MME-BASED ATTITUDE DYNAMICS IDENTIFICATION AND ESTIMATION FOR SAMPEX

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ABSTRACT

A method is described for obtaining optimal attitude estimation algorithms for spacecraft lacking attitude rate measurement devices (rate gyros), and then demonstrated using actual flight data from the Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) spacecraft. SAMPEX does not have on-board rate sensing, and relies on sun sensors and a three-axis magnetometer for attitude determination. Problems arise since typical attitude estimation is accomplished by filtering measurements of both attitude and attitude rates. Rates are nearly always sampled much more densely than are attitudes. Thus, the absence/loss of rate data normally reduces both the total amount of data available and the sampling density (in time) by a substantial fraction. As a result, the sensitivity of the estimates to model uncertainty and to measurement noise increases. In order to maintain accuracy in the attitude estimates, there is increased need for accurate models of the rotational dynamics. The proposed approach is based on the Minimum Model Error (MME) optimal estimation strategy, which has been successfully applied to estimation of poorly-modeled dynamic systems which are relatively sparsely and/or noisily measured. The MME estimates may be used to construct accurate models of the system dynamics (i.e. perform system model identification). Thus, an MME-based approach directly addresses the problems created by absence of attitude rate measurements.

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INTRODUCTION

The attitude of a spacecraft can be estimated by either single-frame deterministic methods (such as TRIAD and QUEST [1–2]) or algorithms which combine analytical models with attitude measurements and, for most spacecraft, attitude rates (such as the Kalman filter [3]). Generally, the use of rate gyros significantly improves the attitude estimation, because the densely-measured rates may virtually eliminate the need for dynamic models. However, the intentional omission of rate gyros in the design of satellites is increasingly likely as resources become more scarce (for example, the Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) satellite does not have rate gyros on board). In addition, existing satellites with rate gyros on board may experience gyro degradation or failure (such as the failure of four of the six rate gyros on the Earth Radiation Budget Satellite (ERBS) [4]). When rate gyro data is either not available or not dependable, the attitude estimation accuracy becomes much more heavily dependent on accurate dynamic models since the attitude measurements are typically much less dense and less accurate than the rate data. In these cases, dynamic models may be required to provide estimates between and/or in addition to the attitude measurements.

Unfortunately, accurate models of spacecraft rotational dynamics are often unavailable. In cases where the spacecraft was launched with rate gyros, the attitude estimation algorithm likely did not require an accurate dynamics model since dense rate measurements were available. Even for spacecraft which do not have rate gyros, determining an accurate rotational dynamics model may be difficult. If an accurate model is necessary in the attitude estimation algorithm, estimation accuracy is compromised. This is especially true for spacecraft launched with rate gyros which subsequently fail.

To circumvent the problem of rate gyro omission or failure, analytical models of gyro biases can be used. An example of a commonly used gyro bias model is the model based on a Markov (exponential decay) process. This simple model has been successfully used in a Real-Time Sequential Filter (RTSF) algorithm in order to propagate dynamic state estimates and error covariances for the SAMPEX satellite (see [5]). A clear advantage to using dynamic models for gyro biases was shown for the case of Sun-magnetic near co-alignment. For this case, the single-frame algorithms, TRIAD and QUEST, showed anomalous behaviors with extreme deviations in attitude estimations. However, since the RTSF propagates an analytical model of the gyro bias, the attitude estimates are improved even when data from only one sensor is available (i.e., only magnetometer measurements).

In theory, perfect, solvable models of the spacecraft rotational dynamics could be used to obtain perfect attitude estimates. When accurate rate gyros are present, they can often take the place of the dynamic models. When rate gyros are either absent or excessively noisy, attitude estimation accuracy becomes critically dependent on the accuracy of the rotational dynamic models. The ERBS studies [6–8] showed, for an existing satellite, that modeling of the attitude dynamics leads to accurate attitude estimation algorithms. However, the authors concluded that in order to be operationally useful, "automatic" methods for determining these dynamic models must be available.

In this paper, a technique is described which directly addresses the problem of attitude estimation when rate data is not available (or severely degraded), regardless of the cause. The method described herein addresses this problem directly in two distinct but related approaches. First, the MME [9–12] method may be used simply to obtain accurate state estimates for dynamic systems which are both poorly modeled and sparsely measured. This is accomplished through explicit accounting for errors in the dynamic model. Thus, attitude estimation using existing satellite dynamic models (which may not be particularly accurate) is possible. However, the MME estimates may also be used to construct accurate models of the system dynamics (i.e., perform system model identification). Thus, the second, and main, thrust of the approach is the use of the MME to create more accurate dynamic models for use in ANY estimation algorithm (batch, sequential, or MME).

An optimal attitude estimation algorithm is described which is capable of robust and accurate estimates for spacecraft lacking both accurate attitude rate measurements and accurate rotational dynamics models. The current approach is based on the Minimum Model Error (MME) optimal estimation strategy, which has been successfully applied to estimation of numerous poorly-modeled dynamic systems which are relatively sparsely and/or noisily measured. The MME–based approach described in this paper has the capability to automatically determine accurate rotational dynamic models, resulting in algorithms which exhibit the high accuracy of estimation using accurate dynamic models, as shown in [9], while eliminating the practical limitations currently imposed by the requirement that the models be determined manually for each orbit.

The organization of this paper is as follows. First, a brief description of the SAMPEX satellite and associated (model) equations of motion is shown. Then, a brief summary of the MME estimation algorithm for nonlinear systems is shown. An MME estimator, which incorporates the SAMPEX model, is next applied to estimate the dynamics (attitudes, angular rates, and angular momentum) of SAMPEX using actual telemetry measurements. Lastly, candidate functional forms for the model error trajectories given by the MME estimator are investigated. Results are compared with actual telemetry data.

SAMPEX MISSION DESCRIPTION

The Goddard Space Flight Center (GSFC) Small Explorer (SMEX) program was developed to provide relatively inexpensive, frequent space science missions. The Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) satellite is the first of the SMEX class missions. The SAMPEX [5] general mission is to study energetic particles and various types of rays. The duration of the mission is 3 years with a possible extension of up to 3 more years.

The spacecraft is three-axis stabilized in a 550 by 675 km elliptical orbit with an 82° inclination. The nominal mode is a one rotation per orbit about the Sun vector. The body z-axis is defined by the instrument boresights and is required to be within 15° of zenith near the magnetic poles. The body y-axis nominally is aligned with the Sun vector.

The attitude determination hardware consists of five coarse Sun sensors (CSS) (primarily for Sun-acquisition), one fine Sun sensor (FSS) (for roll and yaw), and a three-axis magnetometer (TAM) (for pitch). The attitude control hardware consists of a magnetic torquer assembly (MTA) (for roll and yaw), and a reaction wheel assembly (RWA) (for pitch). The nominal attitude determination accuracy is $\pm 2°$.

SRTADS

The SAMPEX Real-Time-Attitude-Determination-System (SRTADS) is a graphical-user-interface program which computes and displays attitude solutions along with telemetry measurements in a real-time mode during real-time contacts. One of the functions of the system is to serve as a testing platform for filtering methods used in attitude determination.

The current version of the SRTADS program implements three different attitude determination methods: 1) the TRIAD algorithm; 2) the QUEST algorithm; and 3) the Real-Time Sequential Filter (RTSF). Both TRIAD and QUEST are single-frame deterministic methods which primarily rely on a pair of measured vectors for attitude determination. The RTSF is an extended Kalman filter which combines both measured data and a system model to obtain an attitude solution.

THE MME APPROACH

The Minimum Model Error (MME) estimation algorithm was developed for optimal state estimation of poorly modeled dynamic systems ([9]). Motivated by problems in satellite orbit/attitude determination (see [13]), in which significant unmodeled dynamics may be present, the MME was formulated to rigorously account for both significant modeling error and significant measurement noise.

The MME state and model error estimations have been shown to be extremely accurate in previous work [9–10], [12] and the algorithm shown to be robust with respect to modeling errors, measurement errors, and measurement sparsity [12]. The true state trajectories are accurately estimated, $\hat{x}(t) \approx x(t)$, and, most important for the realization/identification problem, $\underline{d}(t)$ approaches the correct model error trajectory. Another key feature of the MME (explained in [10]) is that the state estimates are free of jump discontinuities evident in Kalman filters, for example.

The MME solution yields the optimal state estimates $\underline{x}(t)$ and the optimal model error estimates $\underline{d}(t)$. Results presented in [12], [13] showed that for poorly modeled systems, the MME state estimates are of considerably higher accuracy than those obtained using standard approaches based on Kalman filtering. In addition, the MME has been used as the basis for highly accurate and robust system identification algorithms, both linear [11], [14–16], and nonlinear [12], [17–18], based on the combination of state and model error estimates.

MODEL EQUATIONS

The following is a brief summary of the kinematic and dynamic equations of motion for a three-axis stabilized spacecraft. The rotational orientation of the spacecraft (kinematic equations) may be represented by the quaternion attitude parameterization as

$$\dot{\underline{q}} = \frac{1}{2}\Omega\underline{q} \qquad (6)$$

where

$$\Omega = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \qquad (7)$$

The elements of Ω are the components of the instantaneous spacecraft angular velocity defined relative to the body frame.

The dynamic equations of motion (Euler's Equations of Motion) for a non-rigid spacecraft (SAMPEX is not modeled as a rigid body because it contains a reaction-wheel assembly), may be defined as

$$\frac{d\underline{L}}{dt} = \underline{N} - \underline{\omega} \times \underline{L} \qquad (8a)$$

$$\underline{L} = \underline{L}_{body} + [I_{rw} \times w_{rw}]\hat{j} \qquad (8b)$$

where

$$\underline{N} = \text{total external torque}$$
$$\underline{\omega} = \text{instantaneous angular velocity}$$
$$\underline{L} = \text{total angular momentum}$$
$$\underline{L}_{body} = \text{angular momentum of the body}$$
$$I_{rw} = \text{inertia of the reaction wheel}$$
$$\omega_{rw} = \text{angular velocity of the reaction wheel}$$

Here, again, all vectors are resolved in a body-fixed coordinate system. The angular momentum of the reaction wheel only acts along the y-body axis. The nonlinear state-

space representation of the dynamic equations of motion is given by

$$
\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{L}_1 \\ \dot{L}_2 \\ \dot{L}_3 \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 & 0 & 0 & 0 \\ -\omega_3 & 0 & \omega_1 & \omega_2 & 0 & 0 & 0 \\ \omega_2 & -\omega_1 & 0 & \omega_3 & 0 & 0 & 0 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\omega_3 & -2\omega_2 \\ 0 & 0 & 0 & 0 & -2\omega_3 & 0 & 2\omega_1 \\ 0 & 0 & 0 & 0 & 2\omega_2 & -2\omega_1 & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ L_1 \\ L_2 \\ L_3 \end{Bmatrix}
$$

$$
+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix} \tag{9}
$$

The body angular rates can be determined using the angular momentum, reaction wheel momentum, and known inertia:

$$
\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} = I^{-1} \begin{Bmatrix} L_1 \\ L_2 - I_{rw} \times w_{rw} \\ L_3 \end{Bmatrix} \tag{10}
$$

where I represents the inertia tensor of the satellite.

SRTADS provides time histories for all external torques (aerodynamic, gravitational, solar, magnetic, etc) and the reaction wheel input. The nominal numerical values for the SAMPEX inertia tensor and reaction wheel inertia are given by [5]

$$
I = \begin{bmatrix} 15.516 & 0 & 0 \\ 0 & 21.621 & -0.194 \\ 0 & -0.194 & 15.234 \end{bmatrix} \text{kg} - \text{m}^2 \qquad I_{rw} = 0.0041488 \ \text{kg} - \text{m}^2
$$

RESULTS

A nominal satellite pass is used to compare the MME estimator to the deterministic method (e.g., TRIAD). Nominally, both FSS and TAM data is available throughout the orbit. Anomalous behavior occurs during either sun occultation and/or co-alignment of the measurement vectors. However, this test case involves a non-event pass (i.e., no anomalous behavior). SRTADS utilizes telemetry and ephemeris data from an orbital pass and determines the Euler attitudes using TRIAD, QUEST, and the RTSF in 5 second intervals.

The MME estimator uses a priori values from the single-frame solutions (i.e., the TRIAD solution for the spacecraft's attitude). The MME estimator is then used to obtain

estimates for both the three-axis attitudes and the dynamic rates. Since this test case involves a non-event pass, the TRIAD solution for the spacecraft's attitude is assumed to be the actual (nominal) solution. Figures 1-3 show the TRIAD solutions for the SAMPEX Euler attitudes. These attitude time histories are assumed to be the "true" attitudes for this test case.

The MME model error term $\underline{d}(t)$ is added only to the last three (angular momentum) states of the dynamic model, represented by Equation (9). The first four states (i.e., the quaternions) are assumed to be perfect kinematic relationships so that no model correction is added to these states. This formulation avoids any difficulties encountered by the normalization constraint of the quaternions [2]. Therefore, no pre-conditioning of the estimator model for the normalization constraint of the quaternion states is required. This formulation has clear advantages over the Kalman filter method for attitude estimation (see Reference [3]).

Results indicate that the attitude time estimates given by the MME estimator are exactly identical to the TRIAD solutions, shown in Figures 1-3. Therefore, the MME estimator provides accurate attitude estimates in this non-event case (i.e., the estimates parallel the TRIAD solution throughout the entire time interval).

The angular momentum estimates from the MME are shown in Figures 4-6. These trajectories are used to determine the instantaneous spacecraft angular rates resolved along the body frame, which propagate the quaternions. The associated model error trajectories $(\underline{d}(t))$ from the MME estimator are shown in Figure 7. It is important to note that the correction is only applied to the angular momentum states. This formulation provides accurate MME state estimates of the Euler angles (see Figures 1-3), and also maintains the quaternion normalization constraint.

The model error trajectories can now be used to correlate a linear or nonlinear correction to the SAMPEX dynamic model. To identify mathematical expressions that describe these trajectories, the Least-Squares-Correlation (LSC) algorithm is used [19]. This algorithm develops a set of mathematical expressions that describe the model error histories as a combination of the state estimates. This algorithm can be implemented in two ways: (1) the code can be allowed to form combinations of simple mathematical functions, or (2) a library of functions may be supplied by the user to augment the search process (i.e., by supplying known functions from intuitive implementations or past studies). This library is a list of functions which the user expects will appear in the system under investigation. These functions may be common occurring functions from initial runs of the LSC algorithm.

A method for identifying possible library functions involves plotting the model error trajectories versus the state estimates. These plots may offer significant mathematical insight on how to formulate library expressions. Plots of the second model error versus the first and second state estimates are shown in Figures 8-9. From these figures, a possible functional form may be a Lemniscate geometric function with internal oscillations. This geometric function is implemented into the library set (along with previous internal

functions). Table 1 contains example candidate functions obtained to this point. The second model error candidate expression shows a high correlation coefficient of 0.99. Figure 10 shows the second model error and the example candidate function that describes it. From the high correlation value, and the presence of only minor discrepancies in this figure the second model error is assumed to be correctly identified by the function shown in Table 1. Note that this model error candidate expression has been identified without the use of any library terms.

The next step in the study is to create a library starting with known attitude dynamical model components and external disturbances (e.g., aerodynamic torque, radiation torque, orbit maneuvering torques, solar radiation pressure, etc). These can be used to further and more accurately identify nonlinear terms for the remaining model error trajectories (i.e., to obtain higher correlations). Once dynamical error models are obtained, they can be used to determine actual trajectories during anomalous periods such as Sun occultation and/or measurement vector co-alignment.
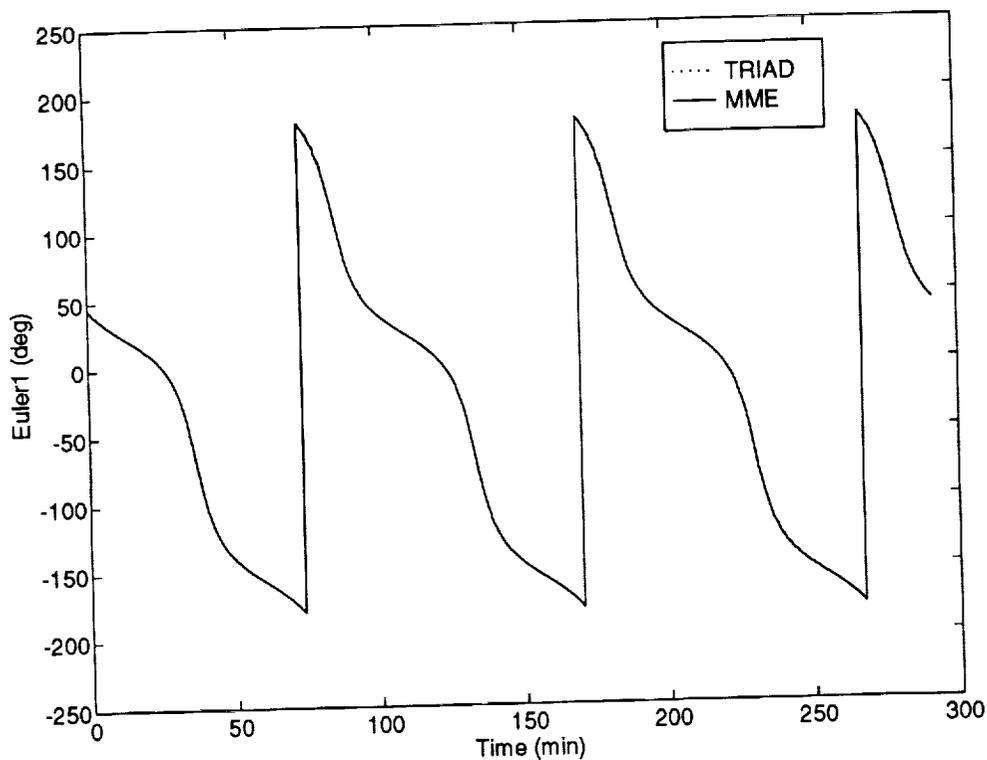
Figure 1 TRIAD and MME Euler1 Angle Solutions
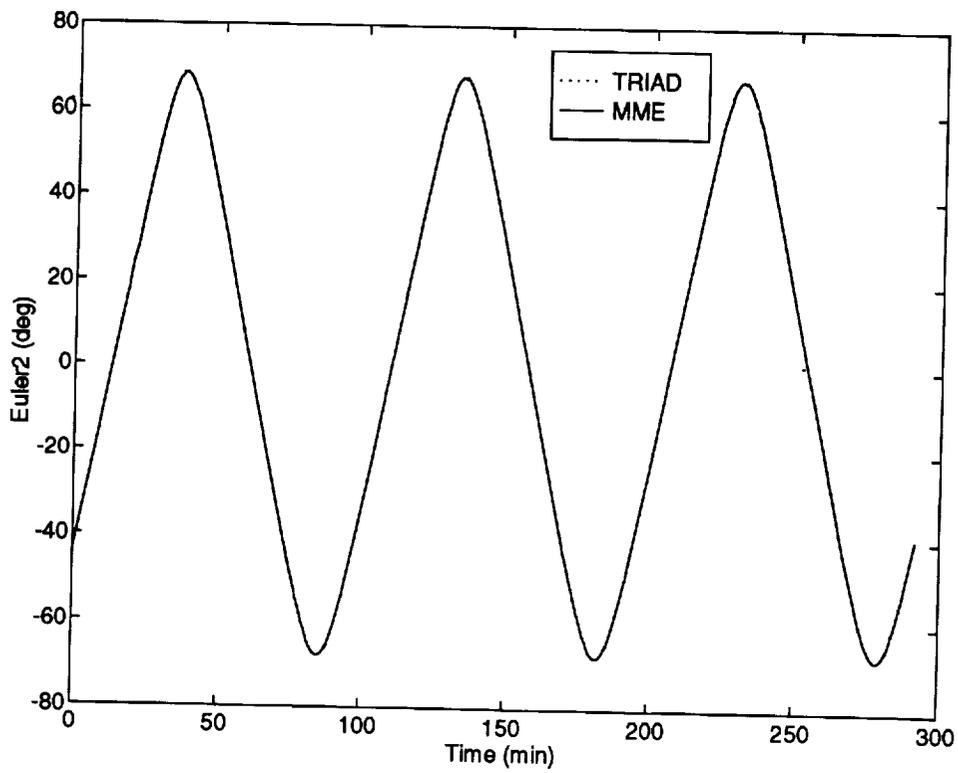
Figure 2 TRIAD and MME Euler2 Angle Solutions
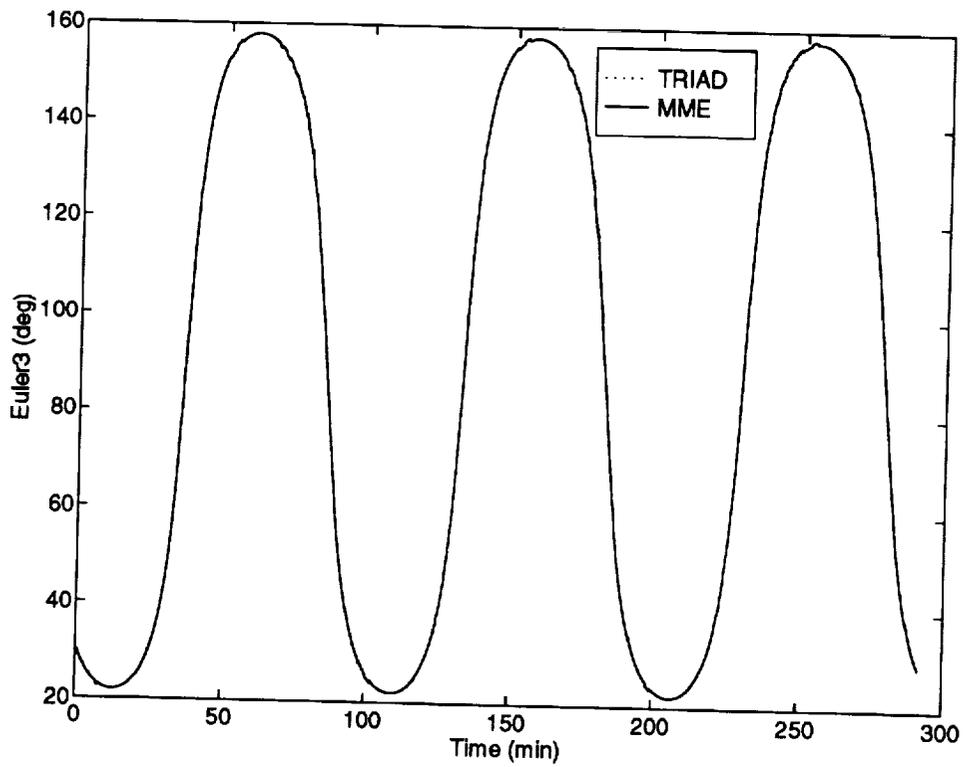
Figure 3 TRIAD and MME Euler3 Angle Solutions
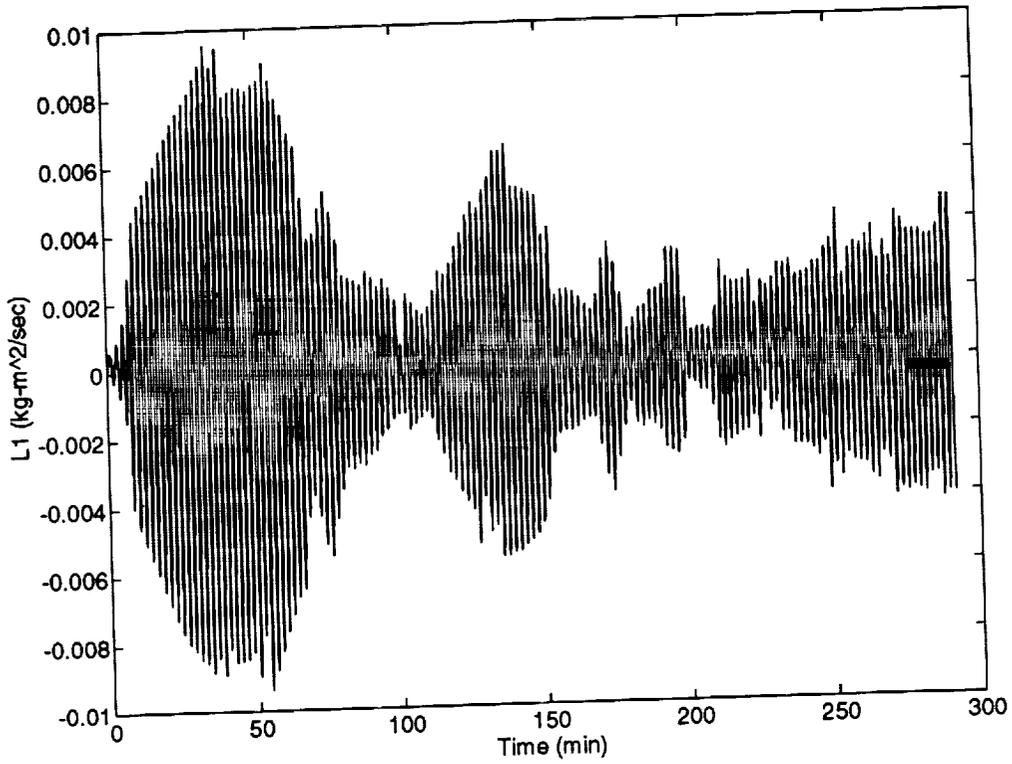
Figure 4 MME L_1 Angular Momentum State Estimate
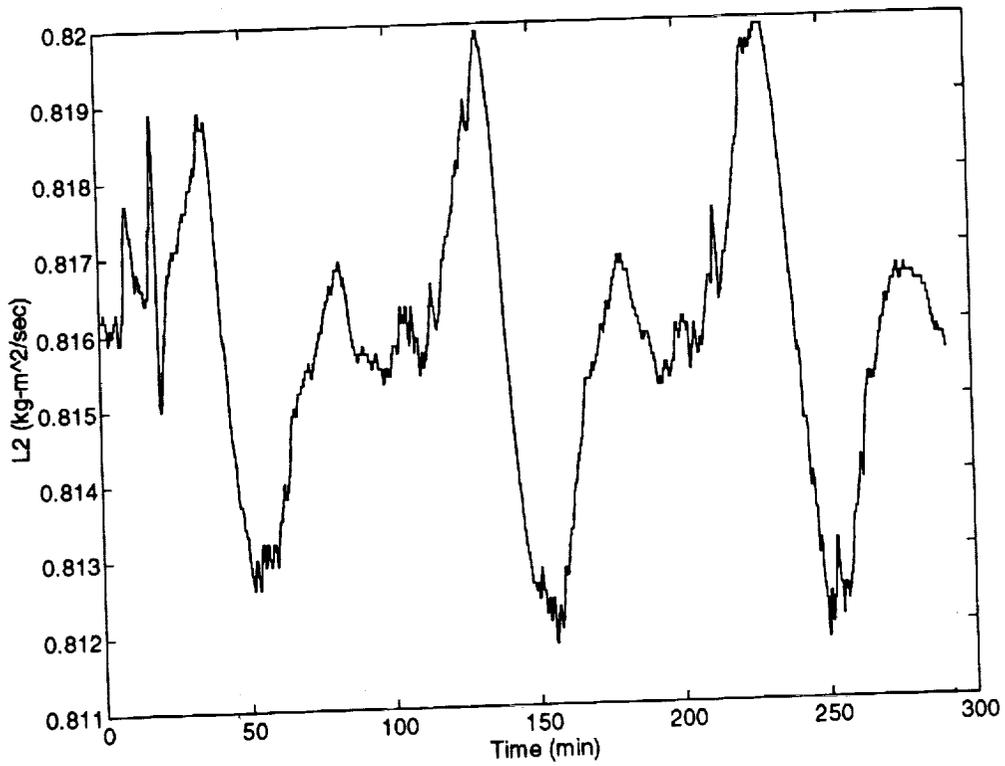
Figure 5 MME L_2 Angular Momentum State Estimate

506
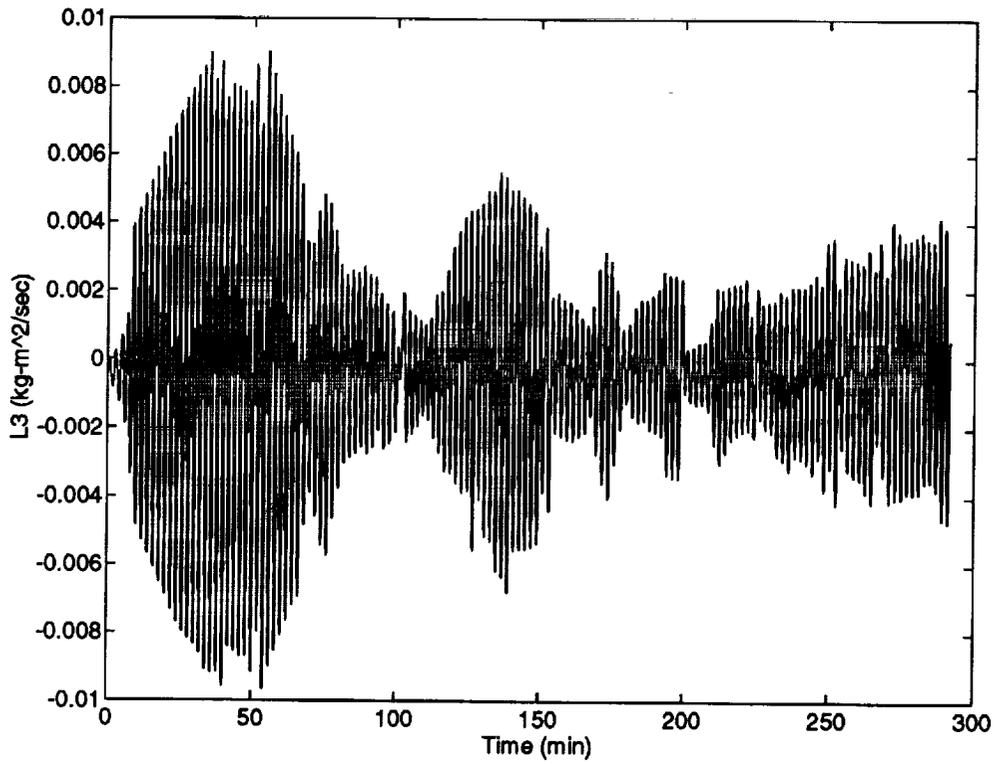
Figure 6 MME L_3 Angular Momentum State Estimate
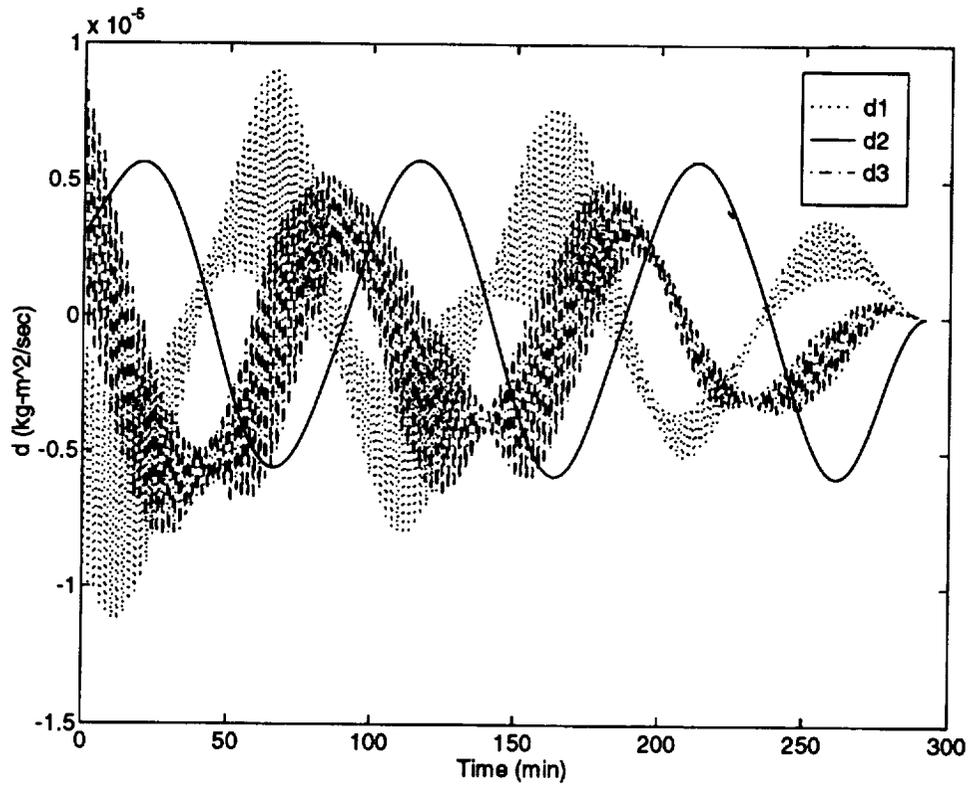
Figure 7 MME Angular Momentum Model Error Trajectories
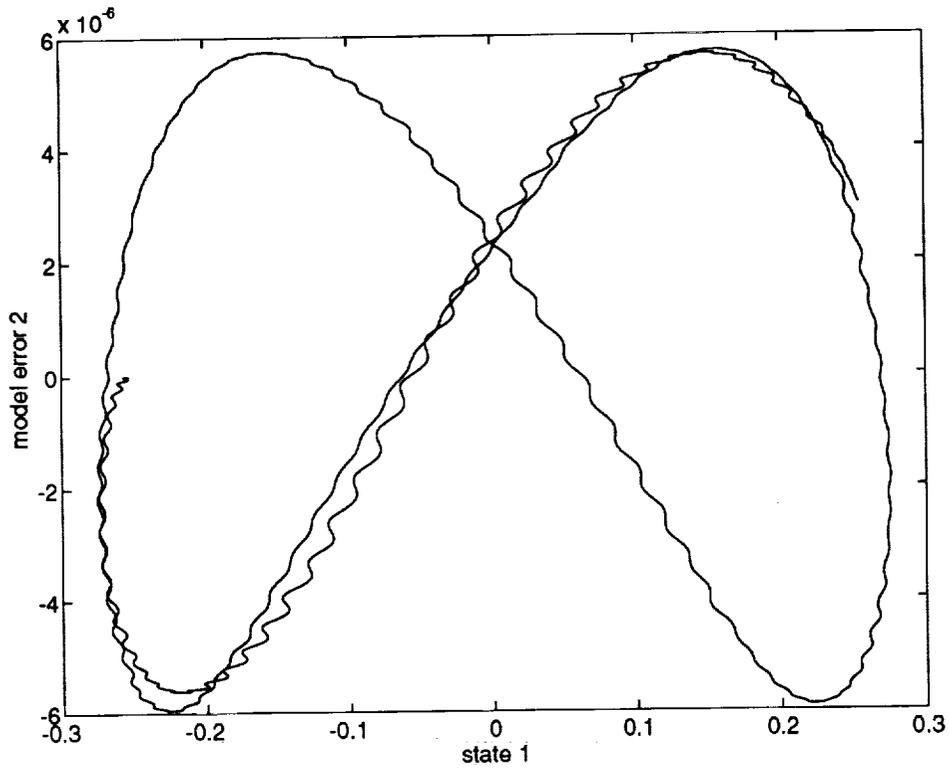
Figure 8 Second Model Error versus First Estimated State
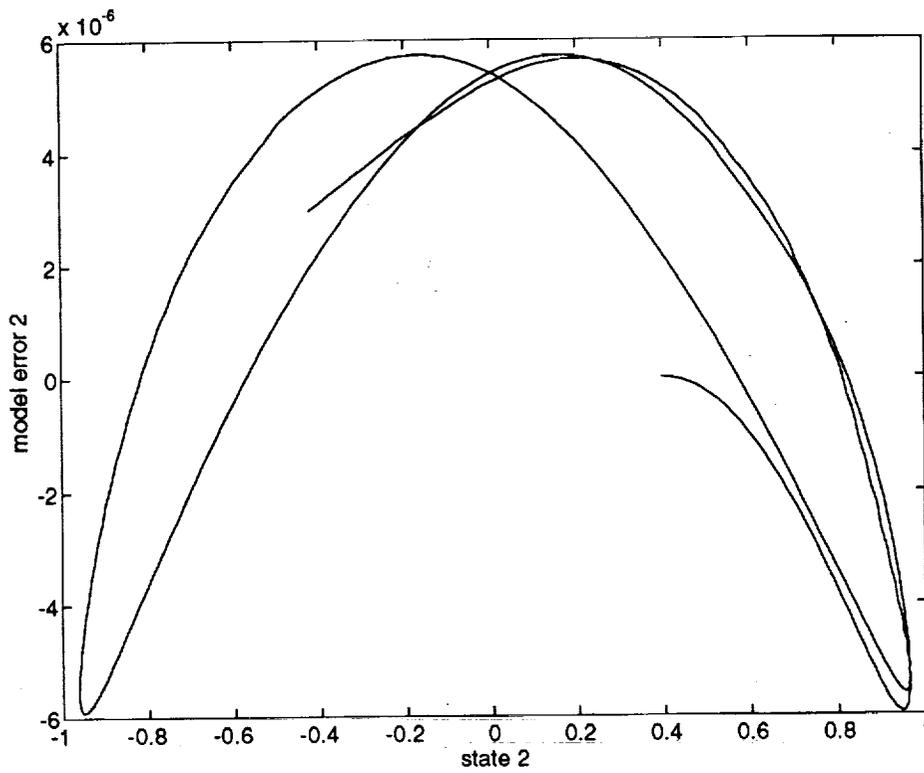
Figure 9 Second Model Error versus Second Estimated State
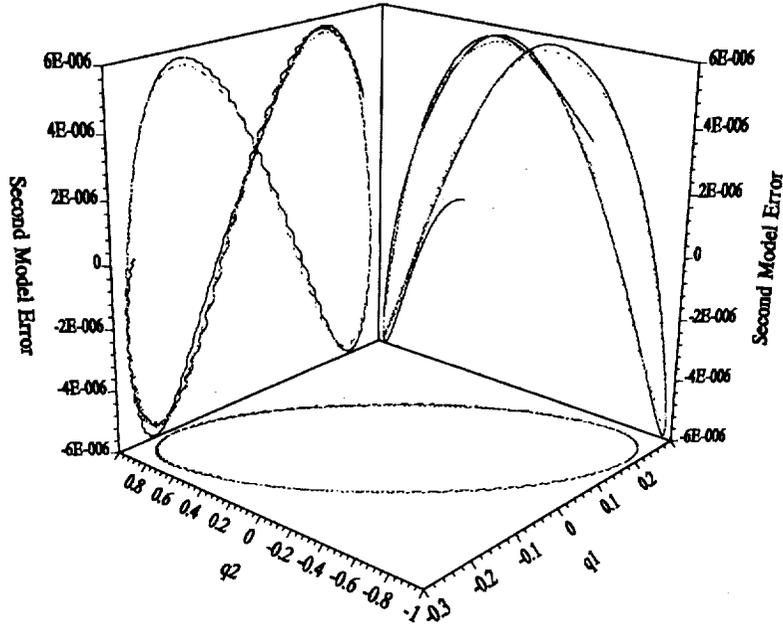
Figure 10 Identified Candidate Function and Second Model Error

Table 1 Example Candidate Functions for the Model Error Histories

Error	Candidate Function	Correlation
d_1	$-2.416 \times 10^{-6} e^{[q_2]^2} e^{[q_4]^4} - 2.120 \times 10^{-3} e^{[L_1]^4} e^{[L_3]^2} + 2.130 \times 10^{-3}$	0.8647
d_2	$-4.581 \times 10^{-6} e^{[q_2]^2} q_2^2 - 9.737 \times 10^{-3} q_1^6 + 5.780 \times 10^{-6}$	0.9963
d_3	$5.250 \times 10^{-4} q_4^2 L_3^2 - 5.163 \times 10^{-5} e^{[L_3]^2} e^{[q_3]^2} + 5.163 \times 10^{-5}$	0.8074

SUMMARY

A technique has been described which leads to algorithms capable of accurate attitude estimation in the presence of significant model error and/or sparse/noisy data. In many satellites, such as SAMPEX, attitude rate measurements are not available either by design or by failure of existing rate gyros. The absence of rate measurements increases the estimation sensitivity to modeling uncertainty and measurement noise in the remaining, relatively sparse attitude measurements. The technique described directly addresses the problem of attitude estimation without rate gyro data.

Results using actual SAMPEX data and corrected models indicate that the technique described in this paper produces accurate estimates for both the spacecraft's position and attitude rate. Also, the formulation described in this paper avoids any difficulties encountered when using quaternions to represent the attitude of the satellite. The new technique may be used directly as an estimator, or, as described in the paper, as a robust method of automatically obtaining accurate dynamic models for existing satellites. Nonlinear candidate functions have been identified with fairly high correlation for SAMPEX. Later studies will utilize more candidate functions in order to obtain unity correlations for all model error trajectories. Therefore, these identified functions can be used to propagate the model accurately in order to determine attitude and rate motion during anomalous conditions.

ACKNOWLEDGMENT

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