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APPLICATION OF A MONTE CARLO ACCURACY ASSESSMENT TOOL TO TDRS AND GPS

Michael S. Pavloff

The MITRE Corporation, Bedford, MA 01730

ABSTRACT

In support of a NASA study on the application of radio interferometry to satellite orbit determination, MITRE developed a simulation tool for assessing interferometric tracking accuracy. Initially, the tool was applied to the problem of determining optimal interferometric station siting for orbit determination of the Tracking and Data Relay Satellite (TDRS). Subsequently, the Orbit Determination Accuracy Estimator (ODAE) was expanded to model the general batch maximum likelihood orbit determination algorithms of the Goddard Trajectory Determination System (GTDS) with measurement types including not only group and phase delay from radio interferometry, but also range, range rate, angular measurements, and satellite-to-satellite measurements. The user of ODAE specifies the statistical properties of error sources, including inherent observable imprecision, atmospheric delays, station location uncertainty, and measurement biases. Upon Monte Carlo simulation of the orbit determination process, ODAE calculates the statistical properties of the error in the satellite state vector and any other parameters for which a solution was obtained in the orbit determination.

This paper presents results from ODAE application to two different problems: (1) determination of optimal geometry for interferometric tracking of TDRS, and (2) expected orbit determination accuracy for Global Positioning System (GPS) tracking of low-earth orbit (LEO) satellites. Conclusions about optimal ground station locations for TDRS orbit determination by radio interferometry are presented, and the feasibility of GPS-based tracking for IRIDIUM, a LEO mobile satellite communications (MOBILSATCOM) system, is demonstrated.

INTRODUCTION

As part of its effort to assess cost and performance benefits of various emerging technologies, NASA Headquarters sponsored a series of studies on the application of radio interferometry to satellite tracking. Though astronomers had used radio interferometry for decades prior, it was not until the late 1960s that interferometry was proposed for use in satellite orbit determination. In an experiment devised by Irwin Shapiro, Alan Whitney, and others, very long baseline interferometric (VLBI) measurements were made on the TACSAT I communications satellite in geosynchronous orbit (GEO), and the semi-major axis of the orbit was measured with accuracy on the order of several hundred meters [1]. Subsequent experiments performed in the 1980s by Jim Ray, Curt Knight, and others to determine the position of the Tracking and Data Relay Satellite (TDRS) yielded accuracy on the order of 75 meters [2]. Such orbit determination accuracy, which derives from the extremely high precision of the group delay and phase delay observables, make radio interferometry an attractive option for satellite tracking.

Operational considerations are also a benefit of radio interferometry in satellite orbit determination, because the group and phase delay measurements are made completely passively. Whereas the existing Bilateral Ranging Transponder System (BRTS) is taxing on TDRS communications resources, radio interferometry can derive its measurements from any signal, including the signal intended for the TDRS user community. Therefore, an interferometric orbit determination system for TDRS would eliminate traffic for tracking on the TDRS transponder. Because an interferometric tracking system would be passive, it would place no design constraints on the space segment, and it would therefore provide backward compatibility with all generations of TDRS. Thus, NASA found radio interferometry to be an attractive technology to pursue for future TDRS tracking applications.

NASA sponsored a series of studies to investigate whether an operational radio interferometry system could provide TDRS orbit determination services (1) at lower cost, (2) at greater accuracy, and (3) across considerably smaller baselines than BRTS. Contributors to these studies included Interferometrics, Inc., where a Small Business Initiative Research (SBIR) contract with NASA was executed to demonstrate hardware and software that would provide group delay measurements on TDRS with VLBI. CSC performed an assessment for the Goddard Space Flight Center (GSFC) on a variety of TDRS tracking alternatives, including VLBI and Connected Element Interferometry (CEI) systems. The Jet Propulsion Laboratory (JPL) sponsored a series of experiments to determine CEI accuracy from its Goldstone facility. For its part of the effort, MITRE assessed optimal site locations and expected life-cycle costs of an operational interferometric TDRS orbit determination system.

For accuracy assessment purposes, MITRE developed a Monte Carlo simulation tool, the Orbit Determination Accuracy Estimator (ODAE), that initially modeled error sources in orbit determination with VLBI and CEI systems. In ODAE, the user can specify a satellite orbit, any set of ground stations between which group or phase delay measurements are to be made, and the statistical properties of the error in those measurements. Upon each iteration of the Monte Carlo simulation, the orbit of the satellite is determined based on measurements with errors added, and the errors in the resulting satellite ephemerides are recorded. Thus, the user may study the statistical properties of the error in the batch orbit determination process resulting from the use of group or phase delay measurements. The initial application of ODAE was to study the effects of varying satellite and measuring station geometries on orbit determination accuracy and to propose optimal siting for TDRS tracking by radio interferometry. The results of that study are presented in this paper.

Subsequent studies for the United States Air Force (USAF) on Space Surveillance Network (SSN) accuracy and Global Positioning System (GPS) accuracy led to the expansion of ODAE to include range, range rate, azimuth and elevation angle, and satellite-to-satellite measurement types. The potential application of GPS to satellite tracking has been under consideration for a number of years (e.g., [3]). For GEO satellites, which have higher orbits than GPS satellites, the problems of low GPS satellite visibility and weak signal strength present limitations [4]. However, for LEO satellites, which have lower orbits than GPS satellites, visibility and signal strength are greatly improved. Attention has been focused recently on LEO satellites in the arena of mobile communications. A number of LEO MOBILSATCOM systems are currently in planning or development, including Motorola's IRIDIUM, Loral's Globalstar, TRW's Odyssey, and, most recently, Teledesic, a joint venture of Kinship Partners with William Gates and Craig McCaw [5]. We used ODAE to study the accuracy of GPS tracking for the IRIDIUM system, the results of which are shown herein.

THE ODAE MODEL

ODAE models the batch maximum likelihood orbit determination process applied in the Goddard Trajectory Determination System (GTDS) [6]. The user specifies a reference true satellite orbit, a set of observing stations (earth-based or space-based), the observation types, and the times at which measurements are to be made. Given a set of observations on the satellite (e.g., radar measurements, group or phase delay measurements, or pseudorange measurements), ODAE determines the set of parameters (e.g., state vector, clock offsets, or atmospheric parameters) that best fit the observations. Upon each iteration of its Monte Carlo simulation, ODAE injects errors of user-specified statistical properties into various parts of the orbit determination process. ODAE computes the error of the measured parameters at each iteration, and at the end of the simulation, ODAE computes the statistical characteristics of the error.

Error sources that can be modeled by ODAE include inherent measurement imprecision, station location uncertainty, atmospheric delays, and clock offsets. The user must specify the statistical properties of the error sources. Trajectory propagation schemes for dynamic orbit determination range from the two-body approximation to numerical integration of the fully disturbed equations of motion. A detailed mathematical specification of the coordinate frame, force models, and numerical integration techniques used in ODAE are given in Reference 7. The only significant deviation from the GTDS approach to orbit determination is the use of Bulirsch-Stoer rational function extrapolation for numerical integration [8, 9]. For the numerical integration of the equations of satellite motion, the Bulirsch-Stoer technique has been shown to provide the same precision as more traditional techniques, such as predictor-corrector integration or Runge-Kutta integration, but at reduced computational cost [7, 10]. For short-term dynamic orbit determination accuracy studies to assess the relative effects of changes in station geometry or measurment errors, it is often sufficient to apply simplified trajectory propagation schemes for the sake of reducing computation time.

ODAE was implemented in Mathematica to allow maximum flexibility of the model. Since its initial application to the problem of optimal ground station siting for interferometric tracking of TDRS, MITRE has applied ODAE to a variety of problems. Most recently, MITRE has proposed the use of ODAE for assessment of initial orbit determination accuracy with the HAVE STARE radar. Existing applications include the assessment of orbit determination accuracy for the Space Surveillance Network Improvement Program (SSNIP) for various classes of orbits, and to the determination of GPS accuracy for various scenarios. Although computational time is increased by using Mathematica, it allows for very natural representation of the equations of motion, numerical integration schemes, and the batch orbit determination algorithm. Also, because ODAE is written in Mathematica, it is very quickly adaptable to a variety of problems, including satellite-to-satellite tracking and GPS navigation. After an overview of the group and phase delay measurement functions, applications of ODAE to TDRS tracking and to GPS navigation for a LEO satellite system are described.

GROUP DELAY AND PHASE DELAY MEASUREMENT FUNCTIONS

Consider an interferometric orbit determination scenario in which O is the origin of an earth-centered inertial (ECI) coordinate system, $\mathbf{r}$ is the position vector of a satellite with respect to O, $\mathbf{b}_1$ and $\mathbf{b}_2$ are the position vectors of two ground stations from which measurements are to be made, and $\mathbf{d}_1$ and $\mathbf{d}_2$ are the position vectors of the satellite with respect to those ground stations, as pictured in Figure 1. The position vectors $\mathbf{r}$, $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{d}_1$, and $\mathbf{d}_2$ are all functions of time. The sum of a station position vector, $\mathbf{b}_k$, and the satellite position vector measured from that station, $\mathbf{d}_k$, is simply the satellite position vector $\mathbf{r}$; therefore, $\mathbf{d}_k = \mathbf{r} - \mathbf{b}_k$. If the propagation rate, c, of the signal through the atmosphere is known, then the transit time, T_k, of the signal from the satellite at point P to ground station number k at point B_k will be given by

$$T_k = \tfrac{1}{c}|\mathbf{d}_k| = \tfrac{1}{c}\sqrt{(\mathbf{r}-\mathbf{b}_k)\cdot(\mathbf{r}-\mathbf{b}_k)}$$

The true group delay, τ, between stations 2 and 1 is the differential transit time of the signal between these two sites:

$$\tau = T_2 - T_1 = \tfrac{1}{c}\left(|\mathbf{d}_2| - |\mathbf{d}_1|\right) = \tfrac{1}{c}\left[\sqrt{(\mathbf{r}-\mathbf{b}_2)\cdot(\mathbf{r}-\mathbf{b}_2)} - \sqrt{(\mathbf{r}-\mathbf{b}_1)\cdot(\mathbf{r}-\mathbf{b}_1)}\right] \tag{1}$$

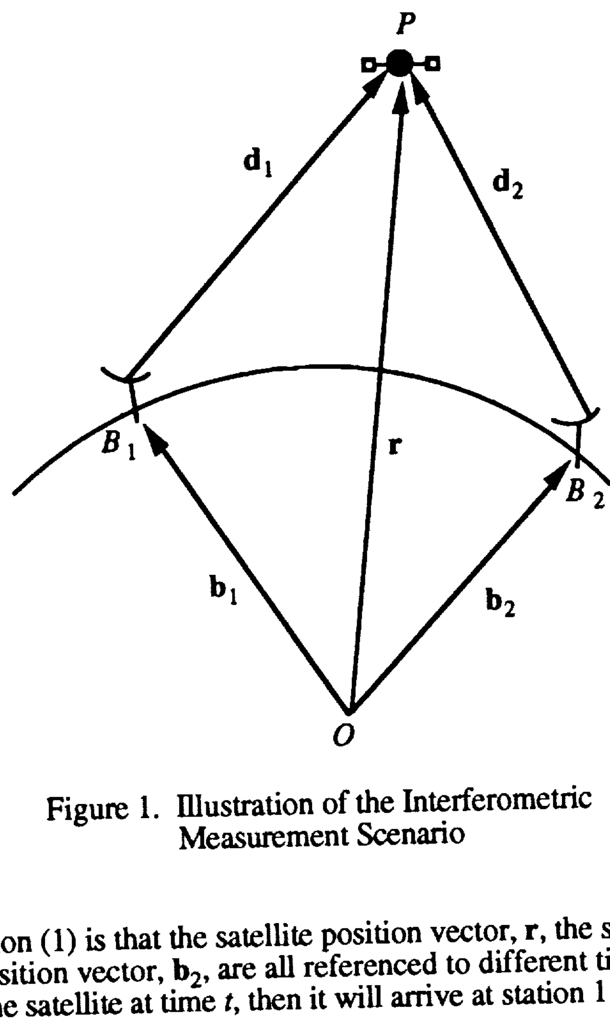

Figure 1. Illustration of the Interferometric Measurement Scenario

A subtlety of equation (1) is that the satellite position vector, $\mathbf{r}$, the station 1 position vector, $\mathbf{b}_1$, and the station 2 position vector, $\mathbf{b}_2$, are all referenced to different times. If the measured signal emanates from the satellite at time t, then it will arrive at station 1 at time $t + T$, and it will arrive at station 2 at time $t + T + \tau$, where T is the signal transit time from the satellite to station 1, and τ is the true group delay between stations 1 and 2. If the satellite position vector $\mathbf{r}$ is measured at time t, then the station 1 position vector is measured at time $t + T$, and the station 2 position vector is measured at time $t + T + \tau$. Thus, we write $\mathbf{r} = \mathbf{r}(t)$, $\mathbf{b}_1 = \mathbf{b}_1(t + T)$, and $\mathbf{b}_2 = \mathbf{b}_2(t + T + \tau)$. The group delay equation (1) is, therefore, more properly written as follows:

$$\tau = \tfrac{1}{c}\sqrt{[\mathbf{r}(t) - \mathbf{b}_2(t+T+\tau)]\cdot[\mathbf{r}(t) - \mathbf{b}_2(t+T+\tau)]} - \tfrac{1}{c}\sqrt{[\mathbf{r}(t) - \mathbf{b}_1(t+T)]\cdot[\mathbf{r}(t) - \mathbf{b}_1(t+T)]} \tag{2}$$

In ODAE, a user specifies a scenario that includes epoch time, satellite state vector at epoch, latitude, longitude, and altitude of interferometer sites, and times at which measurements are to be taken. ODAE must then calculate the true group delay observables from equation (2), but as can be seen, the right-hand side of (2) is a function of τ. Therefore, ODAE solves equation (2) for τ iteratively, as described in Reference 7. During the Monte Carlo simulation, ODAE computes measured group delay by adding measurement or atmospheric fluctuation errors to the true group delay as computed above. The solution of the orbit determination problem on each iteration of the simulation, as described in Reference 7, follows the GTDS maximum likelihood estimation approach, one step of which is the computation of the Jacobian, or matrix of partial derivatives of equation (2) with respect to the state vector parameters at epoch.

For phase delay measurements, ODAE converts phase delay into equivalent group delay. If ν is the reference center frequency of the phase delay measurement, N is an integer number of signal cycles, and ϕ is the true phase delay, then the equivalent true group delay τ can be computed as follows:

$$\tau = \frac{\phi + 2\pi N}{2\pi\nu} \tag{3}$$

This computation can be accomplished so long as the cycle ambiguity N can be determined from *a priori* information about the satellite's position vector. ODAE allows the user to model clock offsets or local oscillator offsets for group or phase delay measurements, respectively. In such cases, the offset is taken as an additional parameter in the orbit determination process.

APPLICATION TO TDRS

In this section, the level of GEO satellite orbit determination accuracy that can be attained with radio interferometry is demonstrated, and conclusions about optimal station-satellite geometry are drawn. The results are applied to recommend optimal ground station siting for orbit determination of TDRS by radio interferometry.

Radio interferometry with baselines the size of BRTS's, which are intercontinental, would translate the high level of observable group delay accuracy into greatly improved TDRS tracking accuracy. However, it was NASA's desire instead to accept only a modest improvement in accuracy while reducing system cost and ameliorating other operational considerations by greatly shortening the baselines. This led naturally to the study of a CEI-based system, where baselines are very short. Because of the requirement for a CEI system to have interferometer sites connected by fiberoptic cable in a temperature-controlled environment, the cost of lengthening baselines is very high. We constrained our baselines to 20 km maximum length for the purposes of this study.

We used ODAE to assess position determination accuracy for a baseline scenario and to determine the effects of varying the relative satellite to ground station geometry. Because the effect only of relative geometry was to be studied initially, it was not necessary to select true TDRS ephemerides or true potential ground station locations. The reference orbit chosen was geosynchronous with a 4° inclination and a subsatellite longitude of 18°W. To provide three independent baselines across which phase delay could be measured, we constrained four CEI sites to lie on the vertices of a square with a 20 km baseline, as shown in Figure 2. The site latitudes, longitudes, and altitudes for this reference scenario are given in Table 1. ODAE modeled simultaneous phase delay measurements across the baselines from station 2 to station 1, station 3 to station 1, and station 4 to station 1 (denoted 2-1, 3-1, and 4-1, respectively). These baselines are illustrated in bold in Figure 2.

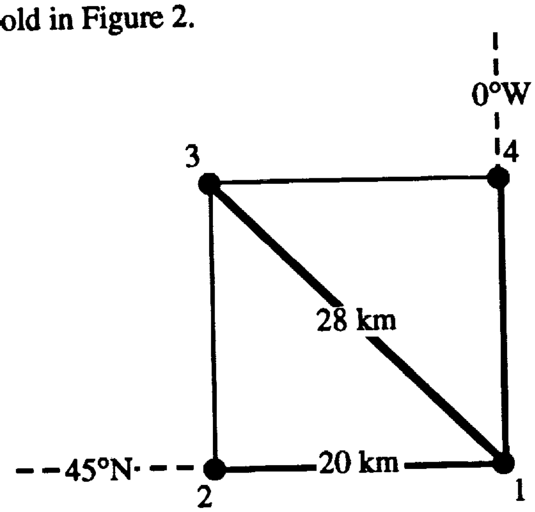

Figure 2. CEI Station Locations

Table 1. CEI Station Locations for Reference Scenario

Station Number	Geodetic Latitude (°)	Longitude (°E)	Altitude (km)
1	45.00000	0.0000	0.1
2	45.00000	-0.2545	0.1
3	45.17997	0.0000	0.1
4	45.17997	-0.2545	0.1

An extension of Alan Whitney's work [11] shows that the theoretically achievable precision of the phase delay observable, σ_ϕ, is given by

$$\sigma_\phi = \frac{1}{2\pi(SNR)\nu} \qquad (4)$$

where ν is the center frequency, in Hz, sampled by the interferometer. Since the TDRS downlink to White Sands is centered at 14 GHz, the theoretically achievable precision of the phase delay observable is 0.23 picosec. While no TDRS tracking experiments were performed with JPL's

CEI equipment at Goldstone, observations were made on natural radio sources at 8.4 GHz to assess the precision of the phase delay observable [12]. The statistical phase error, expressed in radians, is roughly $1/SNR$, and JPL's experiments at Goldstone demonstrated a typical phase error of 0.005 cycles [13]. The achieved SNR was therefore $1/(2\pi\times0.005) \approx 32$. At 8.4 GHz, relationship (4) predicts a phase delay observable precision of 0.59 picosec. JPL demonstrated the standard deviation in the phase delay observable to be approximately 1 picosec [12], which is a factor of 1.7 larger than the theoretically achievable value. Applying this factor to the theoretically achievable phase delay precision for TDRS, we estimated the practically achievable precision to be $0.23\times1.7 \approx 0.4$ picosec. We took this measurement error to be independently normally distributed across each baseline. For the initial study, it was assumed that there were no equipment biases, that there were no atmospheric delay errors, that all station were connected by fiberoptic cable to one clock and frequency standard, that there were no local oscillator offsets between the four stations, and that station positions were known with perfect accuracy. Thus, the pure effect of measurement geometry and observable precision on orbit determination could be assessed.

ODAE Monte Carlo simulation of the orbit determination scenario described above with 200 iterations showed a 1σ root-mean-squared (RMS) position vector accuracy of 3.2 km. We also assessed the accuracy that can be attained with the use of other combinations of baselines. It is practical to have one site in common for all three measurements so that the common site can act as the correlation center at which the phase delay observables are generated. For the particular satellite and ground station locations in this scenario, selection of three measurements where one station is common to each pair (i.e., 2-1, 3-1, 4-1; or 1-2, 3-2, 4-2; or 1-3, 2-3, 4-3; or 1-4, 2-4, 3-4) results in a 1σ RMS position vector accuracy of 3.2 km. Thus, there is no geometrically-preferred common site for the measurements.

The orbit determination scenario described above was the starting point for the assessment of the effects of varying interferometric measurement geometry on orbit determination accuracy. Since only relative geometry matters, and since it would have been more cumbersome to vary the positions of four ground stations, we instead varied the satellite's initial position vector. First, we studied the effect of relative interferometer baseline size on orbit determination accuracy. Satellite range from station 1 was varied while keeping the elevation angle and azimuth angle from that station constant. Because the baseline sizes are small relative to the range to GEO, the range, elevation angle, and azimuth angle from each of the other three stations is close to that of the first. For this particular orbit determination scenario, range from each site to the satellite is approximately 37,850 km, the elevation angle is approximately 39°, and the azimuth angle is approximately 155°. As shown in Figure 3, the smaller the range to the satellite for a constant baseline length (or, equivalently, the longer the baselines across which phase delay is measured relative to the range to the satellite), the greater the position vector accuracy.

Next, we assessed the effect of satellite azimuth angle on orbit determination accuracy. The azimuth angle of the satellite at station 1 in the original scenario was varied while keeping the range and elevation angle from that station constant. Figure 4 shows the variation in position determination accuracy with satellite azimuth angle. The results indicate that for a configuration of four interferometric ground stations at the vertices of a square, position error is maximized when the satellite's azimuth angle is an integer multiple of 90°, and position error is minimized when the satellite's azimuth angle is an integer multiple of 45°.

Finally, we assessed the effect of satellite elevation angle on orbit determination accuracy in this scenario. The elevation angle of the satellite at station 1 was varied while keeping the range and azimuth angle from that station constant. As can be seen in Figure 5, for this particular orbit determination scenario, position error increases monotonically with elevation angle. Thus, based

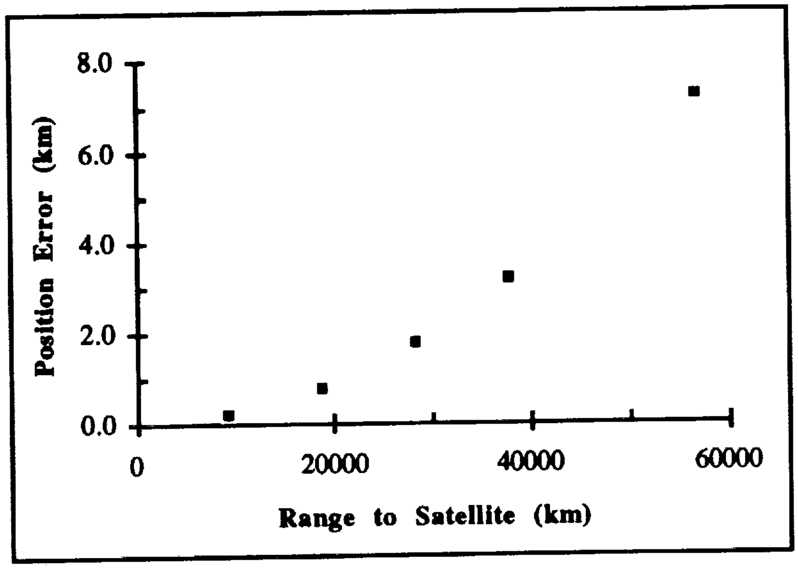

Figure 3. Position Error vs. Range to Satellite

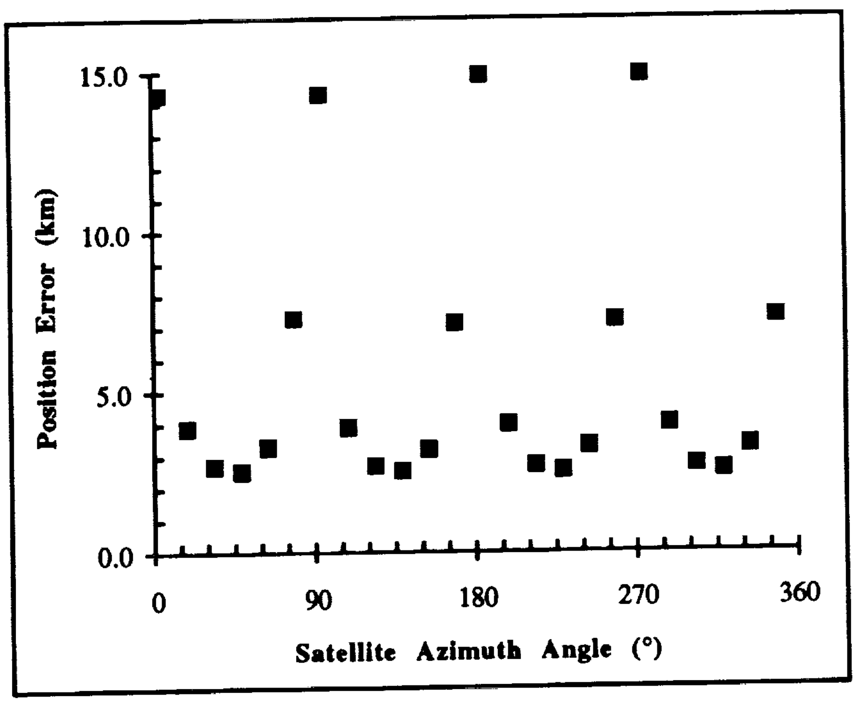

Figure 4. Position Error vs. Satellite Azimuth Angle

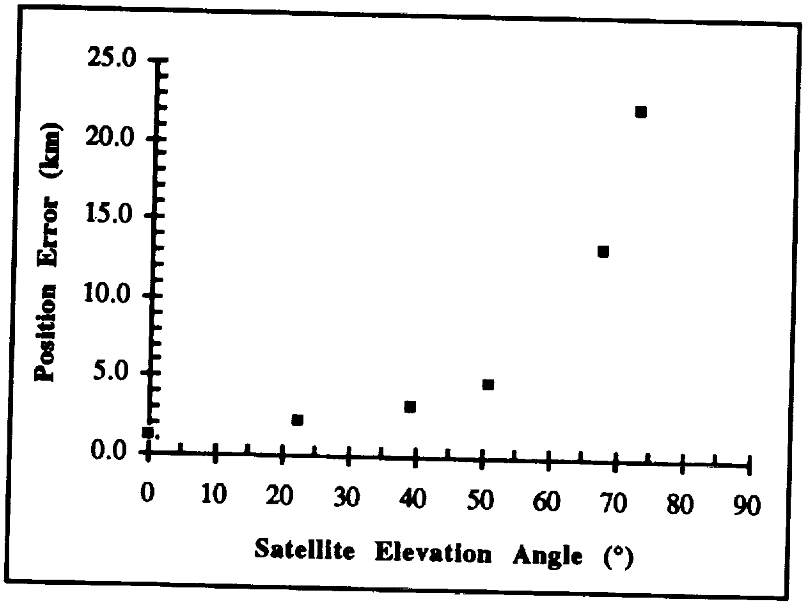

Figure 5. Position Error vs. Satellite Elevation Angle

on the criterion of minimizing ephemeris error due only to error in the phase delay measurement, optimal viewing geometry is at the lowest possible elevation angle, and the scenario becomes degenerate when the satellite is at zenith.

A tradeoff is suggested by the geometrical result that greater orbit determination accuracy is attained at lower elevation angles. The tradeoff arises because statistical models of the variation in signal propagation rate through the troposphere show that errors in predicting signal propagation rate increase as elevation angle decreases [14, 15]. Moreover, errors in predicting propagation rate due to tropospheric fluctuations tend to be the dominant error source in overall accuracy for CEI systems [15]. Thus, we sought to determine the optimal elevation angle for CEI measurements with consideration of both measurement error and tropospheric delay error.

We modeled tropospheric fluctuations between each interferometer site and the satellite as being independently normally distributed. The assumption of independence is based on the fact that water vapor cells can be of several kilometers in diameter, and so tropospheric delay errors from each site can in fact be independent. For an elevation angle of 20° and for a 100 second measurement duration, the magnitude of the standard deviation in tropospheric delay error was estimated to be 4 picosec [14]. The elevation angle dependence of the standard deviation in tropospheric delay error follows the square root of the structure function calculated in Reference [14]. Also, under the assumption of independent errors along each station-to-satellite path, the variance in phase delay error for a particular measurement pair will be the sum of the variances along each path. Since elevation angles are roughly equal along each path, the standard deviation of the phase delay error will, therefore, be $\sqrt{2}$ times the standard deviation along one path. From these assumptions and the results in Reference [14], we computed the values of standard deviation in phase delay error due to tropospheric fluctuations shown in Table 2.

For varying satellite elevation angles, we used ODAE to model error due to tropospheric fluctuations as well as inherent phase delay imprecision. The resulting 1σ position errors are shown in Figure 6. As can be seen, the optimal satellite elevation angle is approximately 30°. In the conclusions section of this paper, we show how these results can be applied to optimally siting a CEI system for TDRS orbit determination.

Table 2. Standard deviation in phase delay error due to tropospheric fluctuations as a function of elevation angle

Elevation Angle (°)	Tropospheric Delay Error (picosec)
10	7.5
20	5.7
30	4.6
40	3.9
50	3.3
60	3.0

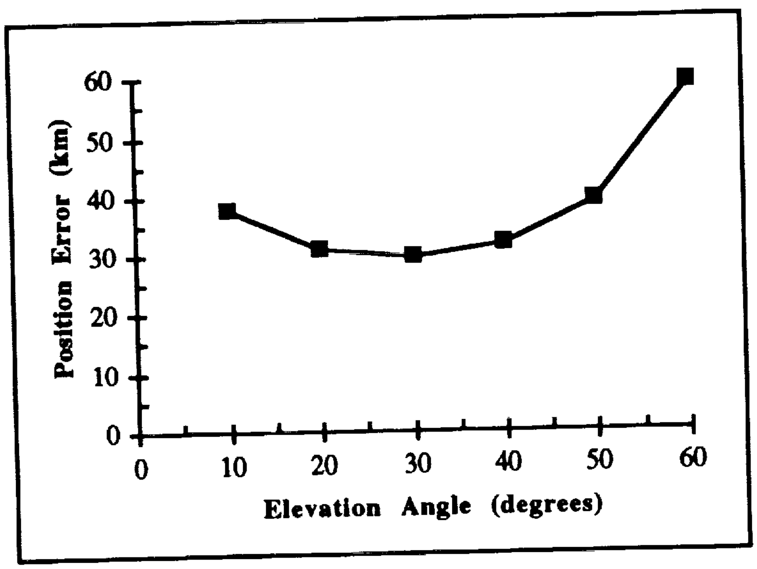

Figure 6. Position Error vs. Elevation Angle with Tropospheric Effects Included

APPLICATION TO GPS

Studies of GPS accuracy for a precision landing system and other GPS-related studies for the Department of Defense led to the application of ODAE to GPS accuracy problems. ODAE models the GPS navigation problem in much the same way as it models the satellite orbit determination problem. In the case of GPS navigation, a "station location" is the position of a GPS satellite at a particular time. GPS almanac data are loaded into ODAE and propagated to the desired measurement times. The unknown receiver position can be on the surface of the earth or on a satellite, and in the case of the former, the satellite orbit is also propagated to the desired measurement time. ODAE tests the visibility of satellites in the GPS constellation from the receiver position, and if the number of visible satellites exceeds the number of available channels in the receiver, ODAE determines the optimal subset of satellites for measurement.

GPS has been considered an attractive option for satellite orbit determination, in part because of the potential for cost and weight savings on a satellite's telemetry, tracking, and command (TT&C) subsystem. In the NASA study of TDRS tracking alternatives, GPS suffered from the problems of limited visibility from GEO. However, for LEO systems, GPS is a more viable alternative for orbit determination. To demonstrate potential accuracy, we considered the tracking of IRIDIUM by GPS.

The IRIDIUM constellation is currently designed to consist of 66 satellites in orbits with a semi-major axis of 7,143 km [16]. Satellites will be divided into six orbital planes spaced by 31.58° in right ascension of the ascending node. Orbital inclinations will be 86.4°, and eccentricities will be 0.0013. Satellites will be spaced equally within each plane, and adjacent planes will be half-way out of phase with one another. We selected an arbitrary satellite from the IRIDIUM constellation (right ascension of the node 31.58°, initial mean anomaly 0°) for analysis. We used ODAE to determine the number of GPS satellites visible to this IRIDIUM satellite as a function of time, and we used ODAE to determine position accuracy as a function of time.

The most recent GPS almanac data (2/13/94 at this time of this study) included the full constellation of 26 satellites. For the purposes of assessing GPS satellite visibility, we assumed a GPS beam width of approximately 27°, which is the angle subtended by the earth from a GPS satellite. Because the beam width is, in fact, larger than 27°, it would be possible to acquire a GPS signal from a satellite on the opposite side of the earth. However, since larger antennas would be required to achieve the necessary gain, the TT&C weight savings would be compromised. Consequently, for the purposes of this study, we considered GPS satellites to be visible to an IRIDIUM satellite only if they are on the same side of the earth.

Figure 7 shows the number of GPS satellites visible as a function of time from the reference IRIDIUM satellite. As can be seen, for four hours, fewer than four GPS satellites are visible; therefore, the GPS system availability to the IRIDIUM satellite would be 0.83. However, with an accurate clock, IRIDIUM could maintain continuous positioning services through GPS because only three visible satellites would be required.

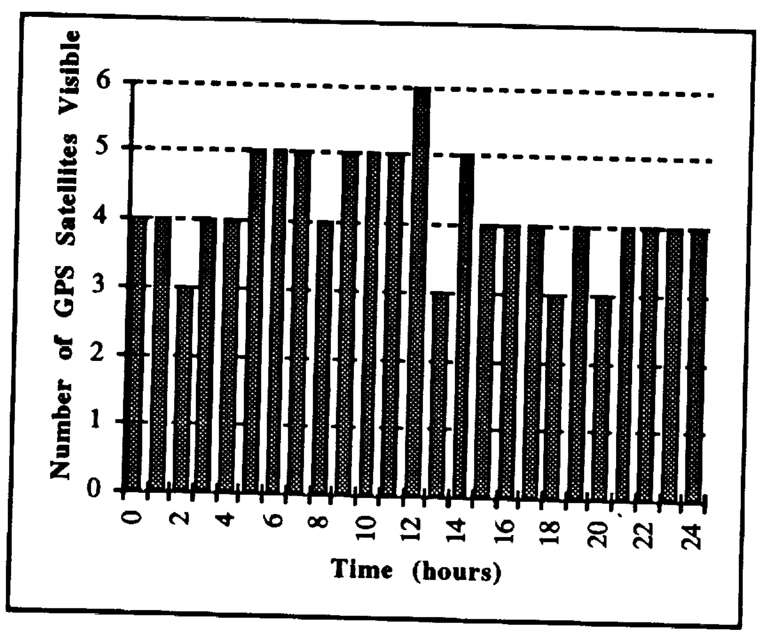

Figure 7. GPS Satellites Visible from an IRIDIUM Satellite Over a One-Day Period

We next computed position accuracy as a function of time for IRIDIUM tracking by GPS. Figure 8 shows position dilution of precision (PDOP) across a 24-hour period for instantaneous position fixes. For the points where only three GPS satellites were visible, we assumed that an accurate clock was available and that the navigation solution could, therefore, be obtained. As expected, Figure 8 shows poorest accuracy at the times when fewer GPS satellites are visible. Except for those times, baseline PDOP appears to be on the order of 2 or 3.

Finally, we used ODAE to compute dynamic orbit determination accuracy for IRIDIUM tracking by GPS. Figure 9 shows PDOP across a 24-hour period where measurements are taken on the hour, added to the previous pool of measurements, and processed in batch. After four hours, PDOP decreases below a value of one, and after 10 hours, a value of roughly 0.5 is obtained. For a low-accuracy situation where the precision of the pseudorange measurement is on the order of 3 meters, the resulting long-term IRIDIUM tracking accuracy would be on the order of $0.5 \times (3 \text{ meters}) = 1.5$ meters. Such accuracy is likely to be sufficient, even with IRIDIUM's stringent formationkeeping requirements [16].

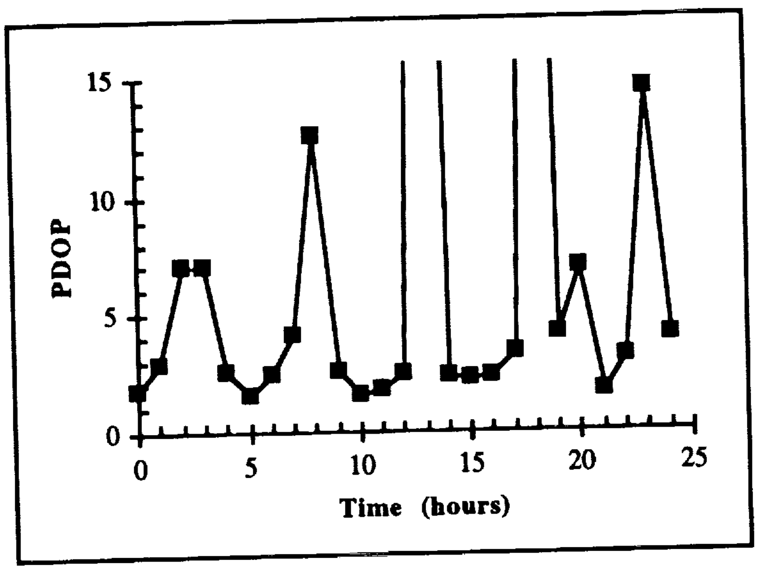

Figure 8. Position Accuracy over a One-Day Period for an IRIDIUM Satellite

CONCLUSIONS

In the first part of this report, we derived conclusions about optimal geometry for orbit determination of a GEO satellite by radio interferometry. Those results can be applied to the problem of optimally siting a CEI system to track TDRS. For a particular TDRS satellite, and for a configuration of four interferometer sites located at the vertices of a square, a geographical position should be chosen so that the satellite's elevation angle is as close to 30° as possible, and the square should be oriented so that the satellite's azimuth angle is an integer multiple of 45°. For TDRS-W at 171°W, the maximum elevation angle visible within the -20 dB contour of the White Sands downlink is in southern California at approximately 20° elevation. For TDRS-E at 41°W, an elevation angle near 30° can be attained within the -20 dB contour of the White Sands downlink by siting a CEI system in eastern Louisiana or western Mississippi.

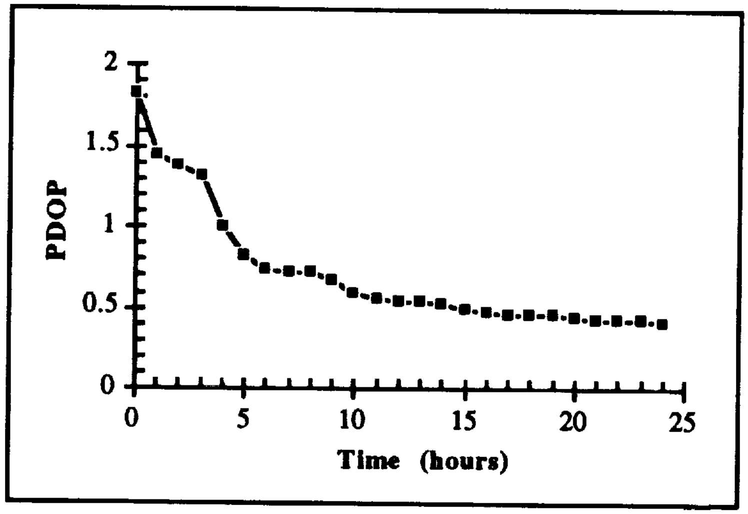

Figure 9. Dynamic Orbit Determination Accuracy over a One-Day Period for an IRIDIUM Satellite

In the second part of this report, we assessed GPS availability and accuracy for tracking a LEO satellite. In particular, for IRIDIUM, where the proposed constellation consists of satellites in near-polar orbits at altitudes of 785 km, instantaneous orbit determination accuracy is available at the level of 9 meters (1σ), and long-term dynamic orbit determination can reduce errors to the level of 1.5 meters. Because GPS receiver equipment has the potential of offering reduced weight and cost by comparison with traditional TT&C equipment, GPS provides an attractive tracking alternative for LEO satellites such as IRIDIUM.

LIST OF REFERENCES

1. Preston, R. A., et al, 27 October 1972, "Interferometric Observations of an Artificial Satellite," *Science*, Vol. 178, pp. 407-409.
2. Ray, J., et al., October-December 1988, "VLBI Tracking of the TDRS," *The Journal of the Astronautical Sciences*, Vol. 36, No. 4, pp. 347-364.
3. Leeuwen, A. V., E. Rosen, and L. Carrier, 1980, "The Global Positioning System and its Application in Spacecraft Navigation," Washington, D.C.: The Institute of Navigation, reprints from *Navigation*, Vol. I.
4. Potash, R., et al, September 1989, *Advanced Tracking Systems Design and Analysis*, CSC/TM-88/6060.
5. Tucci, L., 28 March - 3 April 1994, "Teledesic System Proposal Awaits FCC Approval," *Space News*, Vol. 5, No. 13.
6. Long, A. C., et al (ed.), July 1989, *Goddard Trajectory Determination System (GTDS) Mathematical Theory, Revision 1*, FDD/552-89/001, Greenbelt, MD: Goddard Space Flight Center.
7. Pavloff, M. S., September 1993, *A Monte Carlo Tool for Simulation of Satellite Orbit Determination by Radio Interferometry*, Cambridge, MA: The Massachusetts Institute of Technology, Master's Thesis, Department of Aeronautics and Astronautics.
8. Bulirsch, R., and J. Stoer, 1966, "Numerical Treatment of Ordinary Differential Equations by Extrapolation Methods," *Numerische Mathematik*, Vol. 8, pp. 1–13.
9. Stoer, J., and R. Bulirsch, 1980, *Introduction to Numerical Analysis*, New York: Springer-Verlag Inc.
10. Pavloff, M. S., September 1992, *Analysis of Orbit Prediction Algorithms for the Universal Modem System*, WP-92B0000263V1, Bedford, MA: The MITRE Corporation.
11. Whitney, A. R., January 1974, *Precision Geodesy and Astrometry Via Very Long Baseline Interferometry*, Cambridge, MA: The Massachusetts Institute of Technology, PhD Dissertation, Department of Electrical Engineering.
12. Edwards, C. D., "Angular Navigation on Short Baselines Using Phase Delay Interferometry," *IEEE Transactions on Instrumentation and Measurement*, Vol. 38, No. 2, April 1989.
13. Edwards, C. D., "Development of Realtime Connected Element Interferometry at the Goldstone Deep Space Communications Complex," AIAA 90-2903, *AIAA/AAS Astrodynamics Conference*, August 1990.
14. Treuhaft, R. N., and G. E. Lanyi, "The Effect of Wet Troposphere on Radio Interferometric Measurements," *Radio Science*, Vol. 22, No. 2, March-April 1987, pp. 251-265.
15. Edwards, C. D., "The Effect of Spatial and Temporal Wet-Troposphere Fluctuations on Connected Element Interferometry," TDA Progress Report 42-97, January-March 1989.
16. Gaffney, L. M., et al, February 1994, *A Reevaluation of Selected Mobile Satellite Communications Systems: Ellipso, Globalstar, IRIDIUM, and Odyssey*, MTR-93B0000157, Bedford, MA: The MITRE Corporation.