ position uncertainties were 3 meters, 5 meters, and 14 meters in the radial, cross-track, and along-track directions, respectively. The differences between the GTDS solutions and the POEs are less than the uncertainties obtained by covariance analysis. At the maximum 3σ RSS position uncertainty of 14.9 meters, the major contributors to the errors are the uncertainty in the ionospheric refraction correction at WSGT (11.6 meters) affecting TDRS position accuracy and the geopotential (6.0 meters).

The batch-least-squares procedures used in this analysis are being applied to the processing of a longer (20-day) span of data. Preliminary results indicate that the results presented here are reproducible when moderately good conditions are prevalent, such as when the TDRS spans are undisturbed by significant momentum unloads and maneuvers. More frequent momentum unloads and shorter data spans have been observed to have a significant detrimental effect on the TOPEX/Poseidon orbit determination.

GTDS orbit determination solutions have been obtained using state vectors from the Cycle 5 POE as the measurements. This form of orbit determination solution eliminates all observational and TDRS spacecraft dynamical force modeling, thereby making it possible to estimate the amount of error resulting from the dynamical modeling used in GTDS for TOPEX/Poseidon. The solution span corresponds to the same span used for the TOPEX/Poseidon-only orbit determination solution, which was presented earlier. The solution is 4 days long and spans the period 00:00 hours UTC on November 7, 1992, through 00:00 hours UTC on November 11, 1992, and used state vectors at 12-minute intervals. The RSS position differences between this special solution and the Cycle 5 POE are shown in Figure 12. The average RSS position difference is 0.4 meter, with a maximum difference of 1.1 meters. The maximum radial, along-track and cross-track differences are 0.4 meter, 0.9 meter and 1.0 meter, respectively. The average component differences are all zero.

The differences illustrated in Figure 12 reflect the force modeling errors between the GTDS dynamical force modeling and the Cycle 5 POE. Comparison with Figure 8 reveals that the force modeling errors and the measurement modeling errors both contribute approximately 1 meter to the total error, on average. GTDS solutions using 10-day spans from the POEs yielded errors of 2 meters. The error appears to be a function of the solution span, incurring error at the rate of 20 centimeters per day of solution. The nature of the errors implies that GTDS is performing a best average fit to a time-varying term in the dynamics modeling. This is supported by preliminary analysis which has eliminated constant errors in the geopotential terms, including those affected by dynamic polar motion and constant errors in the $C_{2,0}$ term. Likewise, preliminary analysis has indicated that the effect of the $C_{2,0}$ rate term is too small to produce the observed effects.

The validity of the secular trends of the GTDS dynamic modeling was also verified by performing GTDS solutions for arc lengths of 1 day through 10 days, with increasing arc lengths by a day each for Cycle 5. The characteristics of the comparison of the 10 solutions with the POEs did not change from the short (1-day) arc length to the long (10-day) arc length. This demonstrated that the effects of dynamical mismodeling are small compared with the other errors. Corresponding covariance analysis solutions with the same tracking schedules as the 10 GTDS solutions supported the GTDS solutions.

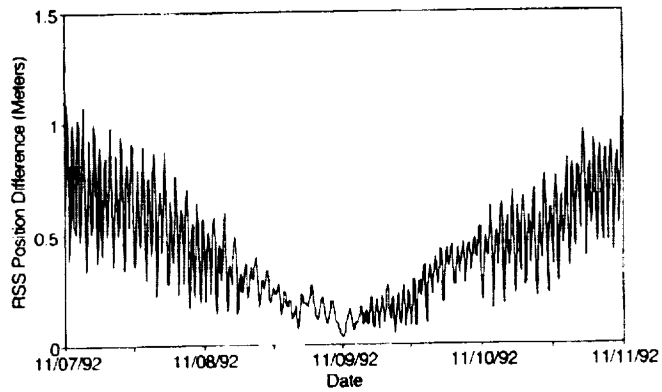

Figure 12. Comparison of Special GTDS Solution With the POE

The accuracies achieved here using batch-least-squares and sequential estimation methods are different, in most part, due to differences in modeling and the effectiveness with which the modeling is used; they are not a reflection of the inherent potential of either estimation technique.

It is important to note that TDRSS tracking does not have a requirement to yield orbit solutions with accuracy comparable to laser-tracked orbit solutions. However, a major objective of this work is to assess the achievable TDRSS orbit determination accuracy.

3.7 Future Analysis

Several areas in the batch-least-squares modeling and orbit determination processing could be improved to yield better results. First, the area modeling of TOPEX itself should be improved. At present, only mean areas are used for the solar radiation and drag force computations. Second, the antenna offset model could be improved to incorporate the effects of the sinusoidal yaw steering mode. The ability to automatically estimate TDRS trajectories through momentum unloads would possibly allow for operational support using the procedures present herein. Finally, better treatment of the unmodeled body-fixed force should help improve the accuracy of the batch-least-squares solutions.

Although the tunable parameters used for the PFS filter runs were close to optimal, the smoother nevertheless provided appreciable improvement in the comparison results. It is thus reasonable to suppose that a similar improvement in ephemeris comparison results would be achieved if a smoother were applied to solutions from an optimally tuned filter.

While appreciable, neither tuning nor smoothing improvements has resulted in POE comparisons commensurate with the inherent filter accuracy implied by the filter's covariance estimates. This indicates that further optimization of the filter's tunable parameters would be worthwhile. The substantial improvement in comparison results for GTDS ephemerides that has been achieved through refinement of the predetermined TDRS solutions suggests that additional analysis involving the simultaneously estimated TDRS orbit solutions would result in further improvement. Other factors limiting agreement with the POEs include dissimilarities in modeling and tracking data types.

4.0 Conclusions

This study analyzed the TDRSS-user orbit determination accuracy using a batch-least-squares method and a sequential estimation method. Independent assessments were performed of the orbit determination consistency within each method, and the estimated orbits obtained by the two methods were compared to the POEs.

In the batch-least-squares analysis, the solution range residuals show an approximately 2-meter, (1σ) mean value for the TOPEX/Poseidon range data, 3 meters for the TDRS BRTS range data, and TOPEX/Poseidon Doppler of the order of 10 millihertz. Virtually all of the observed residual patterns have significant structure and display little noise. These solutions compare with the POEs at less than 2 meters in maximum total position difference. The radial component compares to within 0.5 meter, slightly less than four times the 13-centimeter (1σ) POE accuracy requirement. Dynamical TOPEX/Poseidon modeling errors in GTDS have been shown to cause approximately 1 meter of the observed error in the solutions. Given the observed residuals and the known level of dynamical mismodeling in the current GTDS solutions, it can be stated that the TDRSS tracking measurement data have sufficient quality to support orbit determination to levels better than 2 meters in accuracy, provided issues of sufficient tracking coverage and accurate orbit determination modeling are addressed.

The reduction of the differences, as compared with an earlier analysis (Reference 6) was the direct result of the use of the improved TDRS orbits obtained from the TOPEX/TDRS simultaneous solutions. This demonstrates that the treatment of the relay orbit determination has a significant impact on high-accuracy orbit determination in the TDRSS environment.

After allowance is made for filter settling, the near-optimally tuned filter produced orbit solutions that were within 13 meters of the POEs. Application of a smoother algorithm to these filter solutions reduced the difference with the POEs to within 7 meters. These results demonstrate that smoother postprocessing offers the potential for appreciable improvement in sequential estimation solution accuracy, even when the filter is near-optimally tuned. Additional improvement in sequential orbit determination accuracy would be expected from further refinement of tunable parameters and enhancement of force modeling.

In summary, the differences between the TDRSS/GTDS-derived definitive batch-least-squares ephemerides and the POEs were no larger than about 2 meters. The differences between the smoothed sequentially estimated ephemerides and the POEs were no larger than 7 meters. Further analysis is in progress to understand the magnitudes of the differences. The differences among the POEs, GTDS, and sequential solutions can be traced to differences in modeling and tracking data types, which are being analyzed further.

References

1. Hughes STX Systems Corporation, STX Contract Report, *GEODYN II Systems Description, Volume I*, J. J. McCarthy et al., Lanham, Maryland, February 26, 1993

2. Goddard Space Flight Center, Flight Dynamics Division, FDD/552-89/001, *Goddard Trajectory Determination System (GTDS) Mathematical Theory, Revision 1*, A. C. Long and J. O. Cappellari, Jr. (CSC) and C. E. Velez and A. J. Fuchs (GSFC) (editors), prepared by Computer Sciences Corporation, July 1989

3. Goddard Space Flight Center, Flight Dynamics Division, FDD/554-91/064, *Enhanced RTOD Demonstration System User's Guide*, W. Chuba (ATA), prepared by Applied Technology Associates (ATA), Inc., March 1991

4. J. R. Wright, "Sequential Orbit Determination with Auto-Correlated Gravity Modeling Errors," *Guidance and Control*, Vol. 4, 1981

5. D. H. Oza, T. L. Jones, S. M. Fabien, G. D. Mistretta, R. C. Hart, and C. E. Doll, "Comparison of ERBS Orbit Determination Using Batch Least-Squares and Sequential Methods," NASA Conference Publication 3123, *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, National Aeronautics and Space Administration, Goddard Space Flight Center, Greenbelt, Maryland, May 21–23, 1991

6. C. E. Doll, G. D. Mistretta, R. C. Hart, D. H. Oza, C. M. Cox, M. Nemesure, and D. T. Bolvin, "Accuracy Assessment of TDRSS-Based TOPEX/Poseidon Orbit Determination Solutions," Paper No. AAS 93-572, presented at the AAS/AIAA Astrodynamics Specialist Conference, Victoria, B.C., Canada, August 16–19, 1993

7. B. H. Putney et al., "Precise Orbit Determination for the TOPEX/Poseidon Mission," Paper No. AAS 93-577, presented at the AAS/AIAA Astrodynamics Specialist Conference, Victoria, British Columbia, Canada, August 16–19, 1993

   B. Putney, J. Teles, W. Eddy, and S. Klosko, "Comparison of TOPEX/Poseidon Orbit Determination Results Using Laser, DORIS and TDRSS Data," Paper No. AAS 93-267, presented at the AAS/GSFC International Symposium on Space Flight Dynamics, Greenbelt, Maryland, April 26–30, 1993

   Private communication with Nikita Zelenski, Precision Orbit Determination Team, January 1994

8. R. B. Frauenholz et al., "The Role of Anomalous Satellite-Fixed Acceleration in TOPEX/Poseidon Orbit Maintenance," Paper No. AAS 93-570, presented at the AAS/AIAA Astrodynamics Specialist Conference, Victoria, British Columbia, Canada, August 16–19, 1993

9. A. Gelb (editor), *Applied Optimal Estimation*. Cambridge, Massachusetts: M.I.T. Press, 1974