

# Improved Solution Accuracy for Landsat-4 (TDRSS-User) Orbit Determination\*

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## Abstract

This paper presents the results of a study to compare the orbit determination accuracy for a Tracking and Data Relay Satellite System (TDRSS) user spacecraft, Landsat-4, obtained using a Prototype Filter Smoother (PFS), with the accuracy of an established batch-least-squares system, the Goddard Trajectory Determination System (GTDS). The results of Landsat-4 orbit determination will provide useful experience for the Earth Observing System (EOS) series of satellites. The Landsat-4 ephemerides were estimated for the January 17-23, 1991, timeframe, during which intensive TDRSS tracking data for Landsat-4 were available. Independent assessments were made of the consistencies (overlap comparisons for the batch case and covariances for the sequential case) of solutions produced by the batch and sequential methods. The filtered and smoothed PFS orbit solutions were compared with the definitive GTDS orbit solutions for Landsat-4; the solution differences were generally less than 15 meters.

## 1.0 Introduction

This paper compares the orbit determination accuracy of a prototype sequential orbit determination system with the accuracy achieved using an operational batch-least-squares system for a Tracking and Data Relay Satellite (TDRS) System (TDRSS) user spacecraft. This analysis also evaluates the effect of applying a smoother algorithm to the filter solutions.

TDRSS is a geosynchronous relay satellite network which currently consists of five geosynchronous spacecraft and the White Sands Ground Terminal (WSGT) at White Sands, New Mexico. Of the five TDRSs, three (TDRS-East, TDRS-West, and TDRS-Spare, located at 41 degrees, 174 degrees, and 62 degrees west longitude, respectively) actively support tracking of TDRSS-user spacecraft. Of the two remaining TDRSs, one TDRS (located at 275 degrees west longitude) is used only for satellite communications, while the other TDRS (located at 46 degrees west longitude) is being reserved for future use. TDRSS has the operational capability to provide 85-percent to 100-percent coverage, depending on the spacecraft altitude.

The Bilateral Ranging Transponder System (BRTS) provides range and Doppler measurements for maintaining each TDRS orbit. The ground-based BRTS transponders are tracked as if they were TDRSS user spacecraft. Since the positions of the BRTS transponders are known, their ranging data can be used to precisely determine the trajectory of the TDRSs.

The focus of this paper is an assessment of the relative orbit determination accuracy of the batch-least-squares method, used for current operational orbit determination support, with that of a sequential method implemented in a prototype system, used for analysis in the GSFC Flight Dynamics Facility (FDF). The batch-weighted least-squares

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algorithm implemented in the Goddard Trajectory Determination System (GTDS) estimates the sets of orbital elements, force modeling parameters, and measurement-related parameters that minimize the squared difference between observed and calculated values of selected tracking data over a solution arc (Reference 1).

The sequential estimation algorithm implemented in a prototype system, the Prototype Filter Smoother (PFS), simultaneously estimates the TDRSS user and relay spacecraft orbital elements and other parameters in the force and observation models at each measurement time. The PFS filter is closely related to the Real-Time Orbit Determination/Enhanced (RTOD/E)\* system (Reference 2). PFS performs forward filtering of tracking measurements using the extended Kalman filter with a process noise model to account for serially correlated, geopotential-induced errors, as well as Gauss-Markov processes for drag, solar radiation pressure, and measurement biases. The main features of RTOD/E are summarized in Reference 3.

An orbit determination analysis of Landsat-4 using TDRSS is reported here. Motivation for an orbit determination evaluation of Landsat-4 derives from the fact that the orbital characteristics of Landsat-4 are similar to those of the Earth Observing Satellite (EOS) series of missions, planned for launch starting in 1998. The results of a study for Landsat-4 will provide useful experience and verification of EOS flight dynamics support requirements. Early assessment of conclusions regarding meeting EOS support requirements will provide adequate opportunity to develop comprehensive support scenarios.

The estimated Landsat-4 ephemerides were obtained for the January 17-23, 1991, timeframe. This particular timeframe was chosen because dense TDRSS tracking data for Landsat-4 were available. Independent assessments were made to examine the consistencies (overlap comparisons for the batch case and state error covariances and the measurement residuals for the sequential case) of results obtained by the batch and sequential methods.

Section 2 of this paper describes the orbit determination and evaluation procedures used in this study, and Section 3 presents the results obtained using the batch-least-squares and sequential estimation methods and provides the resulting consistency and cross comparisons. Section 4 presents the conclusions of this study.

## 2.0 Orbit Determination and Evaluation Procedure

This section describes the analysis procedures used in this study. The TDRSS and BRTS tracking data characteristics are presented in Section 2.1, and the orbit determination evaluation methodology and options used are described in Section 2.2.

### 2.1 Tracking Measurements

Landsat-4 was deployed by Delta-3920 in July 1982. It has a nearly circular orbit, an altitude of approximately 715 kilometers, an inclination of 98 degrees, and a period of approximately 99 minutes. The time period chosen for this study was from 0 hours universal time coordinated (UTC) on January 17, 1991, through 10 hours UTC on January 24, 1991. During this interval, unusually dense TDRSS tracking of the Landsat-4 satellite was made available. The tracking consisted of an average of 15 passes of two-way TDRSS range and Doppler observations each day, each pass ranging from 3 minutes to 45 minutes in duration. The normal TDRSS tracking of Landsat-4 (less dense) typically consists of about six 5-minute passes each day. A timeline plot of the TDRSS tracking data distribution is given in Figure 1.

The typical scenario for BRTS tracking of the TDRSs during the period of study included approximately 4 or 9 minutes of range and two-way Doppler measurements from two ground transponders for each relay every 2 to 3 hours, consisting of an average of 12 BRTS passes per TDRS each day. BRTS stations for TDRS-East are located at White Sands and Ascension Island. BRTS stations for TDRS-West are located at White Sands, American Samoa, and Alice Springs, Australia.

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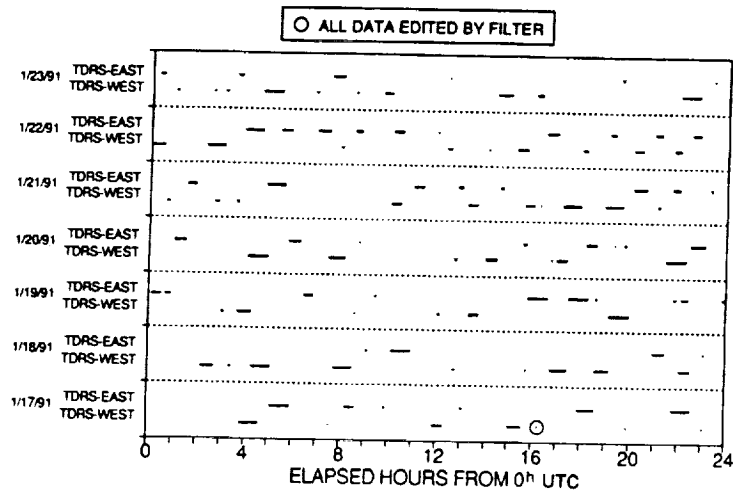


Figure 1. Tracking Data for Landsat-4

## 2.2 Evaluation Methodology

The evaluation methodologies for the batch-least-squares and sequential estimation methods are described below. Since there are some known differences between the GTDS and PFS models (estimation method, solar and planetary ephemerides representation, solid Earth tides, ionospheric refraction corrections to the measurements, and process noise modeling), and since the PFS TDRSS and BRTS measurement models were implemented independently from GTDS, the two systems are not expected to provide identical results. Therefore, this study assumes that each system is used in its optimal configuration. Table 1 gives the parameters and options for the simultaneous solutions of the user and relay spacecraft. Table 2 gives the force and measurement model specifications.

### Batch Least-Squares Method

Except for the variations noted, the computational procedures and mathematical methods used in this study are identical to those used for routine operational orbit determination in the GSFC FDF. The choice to expand the state space of the least-squares solutions to include measurement biases was motivated by the fact that the PFS orbit determination algorithm estimates an equivalent set of bias parameters. The batch-weighted-least-squares algorithm implemented in GTDS (Reference 1) solves for the set of orbital elements and other parameters that minimizes the squared difference between observed and calculated values of selected tracking data over a solution arc. Parameters solved for, other than the spacecraft state at epoch, include free parameters of the force model and/or the measurement model.

A detailed study of the Earth Radiation Budget Satellite (ERBS) with the batch-least-squares estimation method was reported in Reference 4, and it was further refined in Reference 5. The models and options found optimal in the previous study of ERBS are used here for Landsat-4. The options used for the study described in this paper are summarized in columns 2 and 3 of Tables 1 and 2.

To evaluate the orbit determination consistency achievable with a particular choice of options using least-squares estimation, a series of seven 34-hour definitive solutions was performed with 10-hour overlaps between neighboring arcs. The GTDS Ephemeris Comparison Program was used to determine the root-mean-square (RMS) position differences between the definitive ephemerides for neighboring solutions in the 10-hour overlap time period. These "overlap" comparisons measure the adjacent solution consistency, not the absolute accuracy.

### Sequential Estimation Method

PFS has been developed to address future increased TDRSS-navigation accuracy requirements and to provide automation of some routine orbit determination operations. The goal for future orbit determination accuracy is 10 meters total position error ( $1\sigma$ ) for the user and 25 meters total position error ( $1\sigma$ ) for the TDRSSs. PFS provides a

**Table 1. Parameters and Options for the Simultaneous Solutions of User and Relay Spacecraft**

Orbit Determination Parameter or Option	GTDS Values		PFS Values	
	User (Landsat-4)	Relay (TDRS-East & TDRS-West)	User (Landsat-4)	Relay (TDRS-East & TDRS-West)
Estimated parameters	State, drag scaling parameter ( $\rho_1$ ), range and Doppler measurement biases for tracking via each ground station	State, transponder delays for each BRTS transponder, solar reflectivity coefficients	State, coefficient of drag, range and Doppler measurement biases for tracking via each TDRS	State, solar reflectivity coefficient ( $C_R$ ), range and Doppler measurement biases for tracking via each transponder
Integration type	Fixed-step Cowell	Fixed-step Cowell	Variation of parameters	Variation of parameters
Coordinate system of integration	Mean of 1950.0	Mean of 1950.0	Mean of 1950.0	Mean of 1950.0
Integration step size (seconds)	30.0	600.0	60.0	60.0
Tracking data	TDRSS	BRTS	TDRSS	BRTS
Data rate	1 per 20 seconds	1 per 10 seconds	1 per 30 seconds	1 per 10 seconds
DC convergence parameter	0.005	0.005	N/A	N/A
Editing criterion	$3\sigma$	$3\sigma$	$3\sigma$	$3\sigma$
Measurement $\sigma$ 's: Range Doppler	30.0 meters 0.25 hertz	10.0 meters 0.003 hertz	0.4 meter 0.004 hertz	0.4 meter 0.003 hertz
Gauss-Markov parameters: Drag half-life Drag sigma $C_R$ half-life $C_R$ sigma Range bias half-life Range bias sigma Doppler bias half-life Doppler bias sigma	N/A	N/A	840 minutes 0.500 N/A N/A 60 minutes 6 meters 8 minutes 0.034 hertz	N/A N/A 11520 minutes 0.2 60 minutes 4.5 meters 60 minutes 0.02 hertz
Satellite area	12.2644 meters <sup>2</sup>	40 meters <sup>2</sup>	12.2644 meters <sup>2</sup>	40.0 meters <sup>2</sup>
Satellite mass	1900.32 kilograms	1990.76 kilograms (TDRS-East) 1735.46 kilograms (TDRS-West)	1900.32 kilograms	1990.76 kilograms (TDRS-East) 1735.46 kilograms (TDRS-West)

N/A = Not applicable

**Table 2. Force and Measurement Model Specifications**

Orbit Determination Parameter or Option	GTDS Values		PFS Values	
	User (Landsat-4)	Relay (TDRS-East & TDRS-West)	User (Landsat-4)	Relay (TDRS-East & TDRS-West)
Geopotential model	GEM-T3 (50 × 50)	GEM-T3 (8 × 8)	GEM-T3 (50 × 50)	GEM-T3 (6 × 6)
Atmospheric density model	Jacchia-Roberts daily solar flux values (209, 203, 199, 204, 202, 225, 223)	N/A	CIRA 1972 daily solar flux values (209, 203, 199, 204, 202, 225, 223)	N/A
Solar and lunar ephemerides	JPL DE-118	JPL DE-118	Analytical	Analytical
Solar reflectivity coefficient ( $C_R$ )	1.5	Estimated	1.5	Estimated
Coefficient of drag ( $C_D$ )	Estimated	N/A	Estimated	N/A
Ionospheric refraction correction:	Bent Model	Bent Model	No	No
	Ground-to-spacecraft	Yes	—	—
Spacecraft-to-spacecraft	Yes	N/A	—	—
Tropospheric refraction correction	Yes	Yes	Yes	Yes
Polar motion correction	Yes	Yes	Yes	Yes
Earth tides	Yes	No	No	No

GEM = Goddard Earth Model  
 JPL = Jet Propulsion Laboratory  
 N/A = Not Applicable

proof of concept for the use of sequential estimation techniques for orbit determination with TDRSS tracking data and offers the potential for enhanced accuracy navigation. PFS is a research tool for assessing sequential estimation for FDF navigation applications in realistic operational situations.

PFS uses the extended Kalman filter form for sequential orbit estimation. With the sequential estimation method, each tracking measurement can be processed immediately upon receipt to produce an update of a spacecraft's state vector and auxiliary state parameters. This fact makes it well suited for realtime or near-realtime operation. Sequential estimation is particularly well suited to the development of systems to perform orbit determination autonomously on the spacecraft's onboard computer (Reference 6). Spacecraft orbit determination during and just after a maneuver is a critical support function for which orbit determination is needed in near realtime. Therefore, sequential estimation is also well suited for such an application. In addition, the forward filter can be augmented with a backward smoothing filter to further improve the overall accuracy, especially during periods without tracking data.

PFS employs a sequential estimation algorithm with a process noise model to stochastically account for gravity model errors (Reference 7). In addition to the spacecraft orbital elements, the filter estimates free parameters of the force model and the measurement model, treating these parameters as random variables whose behavior is governed by a Gauss-Markov stochastic process. The specific options used in PFS for this study are listed in the last two columns of Tables 1 and 2.

A good indicator of the consistency of the sequential estimation results is provided by the state error covariance function generated during the estimation process (Reference 8). In addition, the relationship of the first predicted measurement residual of each tracking pass to the associated predicted residual variance provides an indication of the physical integrity of the state error covariance of the filtered orbits. These parameters were monitored during the sequential estimation process.

### 3.0 Results and Discussion

The results of this study for the Landsat-4 and TDRSS relay spacecraft are presented in this section, along with an analysis of the results. Greater emphasis is placed on the Landsat-4 results, since the primary objective is to study TDRSS user orbit determination. The orbit determination results using batch-least-squares calculations and sequential estimation are given in Sections 3.1 and 3.2, respectively; the comparisons are presented in Section 3.3.

### 3.1 Batch-Least-Squares Results

In general, all data arcs for Landsat-4 solutions consisted of 34 hours, beginning at 0 hours UTC of each day from January 17 to January 23, 1991, with one exception. The exception was made for the arc beginning at 0 hours UTC on January 20, 1991. There is a long data gap of about 5 hours (see Figure 1) at the end of the nominal 34-hour period, resulting in a predicted solution for the last 5 hours instead of a definitive solution. Therefore, for this particular solution, the arc length was extended by 2 hours to 36 hours so that the next tracking pass was included in the solution.

The RMS values of six Landsat-4 overlap comparisons are summarized in Figure 2. The RMS overlap differences vary from about 3 to 5 meters. The mean and sample standard deviation of this distribution, in the form of mean  $\pm$  standard deviation, is  $3.9 \pm 0.8$  meters. The maximum total position differences over the same distribution vary between 5 and 9 meters, with a mean and standard deviation of  $6.5 \pm 1.3$  meters. The maximum position difference values for Landsat-4 are typically a factor of 1.7 larger than the RMS values.

A batch-least-squares covariance analysis was performed to identify the major sources of error. The actual tracking data distribution was used for the covariance analysis. For the seven covariance analysis solutions, six RMS overlap comparison values were obtained. The mean and standard deviation of the overlap comparisons were  $5.4 \pm 0.5$  meters, which is comparable to the GTDS-based orbit determination overlap comparison results. The agreement between the covariance analysis and the GTDS overlap values establishes confidence in the error models used in the covariance analysis. The dominant orbit determination error source was due to the geopotential model error, with the error magnitude significantly larger than the next largest error source.

The RMS values of six TDRS-East and TDRS-West overlap comparisons are summarized in Figure 3. The overlap values for TDRS-East vary from about 11 to 17 meters. The mean and sample standard deviation of this distribution is  $14.9 \pm 2.3$  meters. The maximum total position differences over the same distribution vary between 14 and 25 meters, with a mean and standard deviation of  $18.7 \pm 4.0$  meters. The overlap values for TDRS-West vary from about 10 to 49 meters. The mean and the sample standard deviation of this distribution is  $20.8 \pm 13.3$  meters. The maximum total position differences over the same distribution vary between 13 and 67 meters, with a mean and standard deviation of  $24.9 \pm 19.1$  meters. The maximum position difference values for the TDRSs are typically a factor of 1.1 larger than the RMS values.

The possible advantage of varying the estimation arc lengths to exclude periods of TDRS angular momentum unloads was evaluated. These momentum unloads are designed to use opposing thrusters so that the effects on the orbit are minimized. However, earlier analysis on the Ocean Topography Experiment (TOPEX) satellite indicated

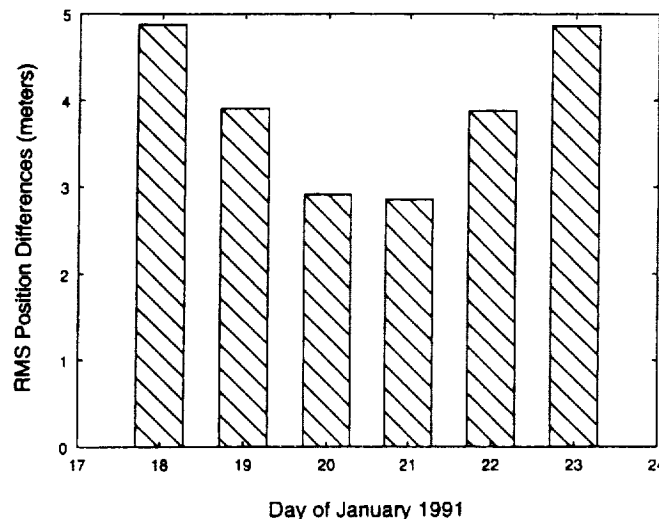
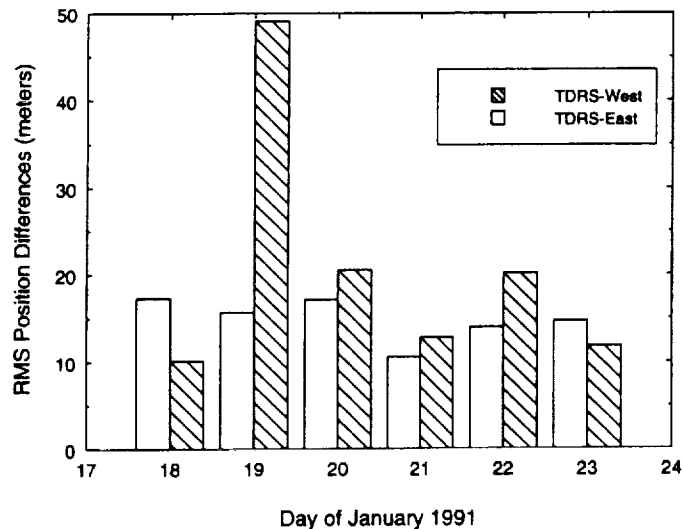


Figure 2. Landsat Overlap Comparisons



**Figure 3. TDRS-East and TDRS-West Overlap Comparisons**

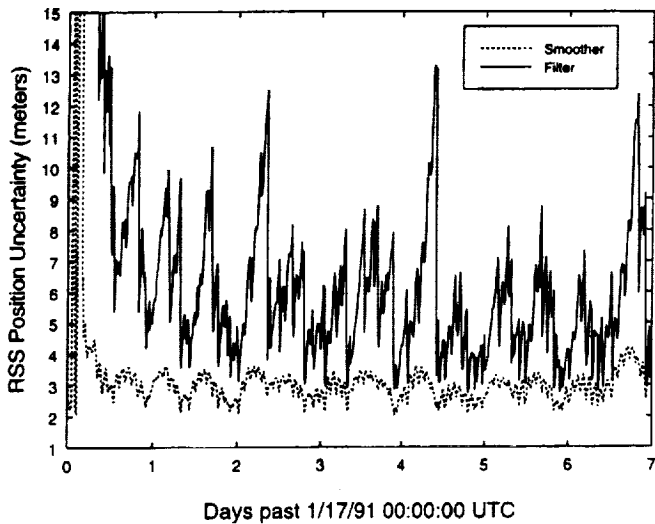
that by excluding momentum unloads from the estimation arcs, solutions with greater accuracy were possible (Reference 9). Five TDRS momentum unloads were performed during the period covered by this study—three by TDRS-East (at UTC 1/17/91 20:25:00, 1/20/91 20:00:00, and 1/22/91 21:00:00) and two by TDRS-West (at UTC 1/19/91 09:20:00 and 1/19/91 12:30:00). These momentum unloads were excluded by performing five solutions with arc lengths of about 20, 37, 31, 49, and 27 hours, respectively. A period of about 3 hours between the TDRS-West momentum unloads was excluded entirely. Predicted periods of 10 hours at both ends of the solution arcs were used in overlap comparisons to judge consistency. These overlap comparisons were less favorable than for similar predicted overlap comparisons for the 34-hour solution arcs presented above. In particular, it appeared that the initial 20-hour arc may have been too short to accurately estimate all 21 parameters in the state vector. The GTDS solutions using 34-hour arc lengths will be used in the comparisons presented in Section 3.3.

### 3.2 Sequential Estimation Results

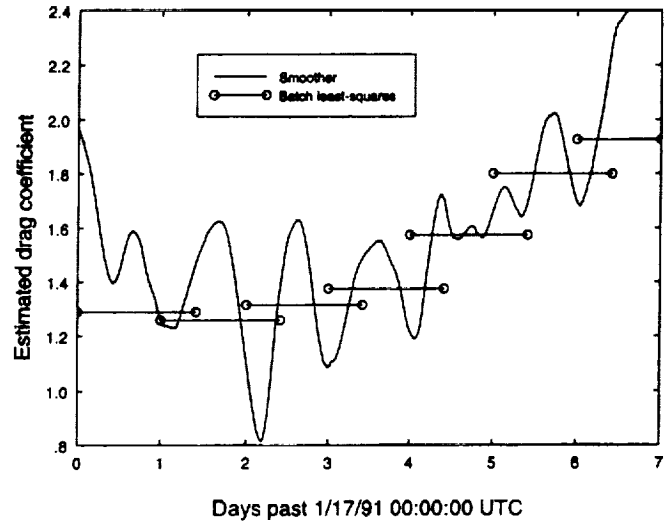
During sequential processing of the TDRSS and BRTS measurements using the PFS filter/smoothen, the position component standard deviations from the state error covariance function ( $\sigma$ ) were closely monitored. The filter was started with high initial diagonal values in the covariance matrix. The smoother was of the Rauch-Tung-Striebel type and was therefore started at the end of the time period of investigation (UTC 1/24/91 00:00:00) with the same covariance as the final filter covariance.

The root-sum-square (RSS) position standard deviations ( $1\sigma$ ) for both the filter and smoother runs for Landsat-4 are plotted in Figure 4. The filter standard deviations initially increase to about 2 kilometers. This is not unusual before the filter has reached steady-state performance, especially considering that there are no TDRSS data for Landsat-4 in the first 4 hours (see Figure 1). After an initial filter settling period (about 24 hours), the  $1\sigma$  values varied from about 2.9 to 13.2 meters in the RSS position for Landsat-4. The  $1\sigma$  values for Landsat-4 dropped to their lowest levels during a tracking pass and then gradually rose to the maximum values during the time update phase (propagation phase). (The duration of the time update phases can be seen in Figure 1). The smoother RSS standard deviation remained fairly constant at about 2.8 meters, with greater values at either end, a result predicted by theory.

Unlike Landsat-4, the filter  $1\sigma$  RSS values for TDRS-East and TDRS-West continued to decline gradually for about 4 days. Near the end of the filter run, the  $1\sigma$  RSS position standard deviations for TDRS-East and TDRS-West remained relatively steady at about 10.8 meters and 7.2 meters, respectively. The smoother RSS standard deviations for the TDRSs were fairly constant at about 6.9 meters for TDRS-East and 6.6 meters for TDRS-West, with slight increases at either end.



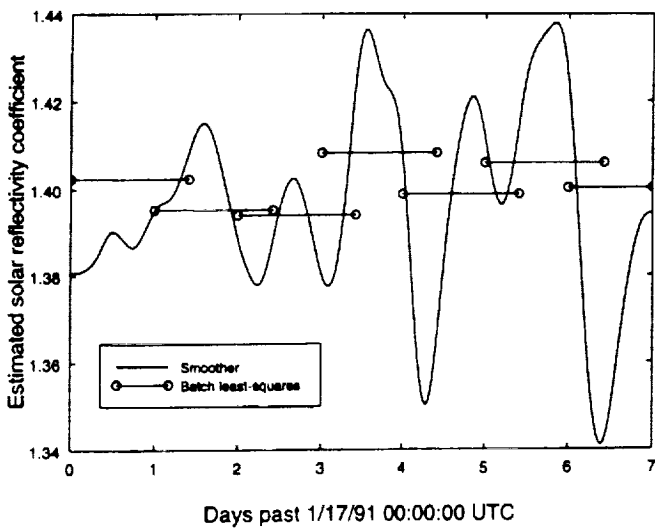
**Figure 4. RSS Position Standard Deviation ( $1\sigma$ ) for Landsat-4 PFS Filter/ Smoother**



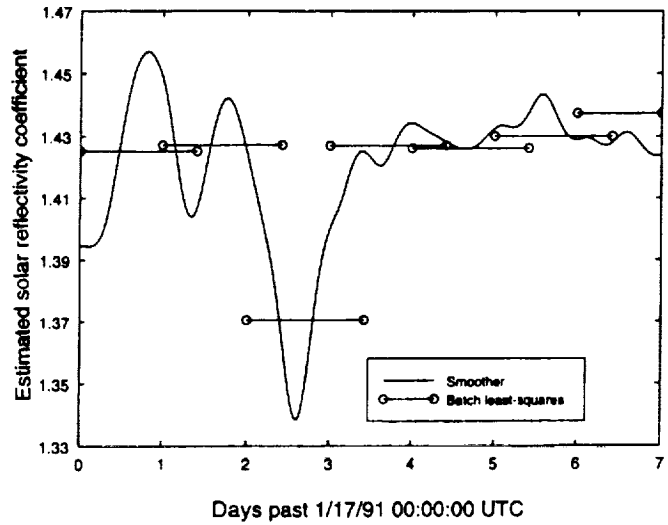
**Figure 5. Coefficient of Atmospheric Drag ( $C_D$ ) for Landsat-4**

The estimated force model parameters varied as a function of time and were updated after each measurement was processed. The time variation of the atmospheric drag coefficient for Landsat-4 is shown in Figure 5 for the smoother solution. The drag coefficients estimated by the batch-least-squares solutions (34-hour arcs) are also indicated in Figure 5. The drag coefficient estimate from the smoother varied from a low of about 0.8 to a high of about 2.5. Throughout most of this interval, the smoother's drag coefficient standard deviation ( $1\sigma$ ) remained fairly constant at about 0.18, increasing to about 0.35 at both ends of the interval. The upward trend in the drag coefficient indicated by the batch-least-squares results is reflected in the smoother results as well.

The time variation of the smoother's estimate of solar reflectivity coefficients for TDRS-East and for TDRS-West are shown in Figures 6 and 7, respectively. The corresponding batch-least-squares results are indicated on these figures as well. The solar reflectivity coefficient varied from about 1.34 to about 1.44 for TDRS-East and from about



**Figure 6. Coefficient of Solar Radiation Pressure ( $C_R$ ) for TDRS-East**



**Figure 7. Coefficient of Solar Radiation Pressure ( $C_R$ ) for TDRS-West**



1.34 to about 1.46 for TDRS-West. During most of this time interval, the smoother's solar reflectivity coefficient standard deviations ( $1\sigma$ ) remained fairly constant at about 0.02 for TDRS-East and 0.02 for TDRS-West. There appears to be fairly good agreement between the smoother and batch-least-squares results for the solar reflectivity coefficient. In particular, an excursion on the 19<sup>th</sup> of January for TDRS-West is reflected in the results of both systems.

The solar flux values are input to the PFS filter/smoother on a daily basis. The time variation of the flux value over the 24-hour period is not input. Therefore, the atmospheric drag coefficient must be adjusted to compensate for the variation (Figure 5). The filter/smoother also models the area of the TDRS to be a constant throughout the day, whereas in actuality the TDRS surface area exposed to the solar flux varies with a 24-hour period. An oscillatory signature of the variation in  $C_R$  values with a period of 1 day is evident in the smoother results.

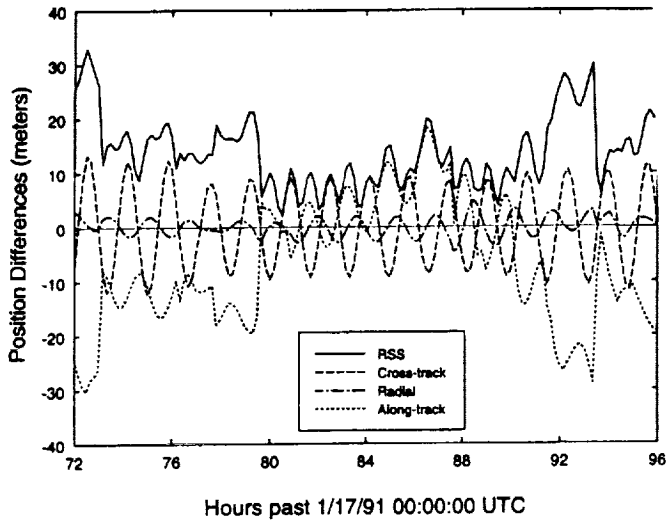
### 3.3 Comparison of Batch and Sequential Estimation Results

Comparisons of the estimated Landsat-4 orbits between GTDS solutions and PFS filter/smoother solutions are presented in Figures 8 and 9. Both figures show the radial, along-track, and cross-track differences, as well as RSS differences over a single day in the middle of the period under investigation (January 20, 1991). Figure 8 shows the comparisons for the filter solution, and Figure 9 shows comparisons for the smoother solution. Both figures are plotted on the same vertical scale so that differences between them are readily apparent. The maximum RSS difference between the filter and the batch-least-squares solution over this time period is about 32 meters, while for the smoother comparison to batch, it is about 12 meters.

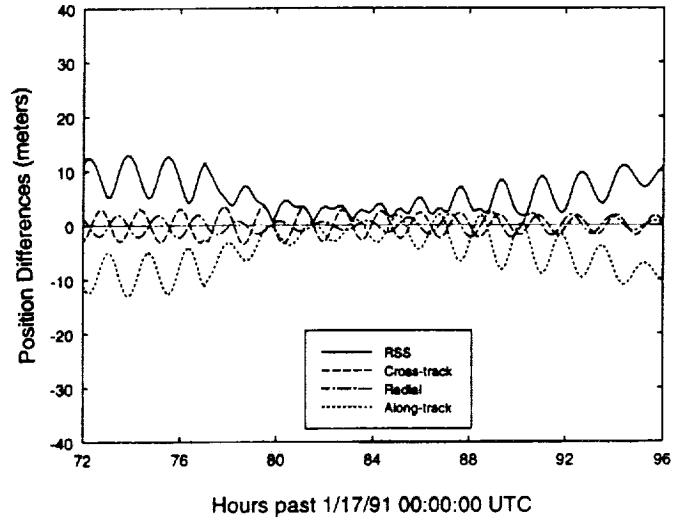
In Figures 10 and 11, comparisons of the estimated TDRS-East orbits between GTDS solutions and PFS filter/smoother solutions are presented. These comparisons are for the same time interval as for the Landsat-4 comparisons. Radial, along-track, cross-track, and RSS comparisons are provided. Figure 10 shows the comparisons for the filter solution, and Figure 11 shows a similar comparison for the smoother solution. The most striking feature is the relatively constant 90-meter along-track offset seen in the filter solution that is not present in the smoother solution. Such an offset ordinarily might have been attributed to coordinate system differences between the two systems or to measurement model discrepancies. Since this offset does not appear in the smoother solution, these explanations are not valid for this case (the smoother uses the same coordinate system and measurement model algorithms as the filter). The origin of the along-track offset in the filter solution for TDRS-East is not known at this time, but further analysis is in progress to identify the cause. The discontinuity in the comparisons at around 5 hours into the day arises because two separate batch-least-squares solutions from different arcs were appended.

Finally, in Figures 12 and 13, comparisons of the estimated TDRS-West orbits between GTDS solutions and PFS filter/smoother solutions are presented. Figure 12 shows the comparisons for the filter solution, and Figure 13 shows a similar comparison for the smoother solution. The along-track offset in the filter solution is smaller than it was for TDRS-East (here it is about 30 meters). The smoother solution also shows an along-track offset, although it is much smaller than for the filter (about 10 meters).

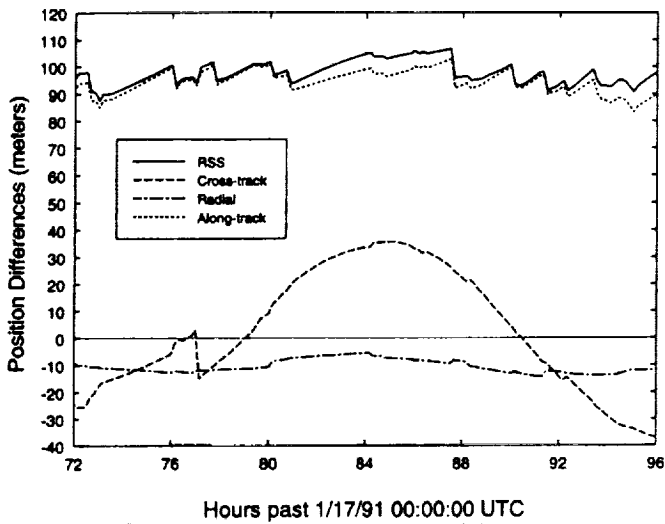
A significant part of the difference between the batch and sequential orbit determination results can be attributed to the differences in the force and measurement models used for GTDS and the PFS filter/smoother. Quantitative estimates for some of these model difference effects are available from previous studies using GTDS. It was reported in Reference 4 that the maximum position differences observed in the definitive ERBS orbits due to the presence and absence of ionospheric refraction correction in the measurement model for the spacecraft-to-spacecraft leg can be  $2.6 \pm 0.9$  meters. The maximum position difference due to solid Earth tide effects on ERBS was measured at  $7.0 \pm 3.2$  meters. A detailed analysis of the influence of polar motion and solid Earth tides on ERBS orbits is given in Reference 10. ERBS is at an altitude of about 600 kilometers, whereas Landsat-4 is at an altitude of about 715 kilometers. Therefore, all the stated effects above for ERBS should be somewhat diminished in magnitude for Landsat-4. However, Landsat-4 has a polar orbit, which has a significant adverse effect on the tracking geometry. Due to the inclusion of a process noise model for geopotential errors in the PFS and its absence in GTDS, the impact of differences in the geopotential models used would be different in the two systems.



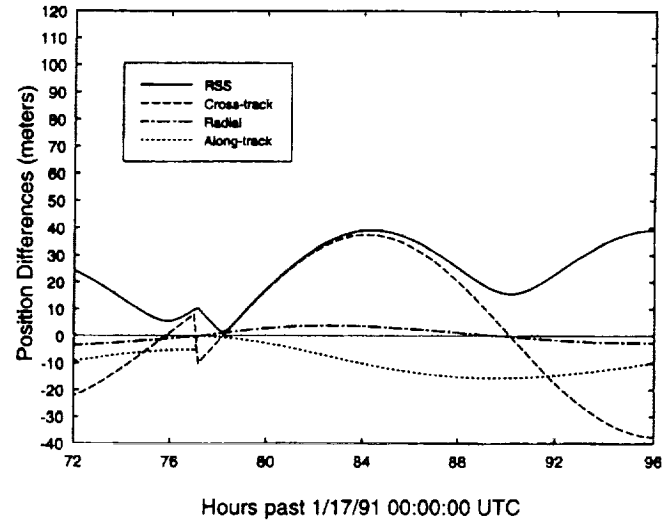
**Figure 8. PFS Filter and GTDS Ephemeris Differences for Landsat-4 (Day 4)**



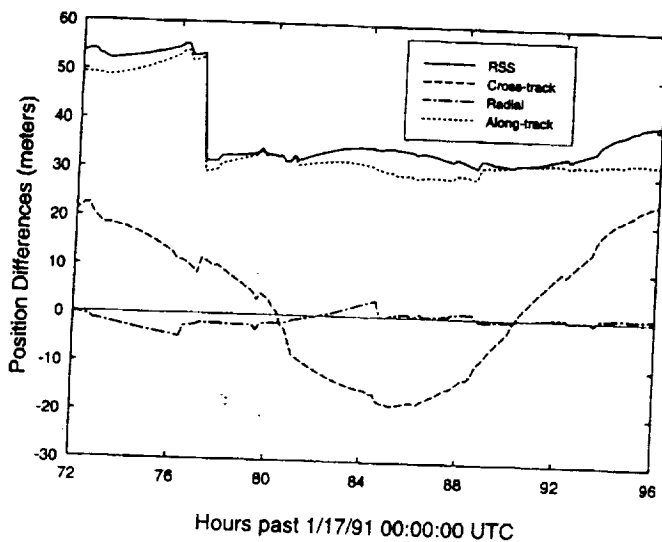
**Figure 9. PFS Smoother and GTDS Ephemeris Differences for Landsat-4 (Day 4)**



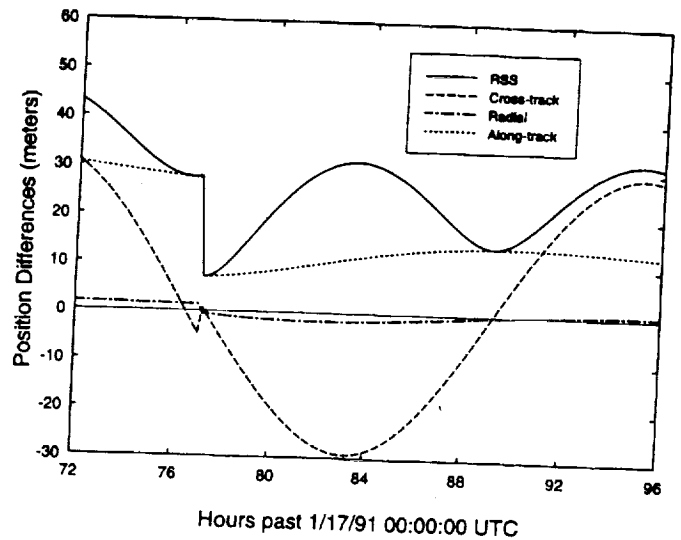
**Figure 10. PFS Filter and GTDS Ephemeris Differences for TDRS-East (Day 4)**



**Figure 11. PFS Smoother and GTDS Ephemeris Differences for TDRS-East (Day 4)**



**Figure 12. PFS Filter and GTDS Ephemeris Differences for TDRS-West (Day 4)**



**Figure 13. PFS Smoother and GTDS Ephemeris Differences for TDRS-West (Day 4)**

Another source of the difference between the GTDS and PFS filter/smoothen estimated ephemerides is due to the fundamental difference in the way the estimated parameters are obtained in the batch-least-squares and sequential estimation techniques. In the batch-least-squares method, a single set of parameter values is estimated over an entire arc. In the sequential estimation process, the set of estimated parameter values is updated at each measurement time. The time variations in selected estimated parameters are shown in Figures 5 through 7.

Based on the magnitude of these differences and the differences in the estimation techniques, the maximum position difference of about 12 meters between the GTDS and PFS smoother results is not unusual. Also, as expected, the smoother provides more accurate solutions since it utilizes data occurring both before and after a given time to estimate the state at that time.

The sensitivity of orbit determination accuracy to varying tracking schedules was reported in Reference 11. The corresponding covariance analysis was also reported in the same paper. The results of a study that successfully processed through orbit-adjust maneuvers were reported in Reference 12.

#### 4.0 Conclusions

This study presented an analysis of TDRSS user orbit determination using a batch-least-squares method and a sequential estimation method. Independent assessments were performed of the orbit determination consistency within each method, and the estimated orbits obtained by the two methods were also compared. This assessment is applicable to the dense-tracking measurement scenario for tracking Landsat-4.

In the batch-least-squares method analysis, the orbit determination consistency for Landsat-4, which was heavily tracked by TDRSS during January 1991, was found to be about 4 meters in the RMS overlap comparisons and about 6 meters in the maximum position differences in overlap comparisons. In the sequential method analysis, the consistency was found to be about 12 meters in the  $3\sigma$  state error covariance function for the smoother and 30 meters for the filter; and, as a measure of consistency, the first residual of each pass was within the  $3\sigma$  bound in the residual space for the filter.

After the filter/smoothen had reached steady state, the differences between the definitive batch-least-squares ephemerides and the sequentially estimated forward filter ephemerides were no larger than 30 meters, and the differences between the batch-least-squares ephemerides and the sequentially estimated smoothed ephemerides

were no larger than 12 meters. The application of a smoother algorithm to the filter solutions consistently reduced the difference with the batch-least-squares solutions. These results demonstrate that smoother postprocessing offers the potential for significant improvement in sequential estimation solution accuracy.

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